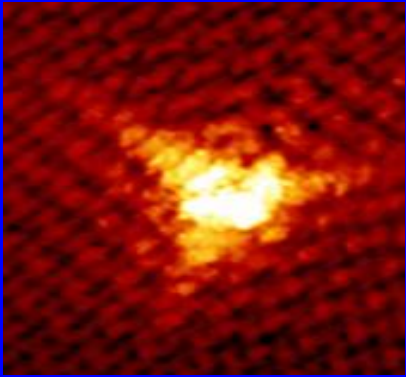
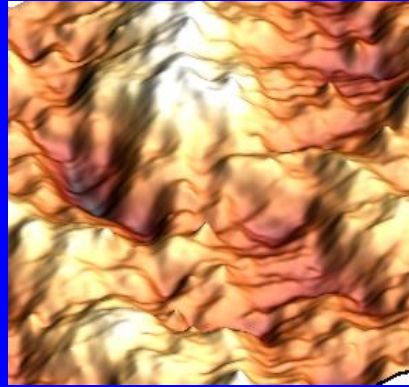


Interaction of Dirac electrons with spins and point charges

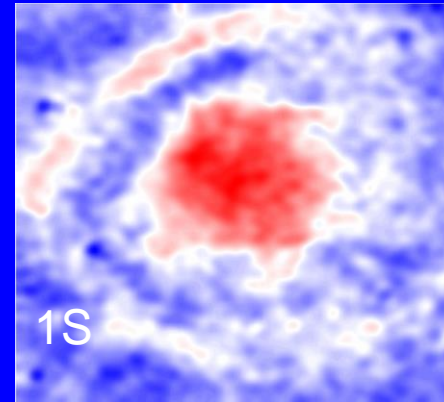
Vacancies in graphene



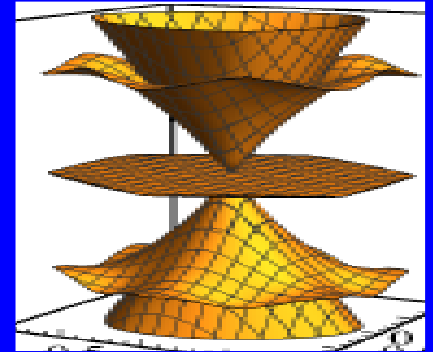
Vacancy Magnetic moment and Kondo screening



Vacancy Charge and Tunable artificial atom



Twisted bilayer graphene



Eva Y. Andrei

LECTURE NOTES POSTED AT:

<http://www.physics.rutgers.edu/~eandrei/links.html#trieste18>



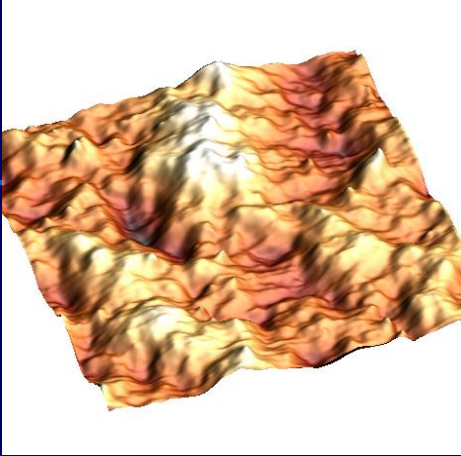
Summer School on
Collective Behaviour in
Quantum Matter



E.Y. Andrei



Scanning tunneling microscopy and spectroscopy



❖ Engineering electronic properties

- Density of states and Landau levels in graphene
- Scanning tunneling microscopy (STM) and spectroscopy (STS)
- Defects:
 - Atomic collapse and artificial atom
 - Kondo effect
- Substrate:
 - Twisted graphene

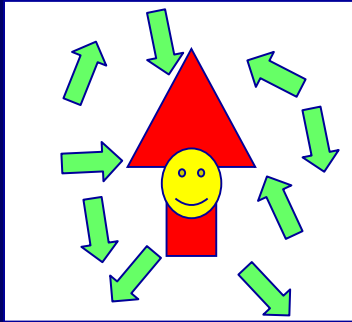




Kondo Screening of Impurity Moments in Metals

$T > T_K$

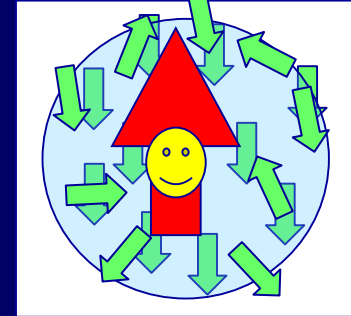
Unscreened



J antiferromagnetic coupling to electron bath

$T < T_K$

Screened



Kondo cloud

$$T_K \propto \exp(-1/\rho J)$$

ρ density of states at E_F

$$\rho(E_F) > 0, J > 0 \rightarrow T_K > 0$$

❖ Normal metals $\rho(E_F) \sim \text{finite}; J \neq 0 \rightarrow T_K > 0$

❖ Insulators $\rho(E_F) = 0$ No Kondo screening

What happens in a pseudogap system?



Kondo Screening in pseudo-gap systems

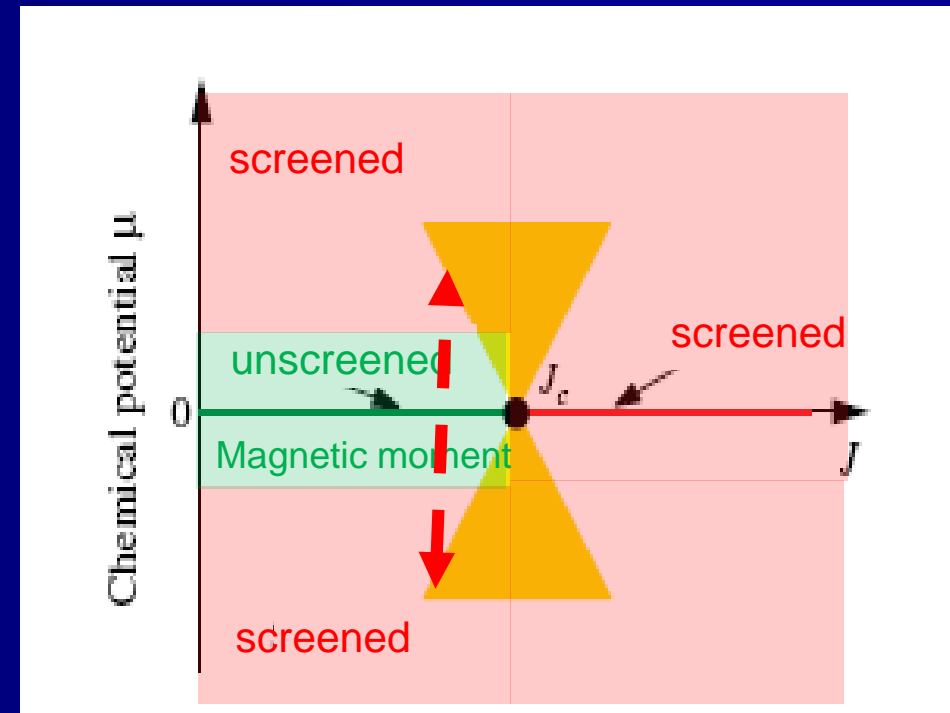
- Pseudo-gap systems $\rho(E) \propto E^r$ screening suppressed.
 - ❖ $r = 1$ (graphene, high T_c superconductors)

$\mu \sim 0$ (undoped)

- Kondo screening only for $J > J_c$
- J_c finite only for asymmetric DOS

$|\mu| \gg 0$ doped

- Normal Kondo screening



- D. Withoff and E. Fradkin, Phys. Rev. Lett. 64, 1835(1990)
- K. Chen and C. Jayaprakash, J. Phys L491 (1995)
- K. Ingersent, Phys. Rev. B54, 11936 (1996)
- C. Cassanello and E. Fradkin, (1996)
- R. Bulla, T. Pruschke, and A. C. Hewson, (1998)
- Polkovnikov A., Phys. Rev. B, 65 (2002) 064503
- Vojta M. and Fritz L., Phys. Rev. B, 70 (2004) 094502.
- Vojta, Fritz, Bulla EPL (2010)
- PW Lo, GY Guo, F. Anders, arXiv:1402.0040

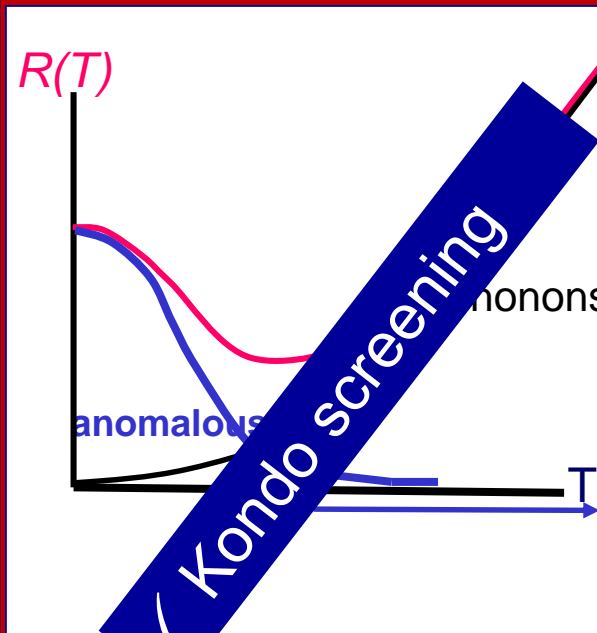
Electrical tuning of magnetic moment





Kondo Screening Experimental Signatures

Resistance minimum



nature
physics

LETTERS

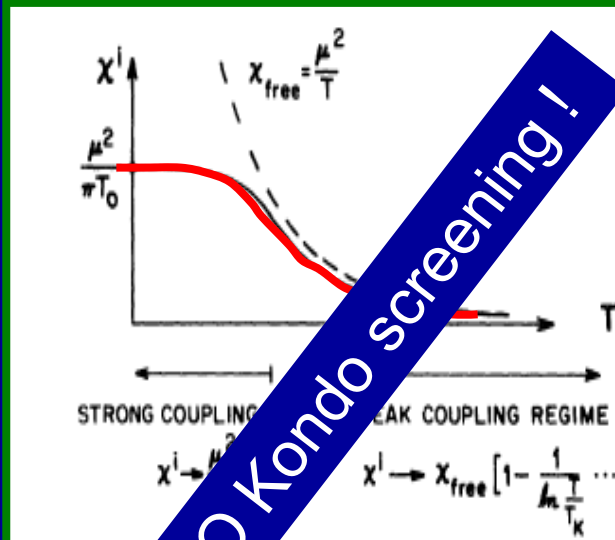
PUBLISHED ONLINE: 3 APRIL 2011 | DOI: 10.1038/NPHYS118

Tunable Kondo effect in graphene with defects

Jian-Hao Chen^{1,2*}, Liang Li², William G. Cullen^{1,2}, Ellen D. Williams^{1,2} and Michael S. Fuhrer^{1,2*}

Measures:
scattering off Kondo cloud

Magnetization saturation



nature
physics

LETTERS

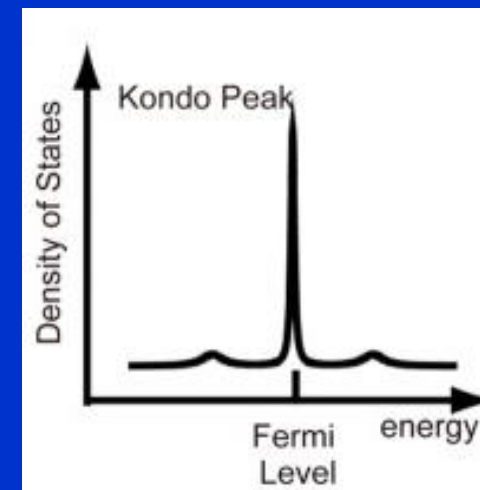
PUBLISHED ONLINE: 10 JANUARY 2012 | DOI: 10.1038/NPHYS218

Spin-half paramagnetism in graphene induced by point defects

R. R. Nair¹, M. Sepioni¹, I-Ling Tsai¹, O. Lehtinen², J. Keinonen², A. V. Krashenninnikov^{2,3}, T. Thomsen⁴, A. K. Geim¹ and I. V. Grigorieva^{1*}

Measures:
Unscreened moment

DOS – Kondo Peak



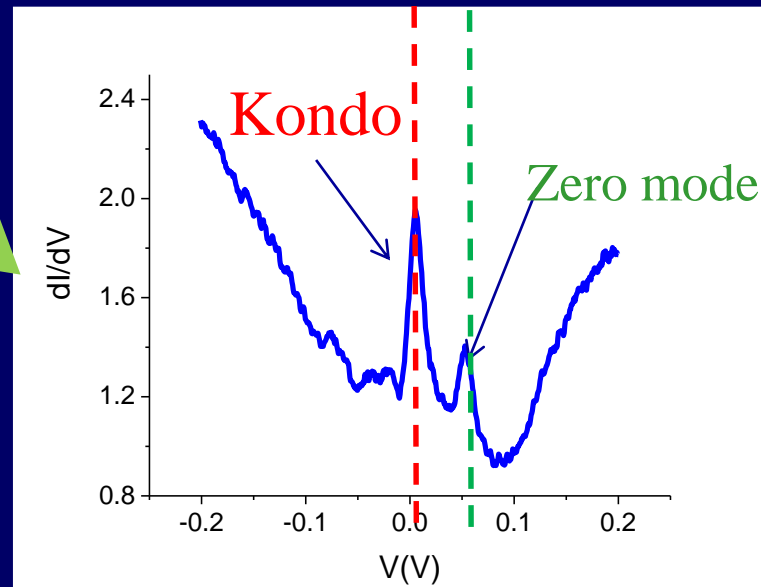
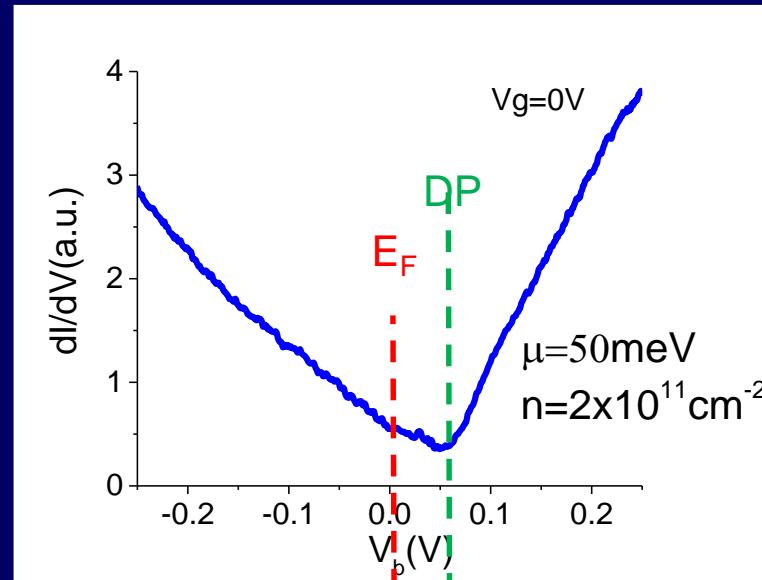
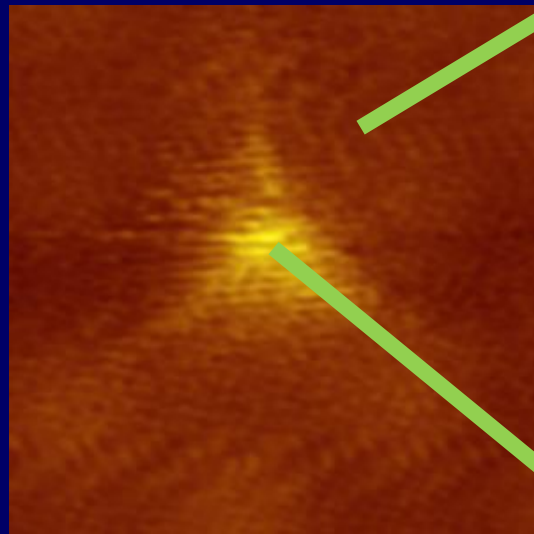
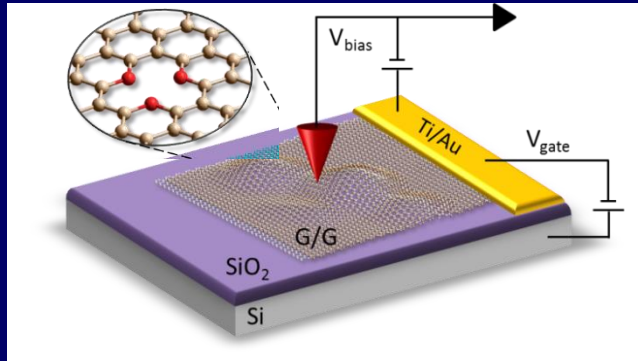
- Kondo Peak at E_F
- Low T linewidth Γ

$$k_B T_K \sim \Gamma/2$$

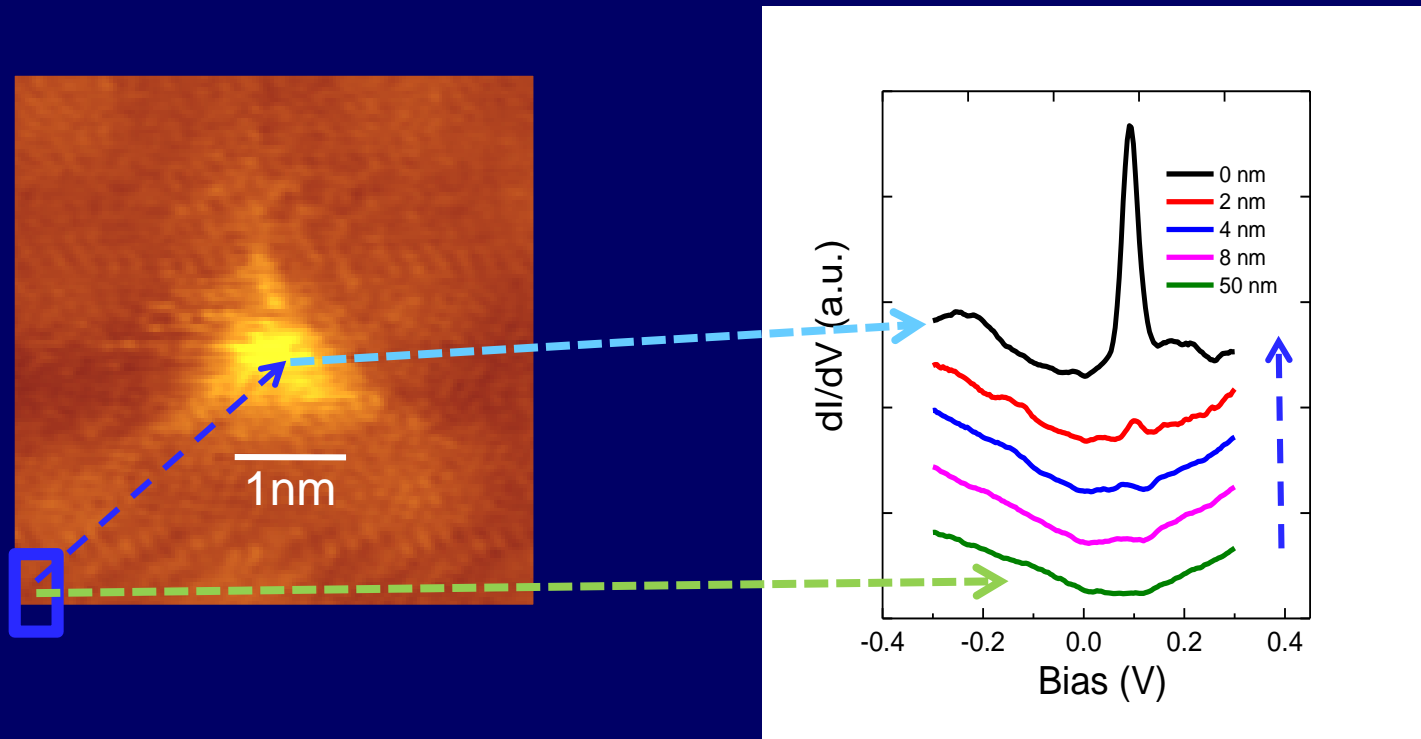
DOS enhancement at E_F



Vacancy in graphene



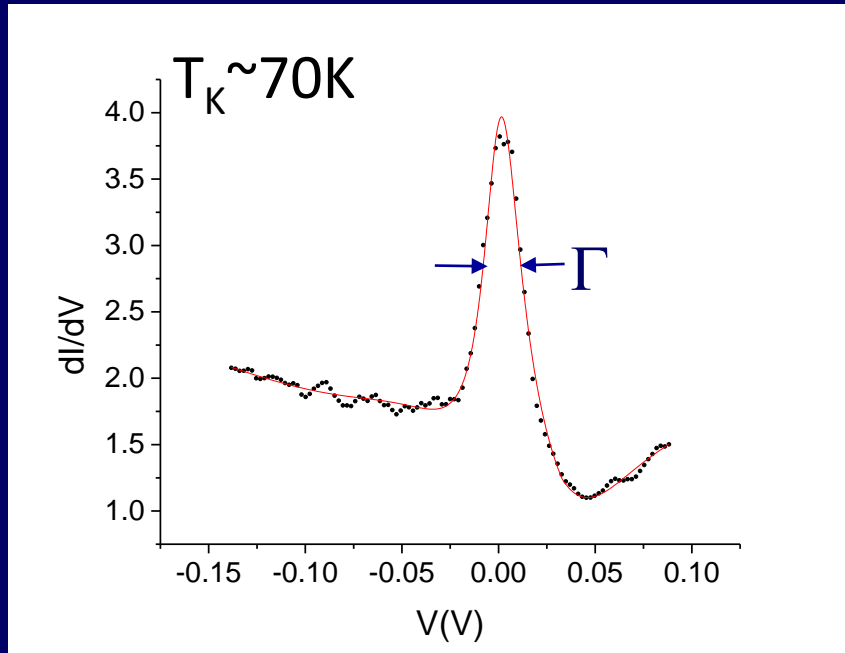
Vacancy Peak



Vacancy Peak

- localized on vacancy site <2nm.
- pinned to the Dirac point

Kondo Temperature

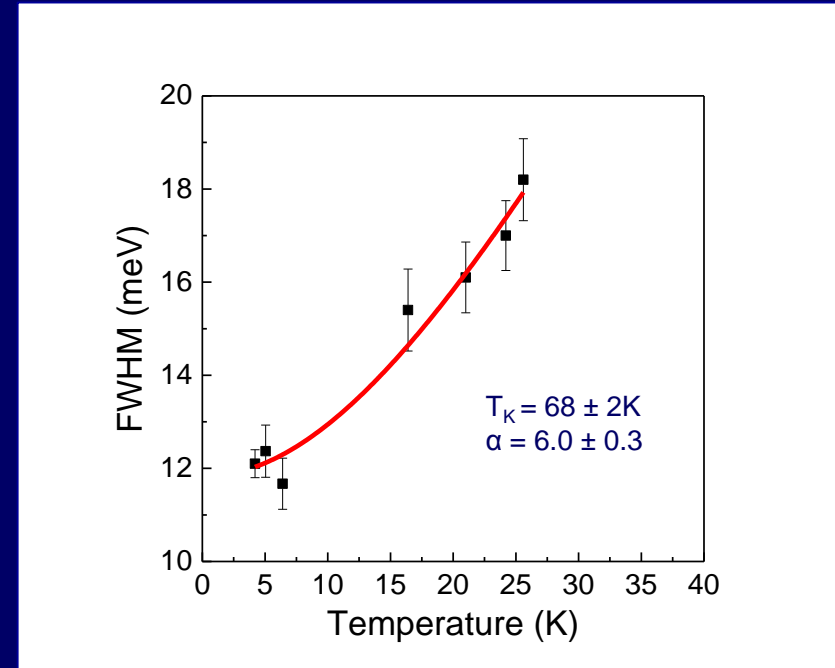


Fit to Fano lineshape

$$\frac{dI(V)}{dV} = A \frac{(\epsilon + q)^2}{1 + \epsilon^2} + B$$

$$\epsilon = \frac{E - \epsilon_0}{\Gamma/2}$$

$$k_B T_K \sim \Gamma/2$$



Fit to T dependence

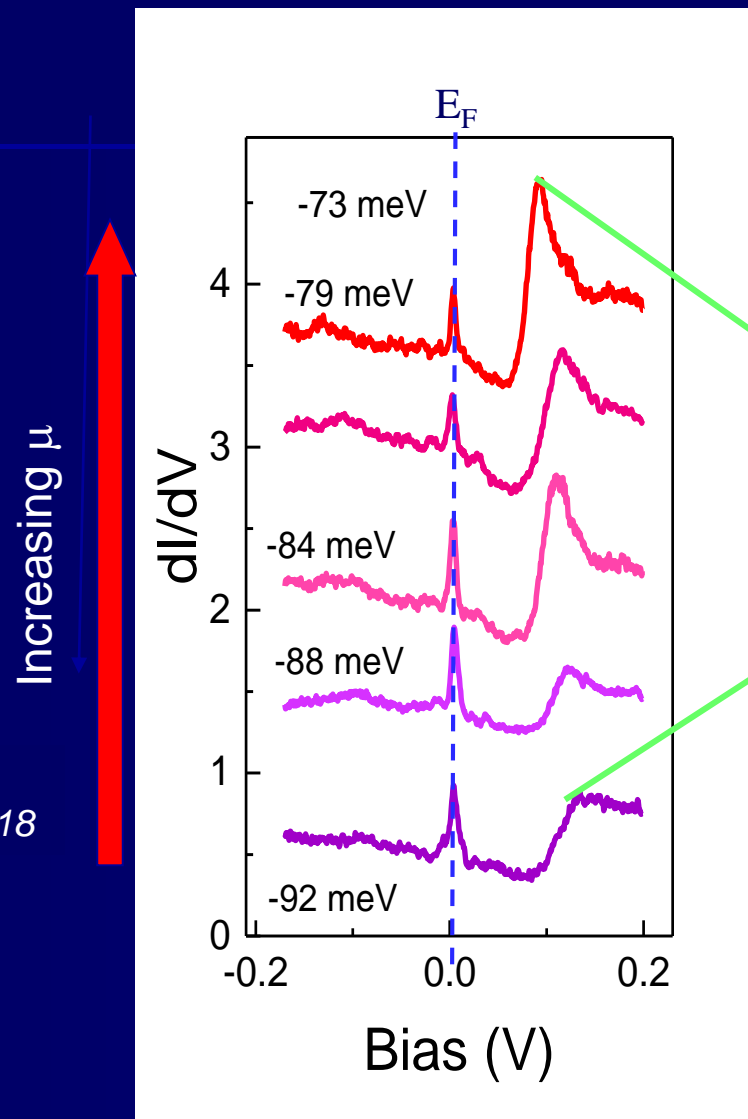
$$\Gamma = \sqrt{(\alpha k_B T)^2 + (2k_B T_K)^2}$$

O. Újsághy, et al. Solid State Commun. **117**, 167(2001)
A.S. Zyazin, et al. Synthetic Metals **161**, 591 (2010)
M. Ternes, et al. J. Phys.: Condens. Matter **21**, 053001, (2009)

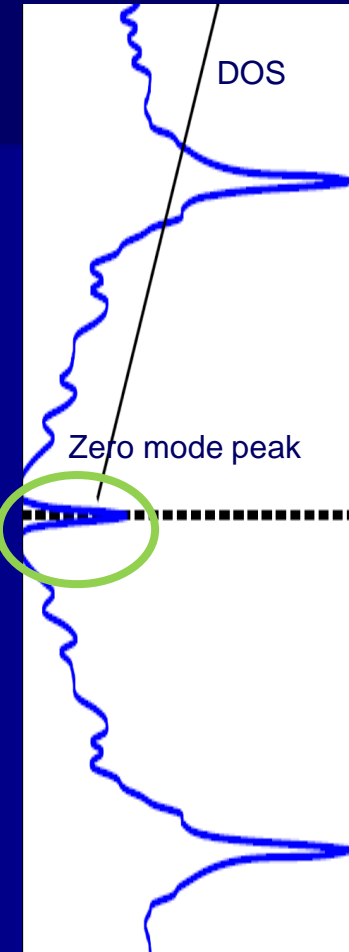


Gate Dependence

Kondo Peak



Zero mode



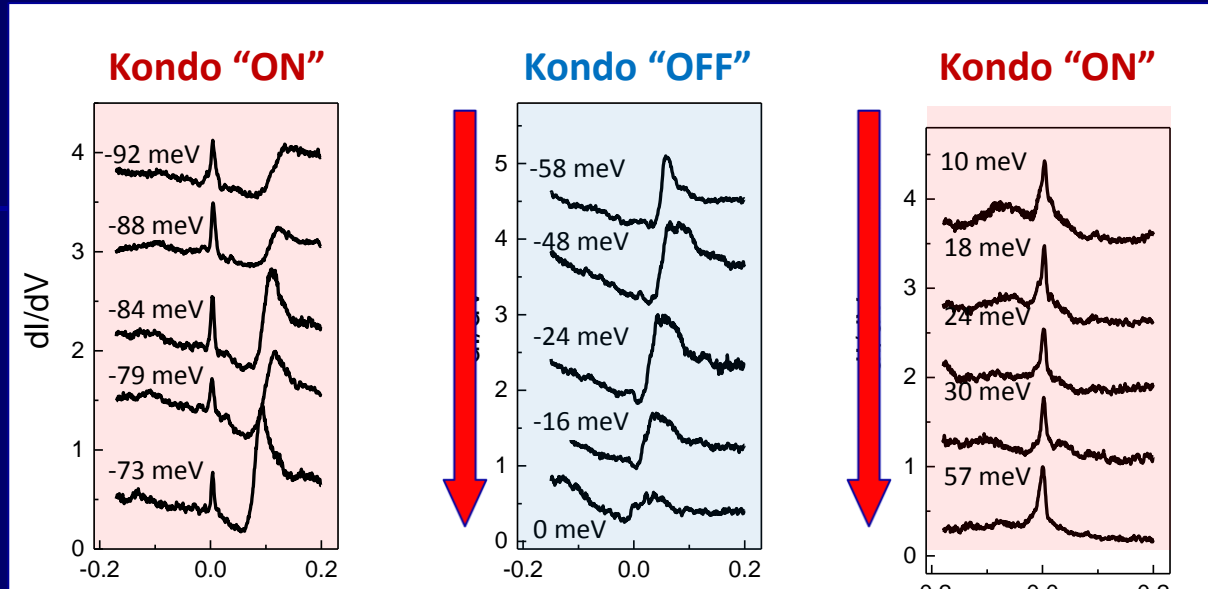
J. Mao et al
Nature Communications 2018

Pinned to E_F

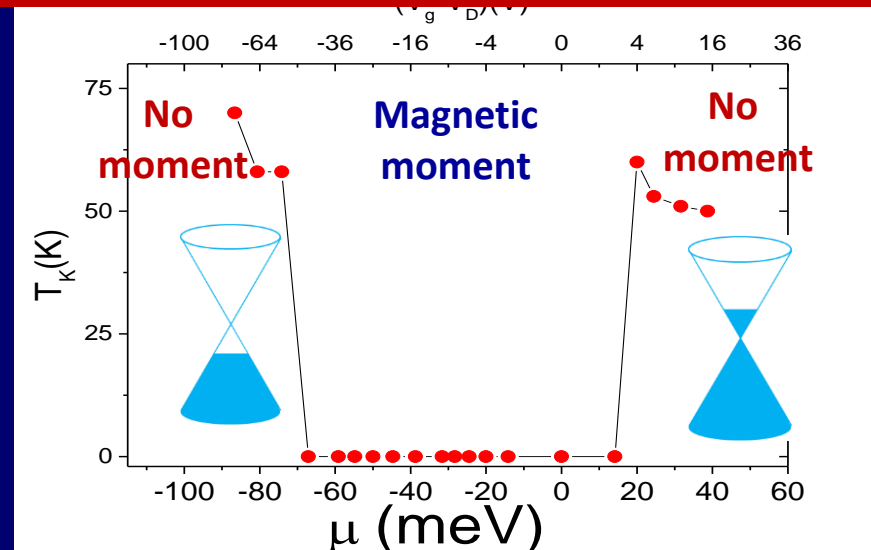


Reentrant Kondo Screening

Increasing μ



Electrically tuned magnetic moment

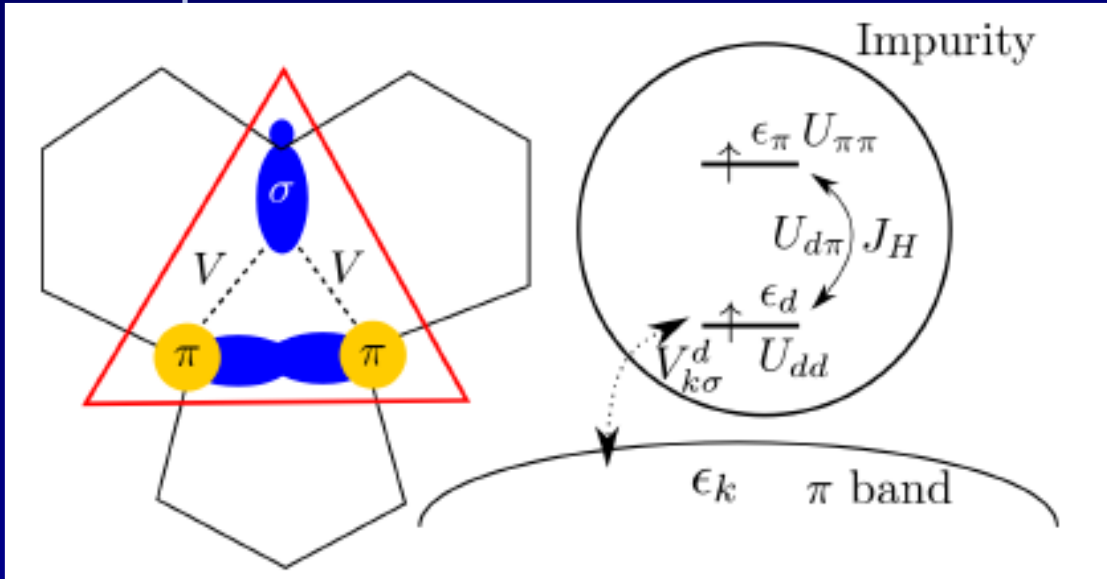


$$J < J_c$$



Model for Kondo screening of vacancy moment

- Anderson impurity model
- Numerical renormalization group calculations



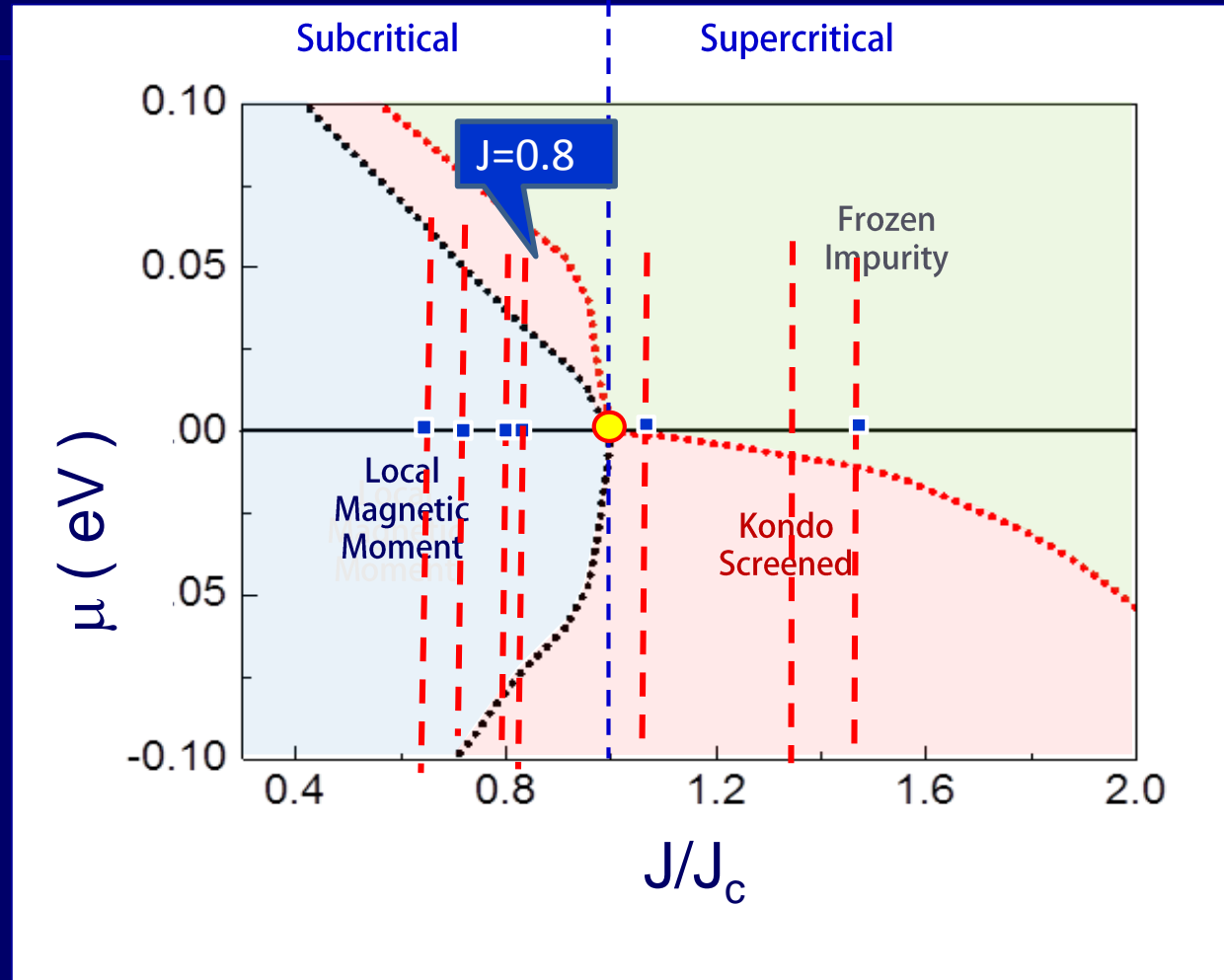
- bare σ -orbital energy $\epsilon_d = -1.6eV$
- On site Coulomb $U_{dd} = 2eV$
- Exchange coupling $U_{d\pi} = 0.1eV$
- Hund coupling $J_H \sim -0.35eV$
- Critical coupling $\Gamma_c = 1.15eV$

Single orbital approximation:
$$U_{eff}(\mu) = \begin{cases} U_{dd} & \mu \leq 0 \\ U_{dd} + \min(U_{d\pi}, \alpha\mu) & \mu > 0 \end{cases}$$



Kondo Screening Phase Diagram

Numerical Renormalization Group



D. May et al *Phys. Rev. B* 97, 155419 (2018)

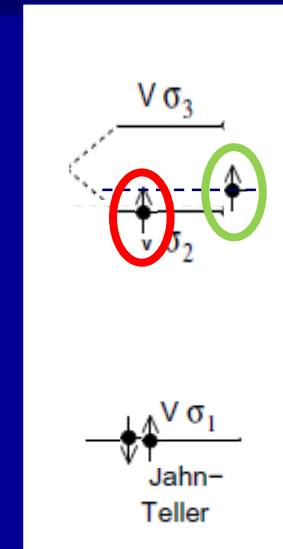
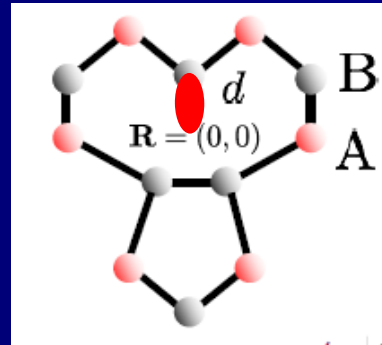
Y Jiang et al *Nature Communications* 2018

J. Mao et al arXiv:1711.06942 (2017)



What determines J ?

- σ Dangling bond \mapsto localized state $\mapsto 1\mu_B$



➤ σ state (in plane) – **orthogonal** to π conduction electrons $\mapsto J=0$

➤ p_z state – **Ferromagnetic** coupling $\mapsto J=0$

$J=0 \mapsto$ NO KONDO SCREENING !!

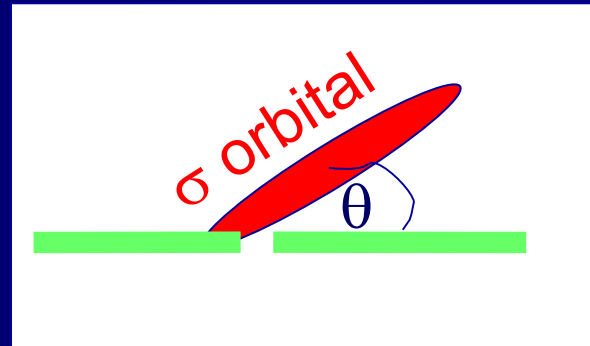
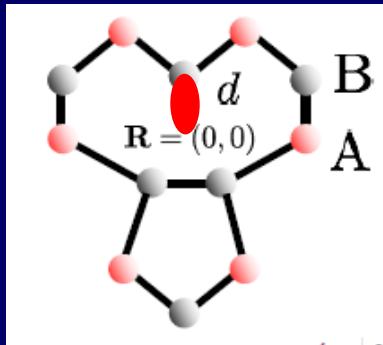


Can J be Finite in Graphene?

Local Moment Formation and Kondo Effect in Defective Graphene

M. A. Cazalilla,^{1,2} A. Iucci,³ F. Guinea,⁴ and A. H. Castro Neto²

- Out of plane distortion of dangling bond
- ↳ Finite AF coupling with conduction electrons ↔ Kondo screening



$$J \sim \sin \theta$$

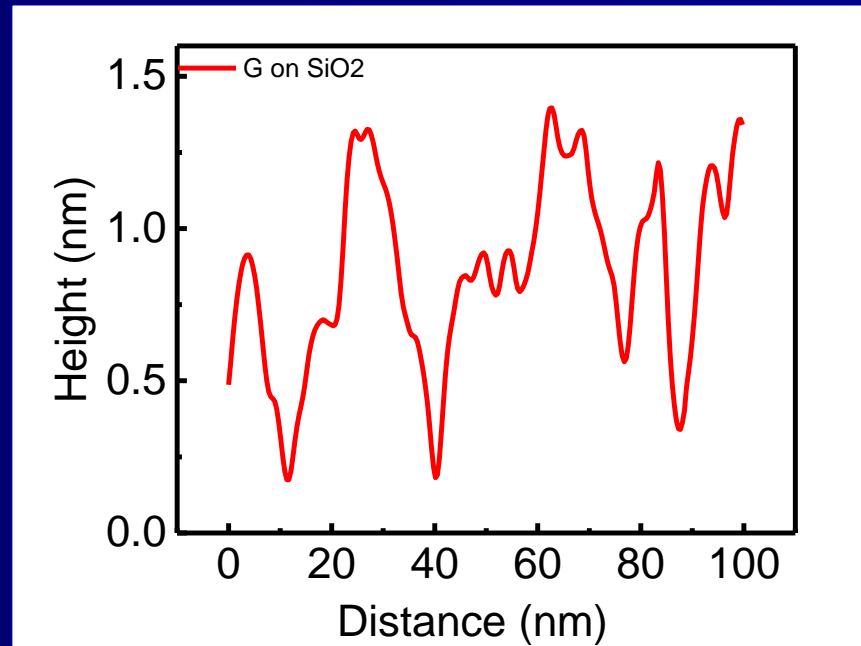
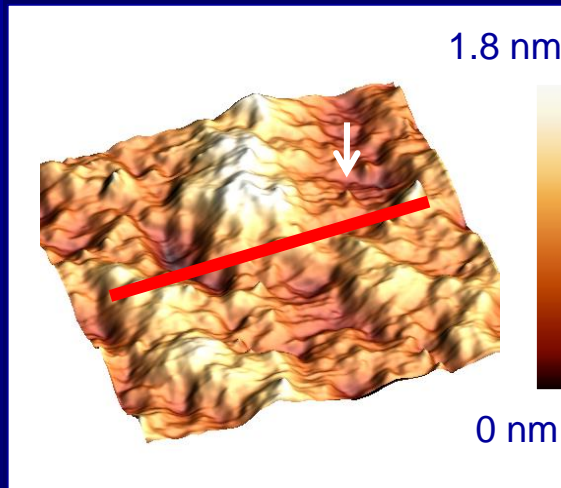
Finite Kondo coupling

Corrugated Substrate ??



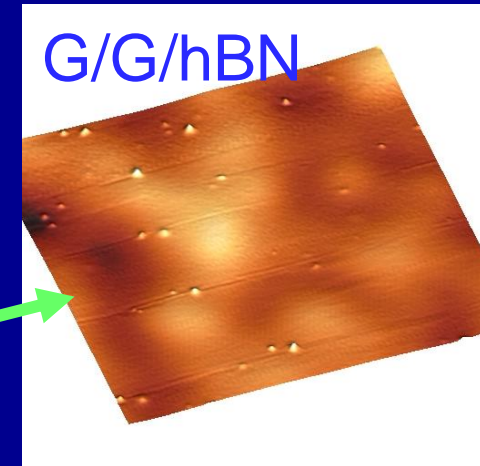
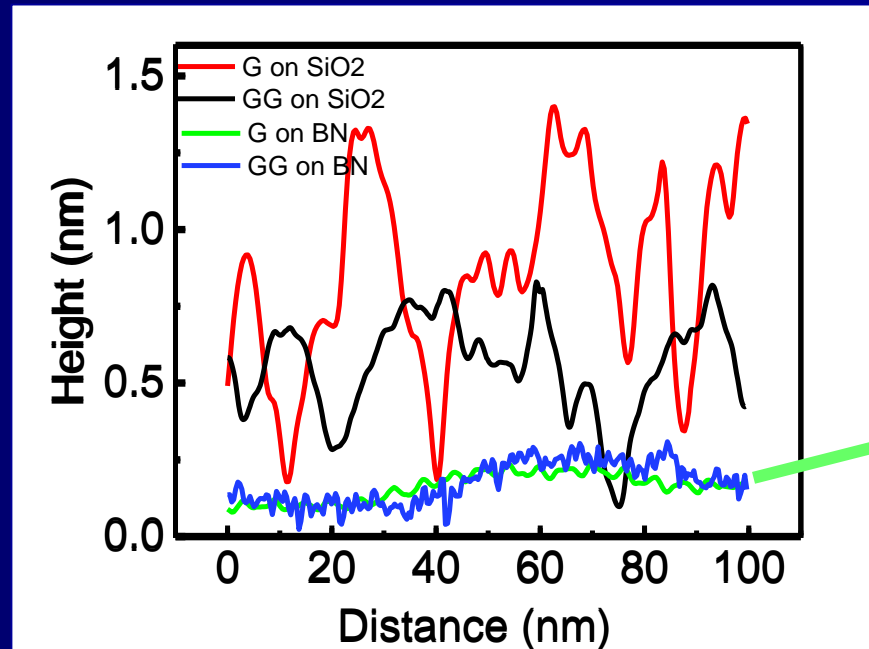
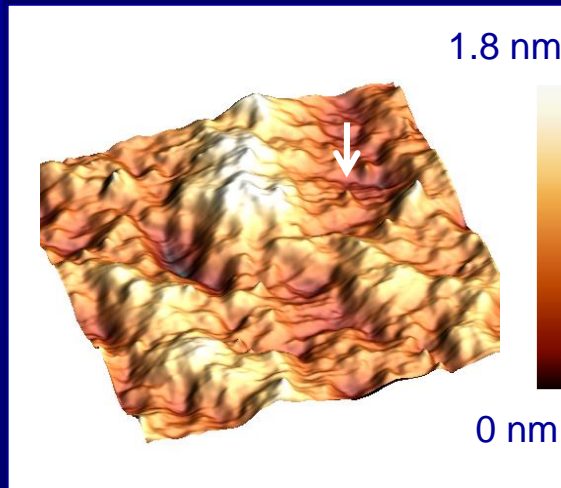
Substrate corrugation and Kondo screening

Substrate Corrugation	G/SiO ₂ 2nm
Maximum T _K	T _K ~180K
% of screened vacancies	Most



Substrate corrugation and Kondo screening

Substrate Corrugation	G/SiO ₂ 2nm	G/G/SiO ₂ 1nm	G/hBN 0.2nm	G/G/hBN 0.2nm
Maximum T _K	T _K ~180K	T _K ~ 70K	No Kondo	No Kondo
% of screened vacancies	Most	30%	none	none



J depends on Local corrugation
 ↳ Mechanically controlled magnetism



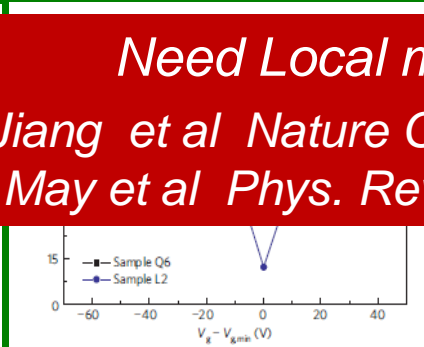
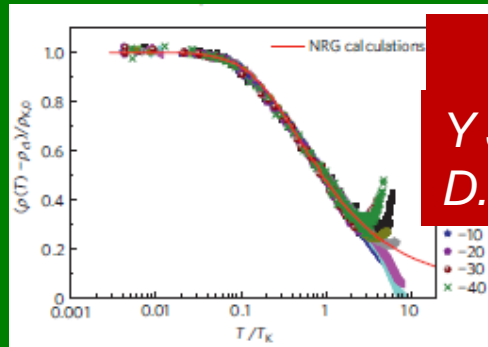
Global Measurements and Conflicting results

nature physics LETTERS
 PUBLISHED ONLINE: 3 APRIL 2011 | DOI: 10.1038/NPHYS1962

Tunable Kondo effect in graphene with defects

Jian-Hao Chen^{1,2†}, Liang Li², William G. Cullen^{1,2}, Ellen D. Williams^{1,2} and Michael S. Fuhrer^{1,2*}

- $R(T) \mapsto$ Kondo screening
 - T_K 20-70K



Need Local measurement
Y Jiang et al Nature Communications 2018
D. May et al Phys. Rev. B 97, 155419 (2018)

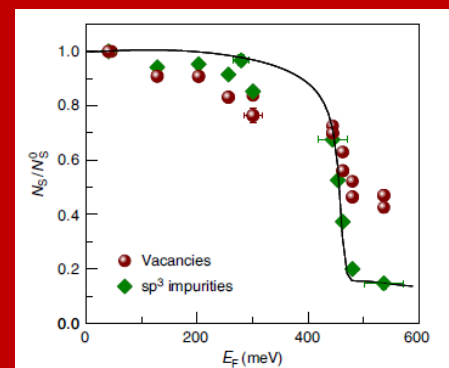
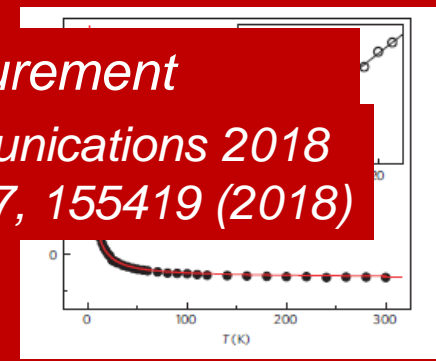
- Measures:
- Scattering off Kondo cloud
 - Sensitive to screened Moments only.

nature physics LETTERS
 PUBLISHED ONLINE: 10 JANUARY 2012 | DOI: 10.1038/NPHYS2183

Spin-half paramagnetism in graphene induced by point defects

R. R. Nair¹, M. Sepioni¹, I-Ling Tsai¹, O. Lehtinen², J. Keinonen², A. V. Krashenninnikov^{2,3}, T. Thomson¹, A. K. Geim¹ and I. V. Grigorieva^{1*}

- $\chi(T) \mapsto$ No Kondo screening
- Moments unscreened at low T

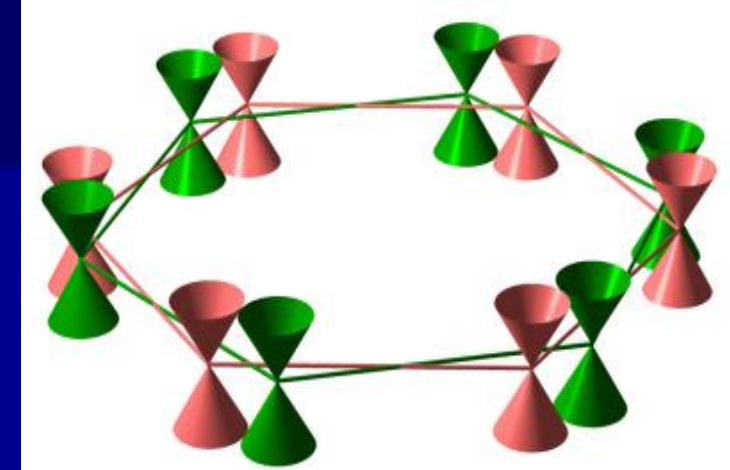
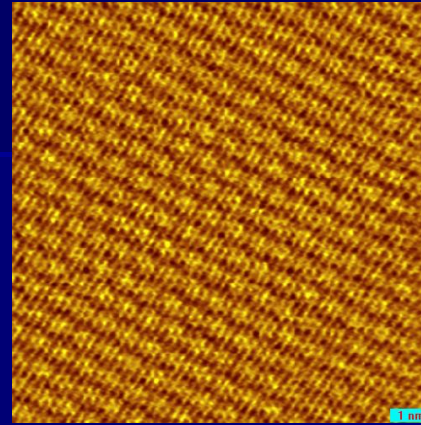
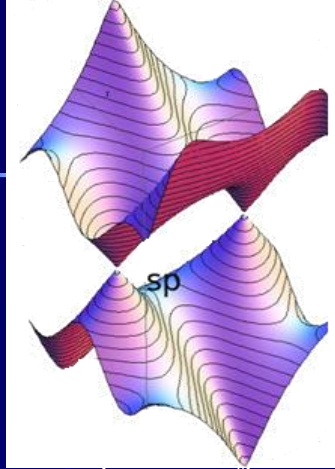


- Measures:
- Magnetic moments
 - Sensitive to unscreened Moments only.

➢ Global measurements probe complementary properties



Graphene with a twist



❖ Engineering electronic properties

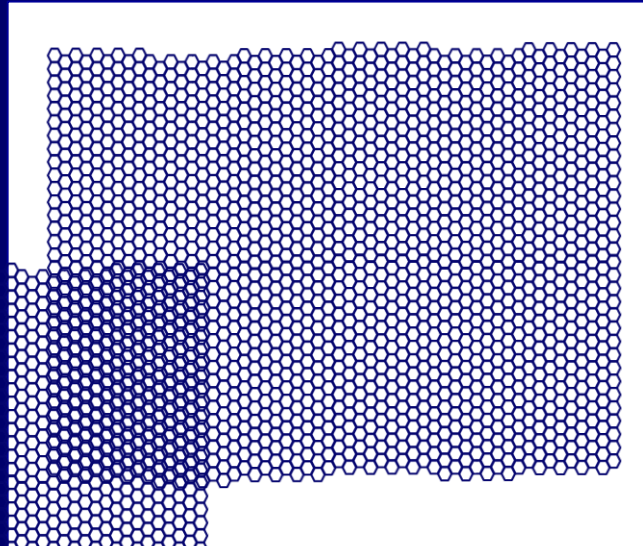
- Density of states
- Landau levels in graphene
- Scanning tunneling microscopy (STM) and spectroscopy (STS)
- Atomic collapse and artificial atom
- Kondo effect
- Twisted graphene



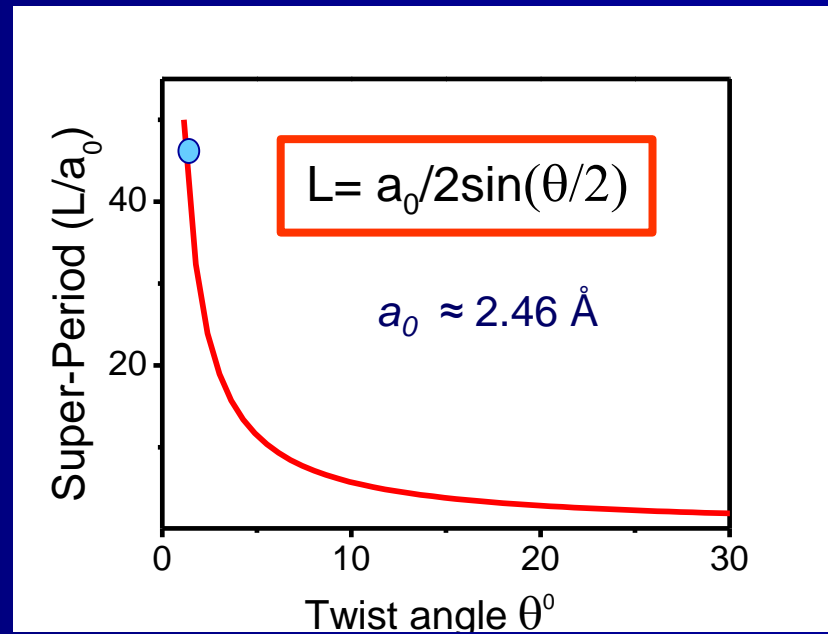
Twisted graphene – Moiré patterns

Twist between layers \mapsto Moiré pattern:

$$\theta = 3^\circ$$



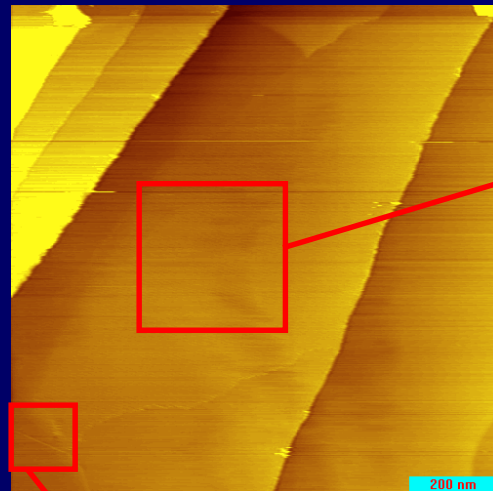
Superstructure with period L



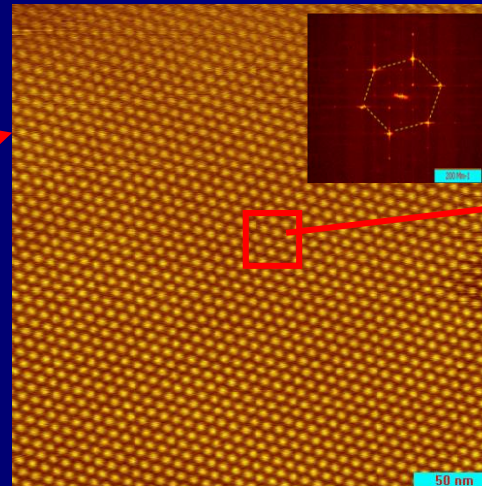
STM topography: Moiré superstructure

G. Li, et al Nature Physics (2010)

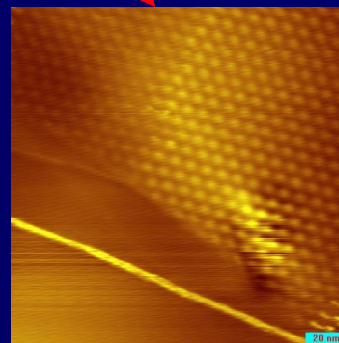
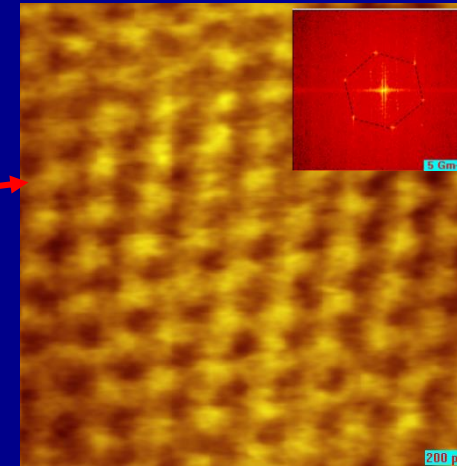
superstructure $L=7.5\text{nm}$ \rightarrow $\theta=1.79^\circ$



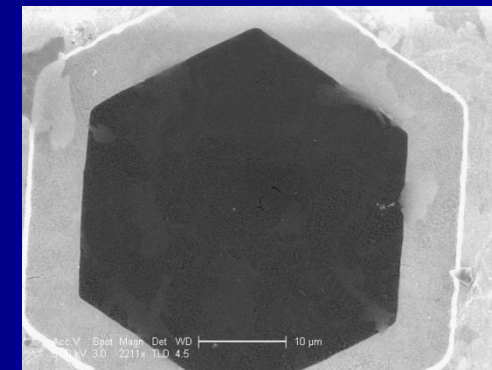
x4



x250

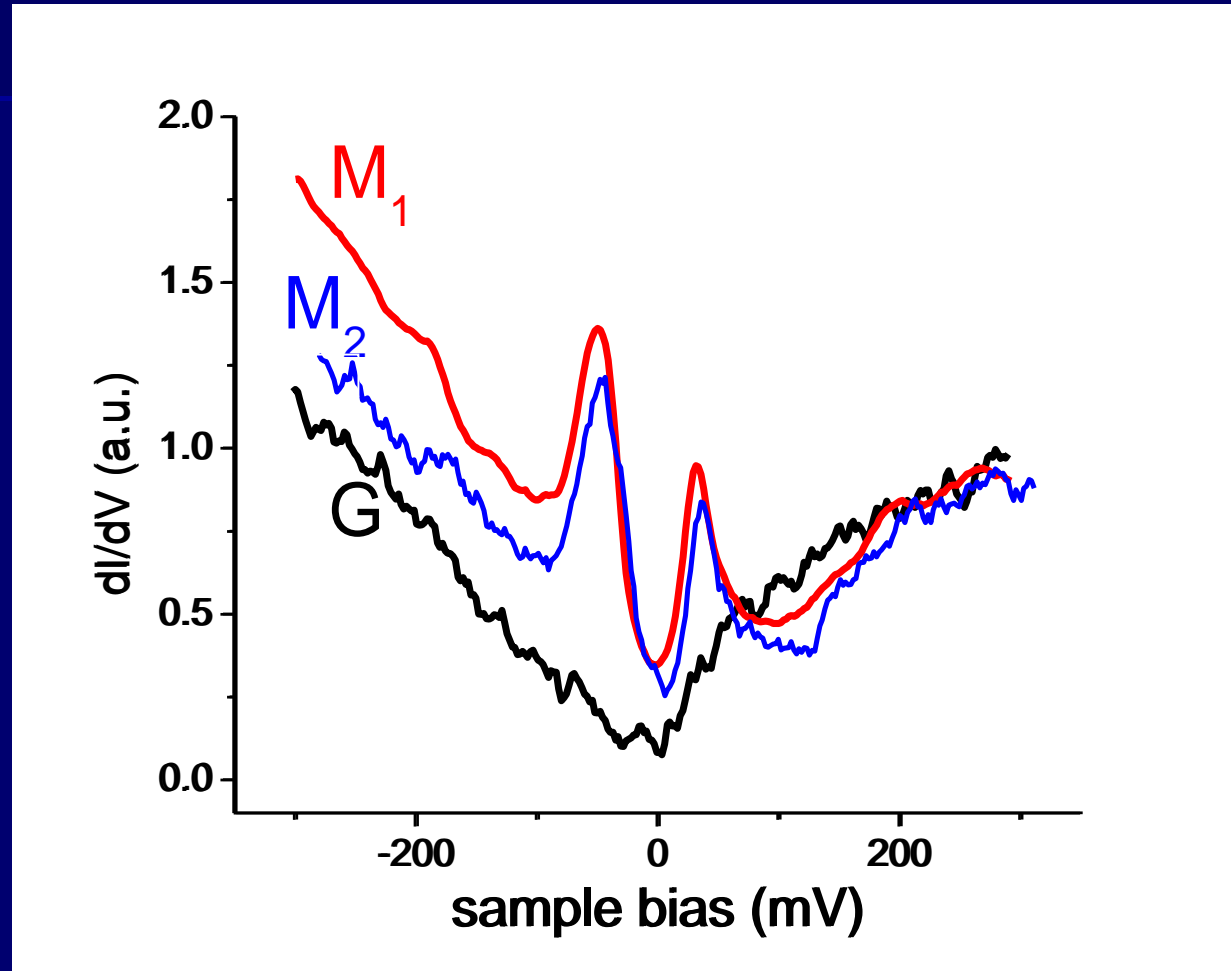
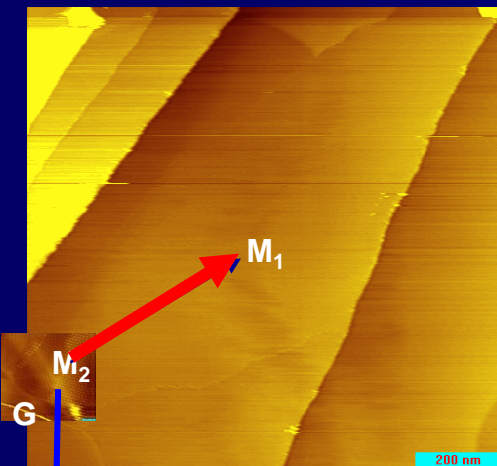
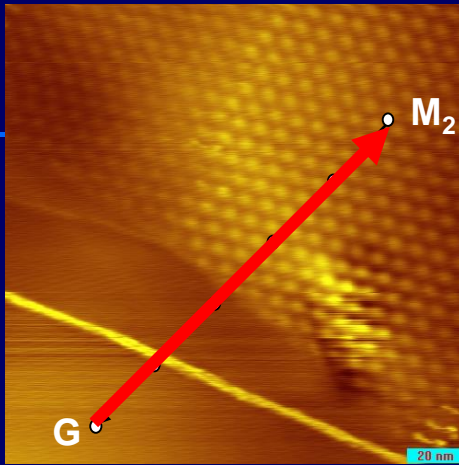


Boundary of area with superstructure



Spectroscopy Surprise

G. Li, et al Nature Physics (2010)



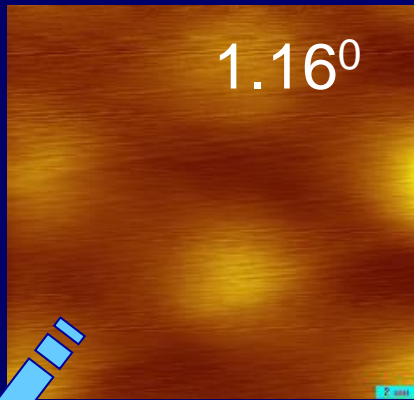
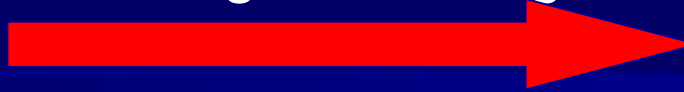
Two peak structure only in twisted region



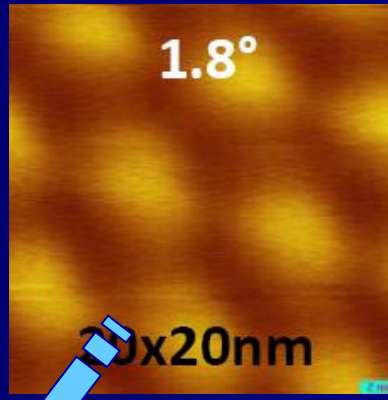
Band structure of twisted graphene

G. Li, et al *Nature Physics* 6, 109 (2010)

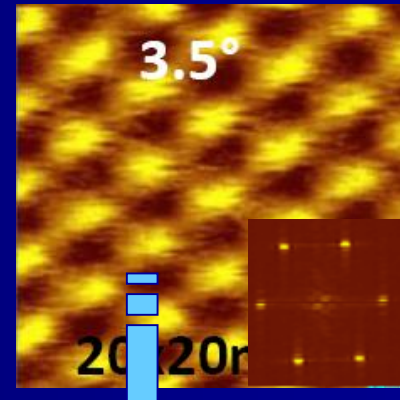
Increasing twist angle θ



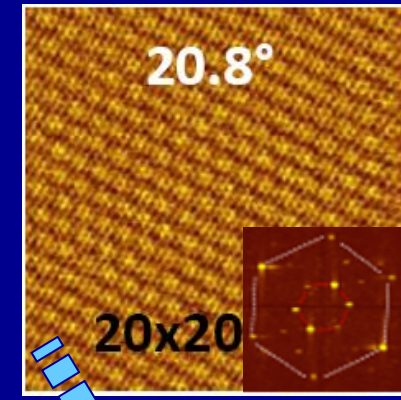
L = 12 nm



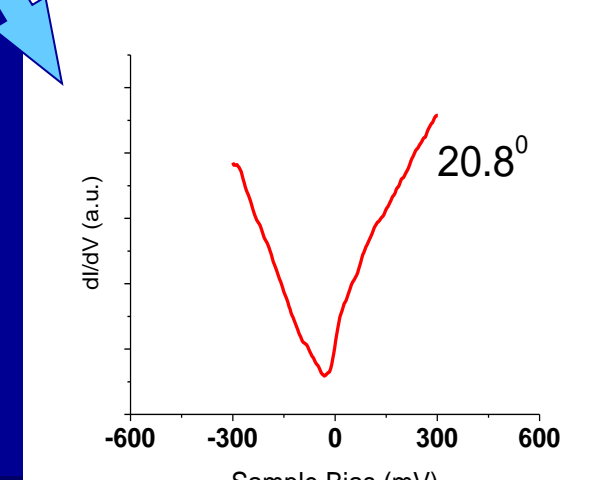
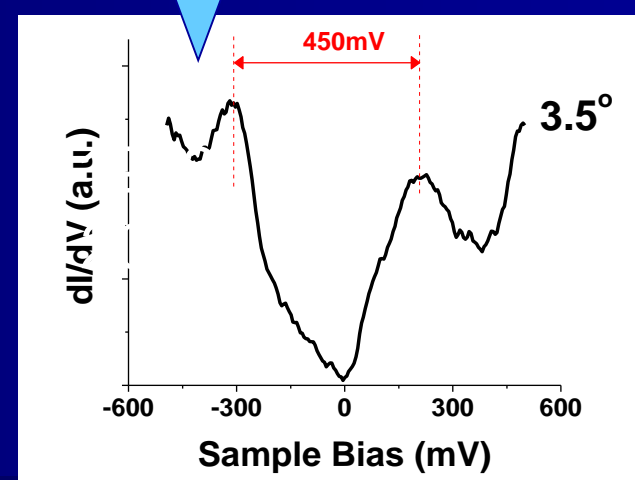
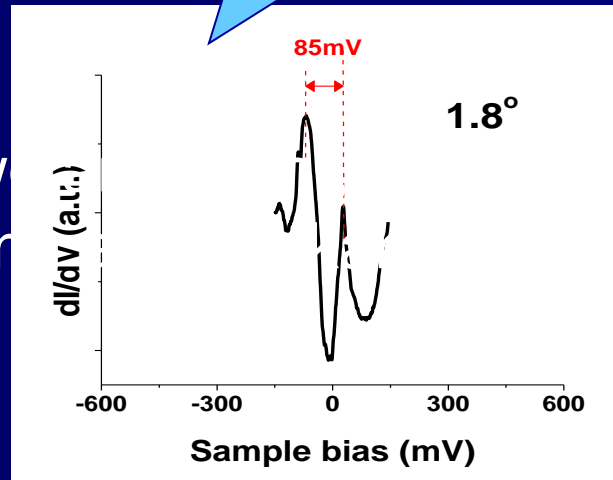
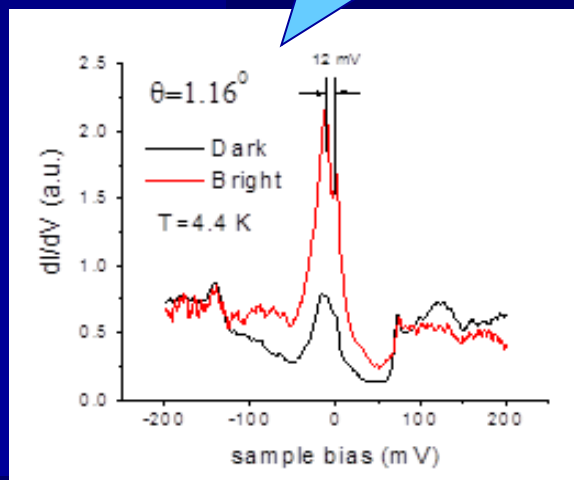
L = 7.8 nm



L = 4.0 nm

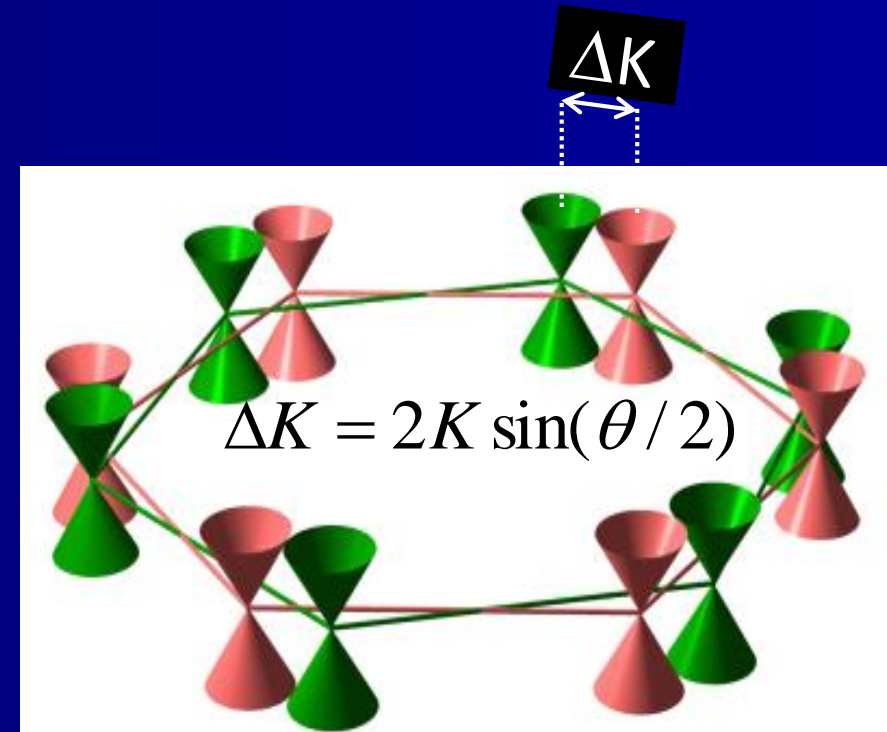
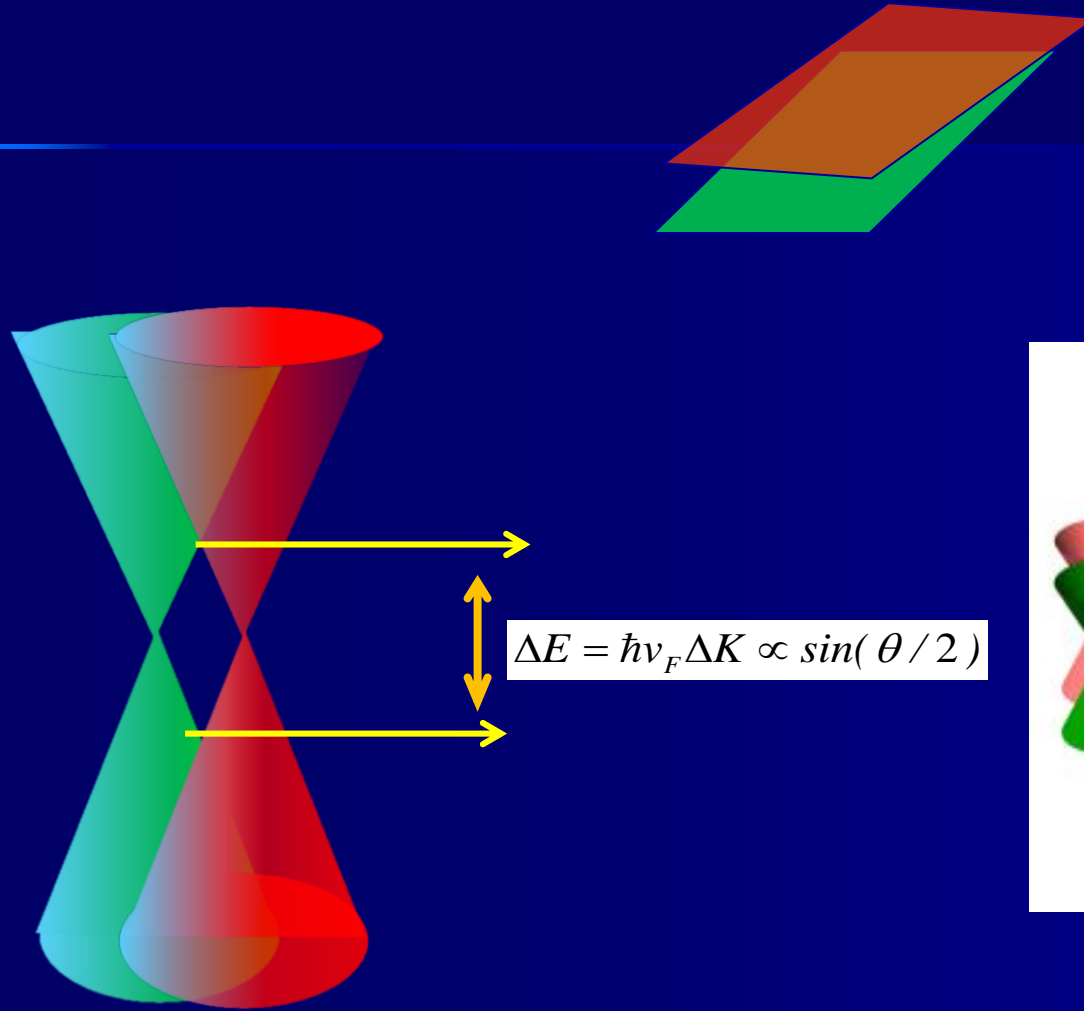


L = 0.7 nm



Band structure engineering with a twist

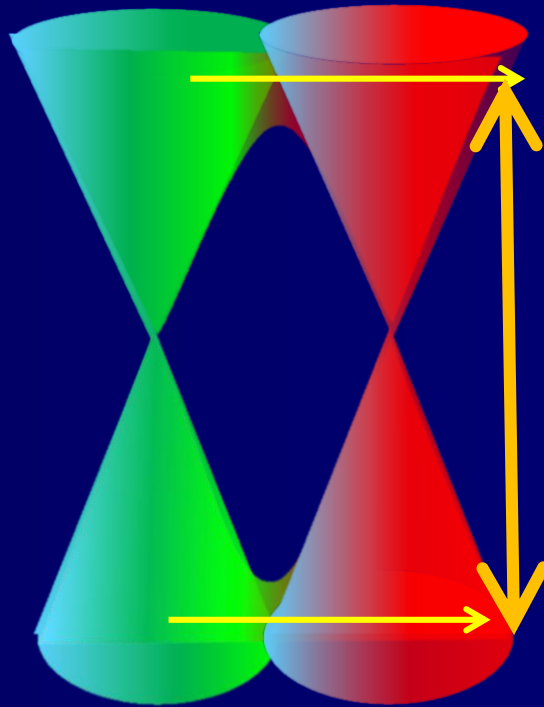
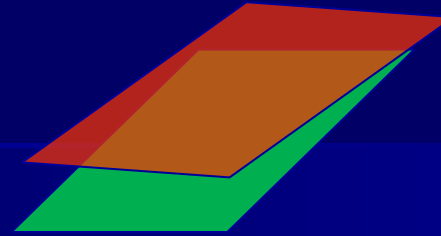
J. Lopes dos Santos, A.H. Castro Neto



Band structure engineering with a twist

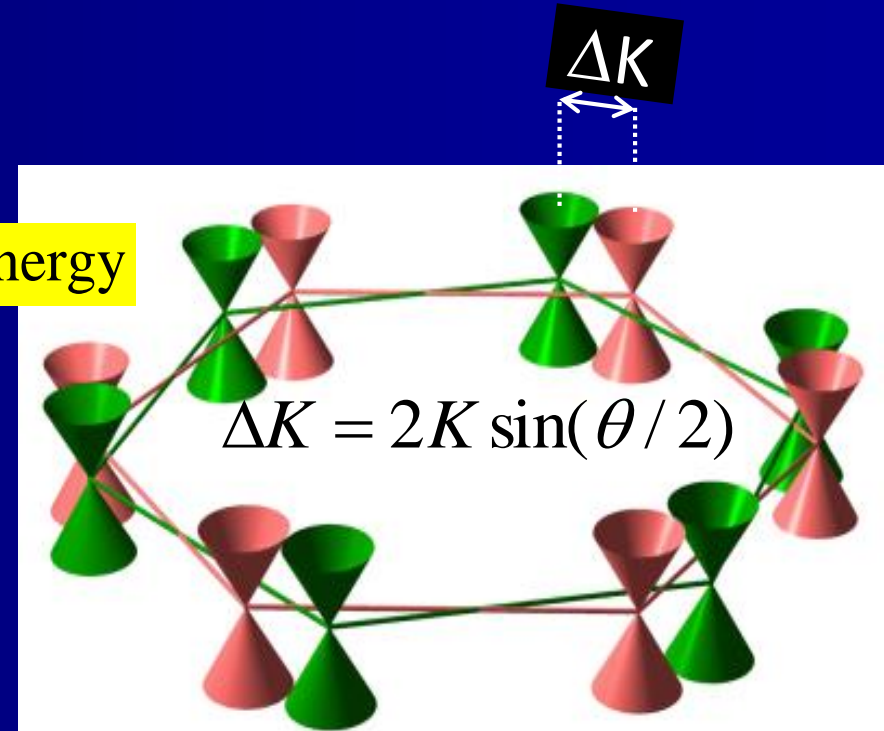
J. Lopes dos Santos, A.H. Castro Neto

Hybridization



w hybridization energy

$$\Delta E = \hbar v_F \Delta K - w$$

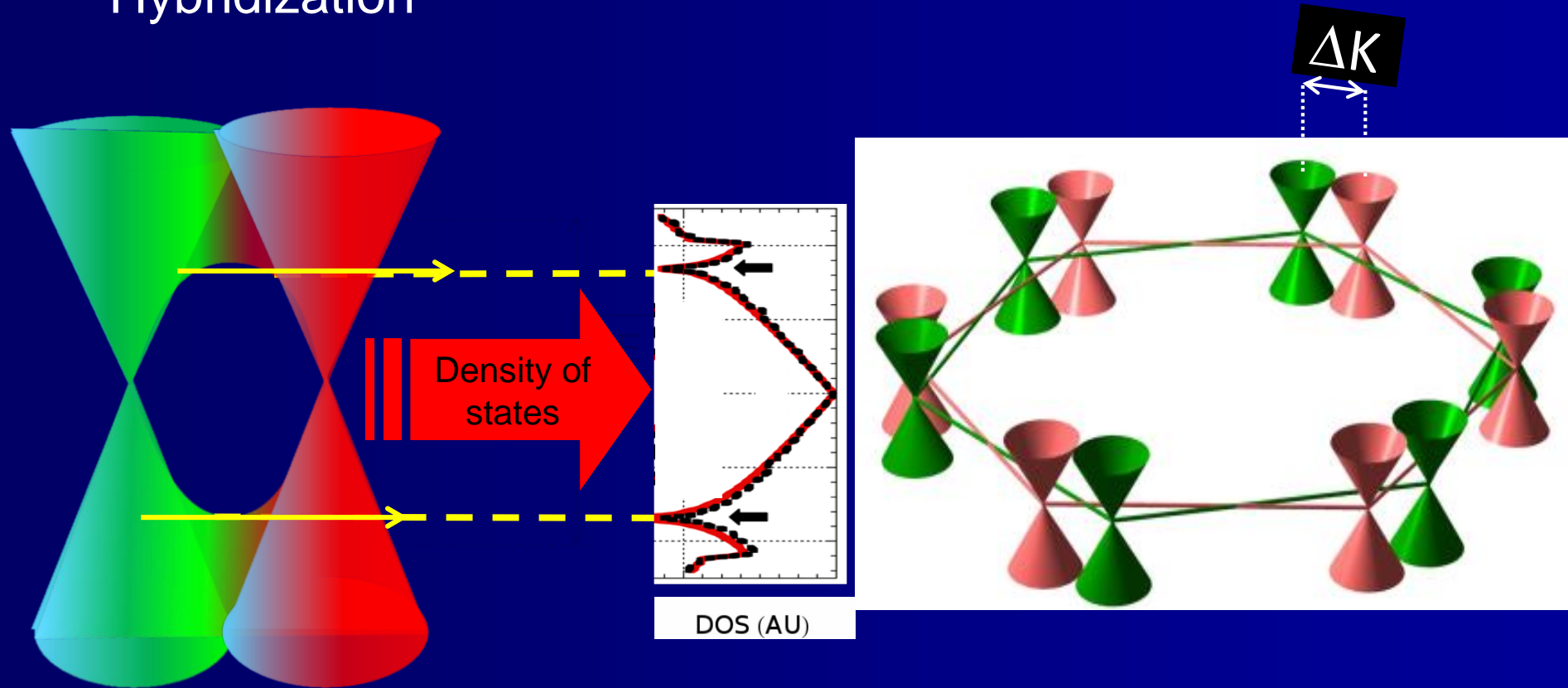


Van Hove singularities

G. Li, et al Nature Physics (2010)

A. Luican, et al PRL 106, 126802 (2011)

Hybridization



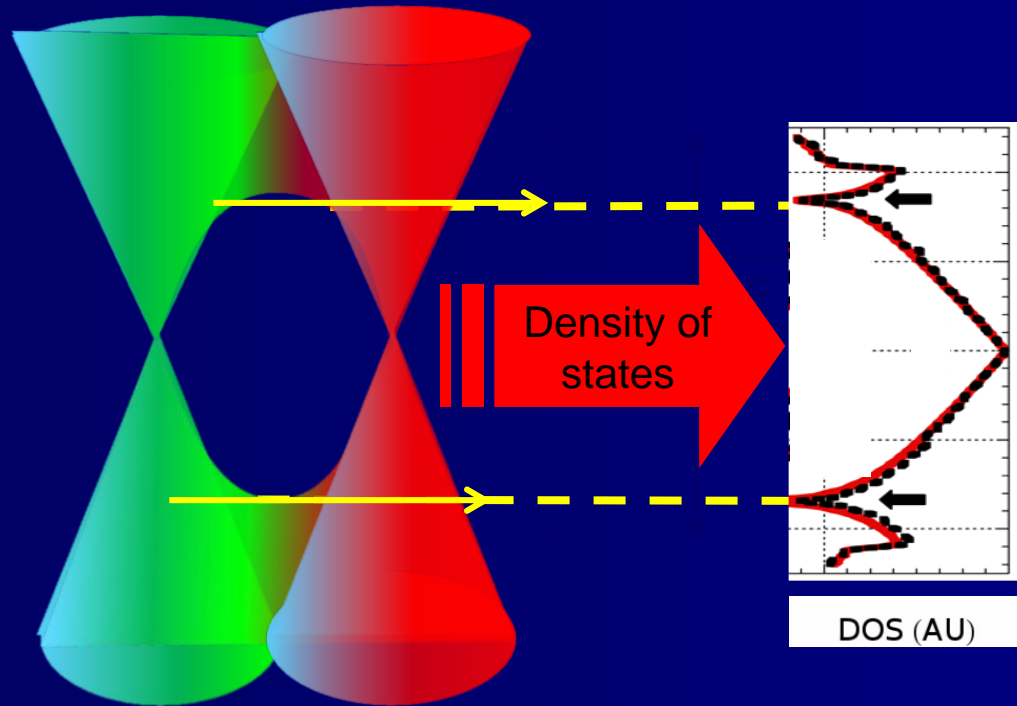
Twisted graphene
develops strong *Van Hove singularities*



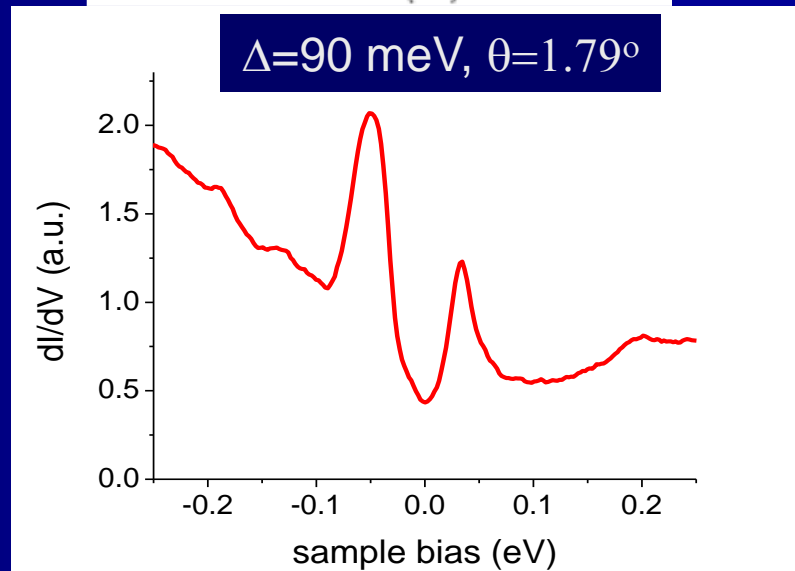
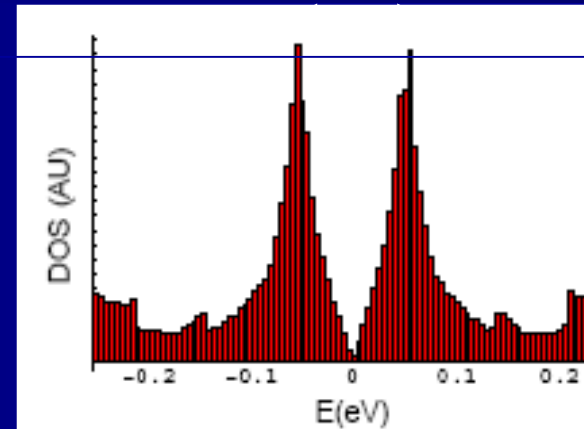
Van Hove singularities

G. Li, et al *Nature Physics* (2010)

A. Luican, et al *PRL* 106, 126802 (2011)

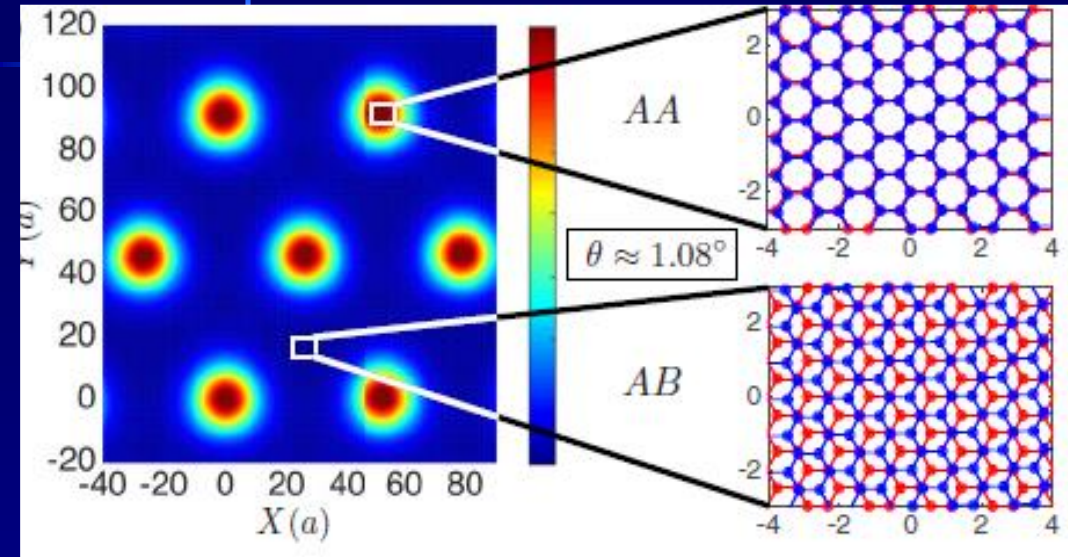


Low energy Band structure and DOS using perturbation theory



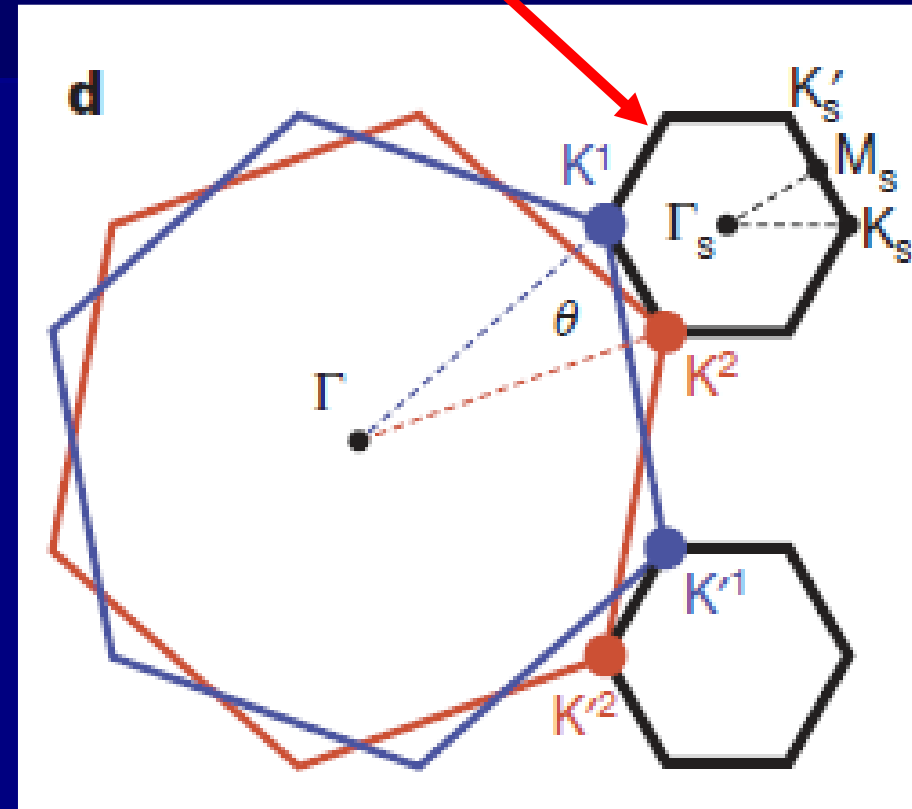
What happens at small twist angles?

DOS map

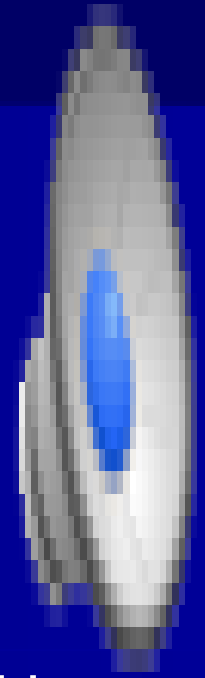
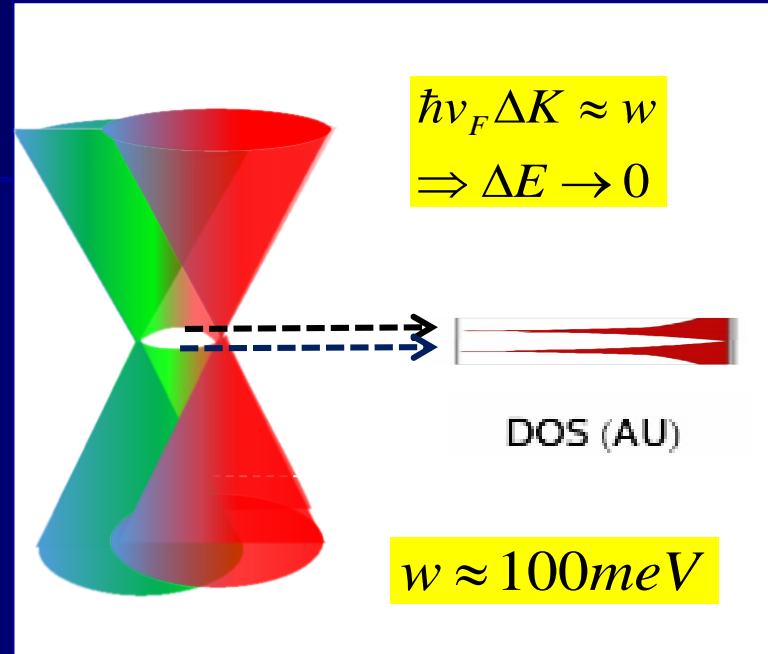
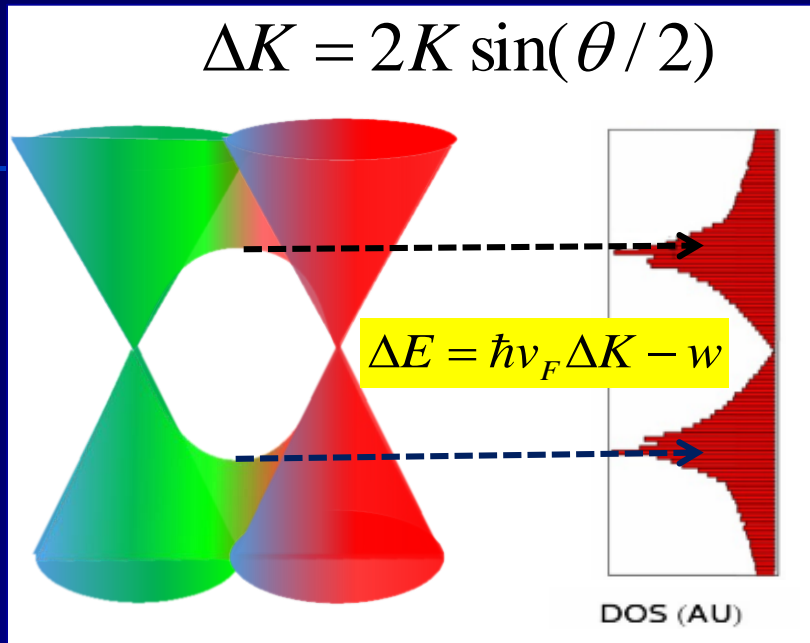


Moire units cell contains 1.3×10^4 atoms!

Moire Brillouin zone



What happens at small twist angles?



For $\hbar v_F \Delta K \approx w$ band flattens \mapsto DOS diverges: Van Hove singularities.

Magic angle:

$$\hbar v_F K \theta_M \approx w$$

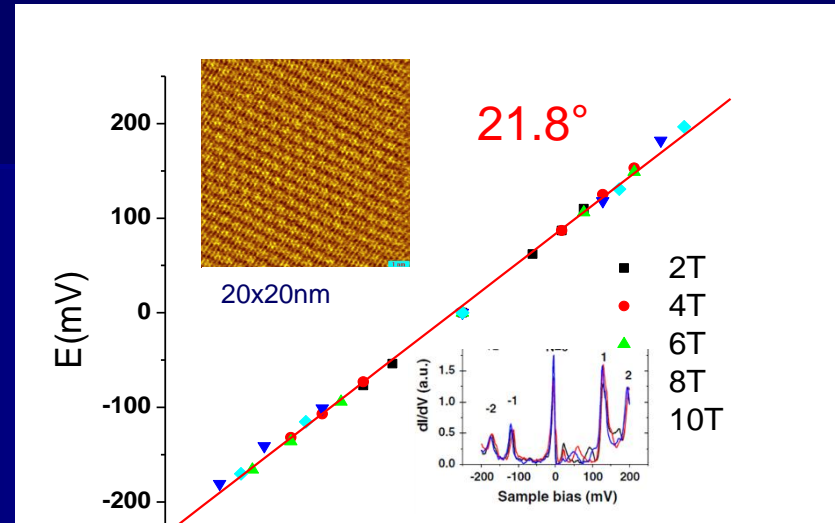
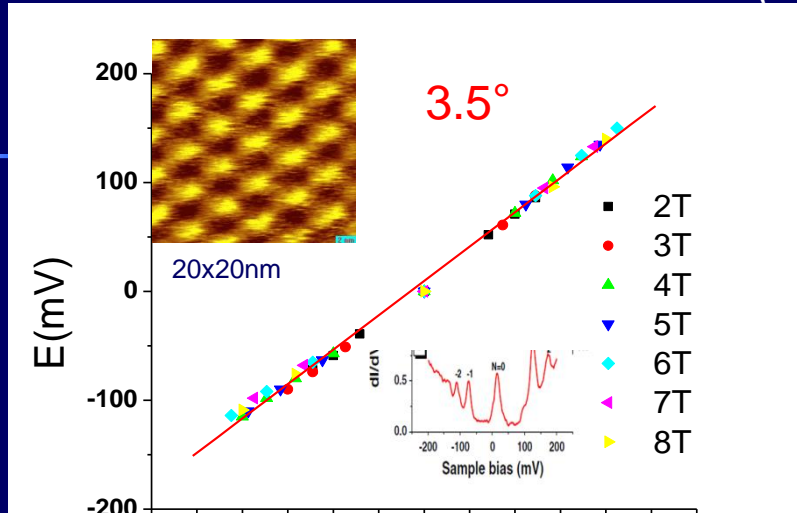
Using $w \sim 100 \text{ meV}$

$$\theta_M \approx 1.09^\circ$$



Fermi velocity : slowdown

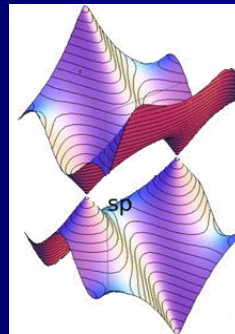
A. Luican, et al PRL 106, 126802 (2011)



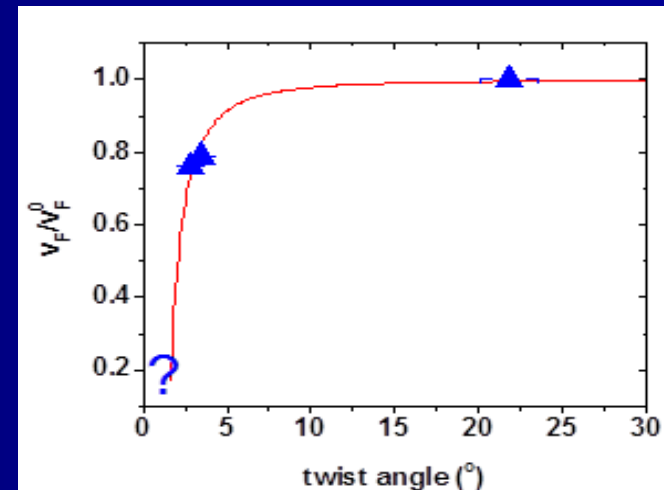
For $\theta > 10^\circ$

low energy band structure of twisted layers is identical to single layer

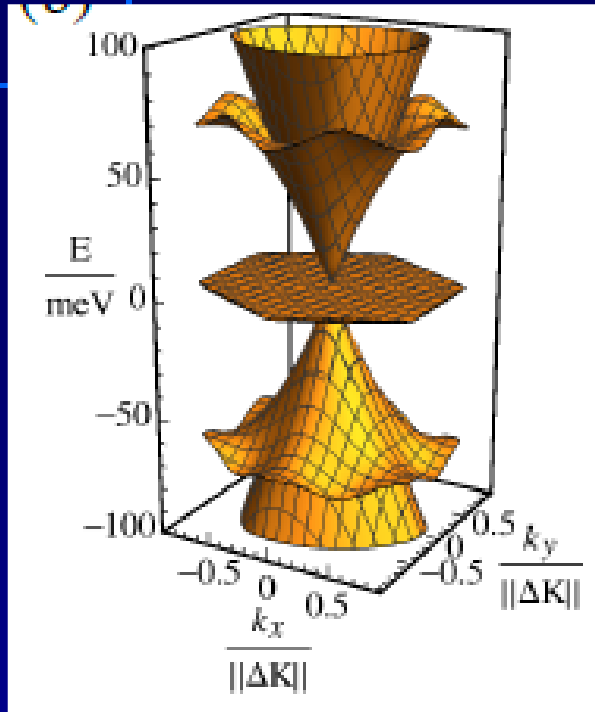
$$\frac{\tilde{v}_F(\theta)}{v_F} = 1 - 9 \left(\frac{w}{\hbar v_F \Delta K} \right)^2$$



- J.M.B.L. dos Santos et al. PRL 99, 256802 (2007).
- G.T. Laissardière et al, Nanoletters ASAP (2009)
- Shallcross et al. PRL. 101, 056803 (2008)
- R. Bistritzer, and A. H. MacDonald, (2010)



Correlation effects



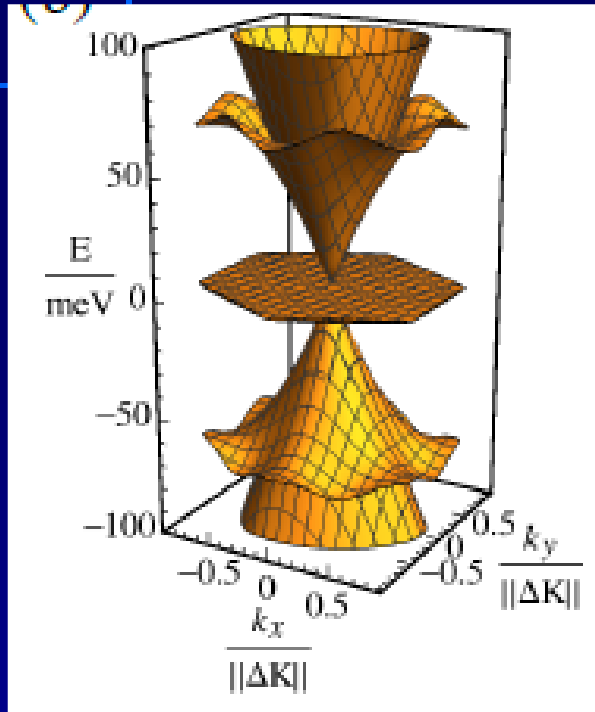
- When the energy scale of electron-electron interactions is comparable to the band-width, correlation effects become important.
- At $\frac{1}{2}$ filling (Fermi energy in middle of gap): Correlated states can emerge: superconductivity, charge density waves, antiferromagnetism, topological insulators etc.

$$E_{\text{coulomb}} = \frac{e^2}{4\pi\epsilon_0\kappa\lambda^2};$$

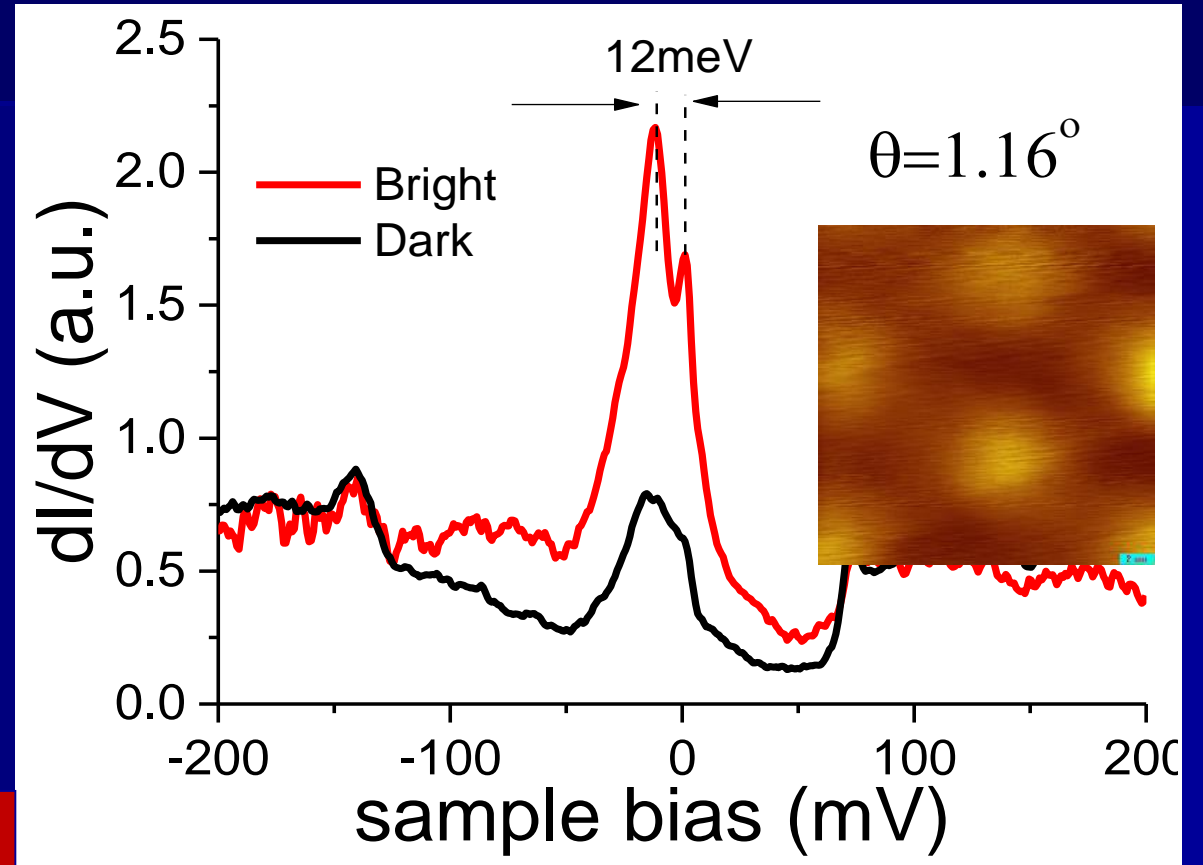
λ moire period;
 κ dielectric constant



Correlation gap



G. Li, et al *Nature Physics* **6**, 109 (2010)



For $\theta = 1.16^\circ$

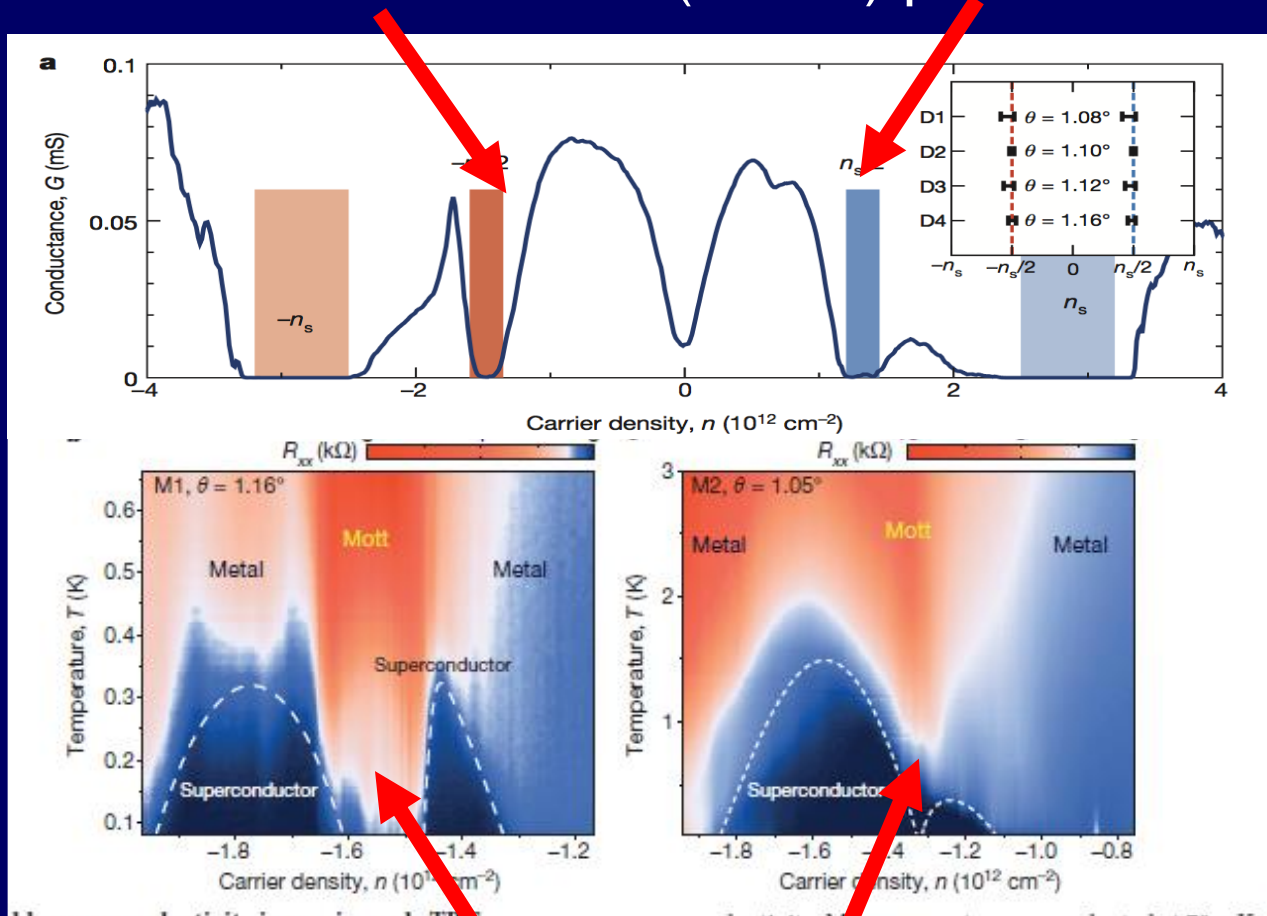
- 12meV Gap opens at the Fermi energy!
- Correlation gap – CDW?



2018 - Magic angle insulator and superconductor

Y. Cao et al., Nature(2018)

- Half Full band – 2 electrons (holes) per moire cell.



- $\frac{1}{2}$ Full band – insulating phase $\sim 4\text{K}$
- Insulating phase flanked by 2 superconducting domes slightly off half-filling
- Maximum $T_c \sim 1.7\text{K}$
- $T_c/E_F \sim 10^{-1} \mapsto$ strong coupling



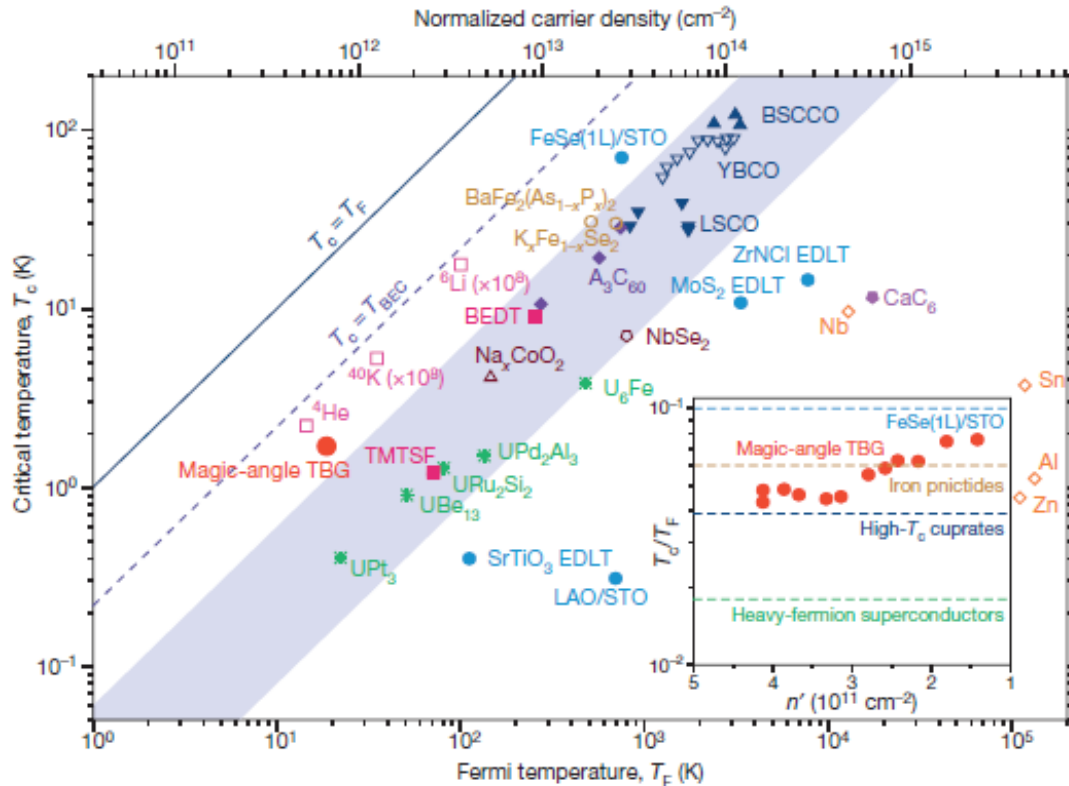
- Resembles high T_c superconductors
- BUT tunable doping and T_c

$\frac{1}{2}$ Full band – 2 electrons (holes) per moire cell.



Strongly coupled superconductor

Y. Cao et al., Nature(2018)



- $\frac{1}{2}$ Full band – insulating phase $\sim 4\text{K}$
- Insulating phase flanked by 2 superconducting domes
- $T_c/E_F \sim 10^{-1} \mapsto$ strong coupling



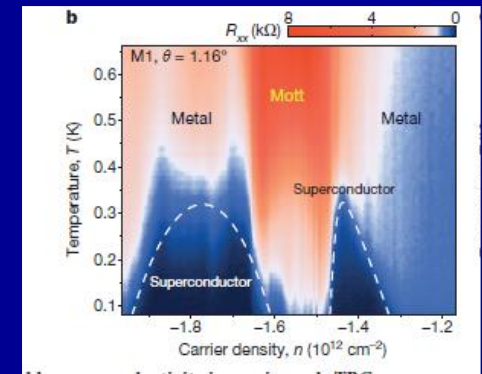
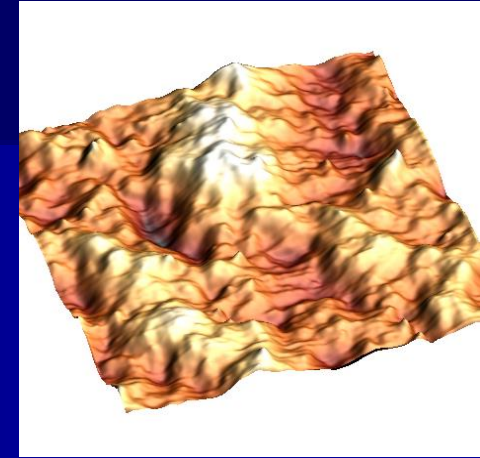
- Resembles high T_c superconductors
- BUT tunable doping and T_c

- OPEN QUESTIONS:**
- Pairing mechanism
 - Gap symmetry
 - Nature of insulating phase



Summary of part IV

- Kondo screening in graphene occurs above a critical coupling strength
- Magnetic moments in graphene can be tuned with gating or local curvature
- If coupling strength is non-uniform global measurements are misleading
- Band structure of bilayer graphene can be tuned with twist angle
- At small twist angles the DOS develops Van-Hove singularities
- At the “magic angle” $\theta \sim 1.1^\circ$ a flat band forms at the charge neutrality point
- At half filling the flat band develops strong correlations resulting in an insulating phase flanked by superconducting domes.



2D materials are cool!

