Physics 602 **Homework Set 1** **Spring 2016**

Due date: Monday, March 21, at the end of the lecture.

Note: you are allowed to use only the textbook (Grosso & Parravicini), lecture notes, materials that I specifically provided, and Mathematica or other similar software to do the homework.

1. *Poisson summation formula.* We used the Poisson summation formula without proof in class to evaluate the free energy (thermodynamic potential) of a free electron gas in a magnetic field. This formula has numerous other applications in physics. A slightly more general version of this formula is

$$\sum_{n=-\infty}^{\infty} f(t+nT) = \frac{1}{T}\sum_{k=-\infty}^{\infty} g(k/T)e^{2\pi ikt/T},$$

where $g(k/T)$ is the Fourier transform of $f(t)$

$$g(k/T) = \int_{-\infty}^{\infty} d\tau e^{-2\pi ik\tau/T} f(\tau).$$

1. Prove the above form of the Poisson summation formula.
2. Evaluate $\sum_{n=1}^{\infty}(n^2+1)^{-1}$.
3. Show that $\sum_{n=-\infty}^{\infty} e^{-n^2\pi z} = z^{-1/2}\sum_{k=-\infty}^{\infty} e^{-k^2\pi/z}$.

2. *Lifshits-Kosevich theory.* In class we derived de Haas-van Alphen (dHvA) oscillations in the case of the free electron gas, i.e. for a quadratic dispersion relation

$$E(p_x,p_y,p_z) = \frac{p_x^2+p_y^2+p_z^2}{2m}. \tag{1}$$

What happens for a different, more complicated band structure? This question was addressed by Lifshits and Kosevich in their famous 1955 paper. I posted a partial copy of their paper on the course website. Read first five pages carefully and then check if it reproduces the answers for the quadratic dispersion (1), which we know from lectures and the textbook. Specifically, apply Lifshits-Kosevich theory to quadratic dispersion (1) and

1. Find the extremal cross-sectional area $S_m(E)$ and its derivatives $\partial S_m/\partial E$ and $\partial^2 S_m/\partial p_z^2$ at energy E.
2. Check if the spacing ΔE_n between Landau levels (see equation below Eq. (1.7) in Lifshits and Kosevich) is what you expect for this case.
3. Evaluate the oscillation period $\mathcal{T} = \Delta(1/B)$ and see if it matches the results we derived in lecture (see Eqs. (15.25b) and (15.26) of Grosso & Parravicini, 2nd edition).

4. Note that the oscillatory part of the magnetization is of the form

$$M_{\text{OSC}} = \sum_{k=1}^{\infty} A_k(T, B) \sin\left(\frac{2\pi k}{\mathcal{T}} \frac{1}{B} + \varphi_k\right).$$

Evaluate the oscillation amplitude $A_k(T, B)$ at zero temperature. Derive the same from the expression for the free energy we obtained in lecture (Eq. (15.26) of Grosso & Parravicini, 2nd edition) and compare.