The Challenge of Heavy Fermions

3rd Super-Pire, REIMEI workshop,
Beijing IOP, March 2014
Frontiers of Condensed Matter Physics.

Piers Coleman[1,2]
(1) CMT, Rutgers U, NJ, USA
(2) Royal Holloway, U. London, UK.

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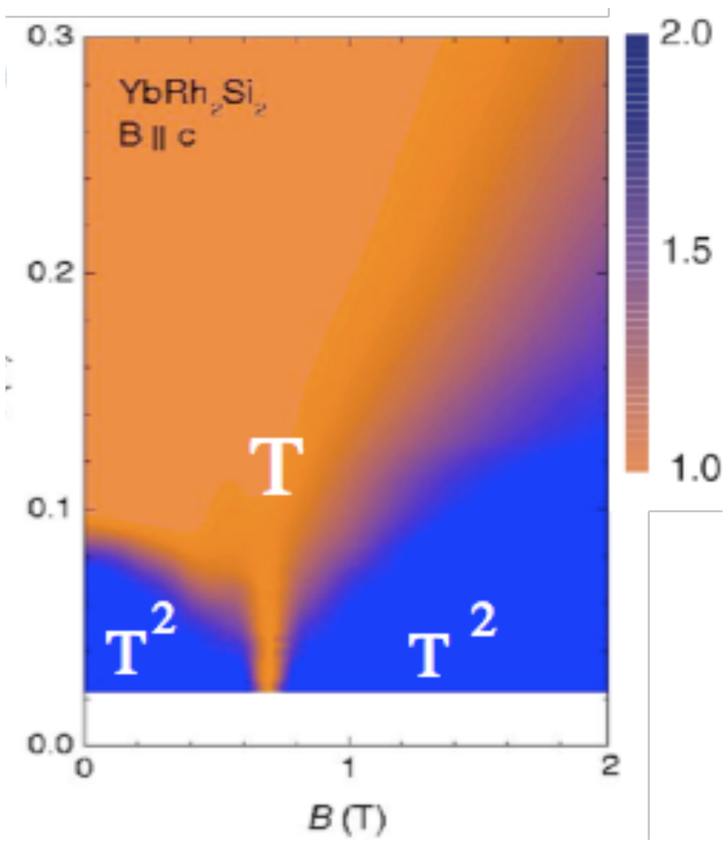
Quantum Criticality & Strange Metals



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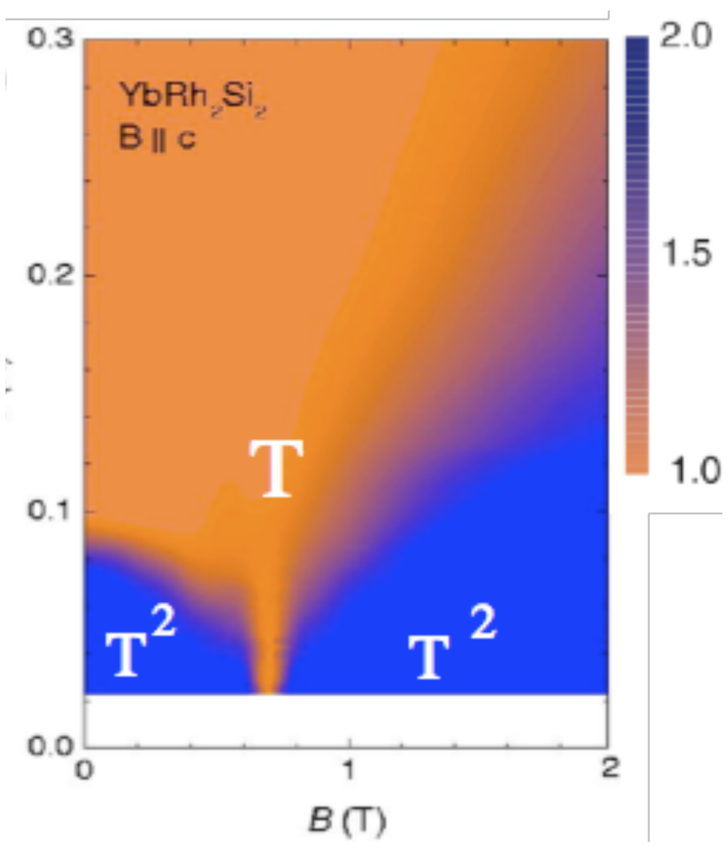
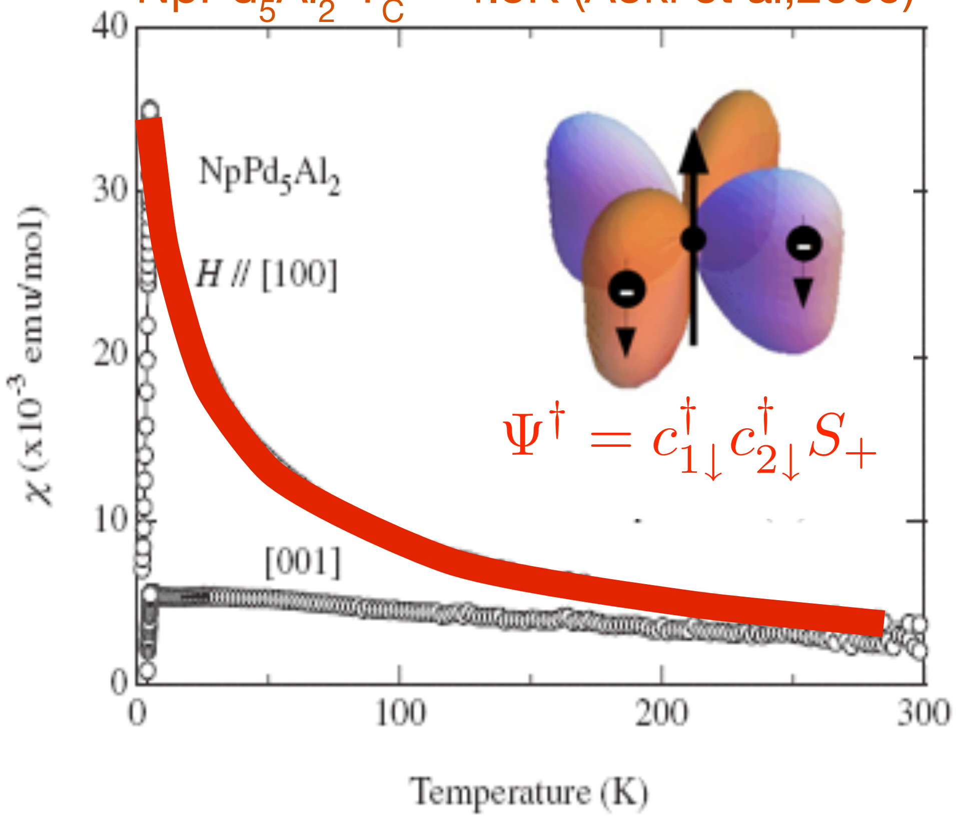


Heavy Fermion SC Composite Pairs

NpPd$_5$Al$_2$ T_C = 4.5K (Aoki et al,2009)



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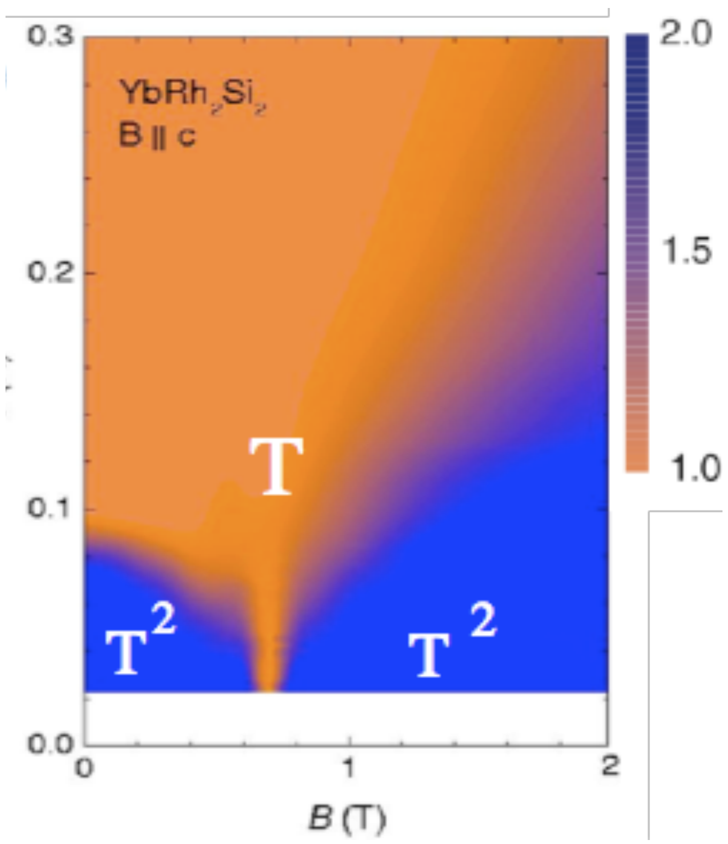

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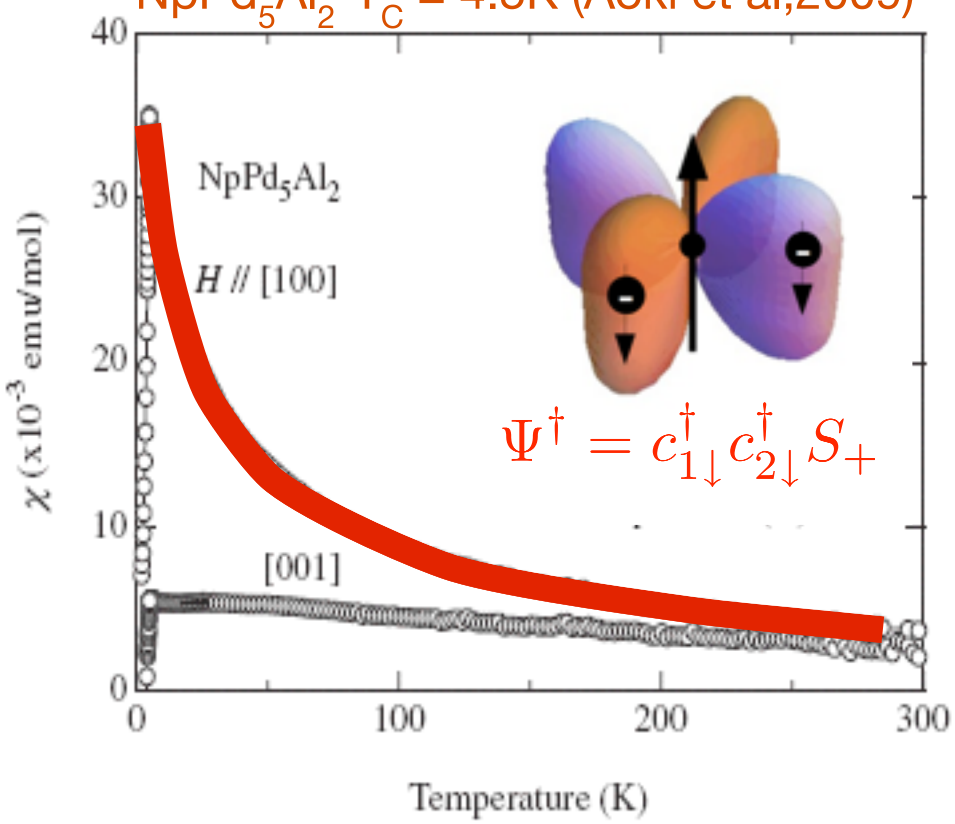
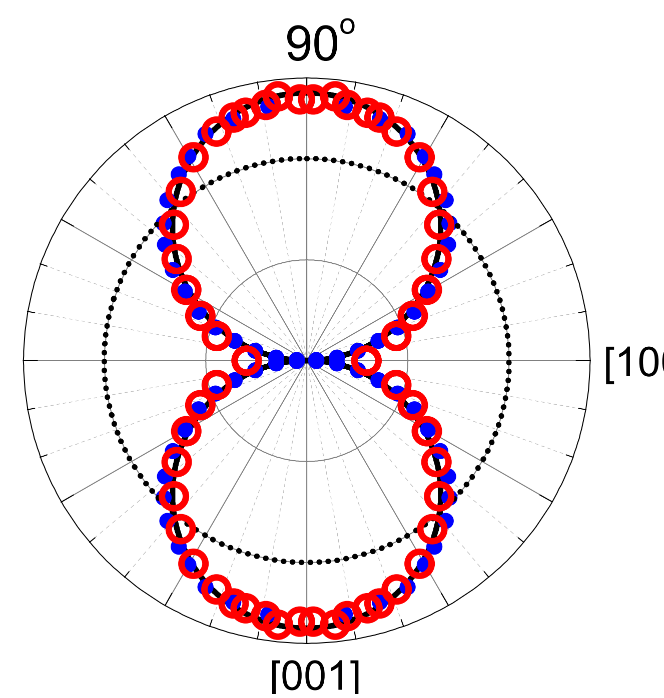


URu$_2$Si$_2$ Hidden Order



Altarawneh et al., (2012)

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Collaborators.

QCP:

Q. Si	Rice
R. Ramazashvili	Toulouse
C. Pepin	CEA, Saclay
Aline Ramires	Rutgers

Composite Order

Rebecca Flint	Iowa State
Maxim Dzero	Kent State
Andriy Nevidomskyy	Rice
Alexei Tsvelik	Brookhaven NL
Hai Young Kee	U. Toronto
Natan Andrei	Rutgers
Onur Erten	Rutgers

Hidden Order

R. Flint	Iowa State
Premi Chandra	Rutgers

Experiment

H. von Lohneysen	Karlsruhe
Almut Schröder	Kent State
S. Nakatsuji	ISSP
G. Lonzarich	Cambridge
F Steglich	Dresden/Zhejiang

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Notes:

"Many Body Physics: an introduction", Ch 8,15-16", PC, CUP to be published (2014). http://www.physics.rutgers.edu/~coleman. Password available on request.

"Heavy Fermions: electrons at the edge of magnetism." Wiley encyclopedia of magnetism. PC. cond-mat/0612006.

"I2CAM-FAPERJ Lectures on Heavy Fermion Physics", (X=I, II, III)
http://physics.rutgers.edu/~coleman/talks/RIO13_X.pdf

General reading:

A. Hewson, "Kondo effect to heavy fermions", CUP, (1993).

"The Theory of Quantum Liquids", Nozieres and Pines (Perseus 1999).

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OUTLINE: CHALLENGE OF HEAVY FERMIONS.

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- Heavy Fermions: intro.

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Heavy Fermions: Introduction

Fruit-Fly : 20th C.

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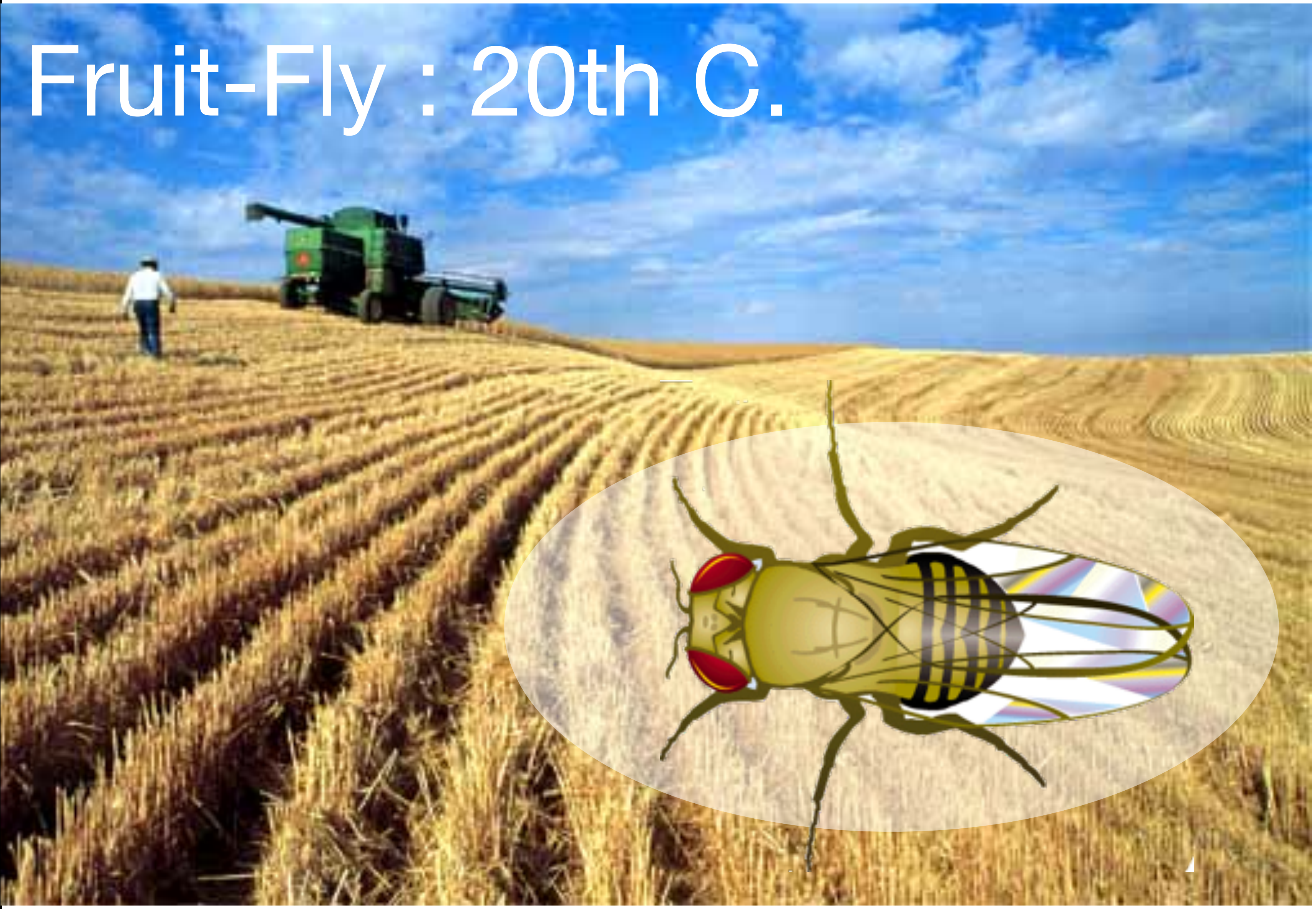

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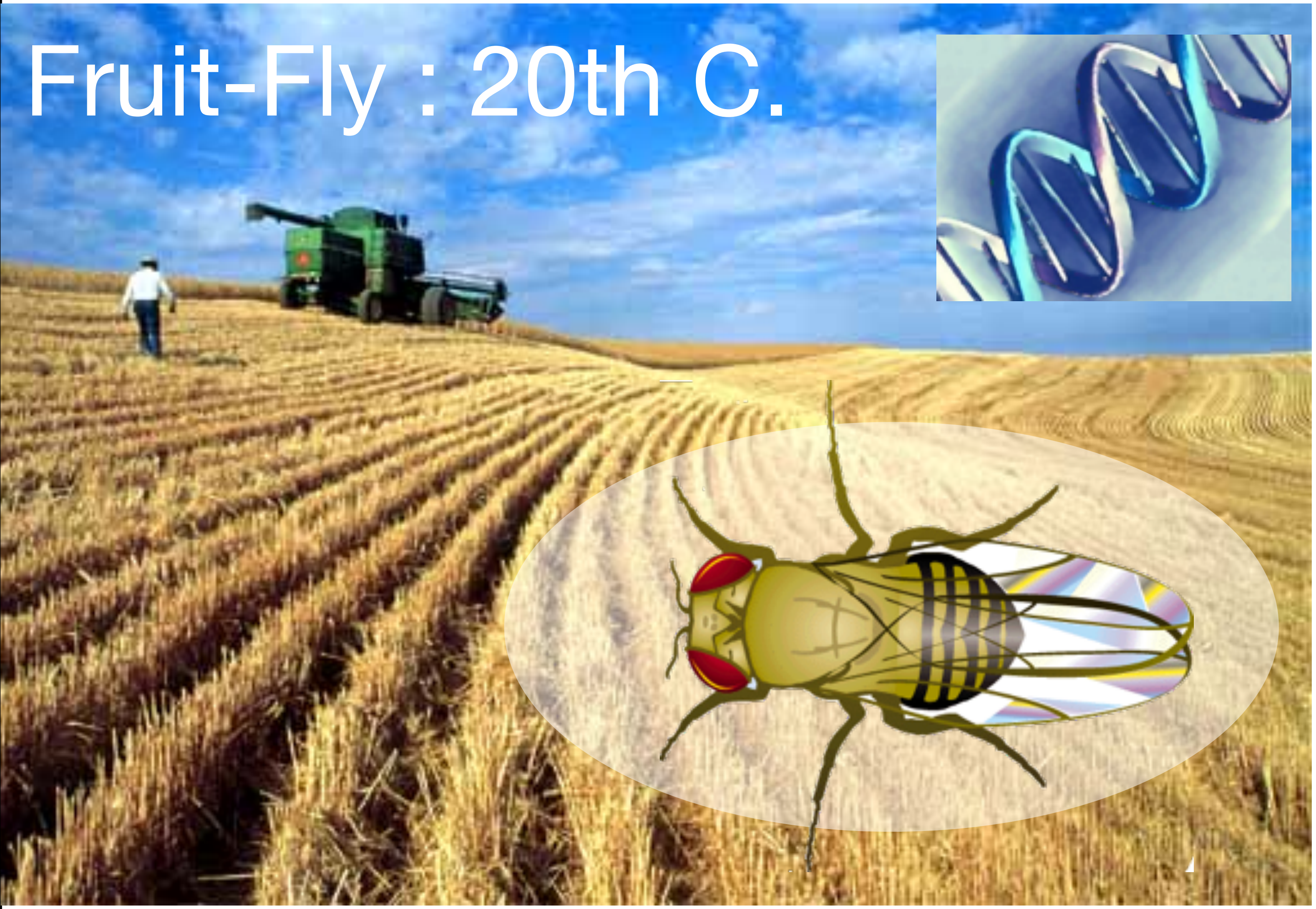

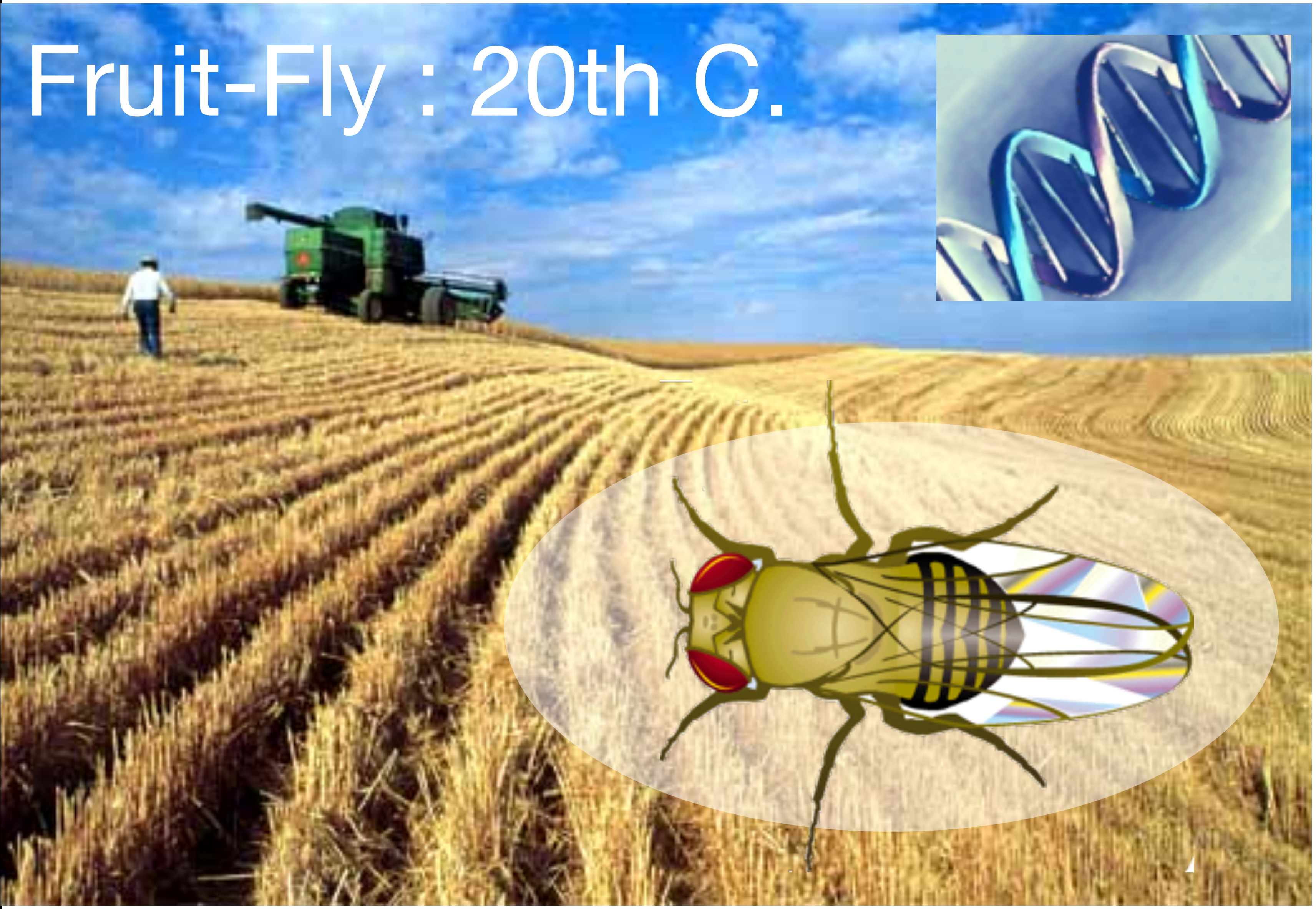
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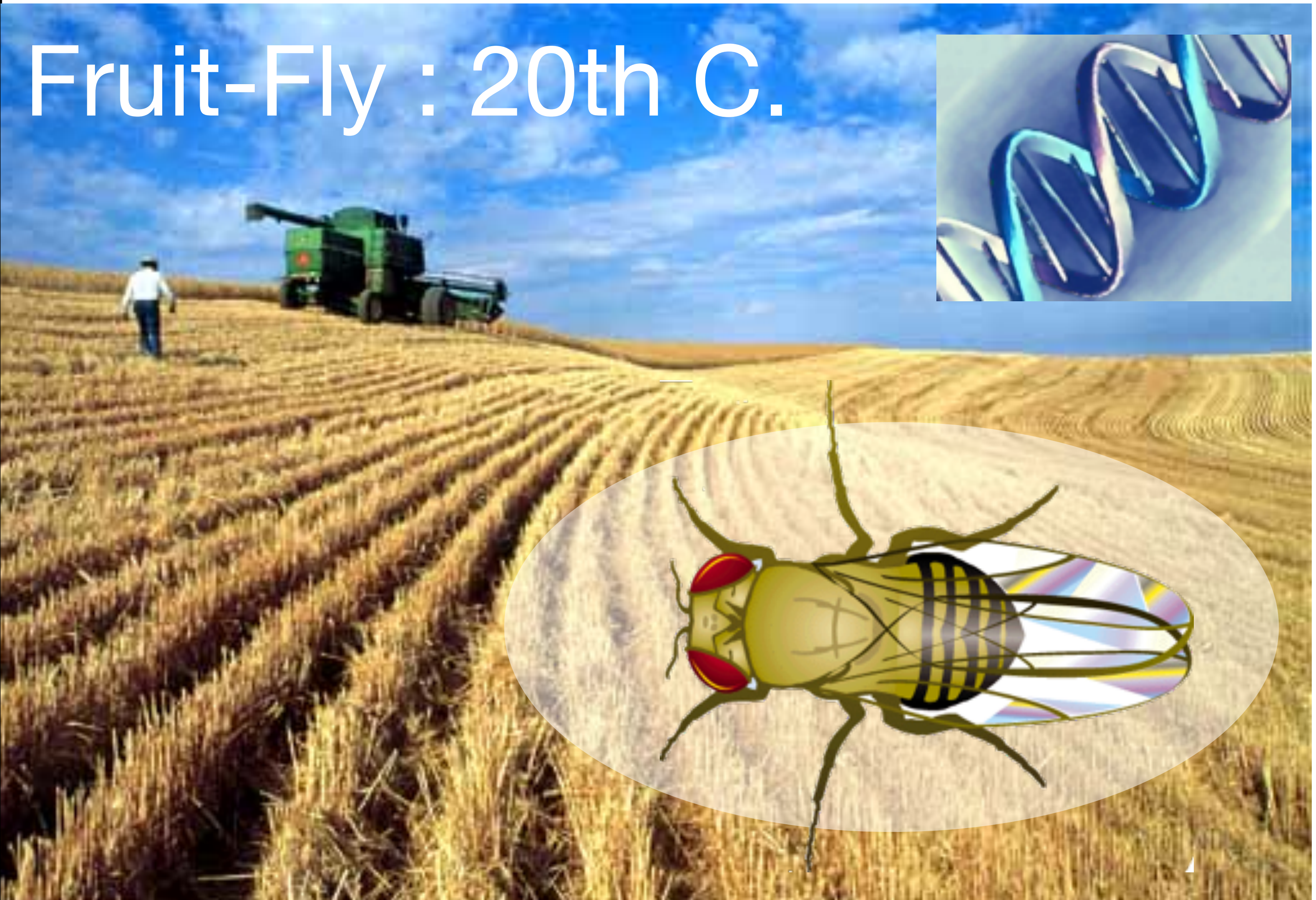
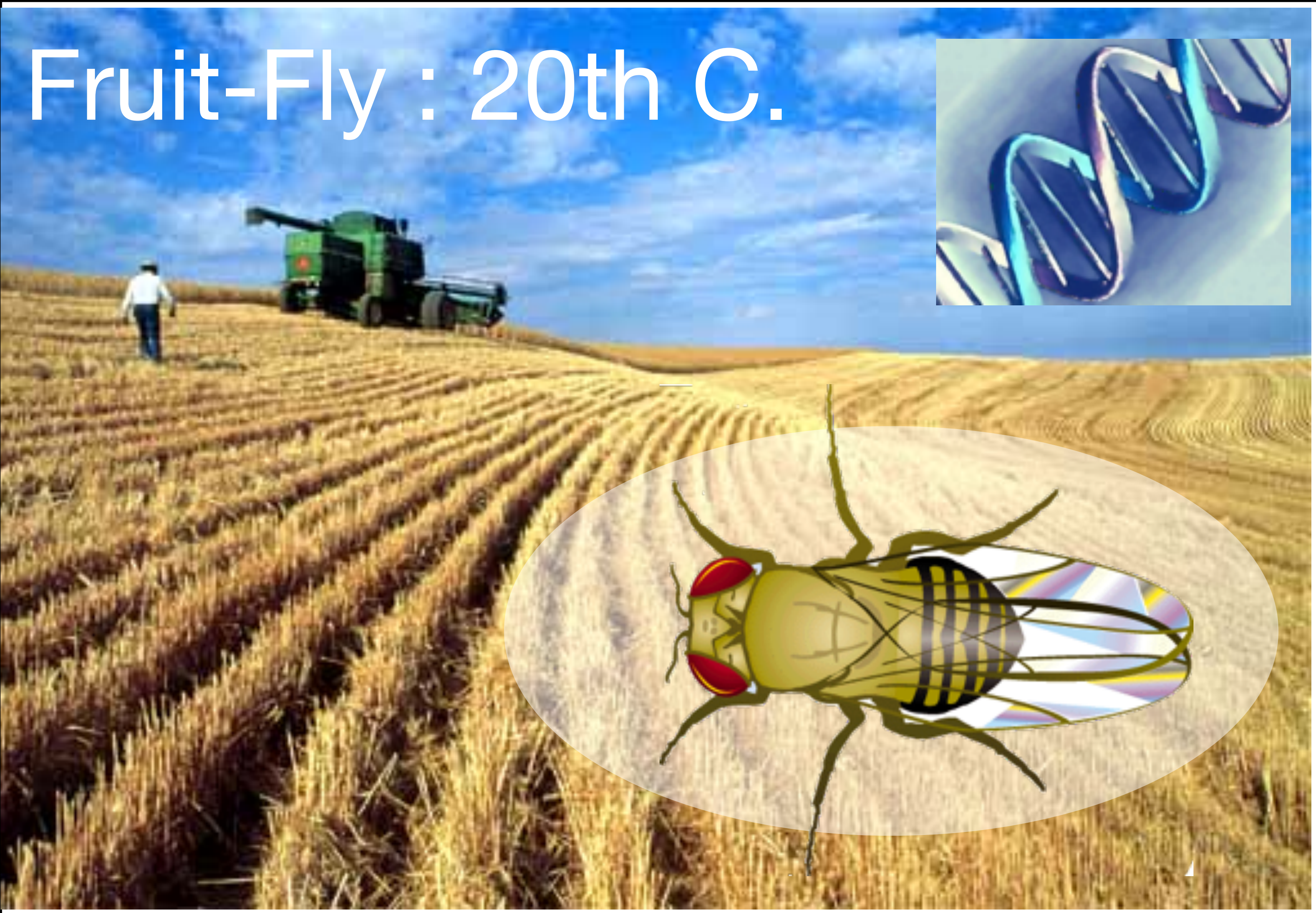
Fruit-Fly of the 21st C


Fruit-Fly : 20th C.


Heavy Electron Physics
PuCoGa5 : 20 K Superconductor
Fruit-Fly of the 21st C


Fruit-Fly : 20th C.

nm µm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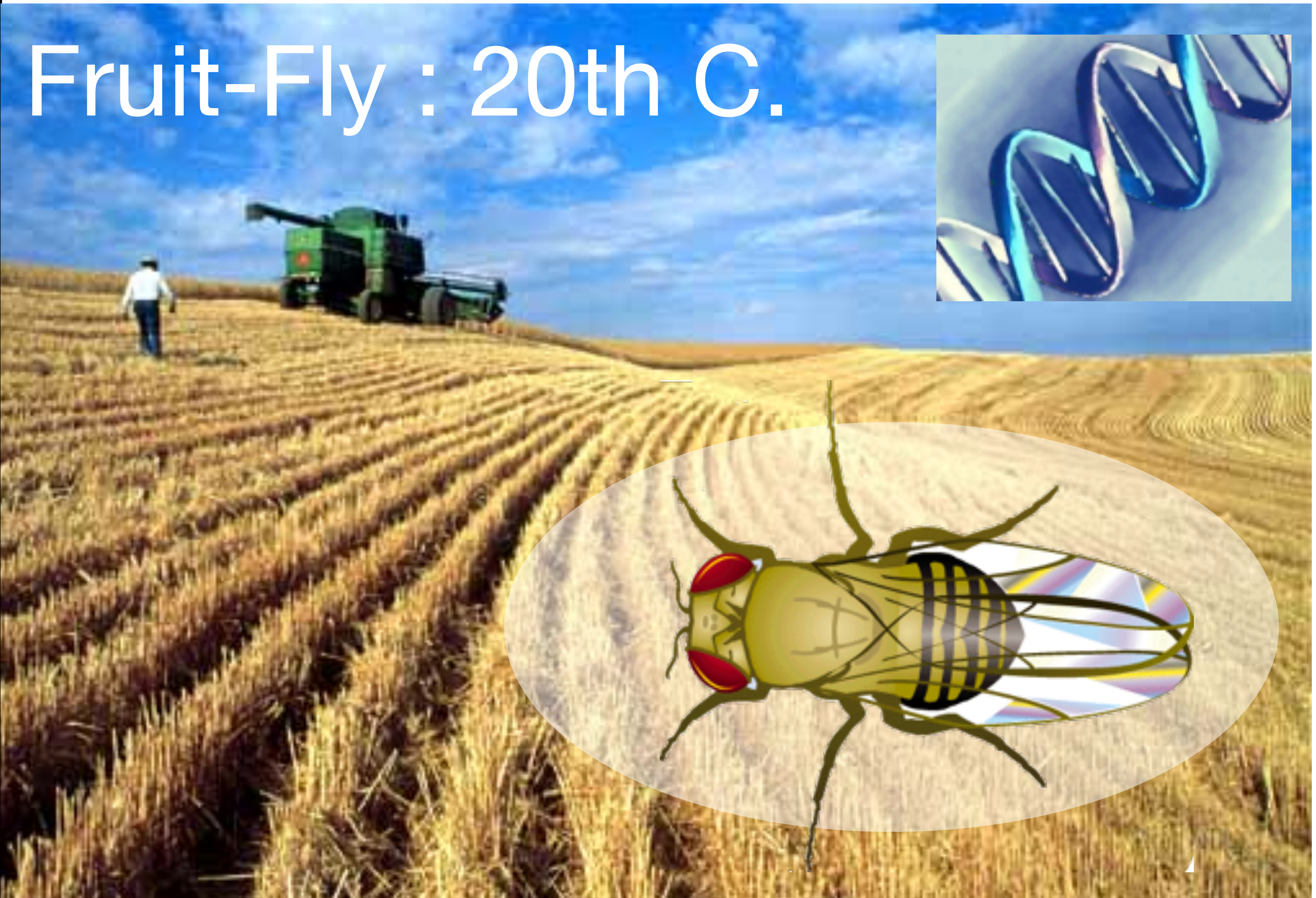
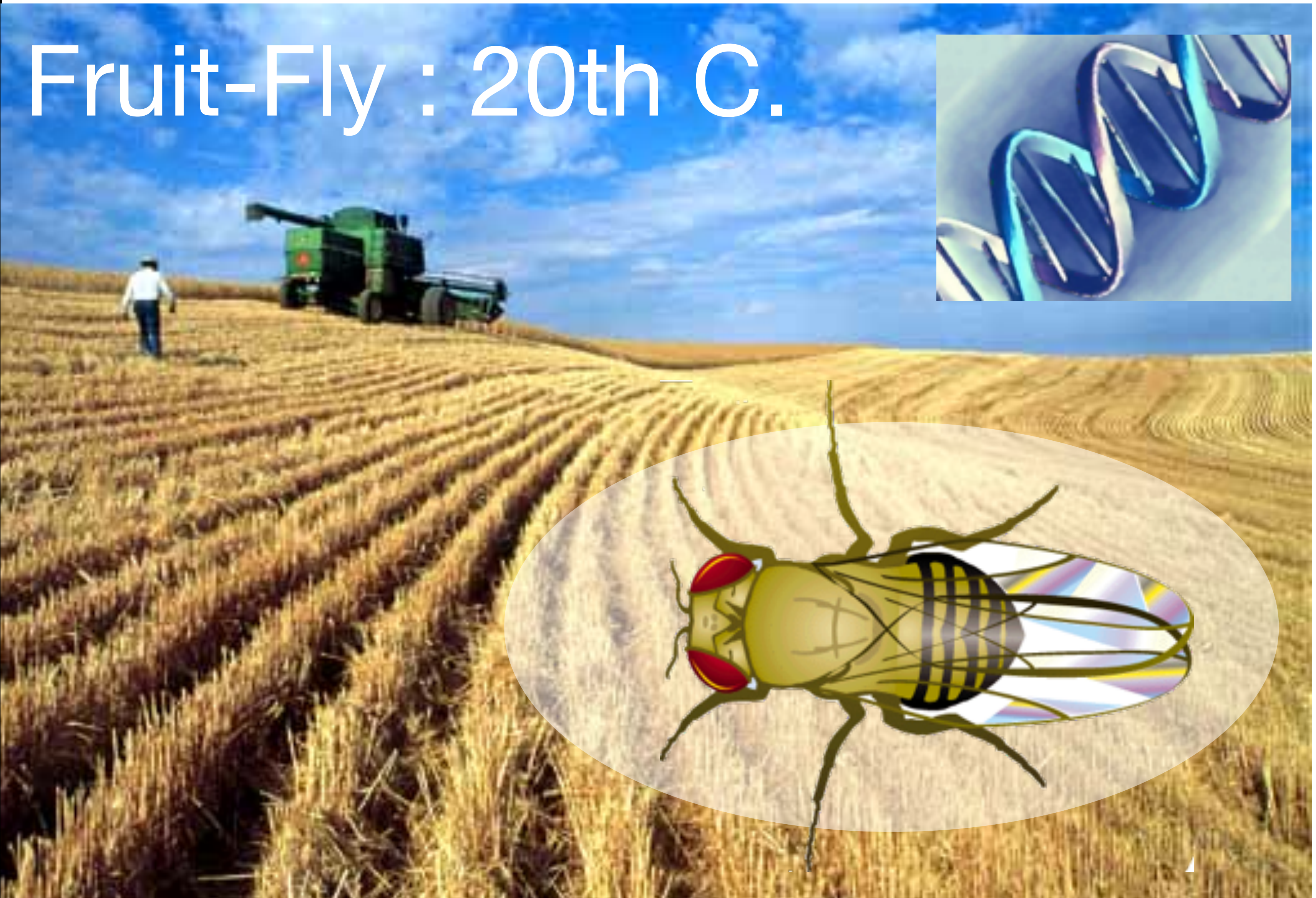
Fruit-Fly of the 21st C



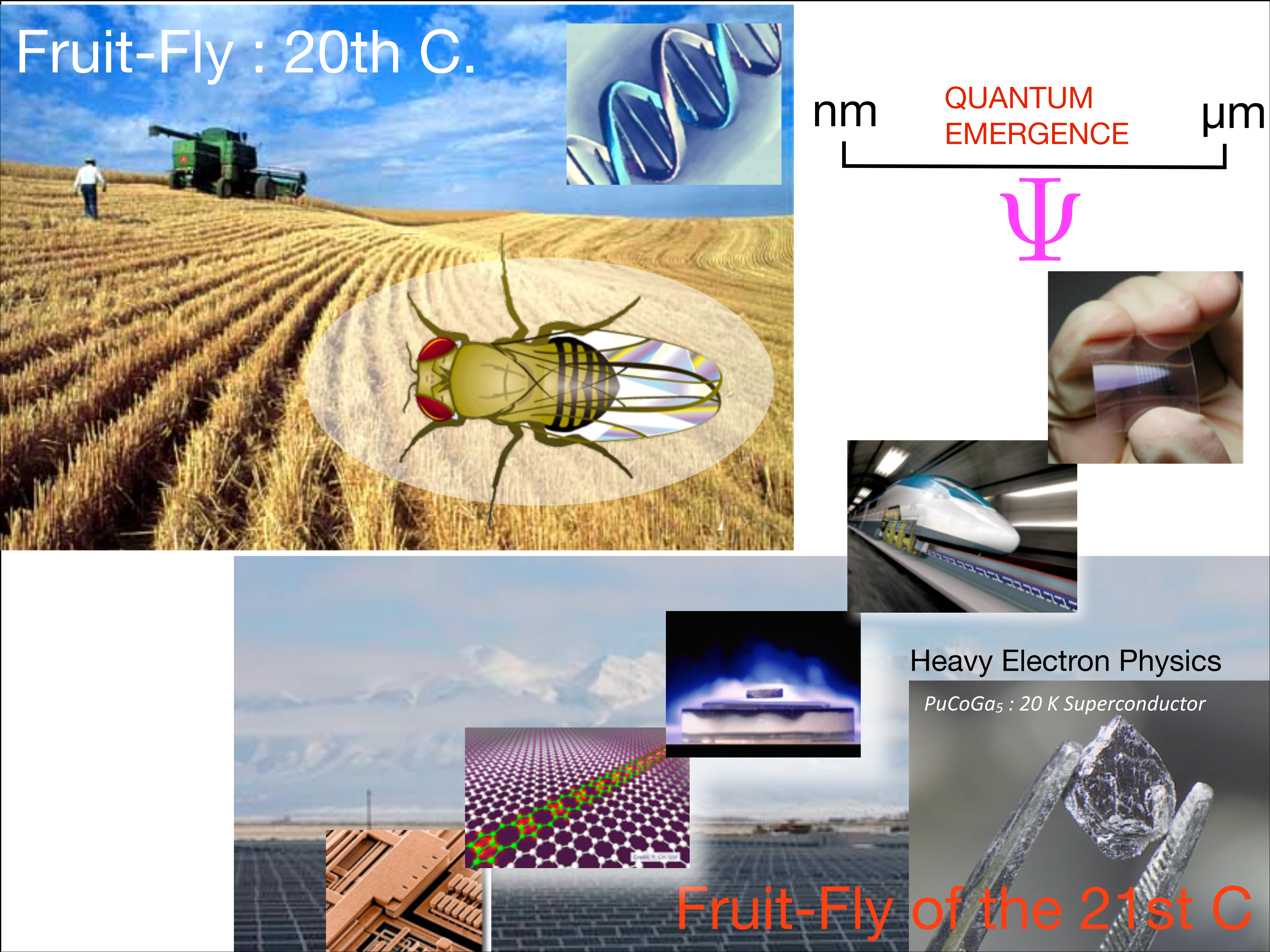
nm QUANTUM EMERGENCE μm

Ψ



nm QUANTUM EMERGENCE μm

Ψ



Fruit-Fly : 20th C.
nm
QUANTUM
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Heavy Electron Physics
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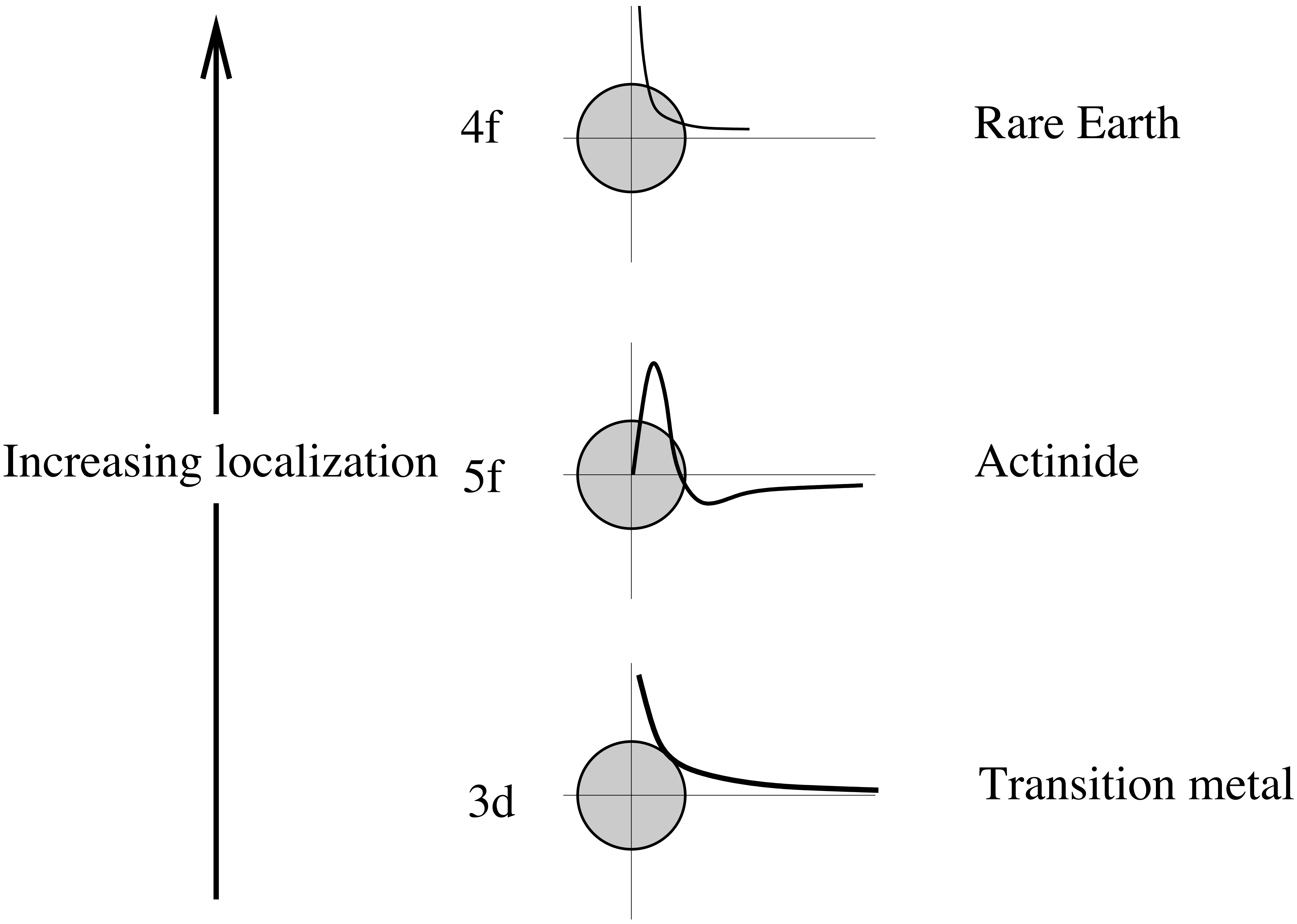
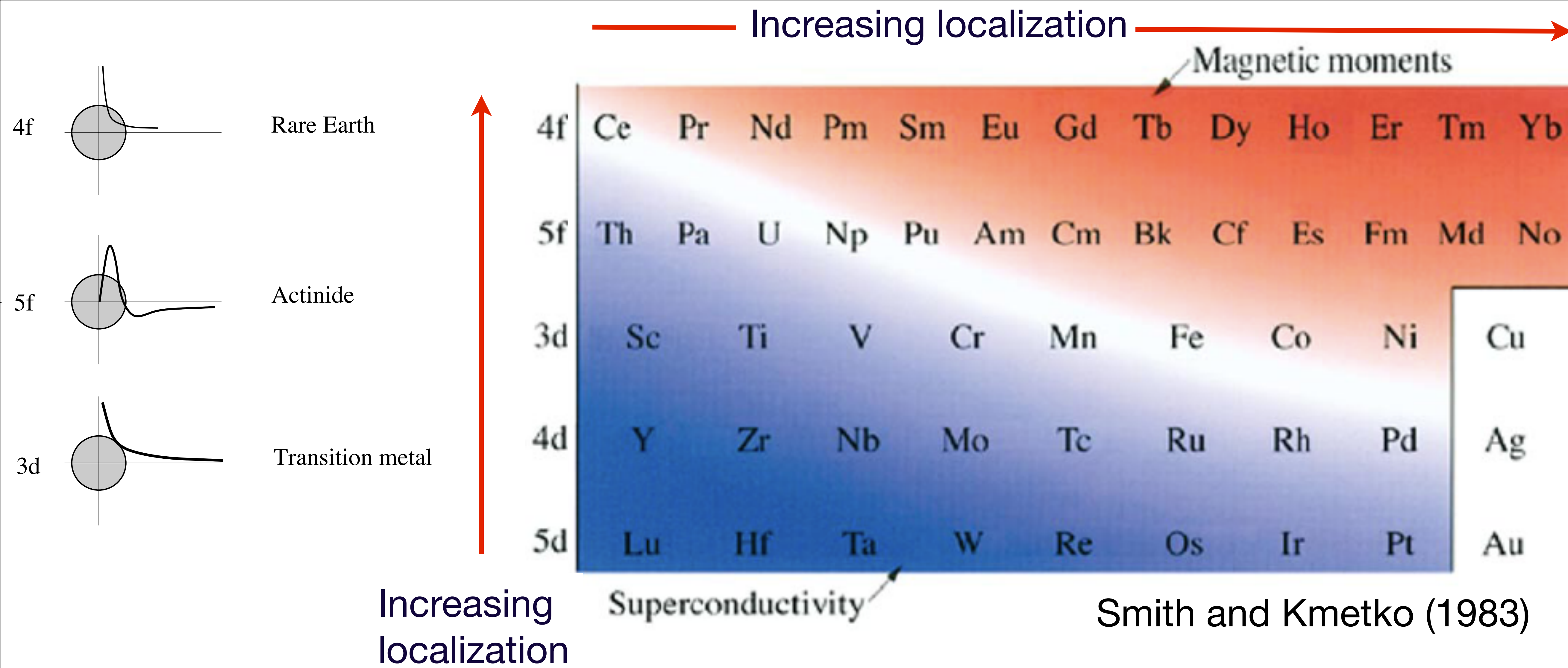
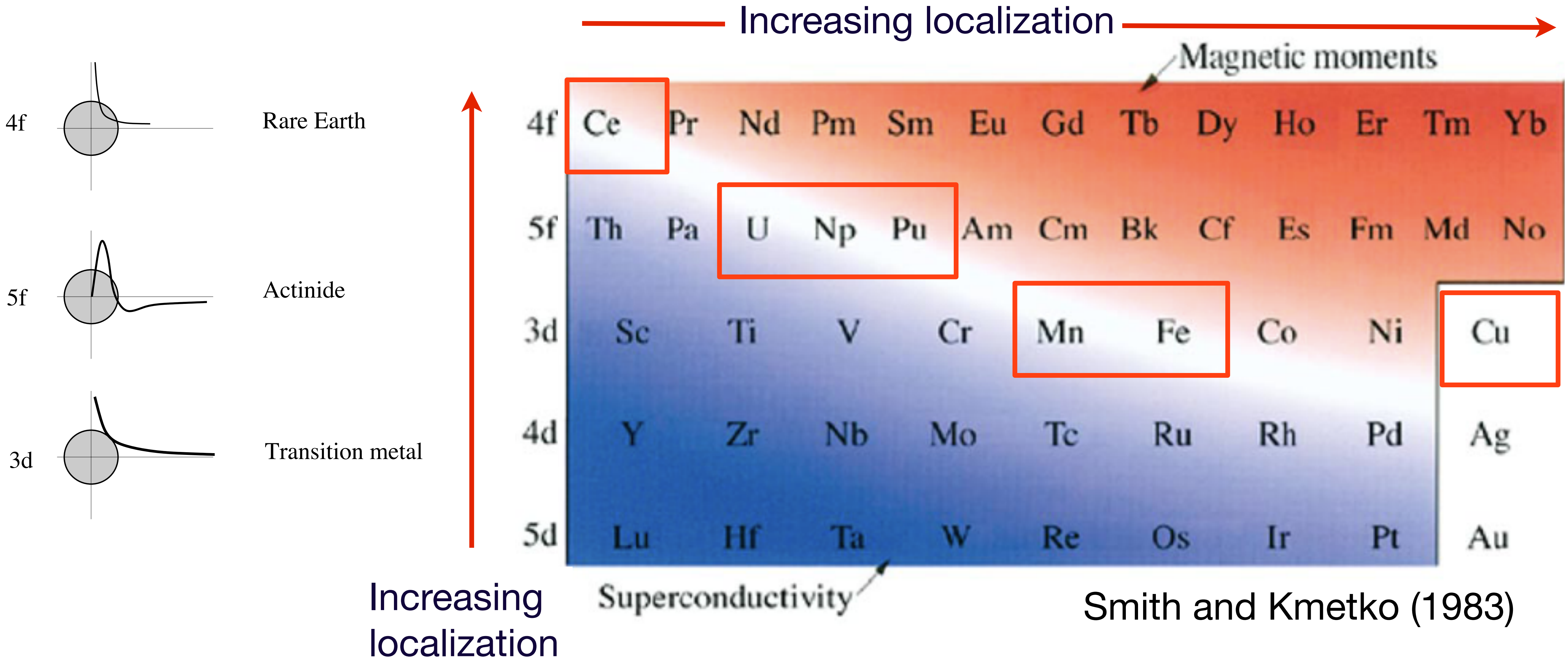

Increasing localization
4f
Rare Earth
5f
Actinide
3d
Transition metal



Smith and Kmetko (1983)



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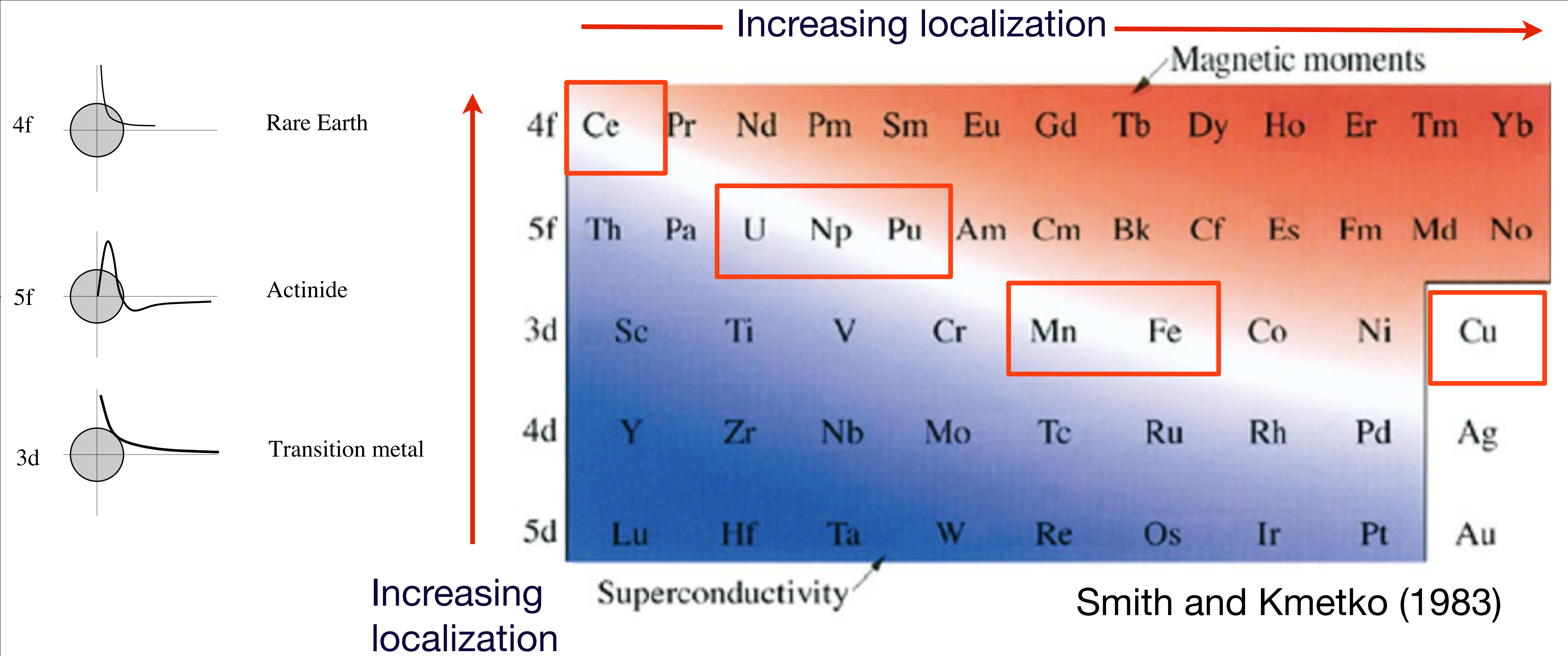
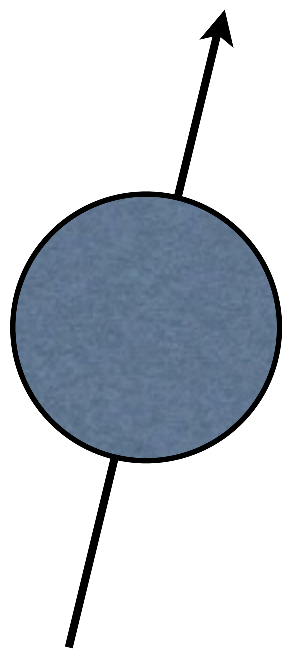


Smith and Kmetko (1983)

Many things are possible at the brink of magnetism.

Heavy Fermions + Kondo



Spin (4f,5f): basic fabric of heavy electron physics.

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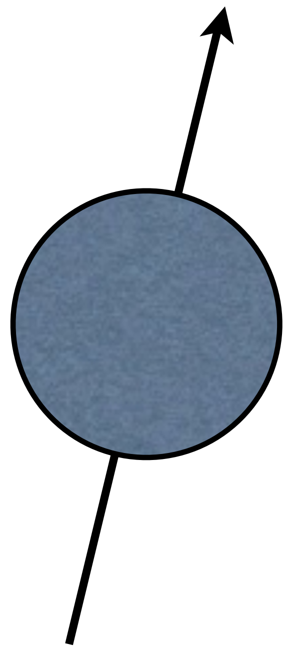

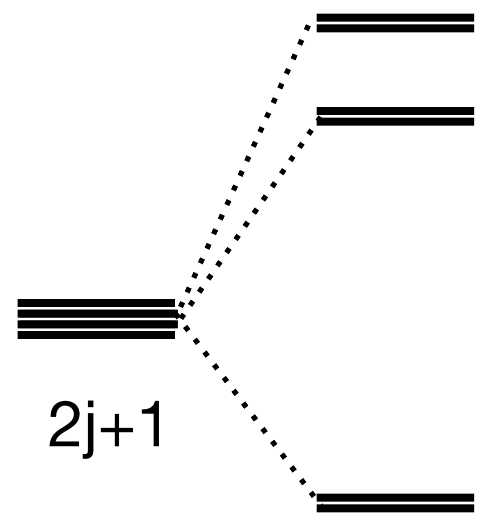

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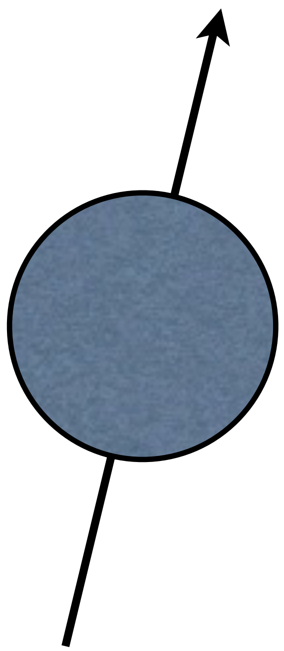

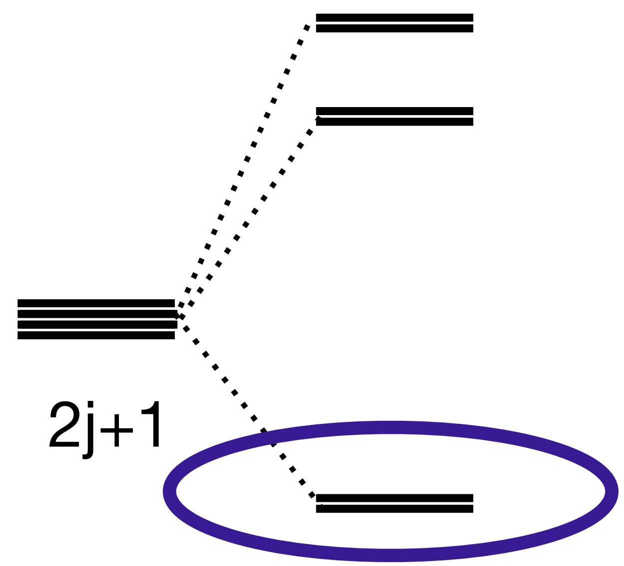

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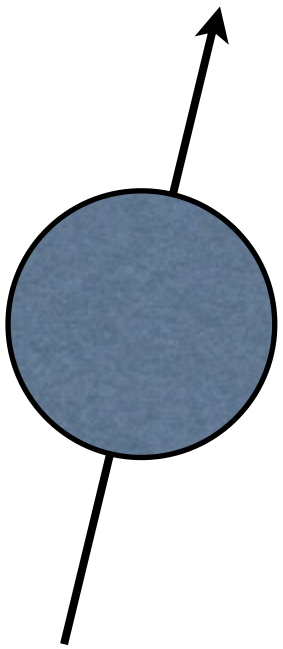

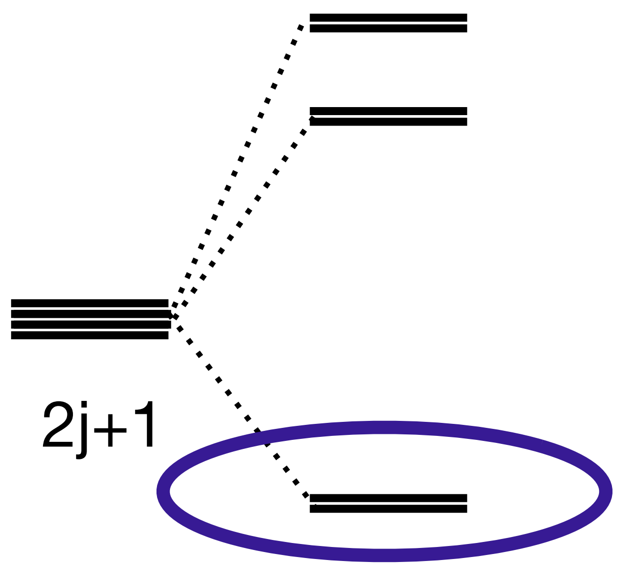

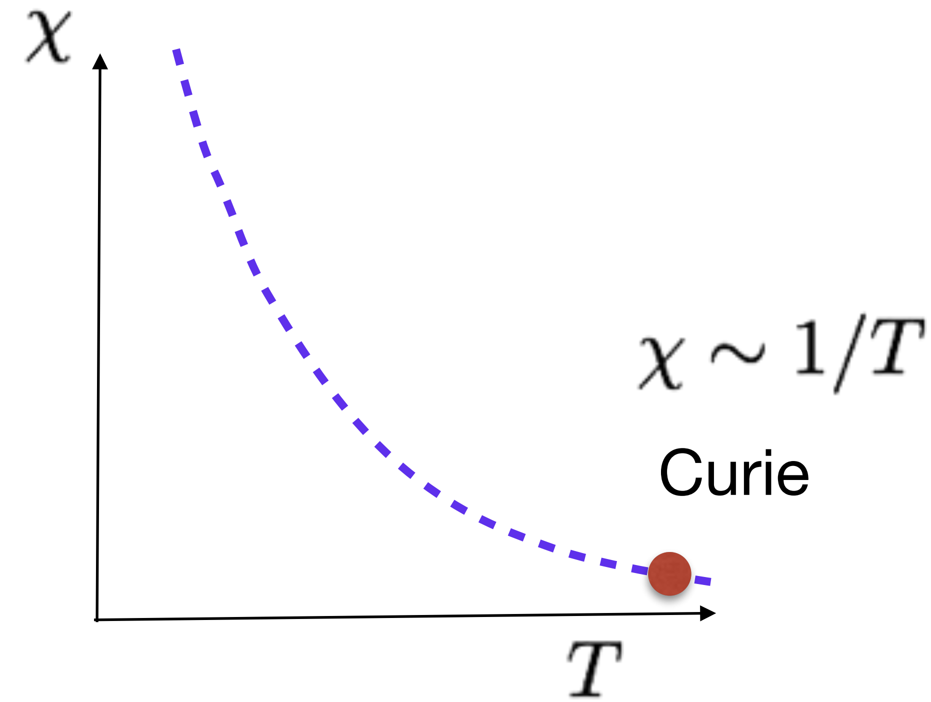

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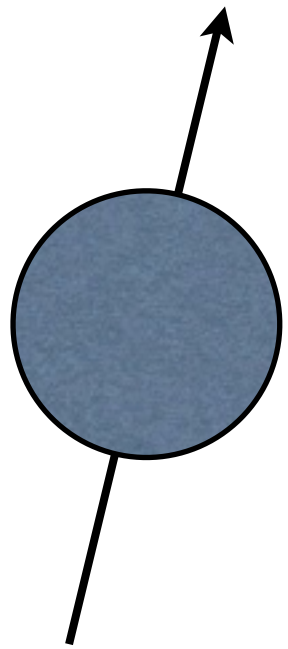

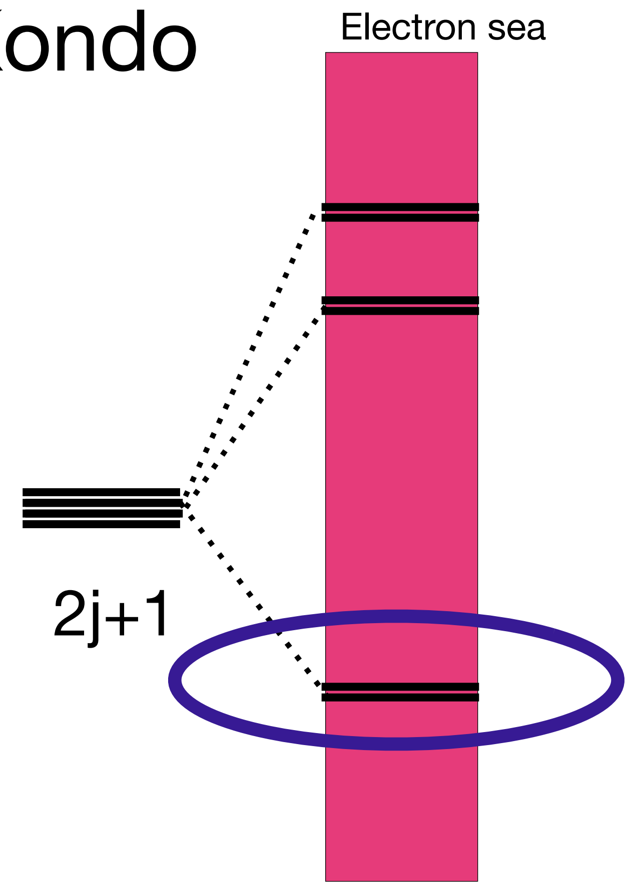

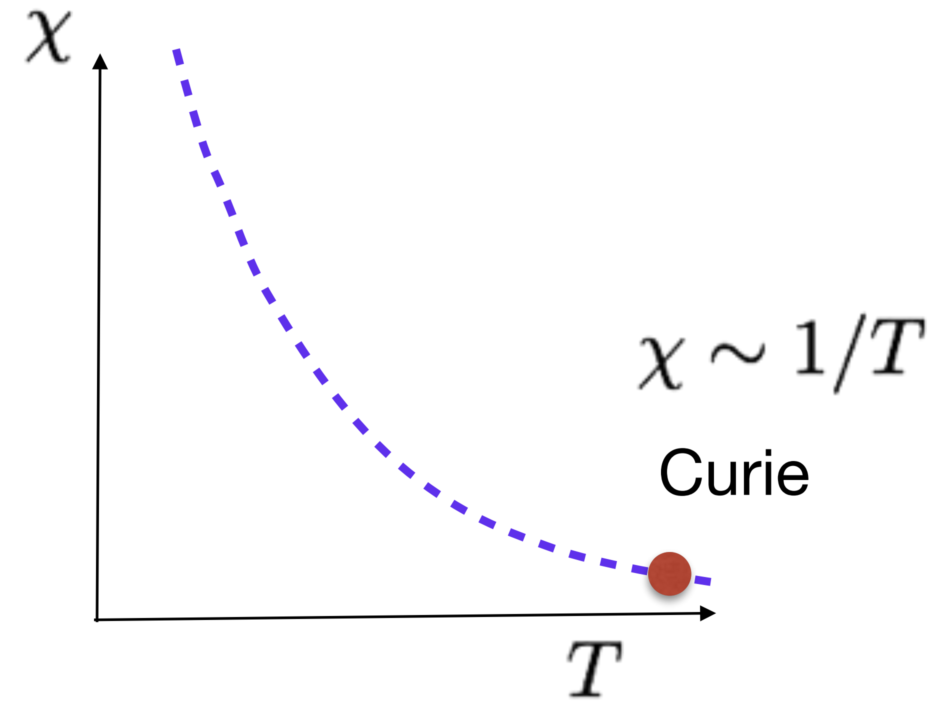

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Scales to
Strong Coupling

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + J \vec{S} \cdot \vec{\sigma}(0)$$

J. Kondo, 1962

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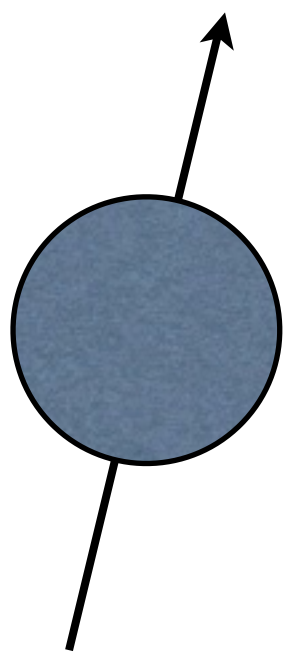

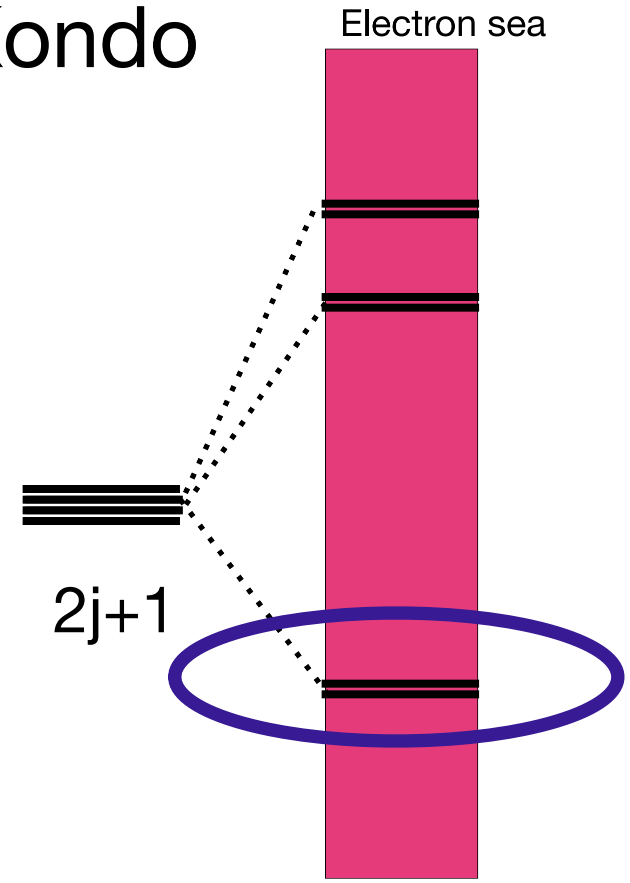

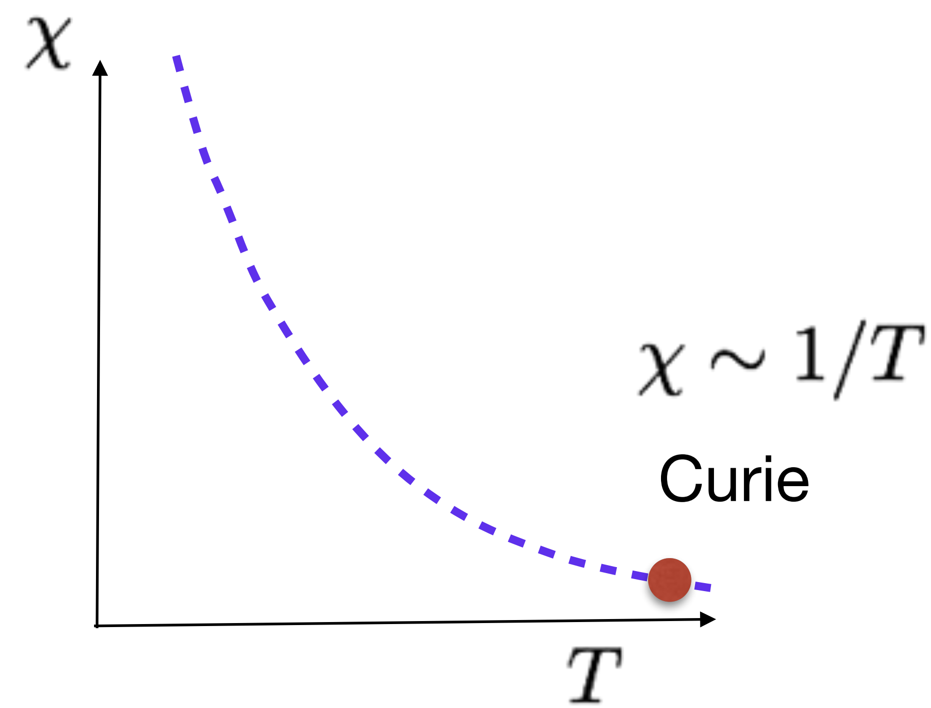

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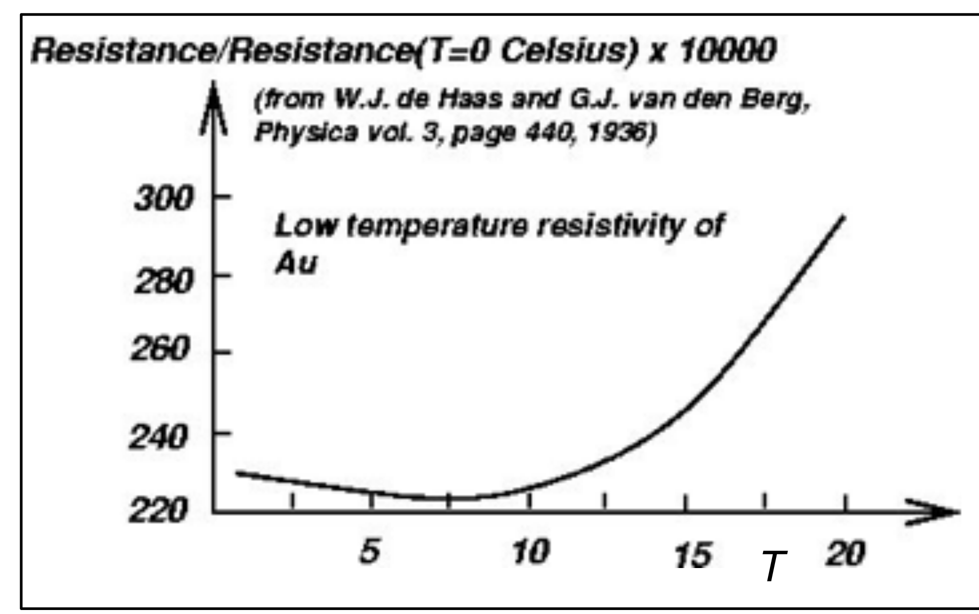

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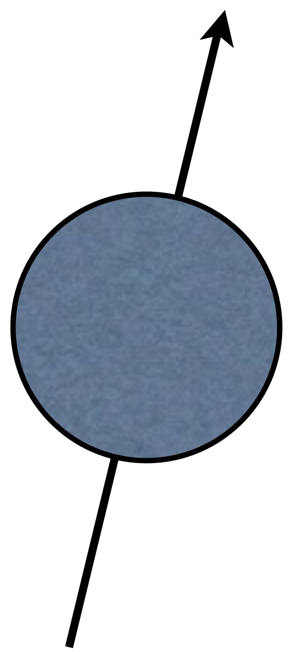

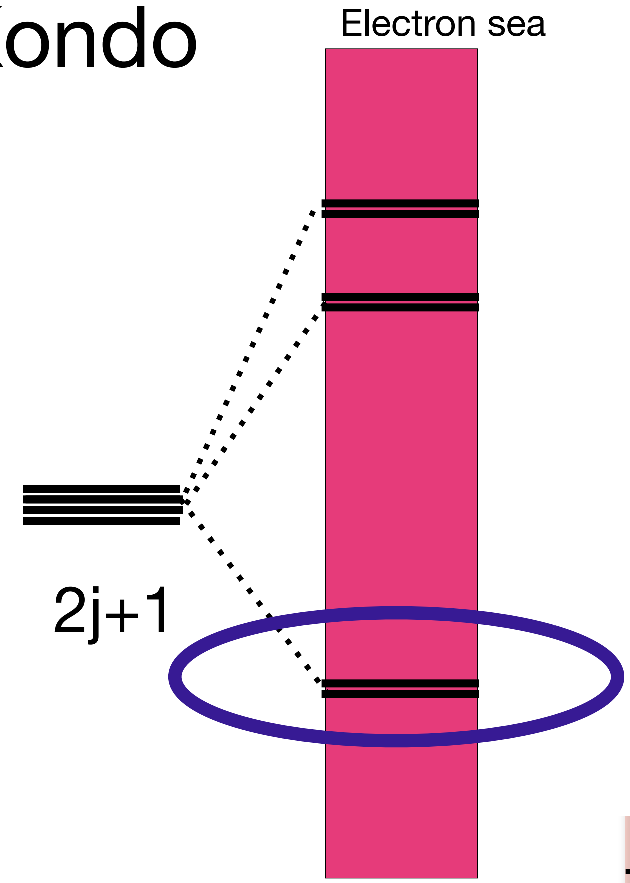

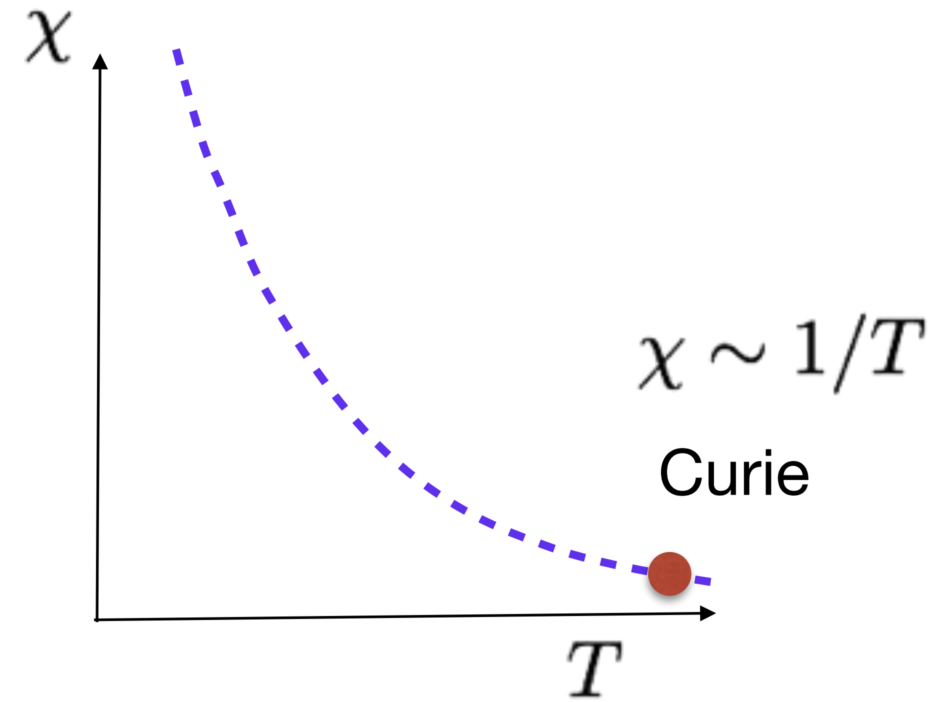

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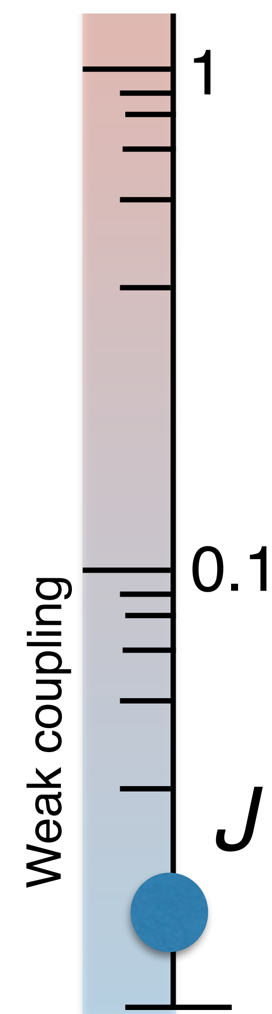

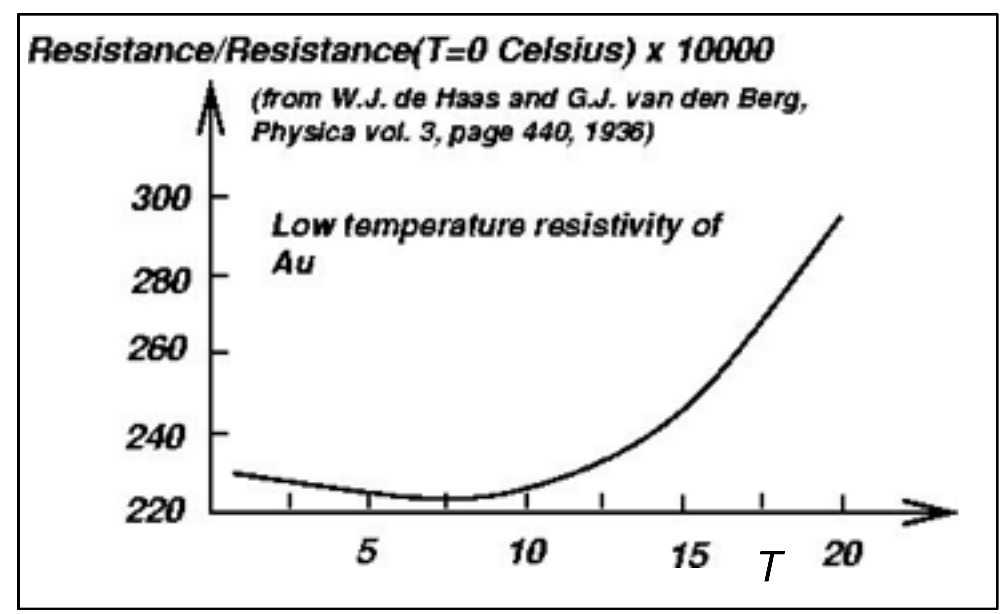

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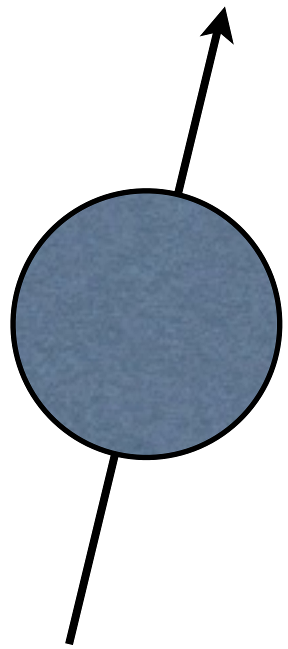
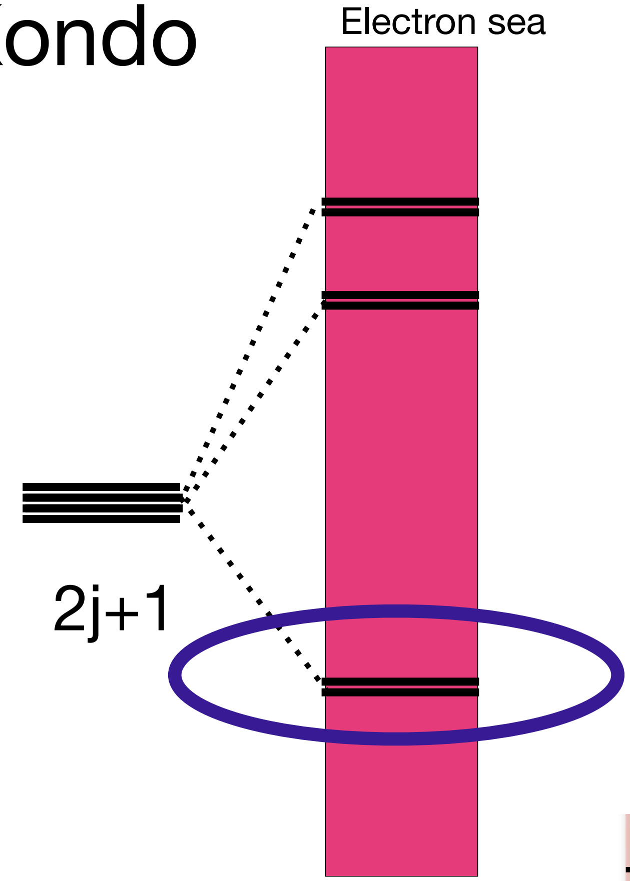
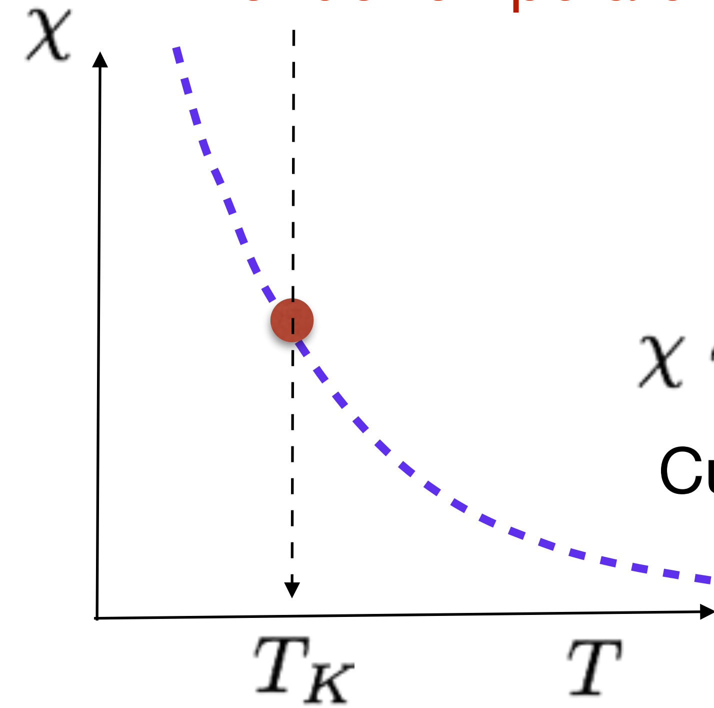




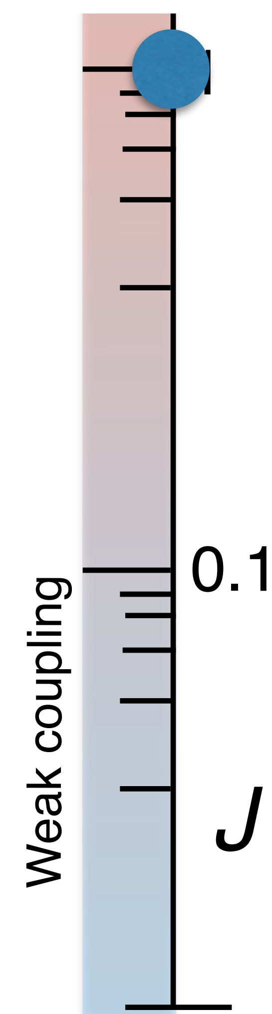
“Kondo temperature”



$\chi \sim 1/T$

Curie

$$T_K = W\sqrt{J\rho}\,e^{-\frac{1}{2J\rho}}$$



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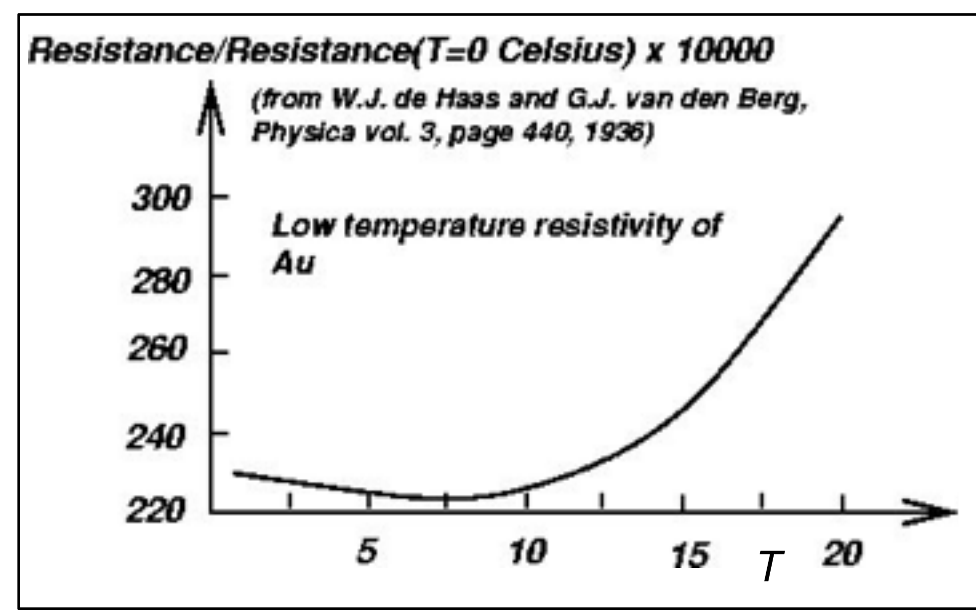

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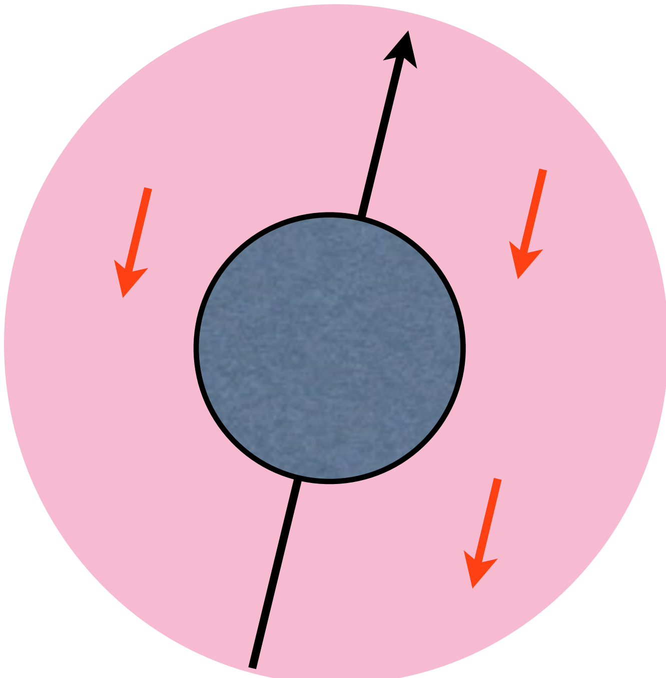
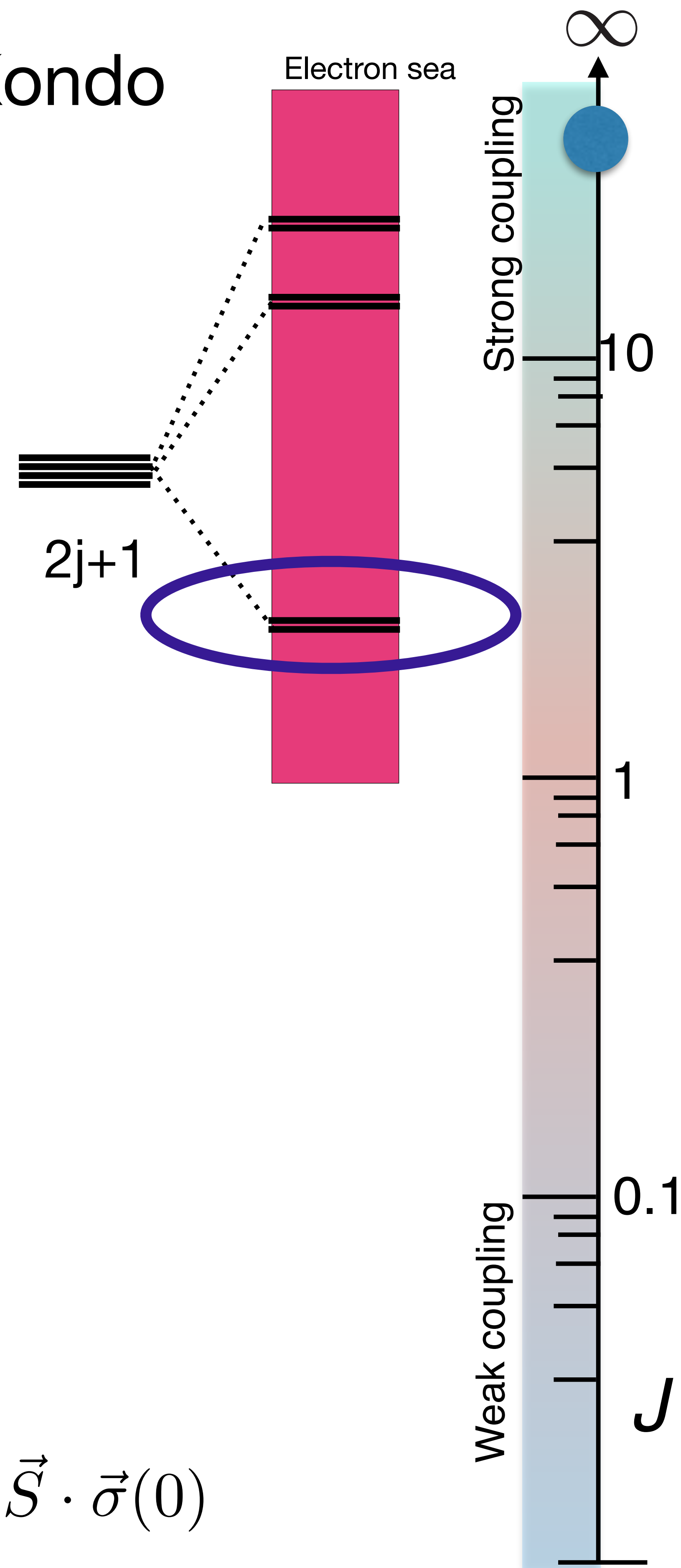
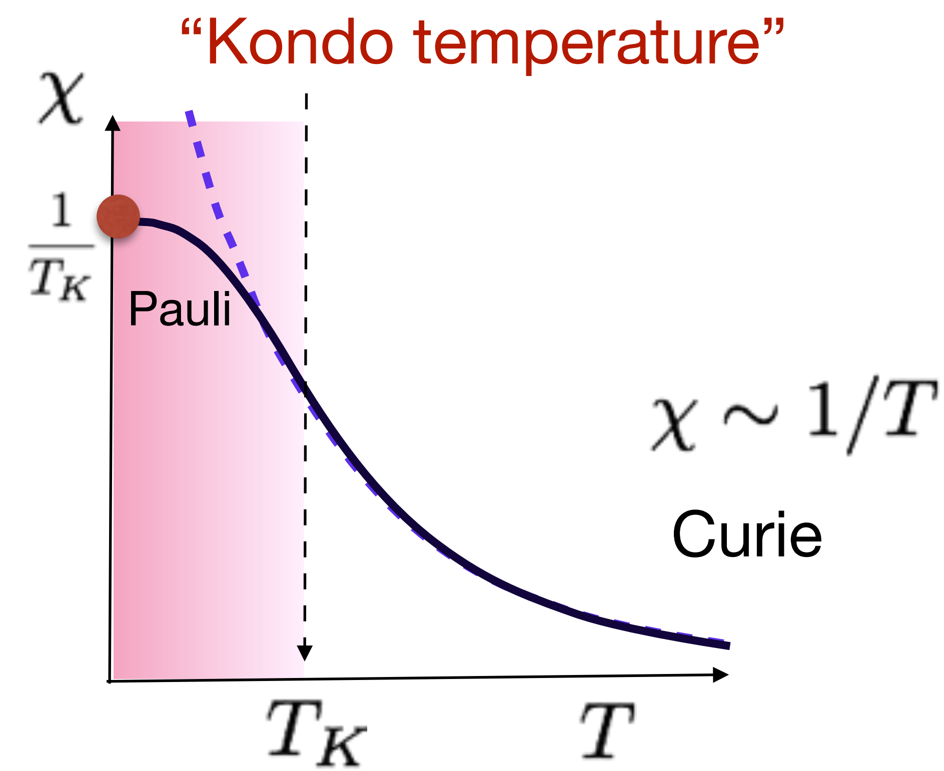
Heavy Fermions + Kondo







$$T_K = W\sqrt{J\rho}\, e^{-\frac{1}{2J\rho}}$$

Spin screened by conduction electrons: entangled

$$\uparrow\downarrow - \downarrow\uparrow$$

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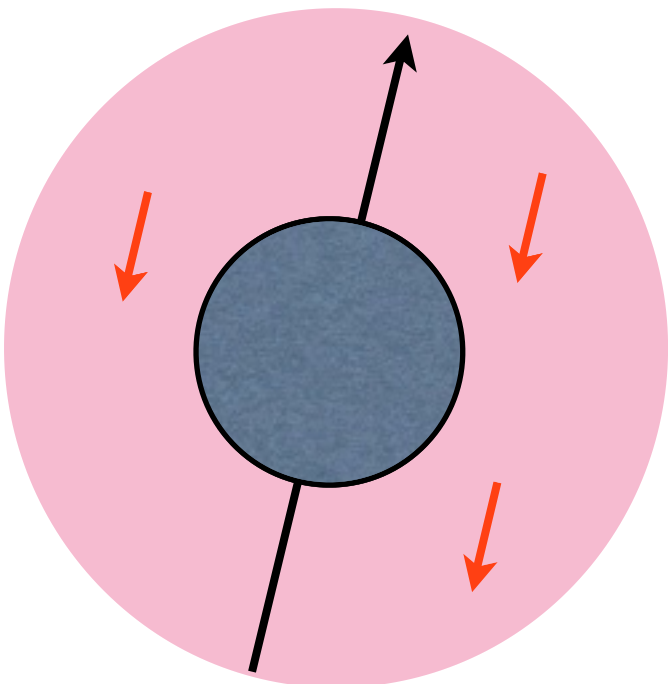

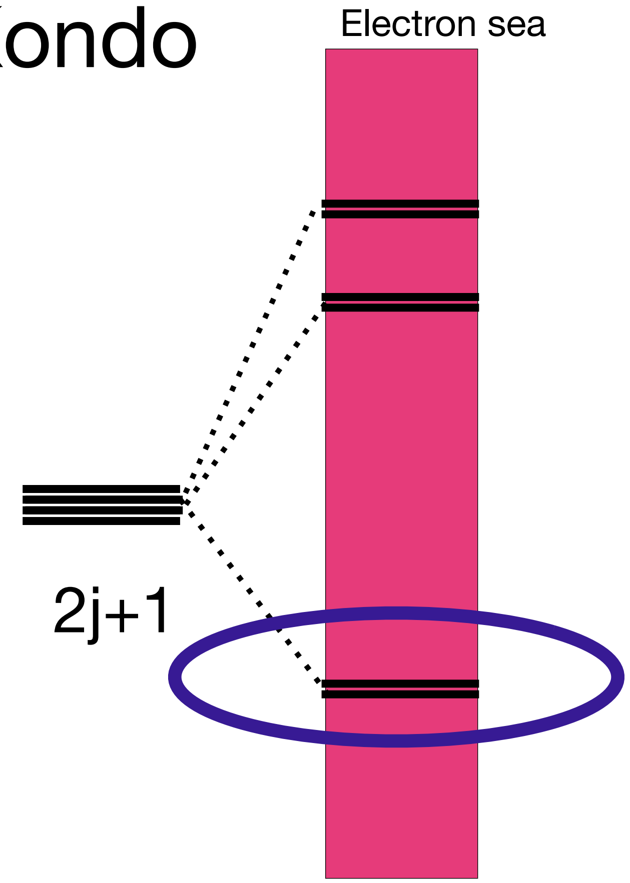

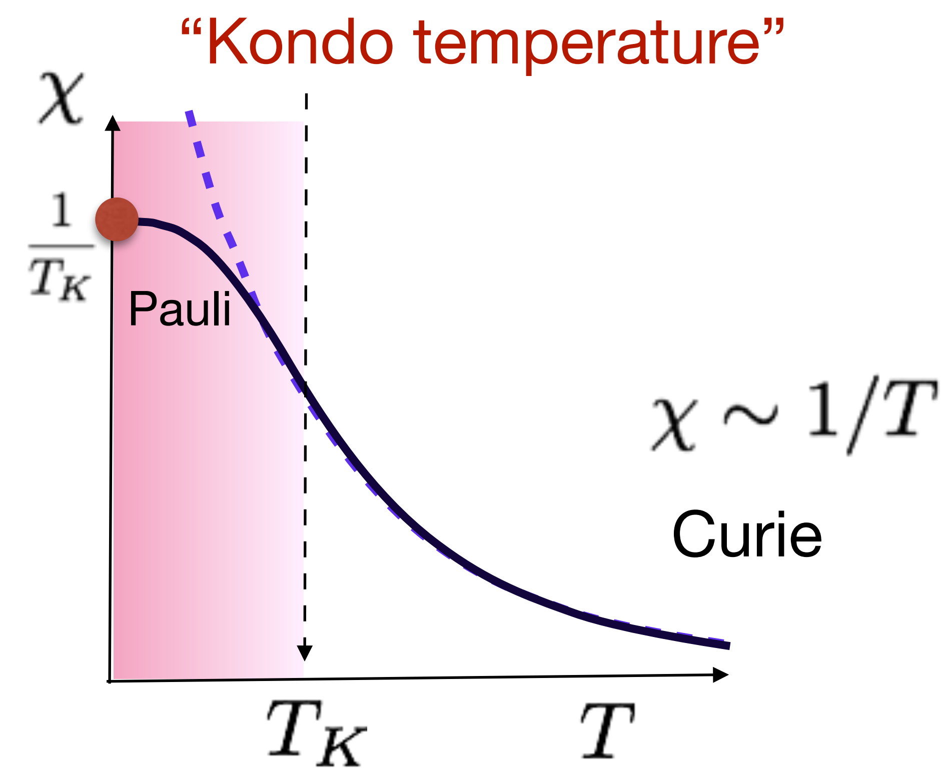

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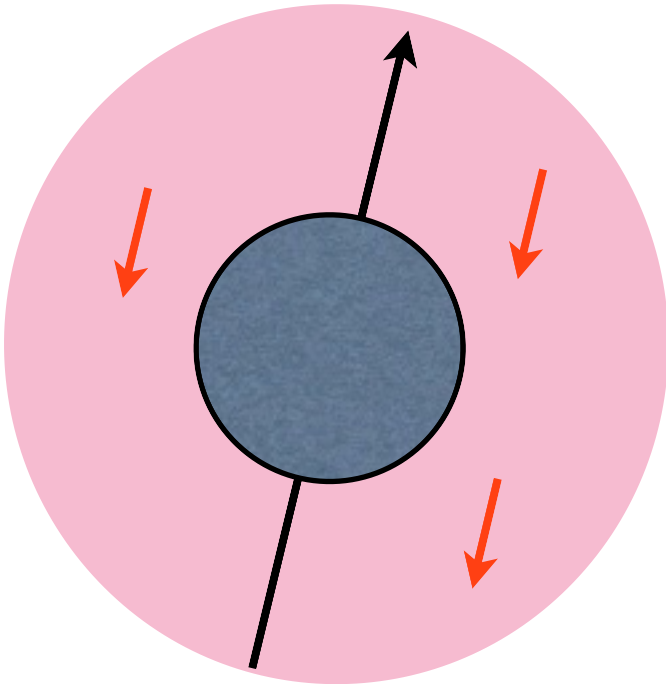

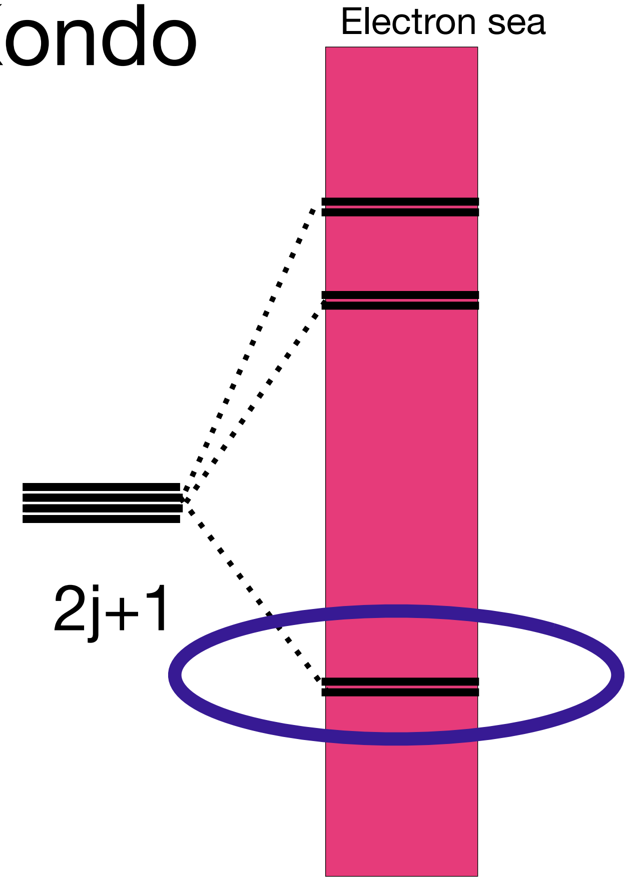

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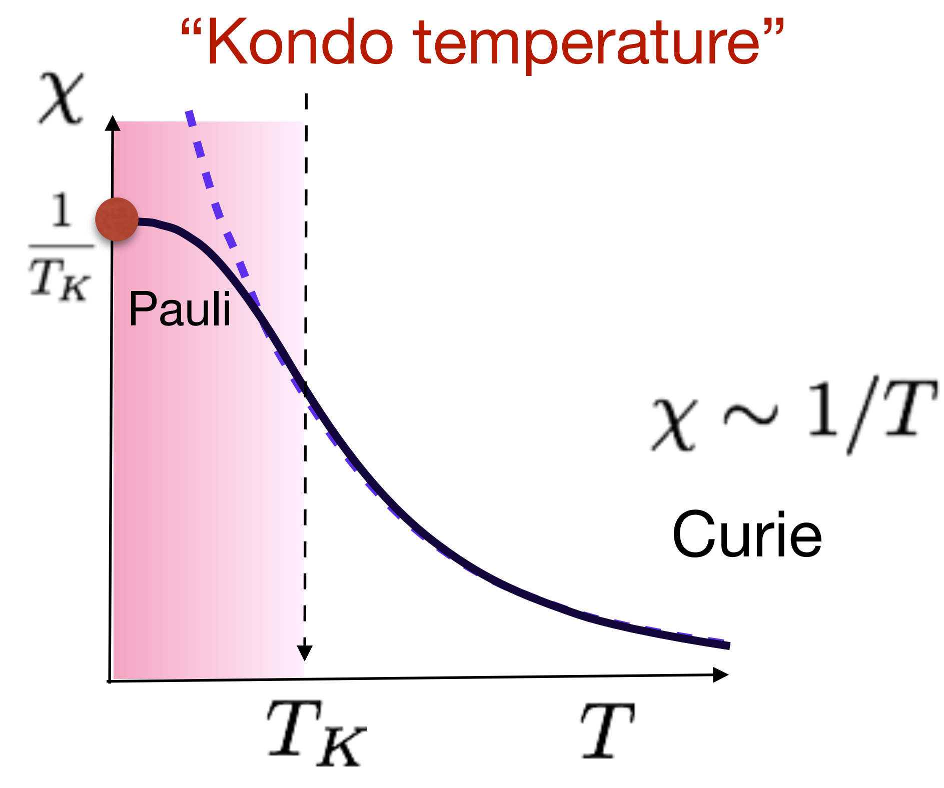
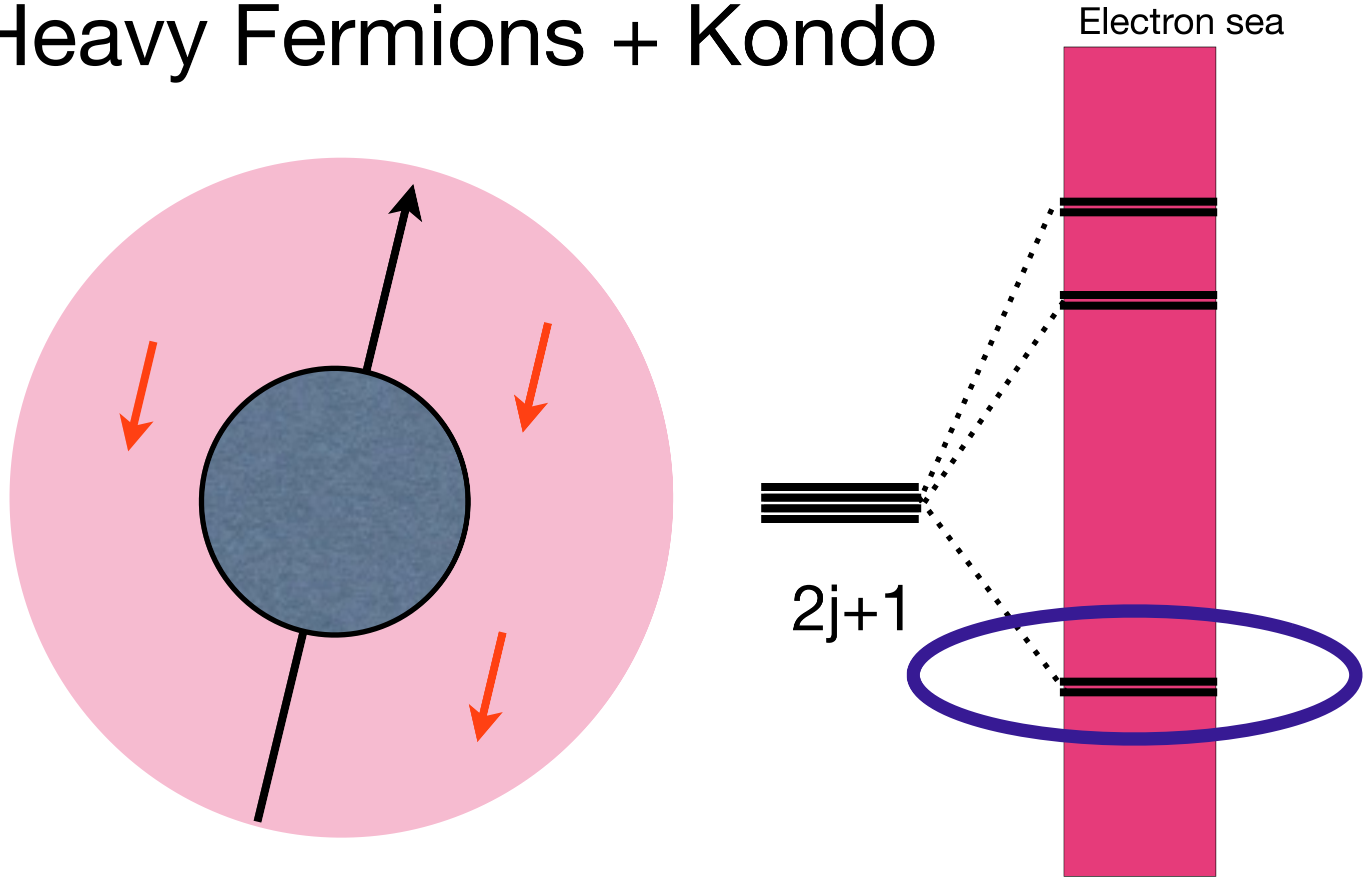


$$T_K = W\sqrt{J\rho}\,e^{-\frac{1}{2J\rho}}$$

$$S(T) = \int_0^T \frac{C_V}{T'}dT'$$

Spin entanglement entropy

Heavy Fermions + Kondo



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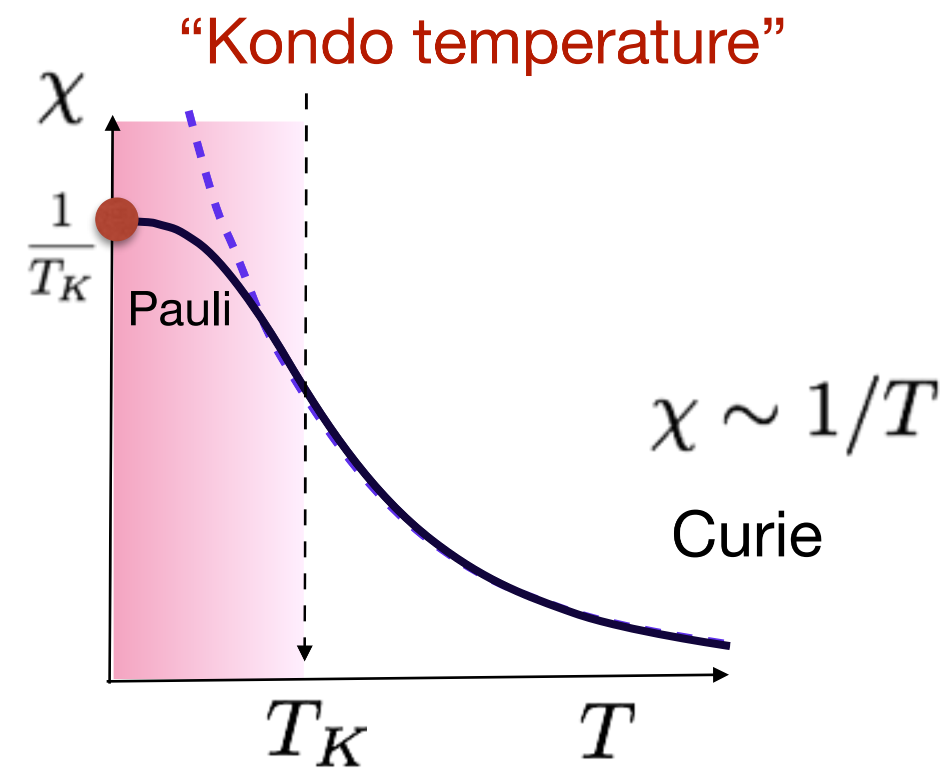

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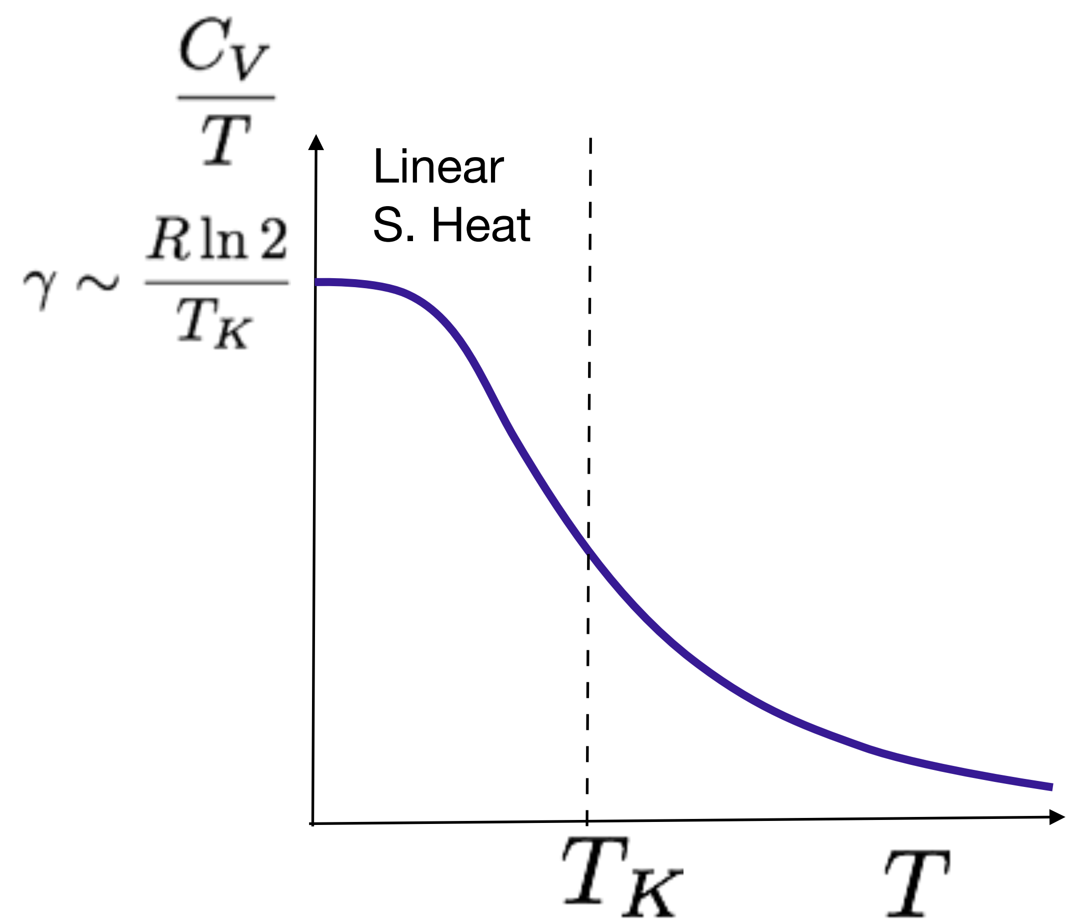

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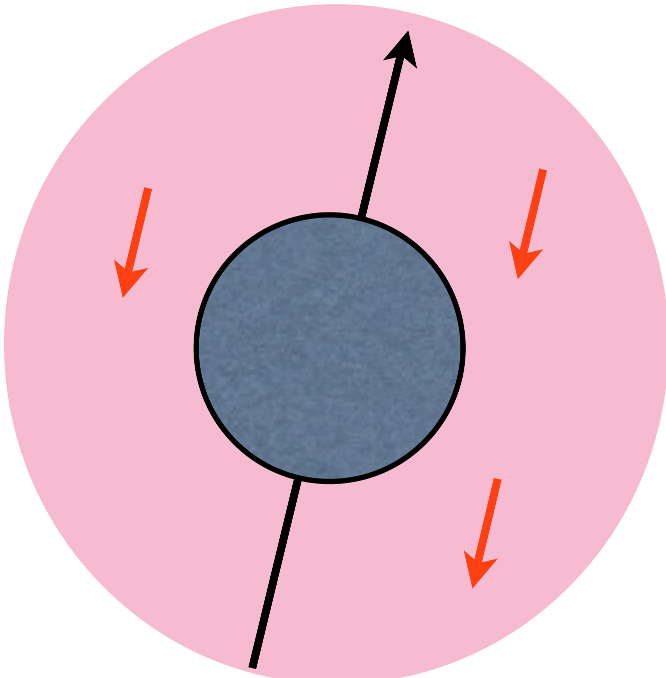
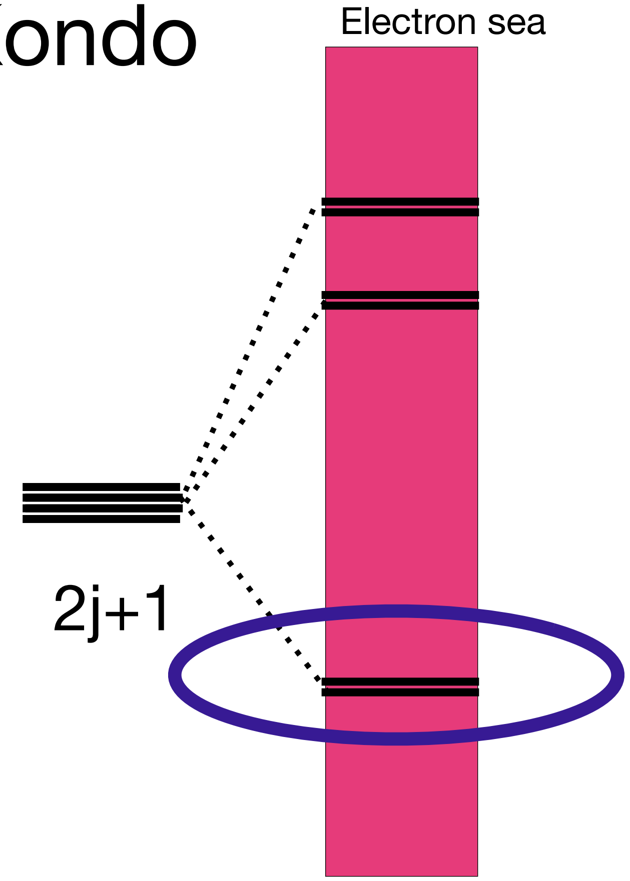
Spin entanglement entropy

Heavy Fermions + Kondo



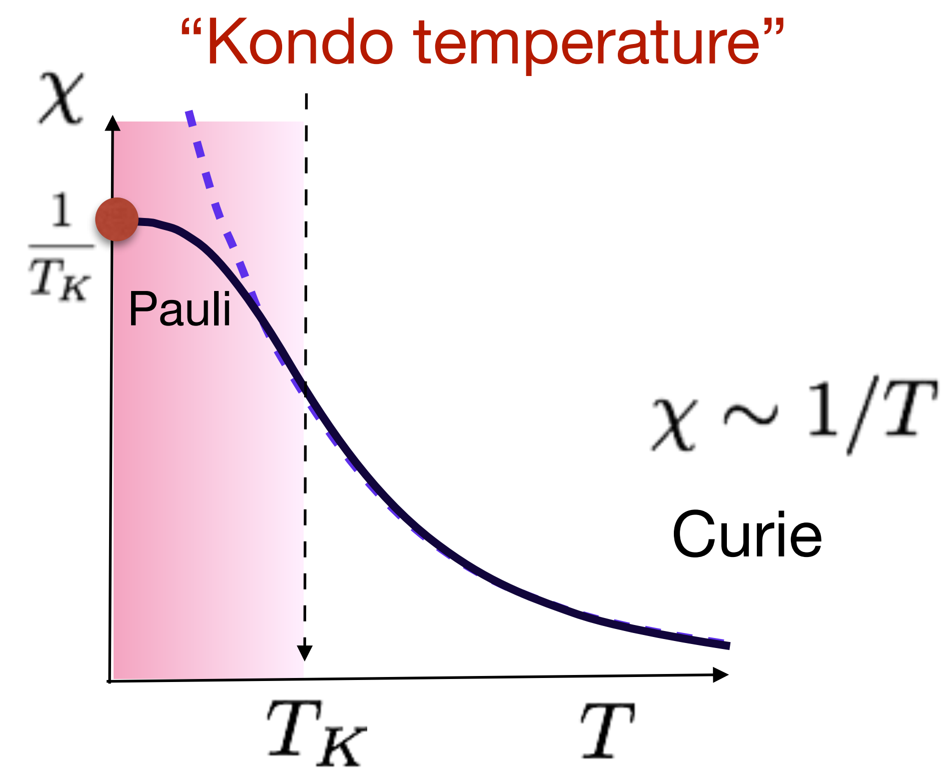


Spin screened by conduction electrons: <u>entangled</u>

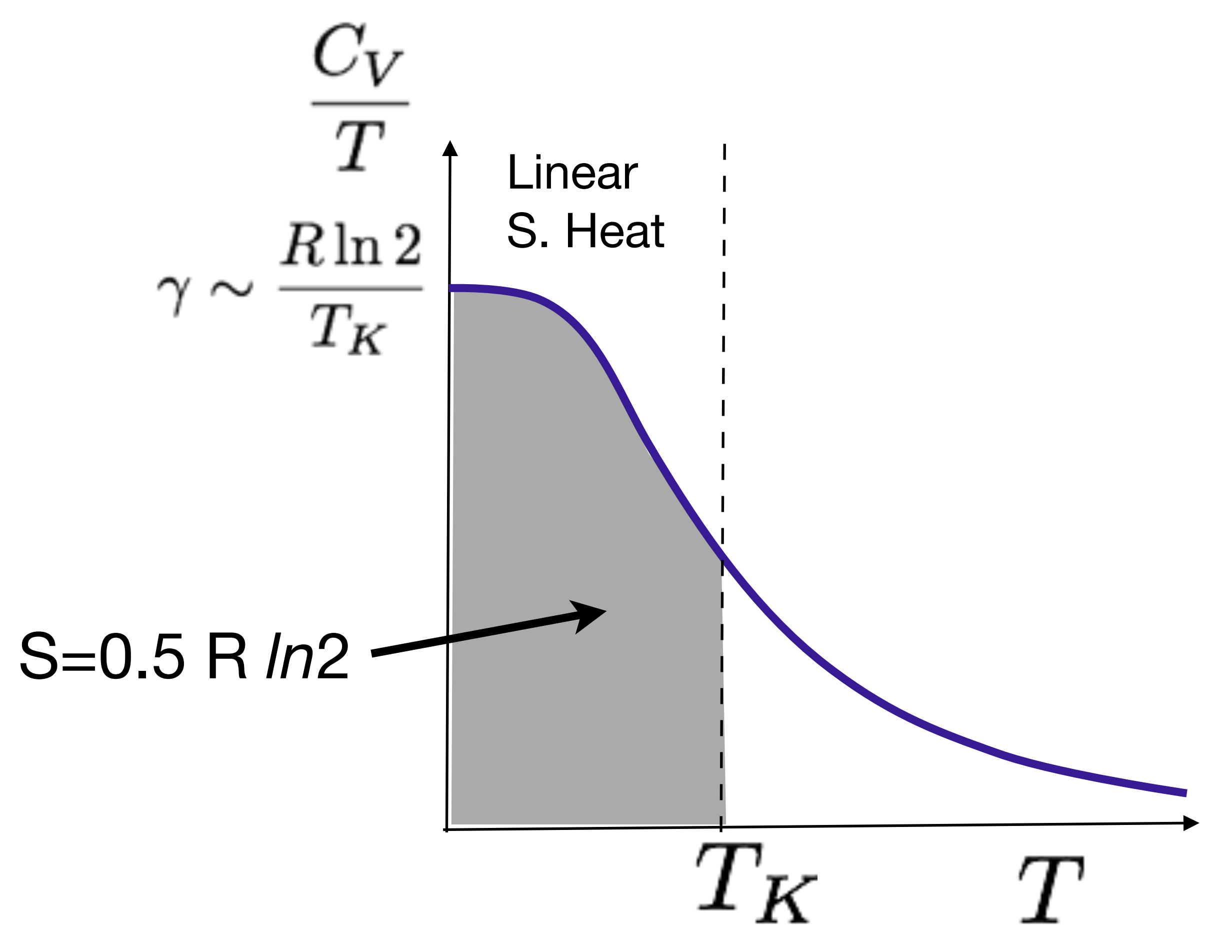
$$\uparrow \downarrow - \downarrow \uparrow$$

$$S(T) = \int_0^T \frac{C_V}{T'} dT'$$

Spin entanglement entropy



$$T_K = W\sqrt{J\rho}\, e^{-\frac{1}{2J\rho}}$$



DONIACH'S

Hypothesis.

Doniach (1977)

$$H = \sum \varepsilon_k c_{k\sigma}^{\dagger} c_{k\sigma} + J \sum_j (\psi^{\dagger}_j \vec{\sigma} \psi_j) \bullet \vec{S}_j$$

Kondo Lattice Model
(Kasuya, 1951)

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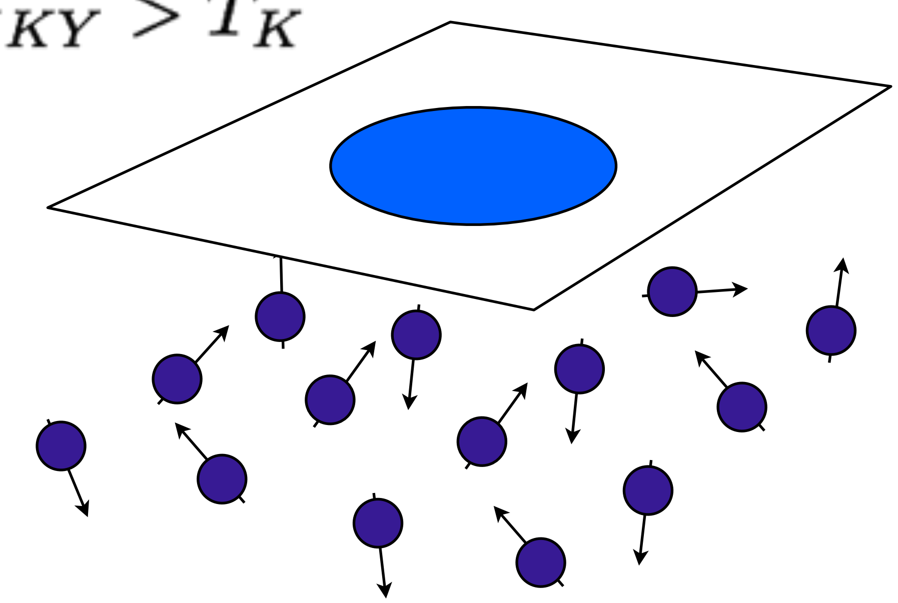
$$T_{RKKY} > T_K$$



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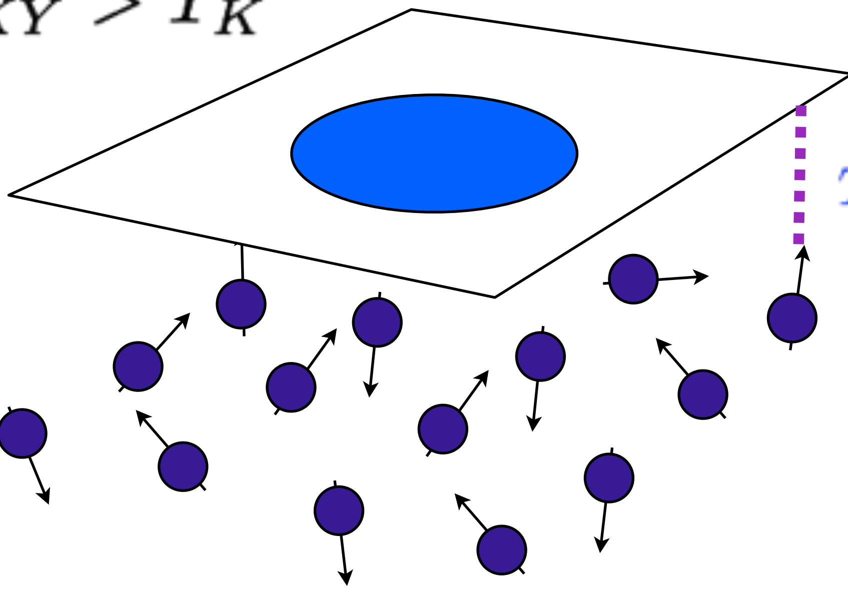

$$T_K \sim D \exp\left[-\frac{1}{2J\rho}\right]$$

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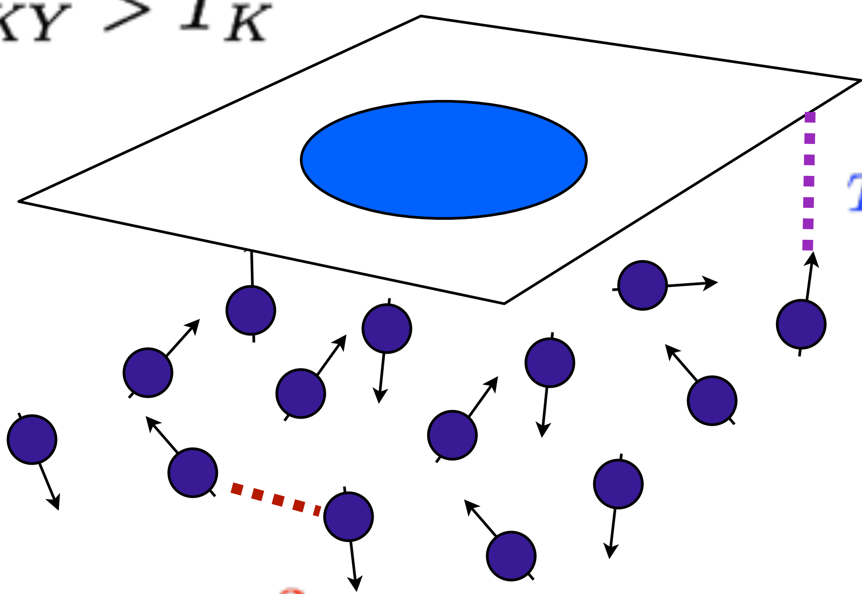

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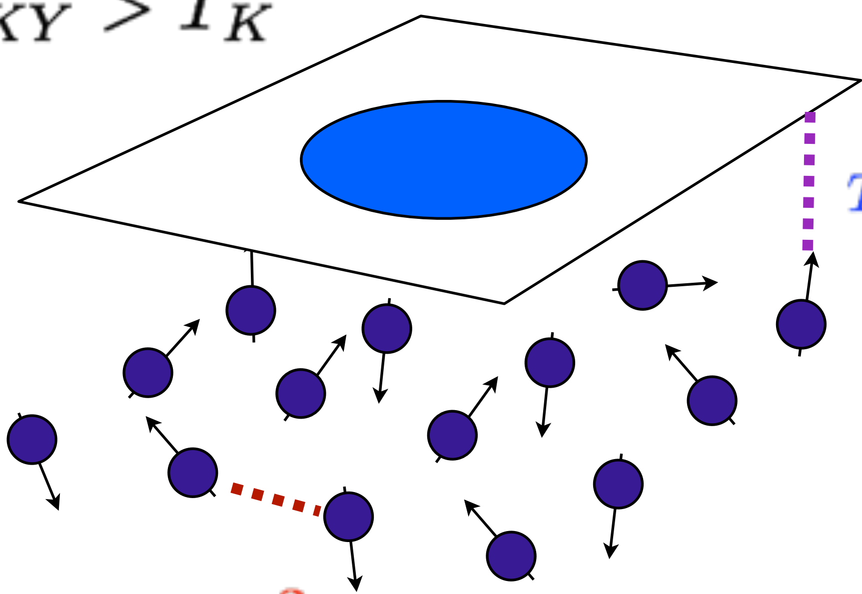

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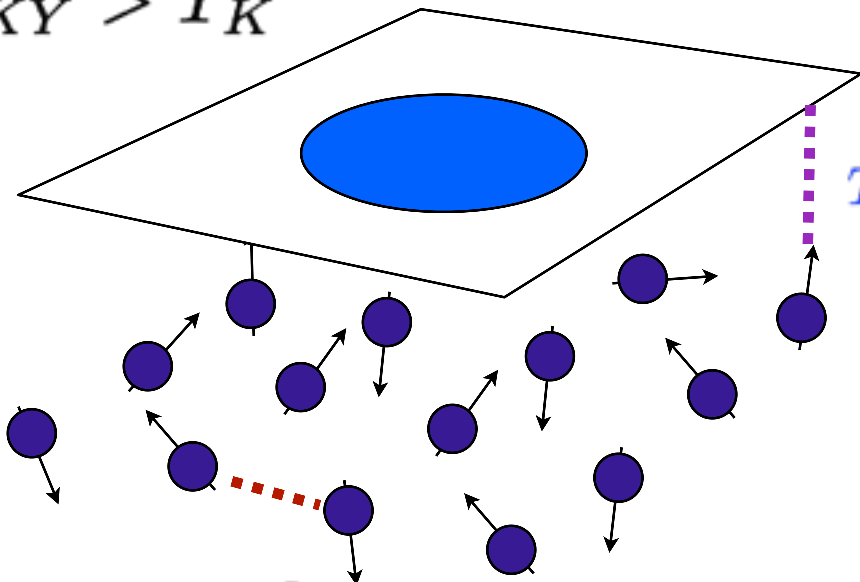

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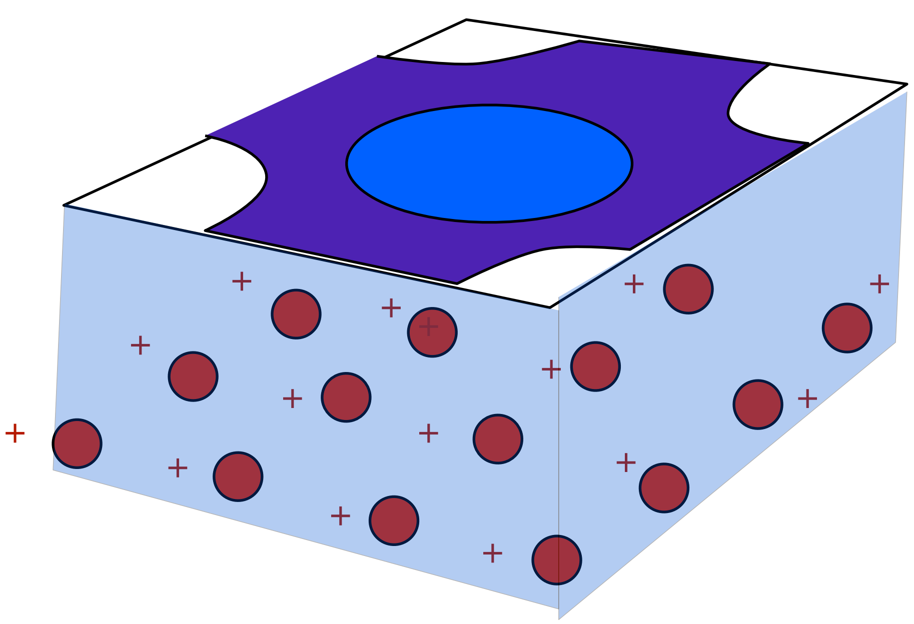
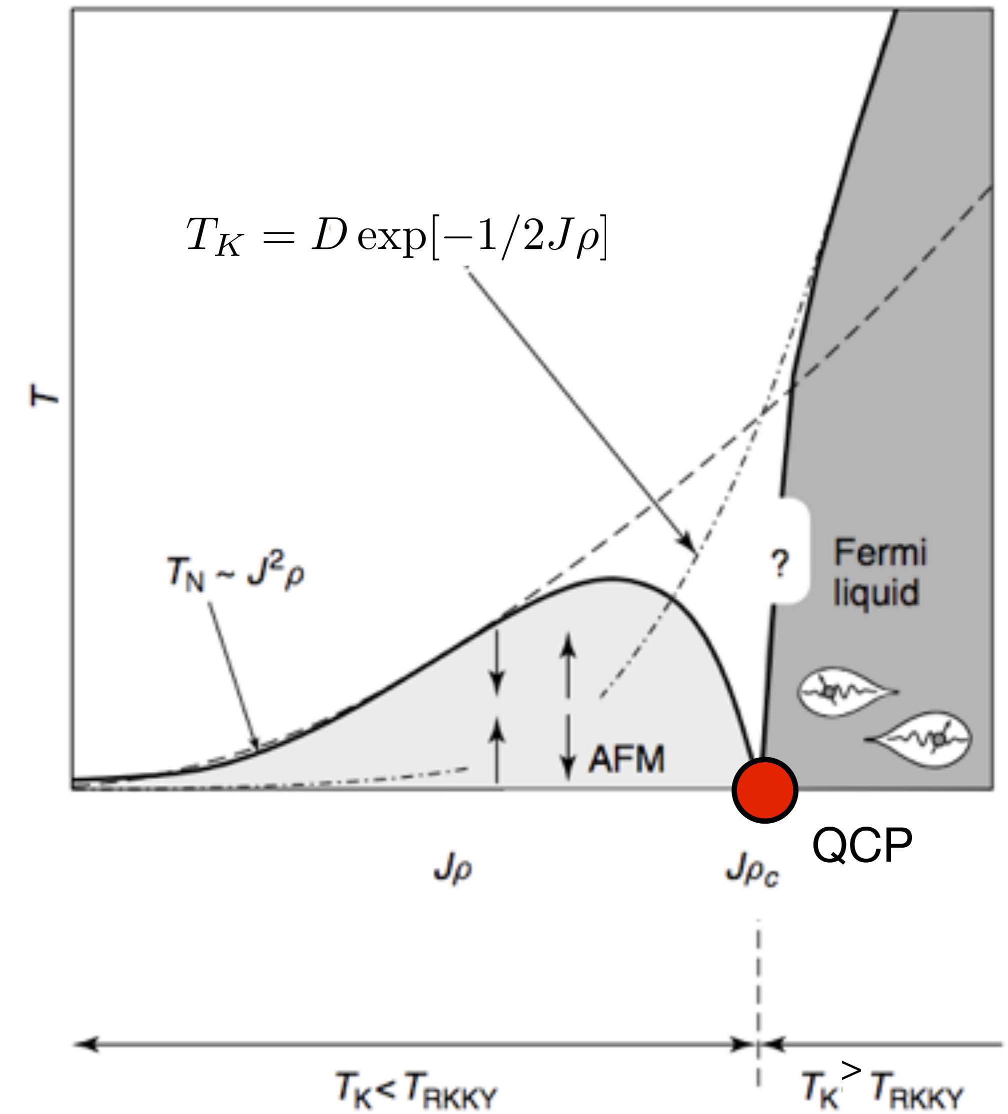
Large Fermi surface of composite Fermions





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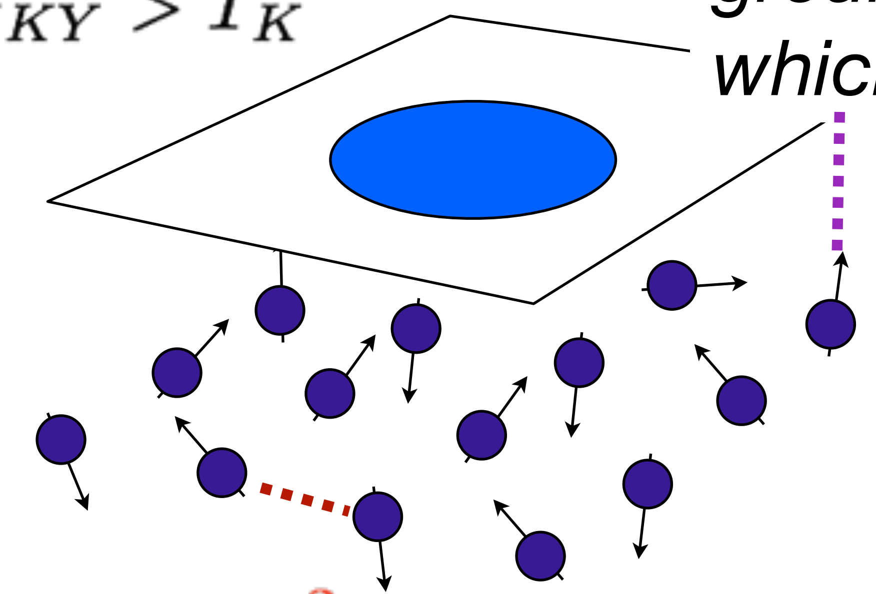

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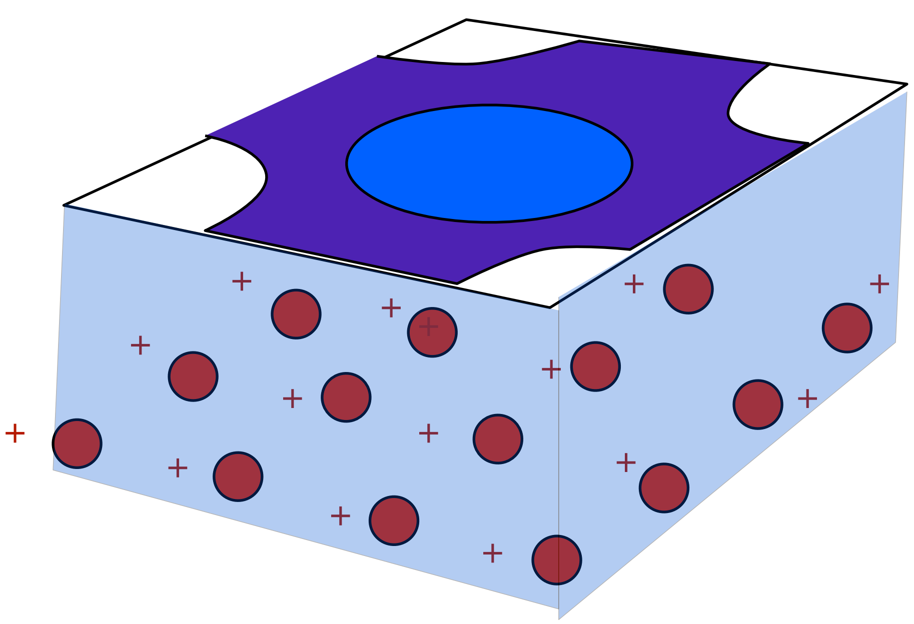

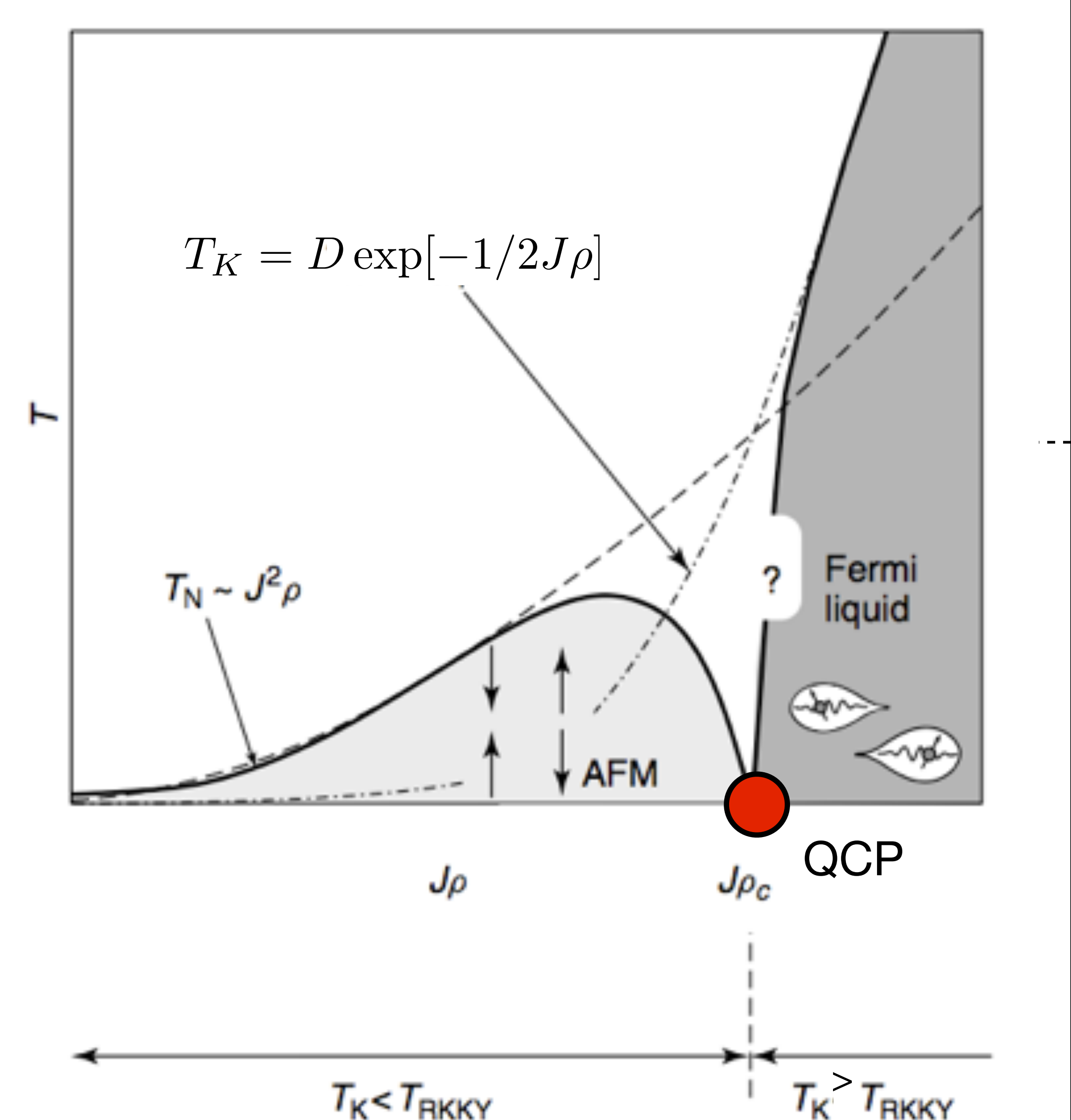

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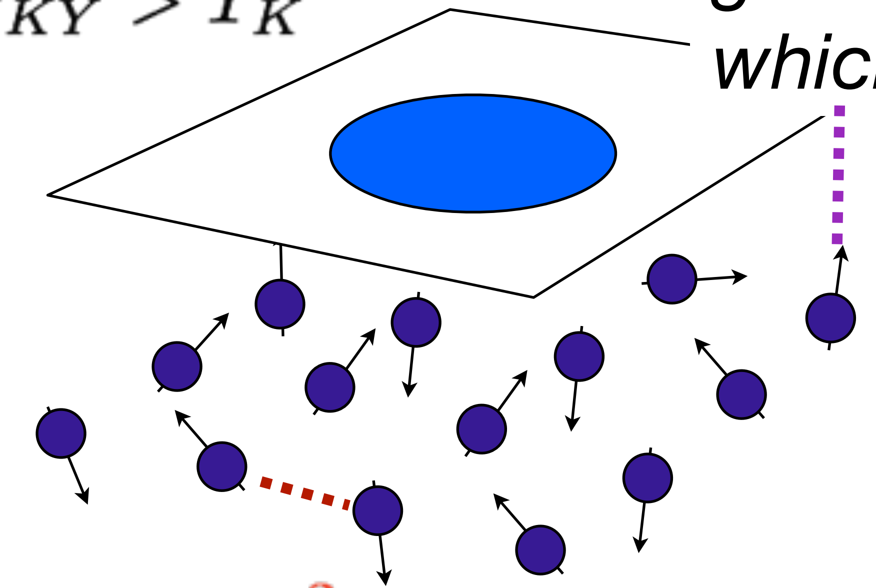

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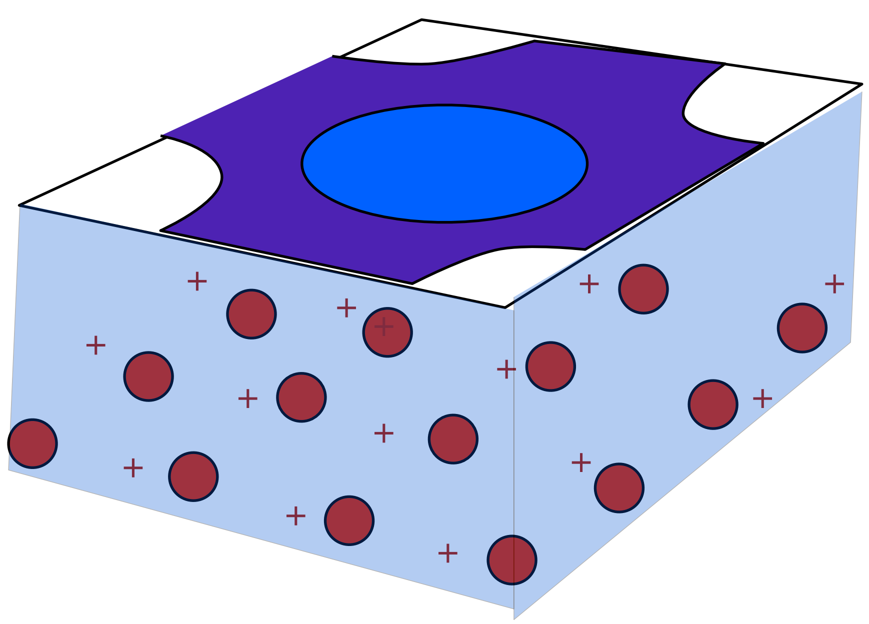
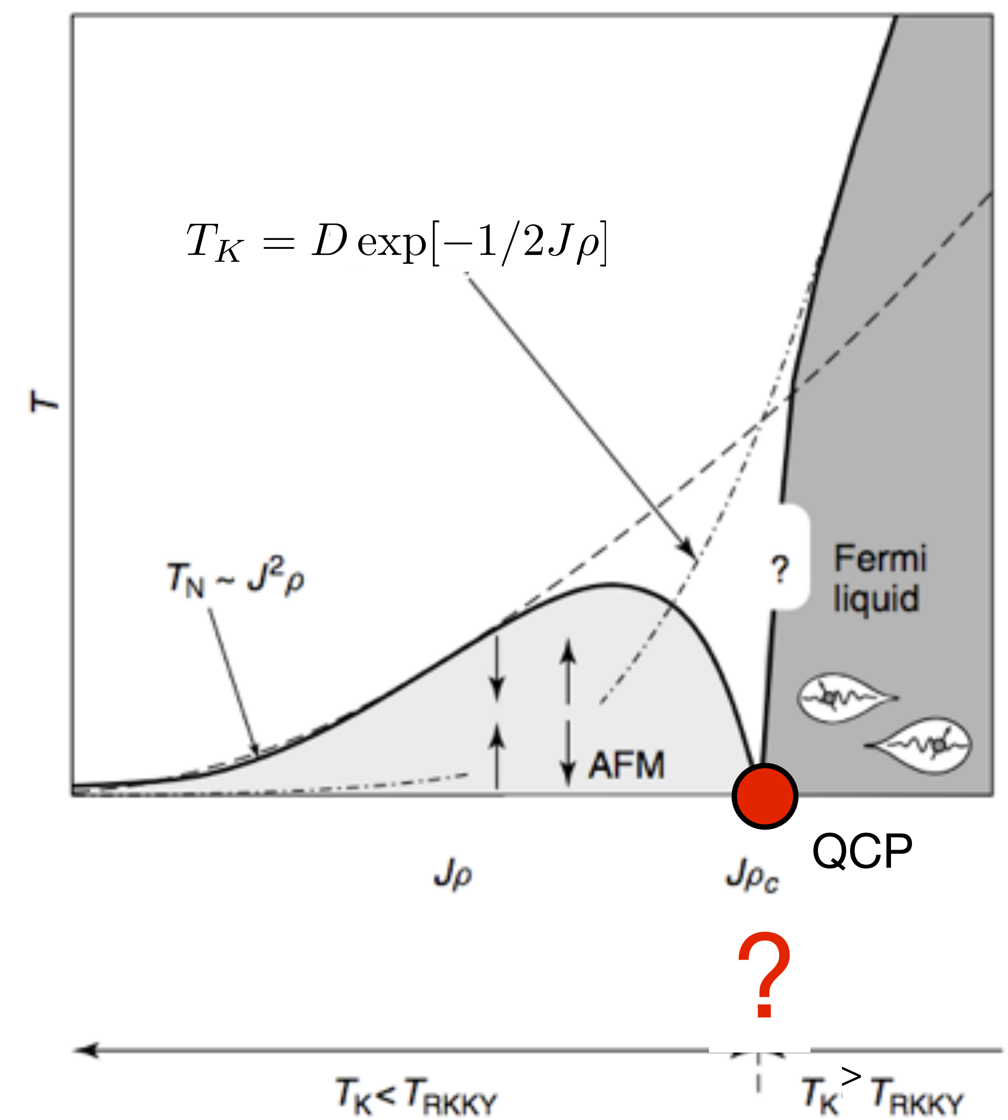
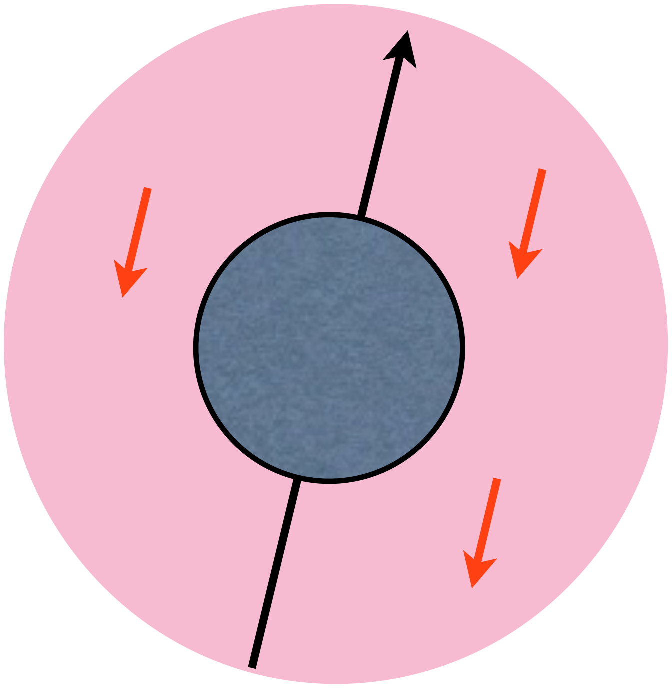
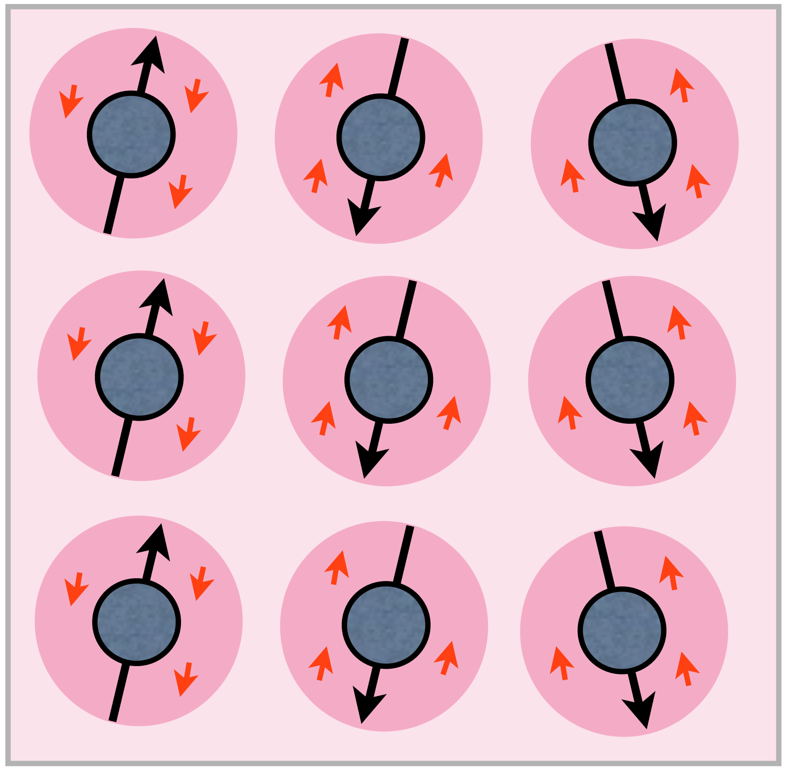




Heavy Fermion Primer



$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + J \sum_{j} \vec{S}_j \cdot \vec{\sigma}(j)$$



“Kondo Lattice”

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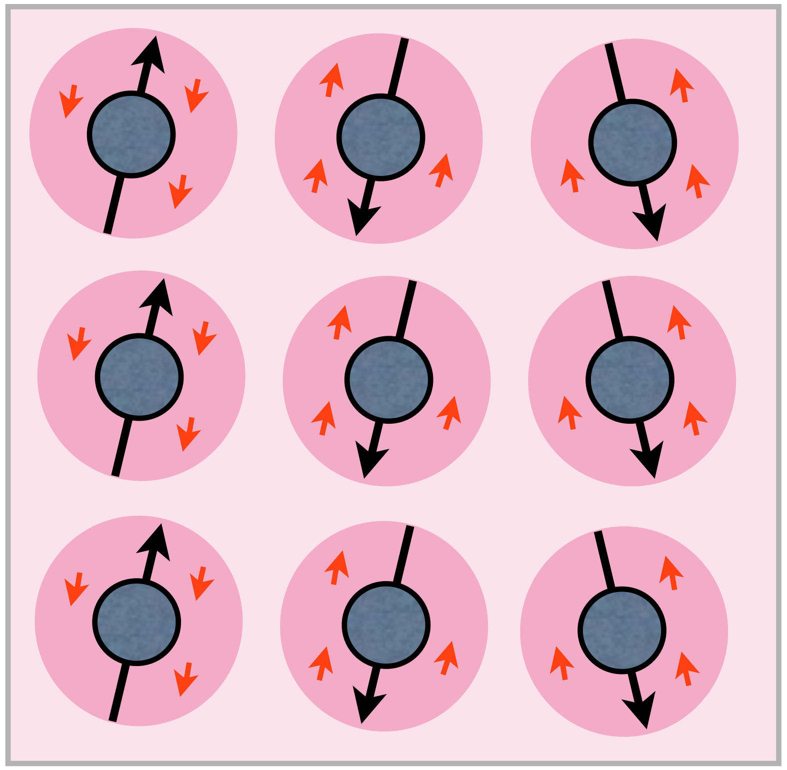

"Kondo Lattice"

Entangled spins and electrons
→ **Heavy Fermion Metals**

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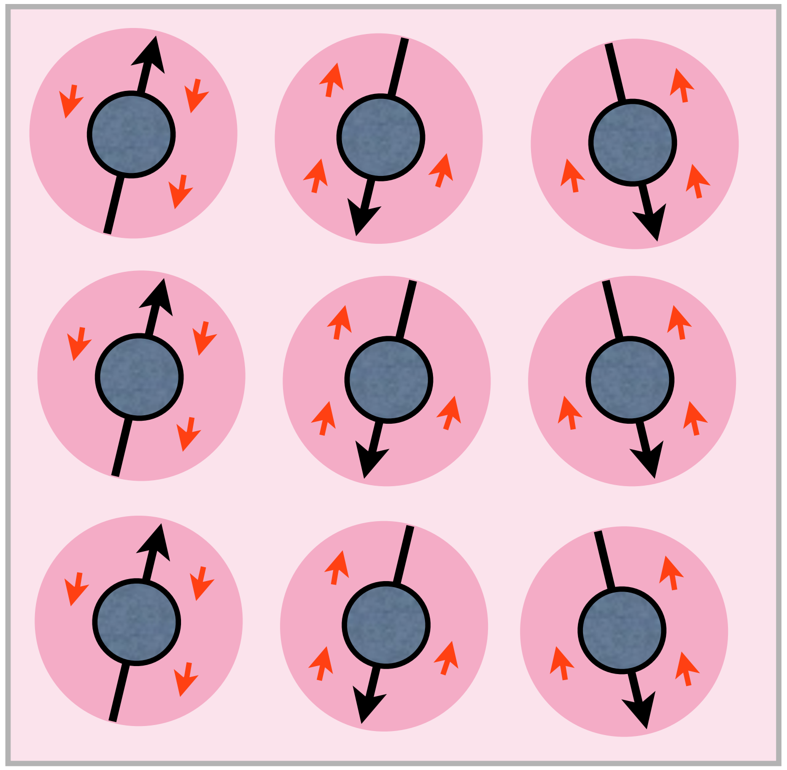

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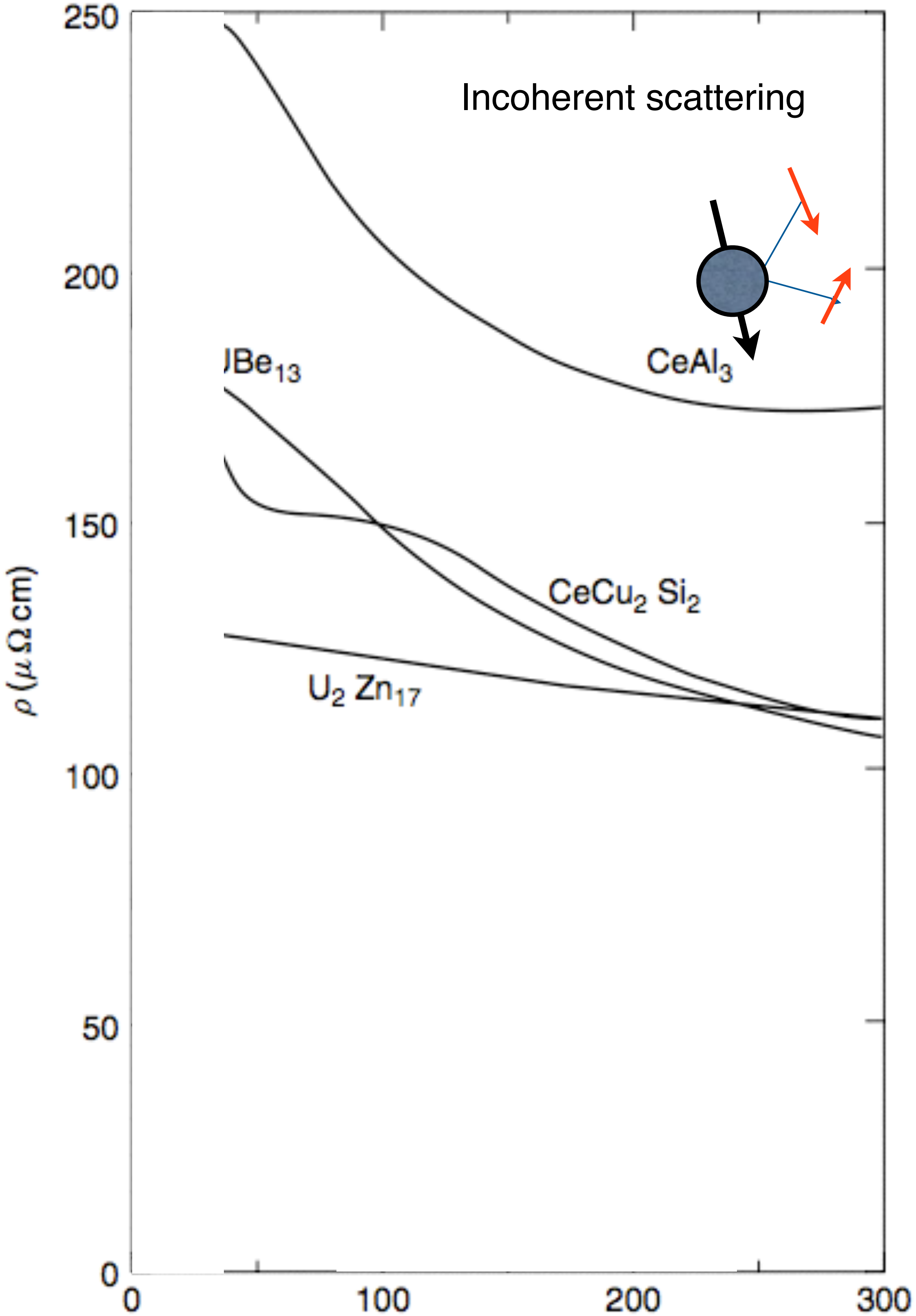

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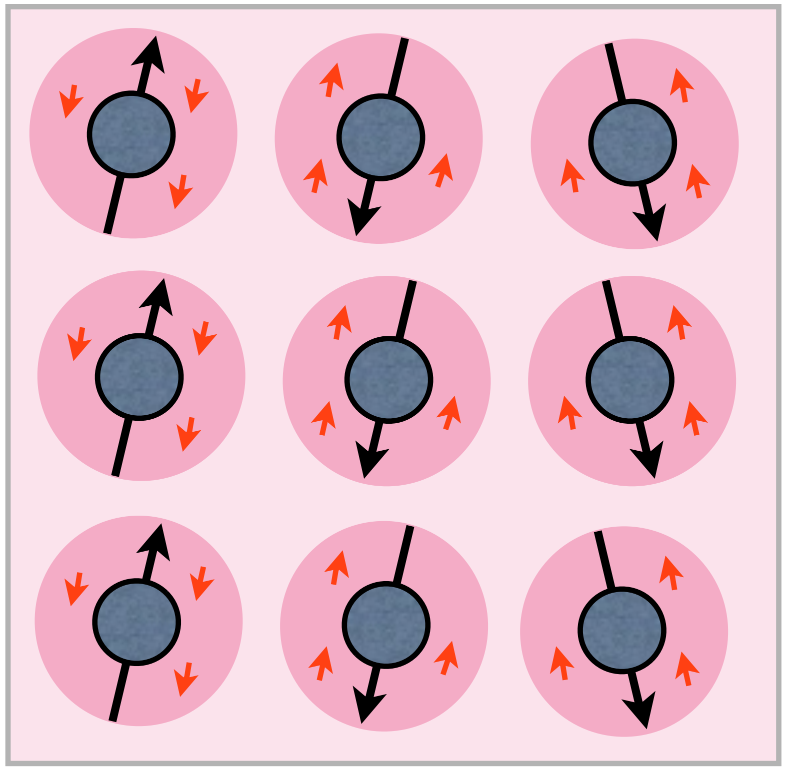

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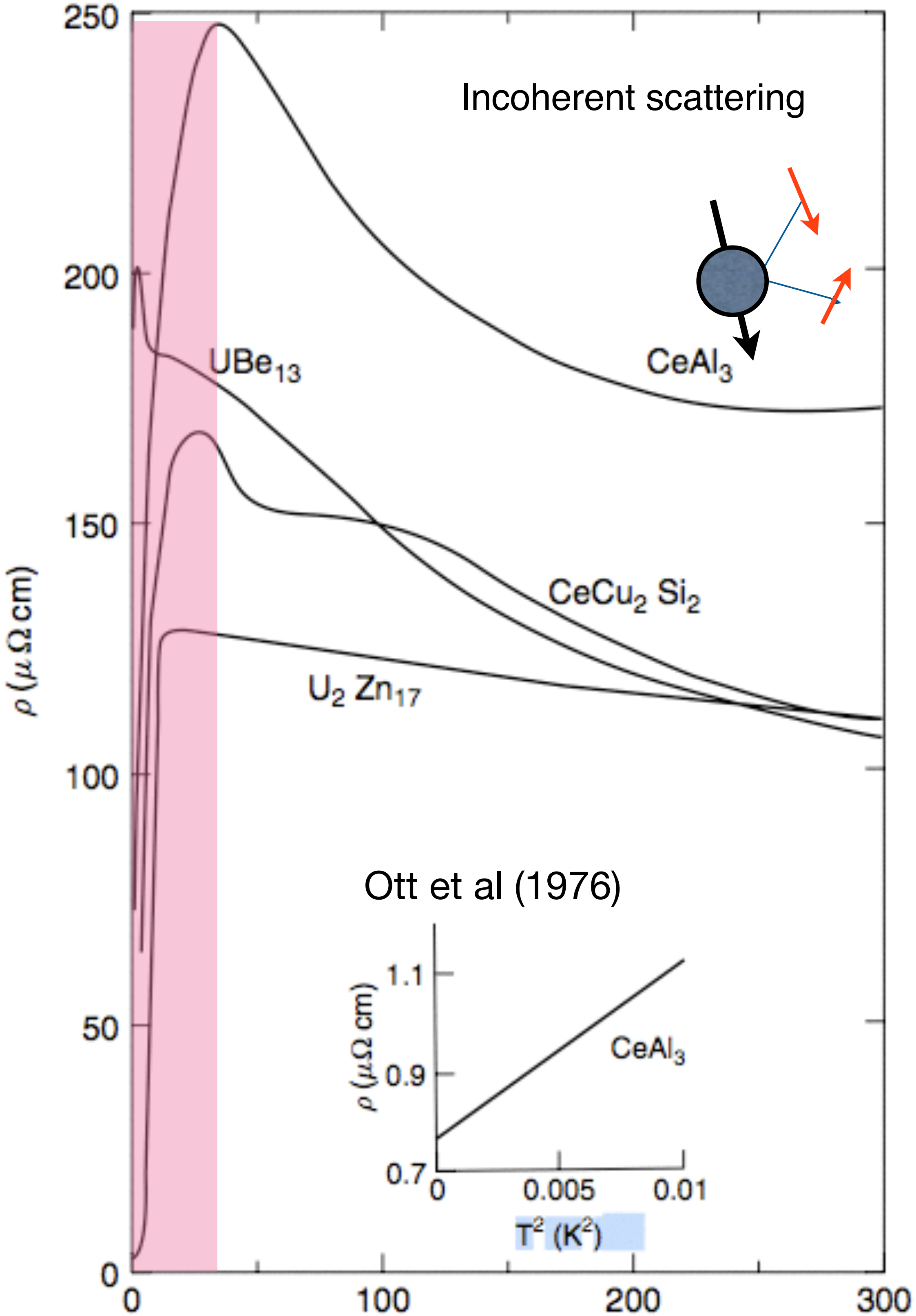

$$\rho(T) = \rho_0 + AT^2$$

Coherent Heavy Fermions

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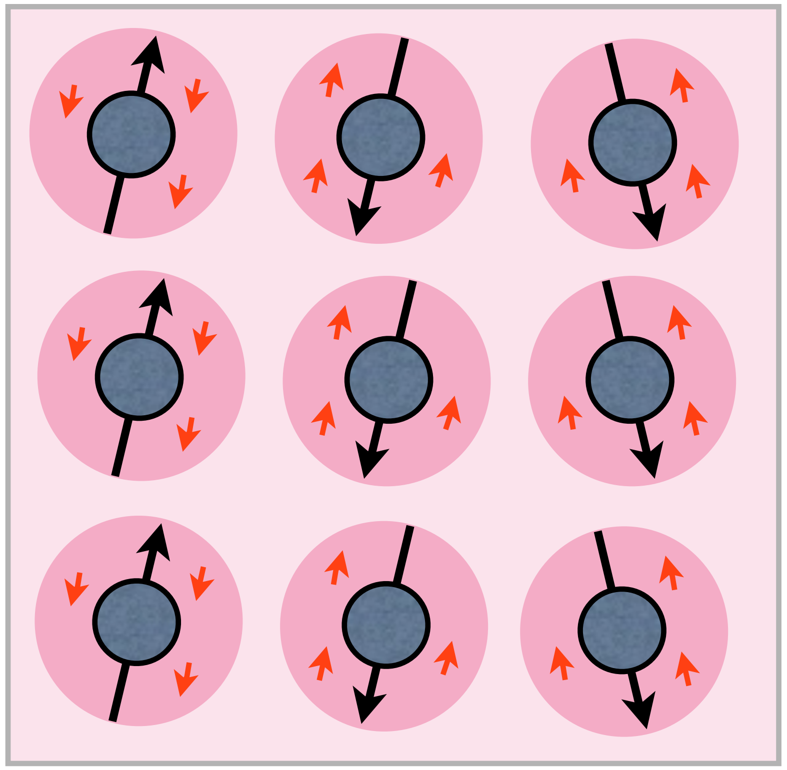
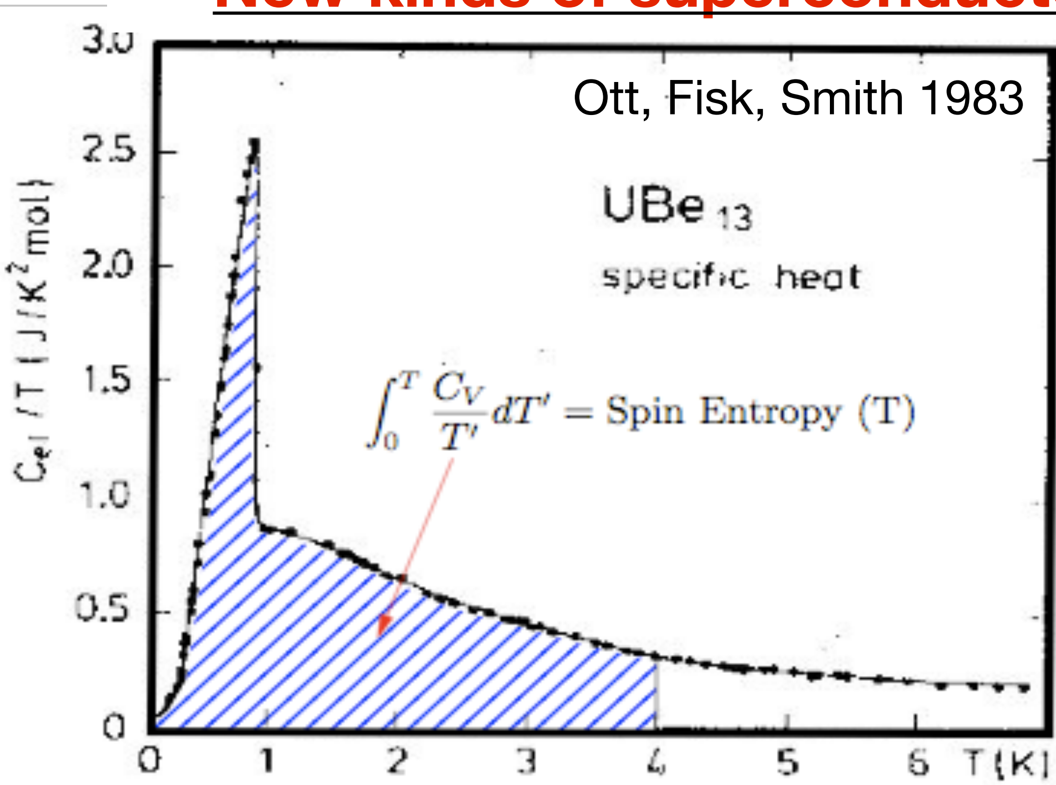
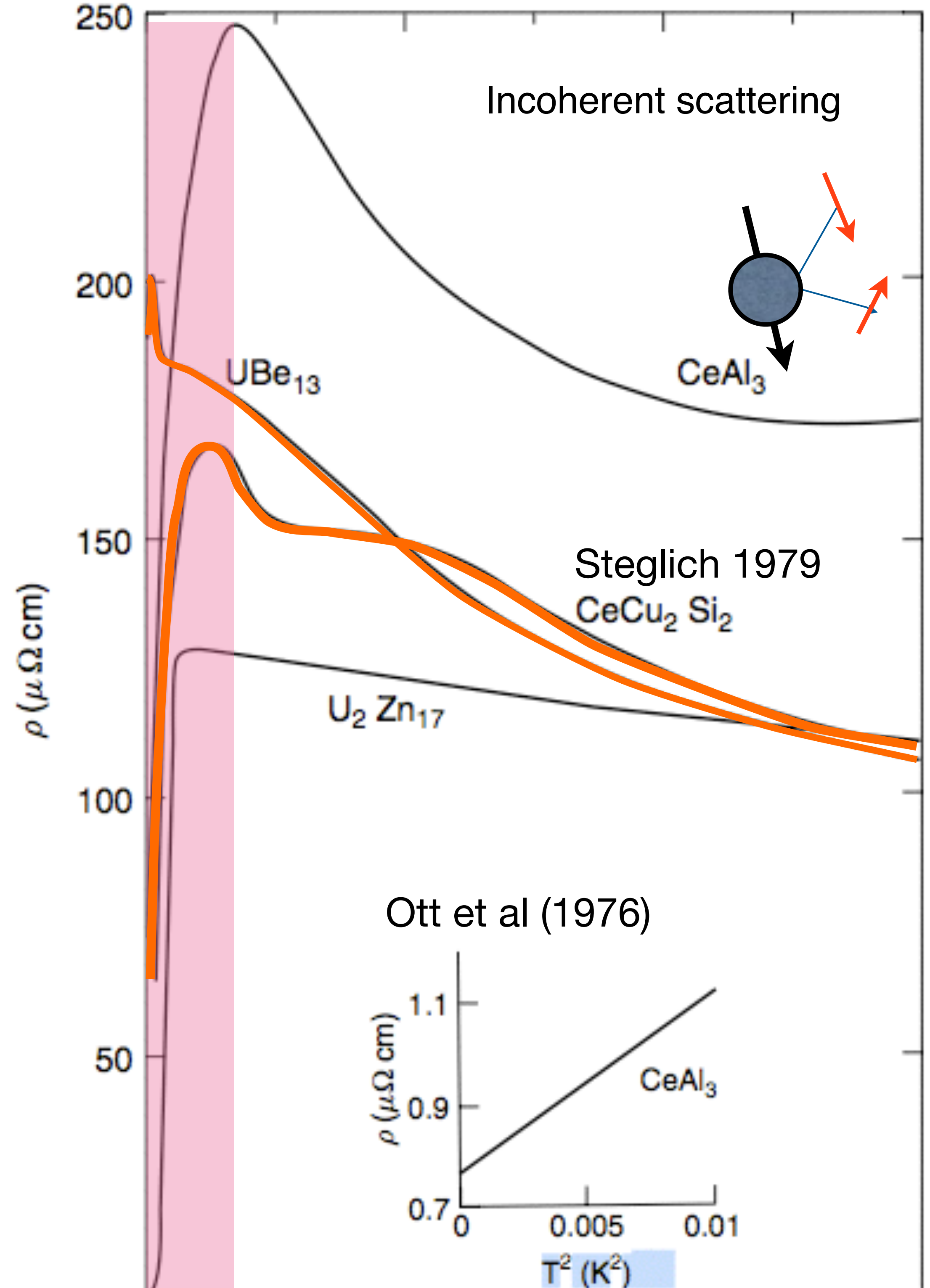


"Kondo Lattice"

Entangled spins and electrons

→ **New kinds of superconductor**





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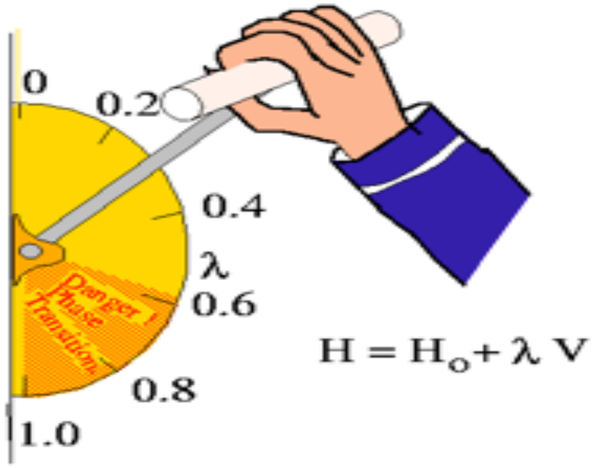
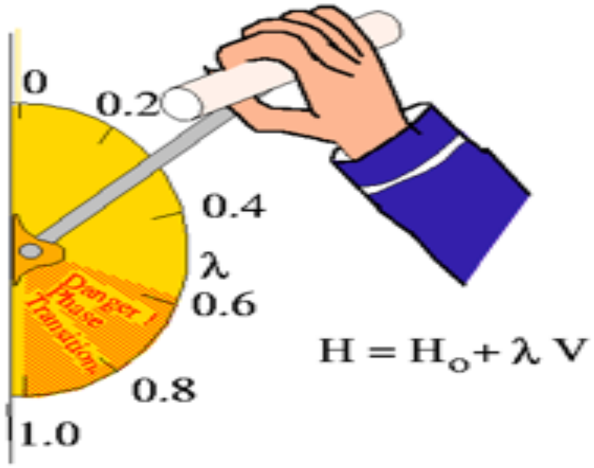
Coherent Heavy Fermions

Digression:
Landau Fermi Liquid Theory



Landau: interactions can be turned on adiabatically, preserving the excitation spectrum.



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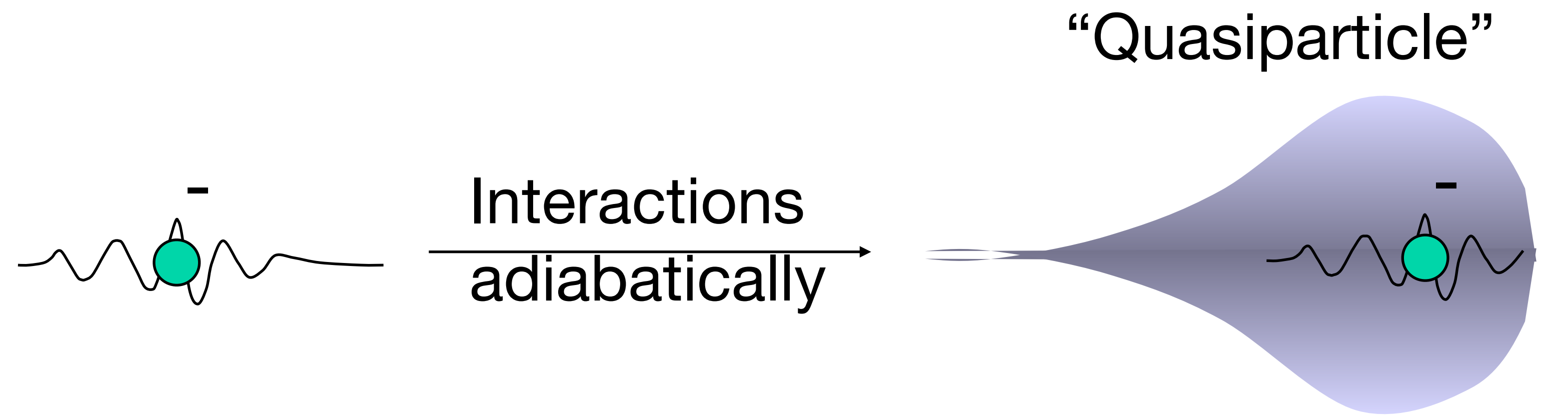
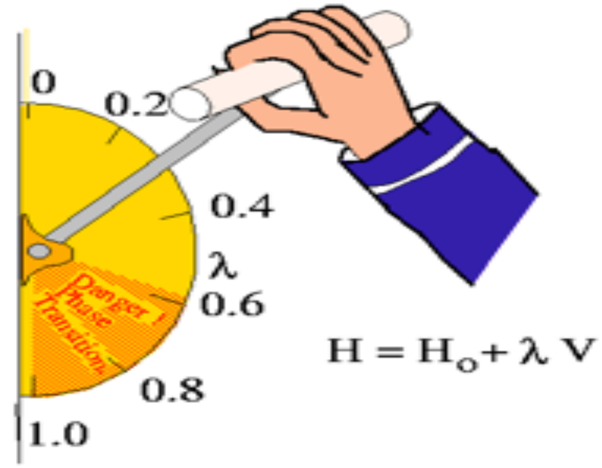


$$|e^-\rangle \qquad\qquad |qp^-\rangle$$



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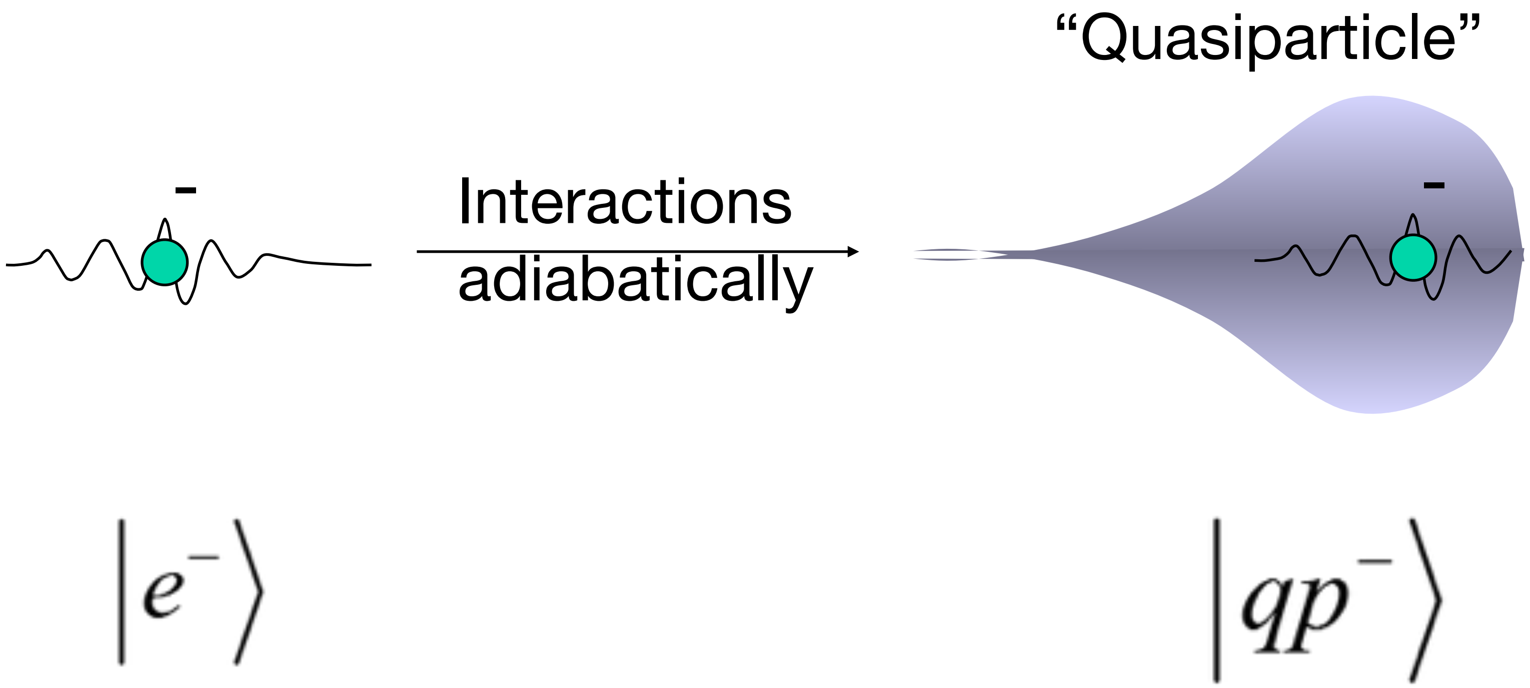

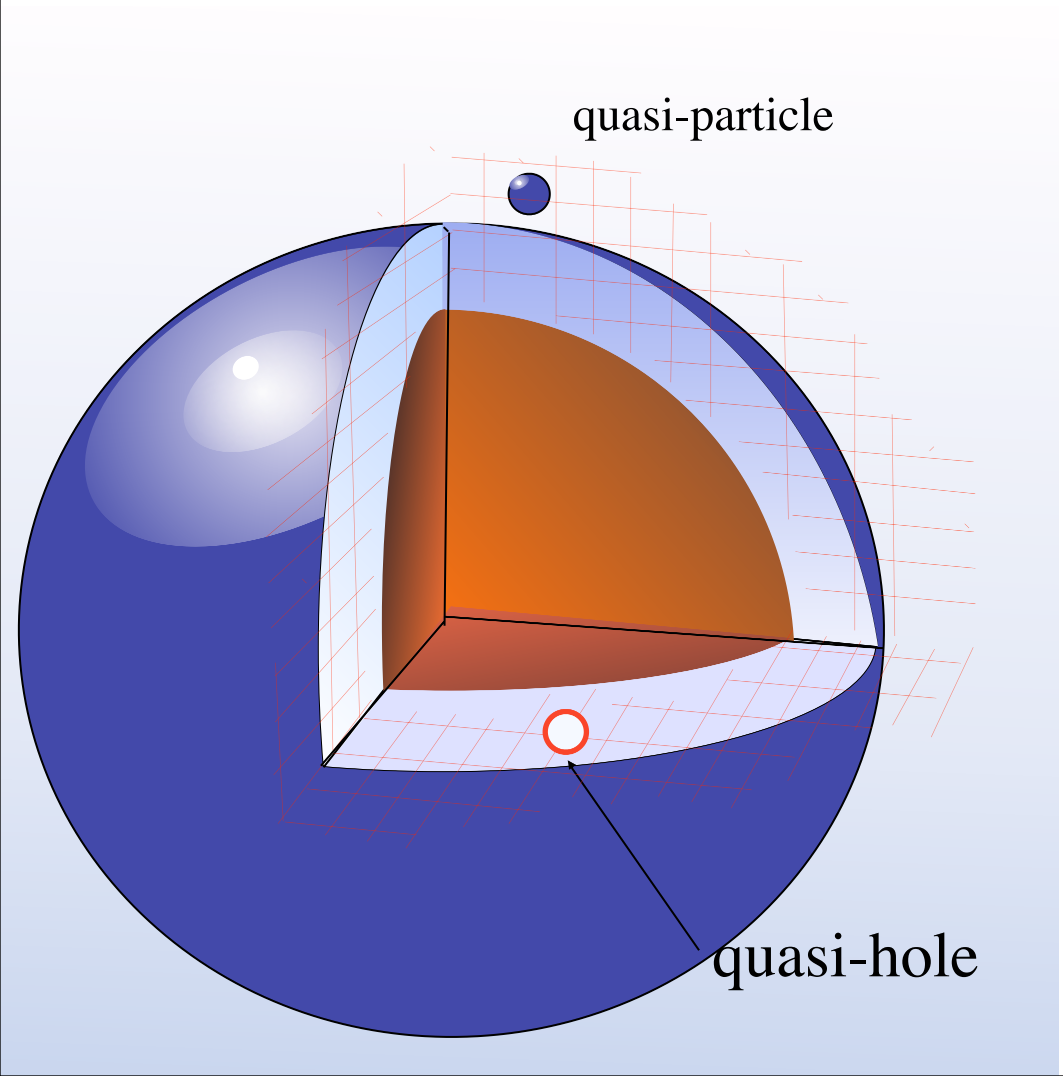

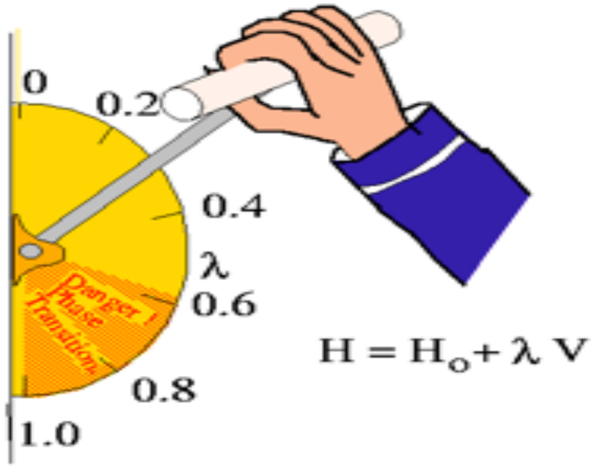

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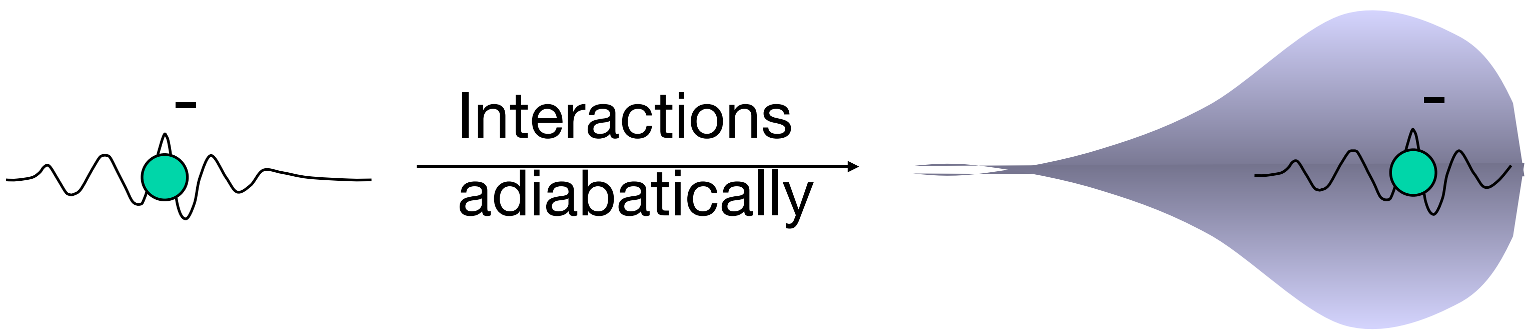
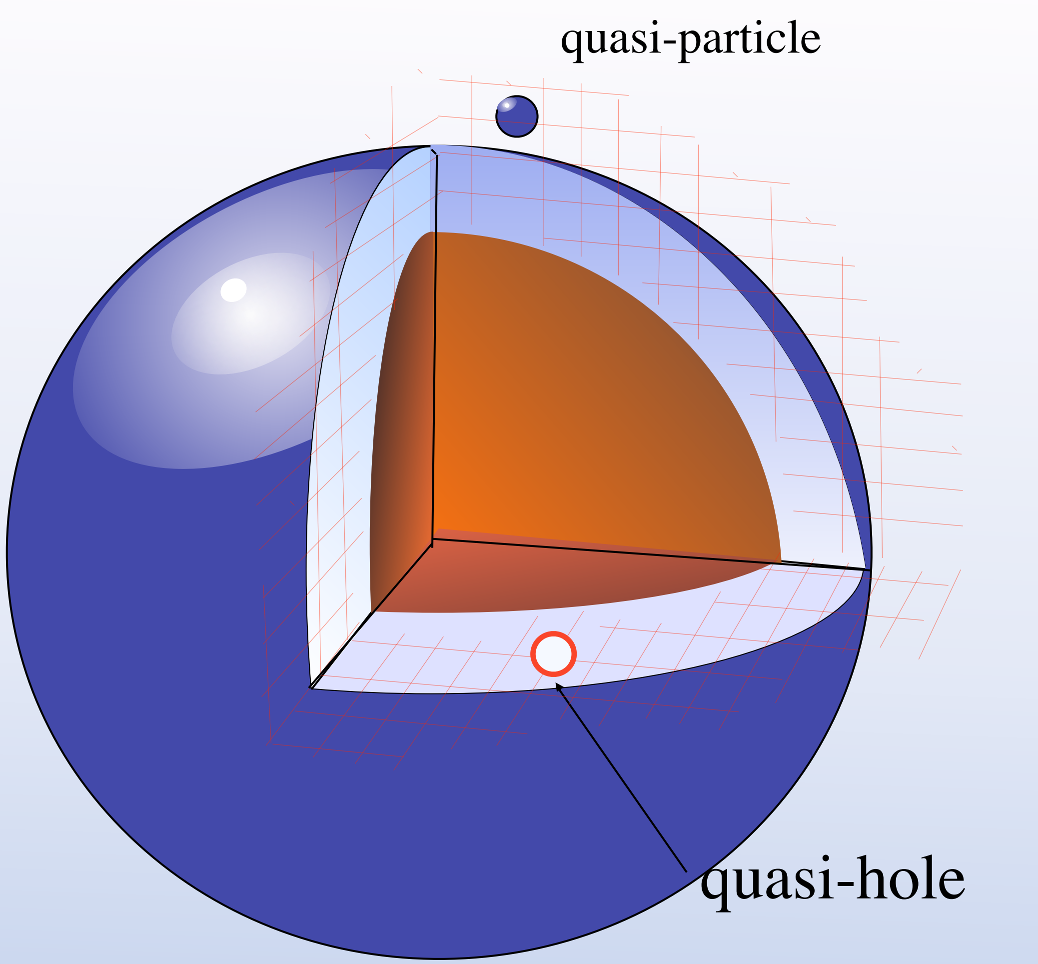


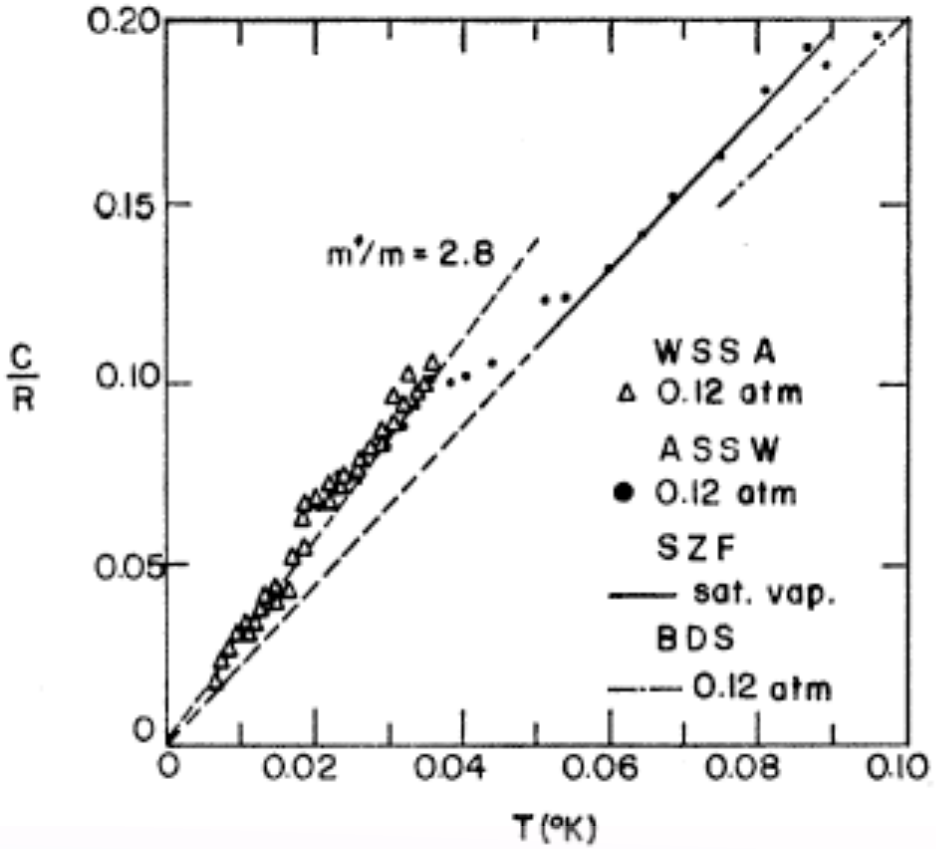
$$\left|e^{-}\right\rangle \qquad \left|qp^{-}\right\rangle$$

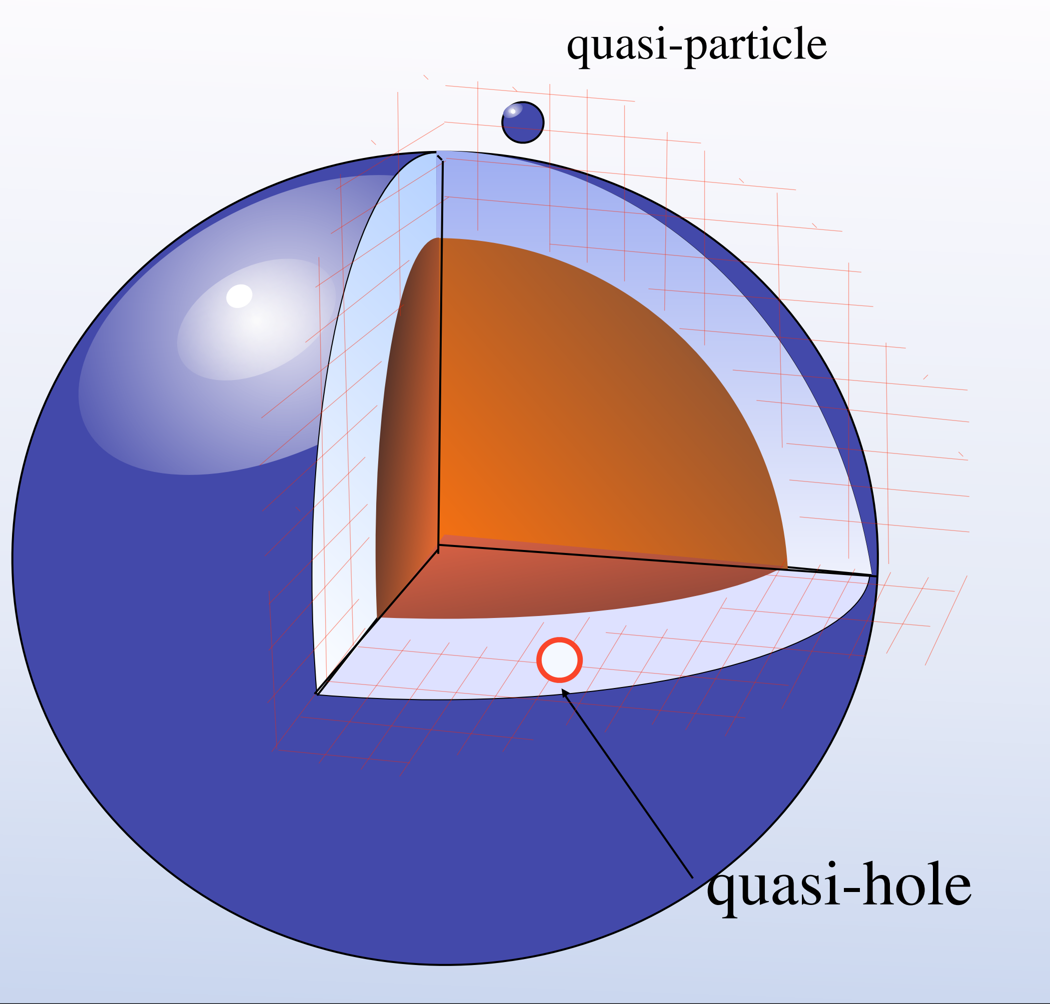


$$\frac{m^*}{m} = \frac{N(0)^*}{N(0)} = 1 + \frac{F_1^s}{3}$$

Landau, JETP 3, 920 (1957)

He-3 (1950/60s)
(Fairbanks, many others)





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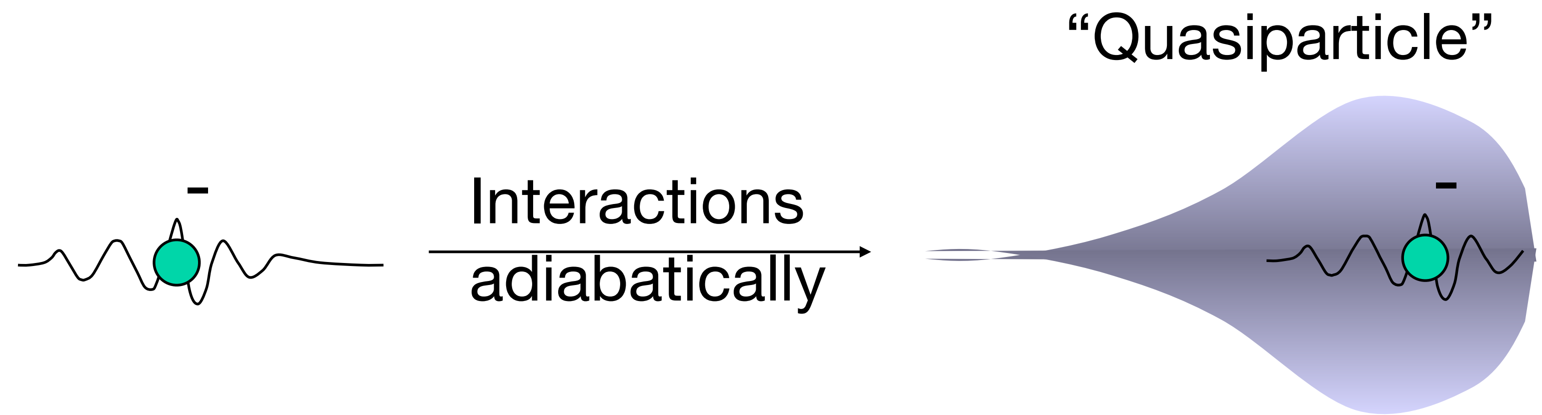

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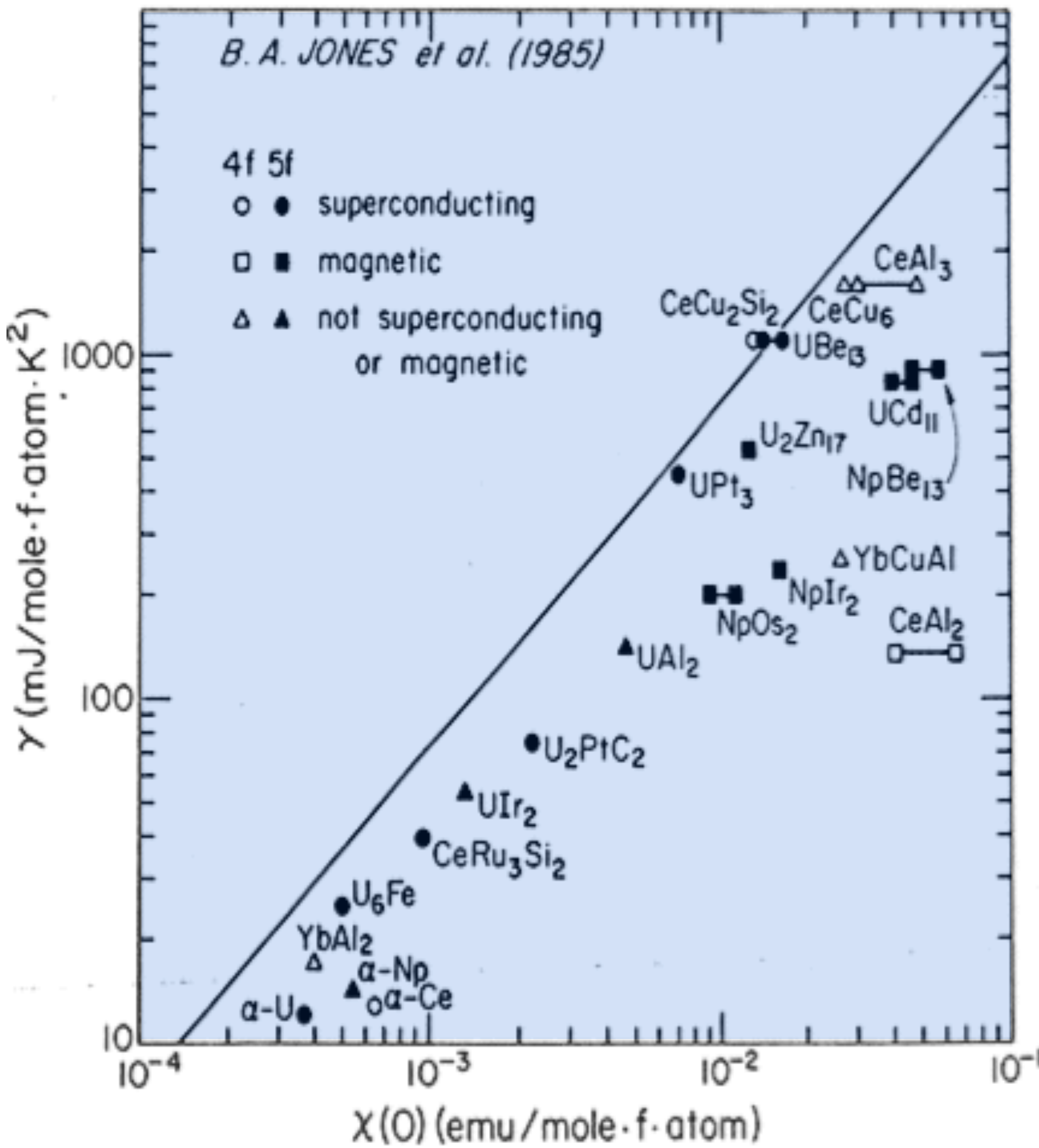

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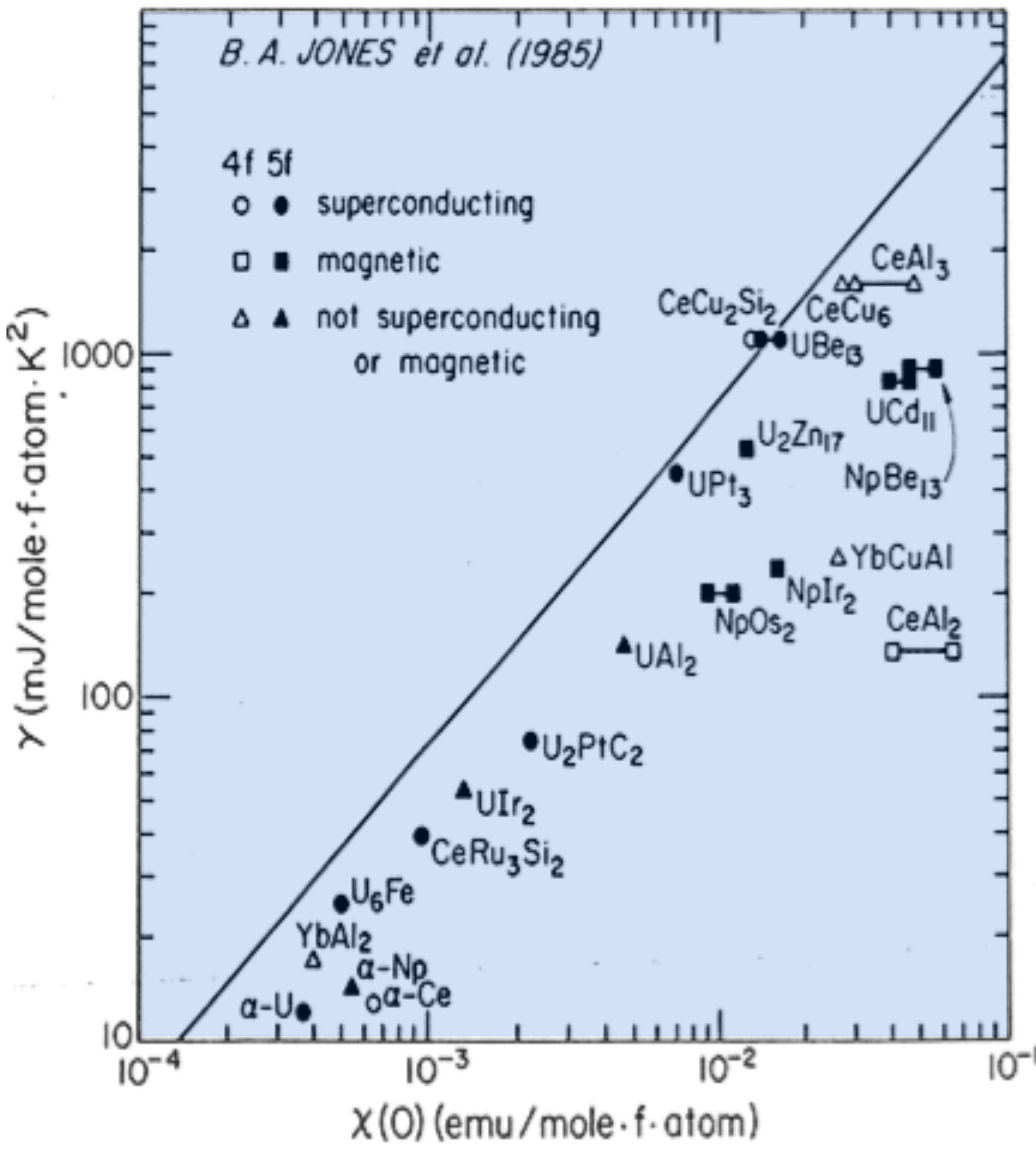

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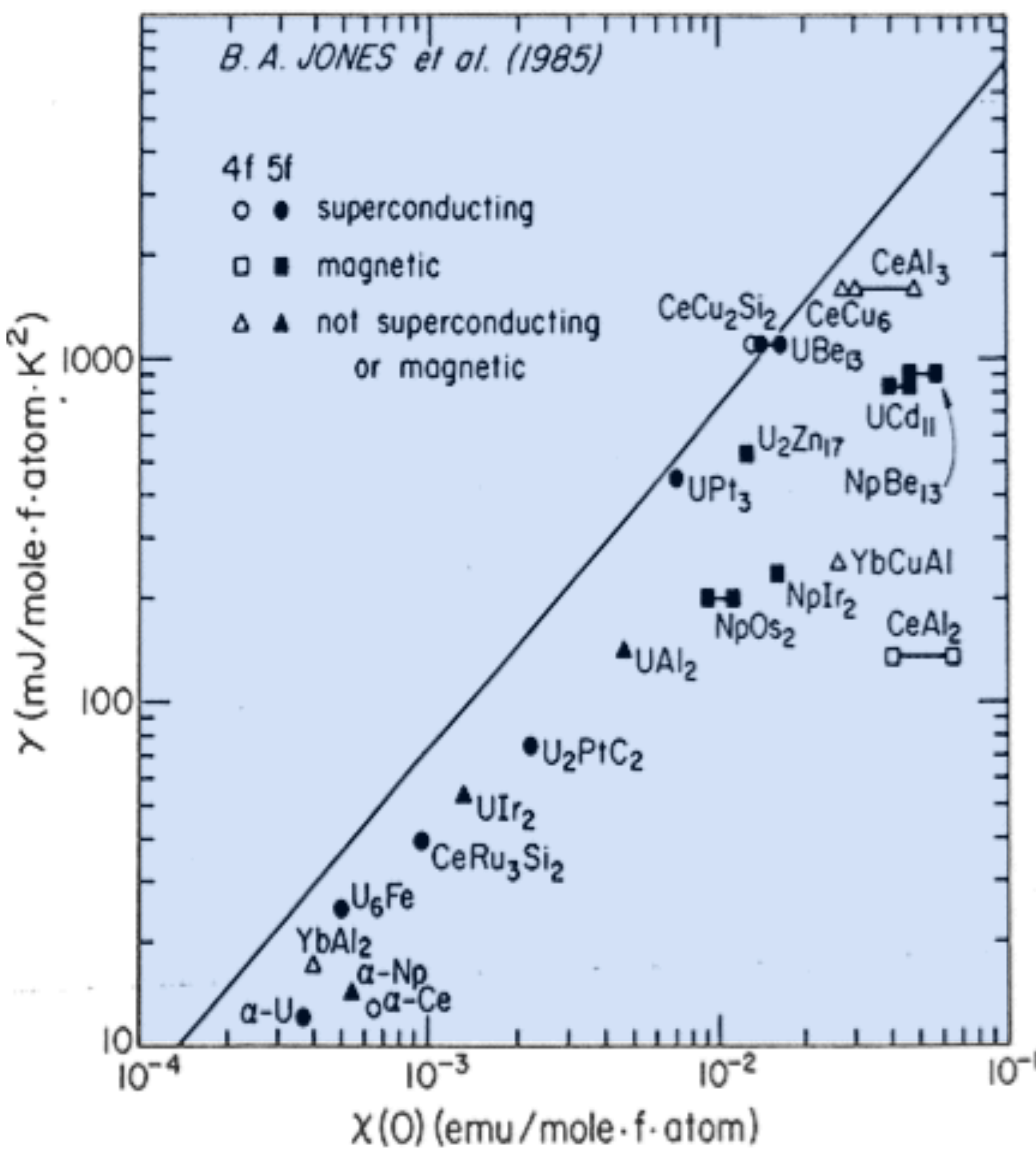

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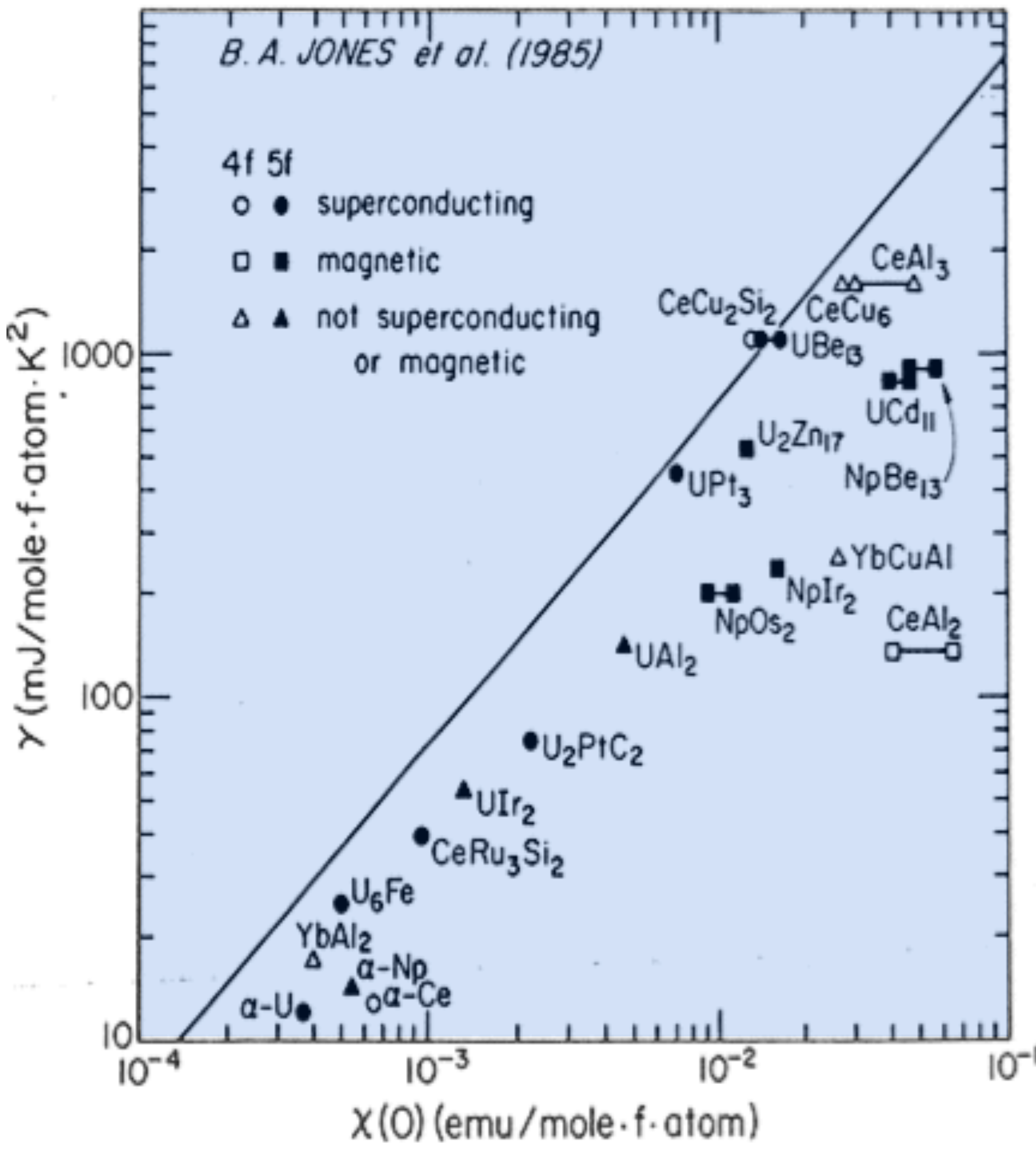
"Wilson" or "Sommerfeld" ratio.



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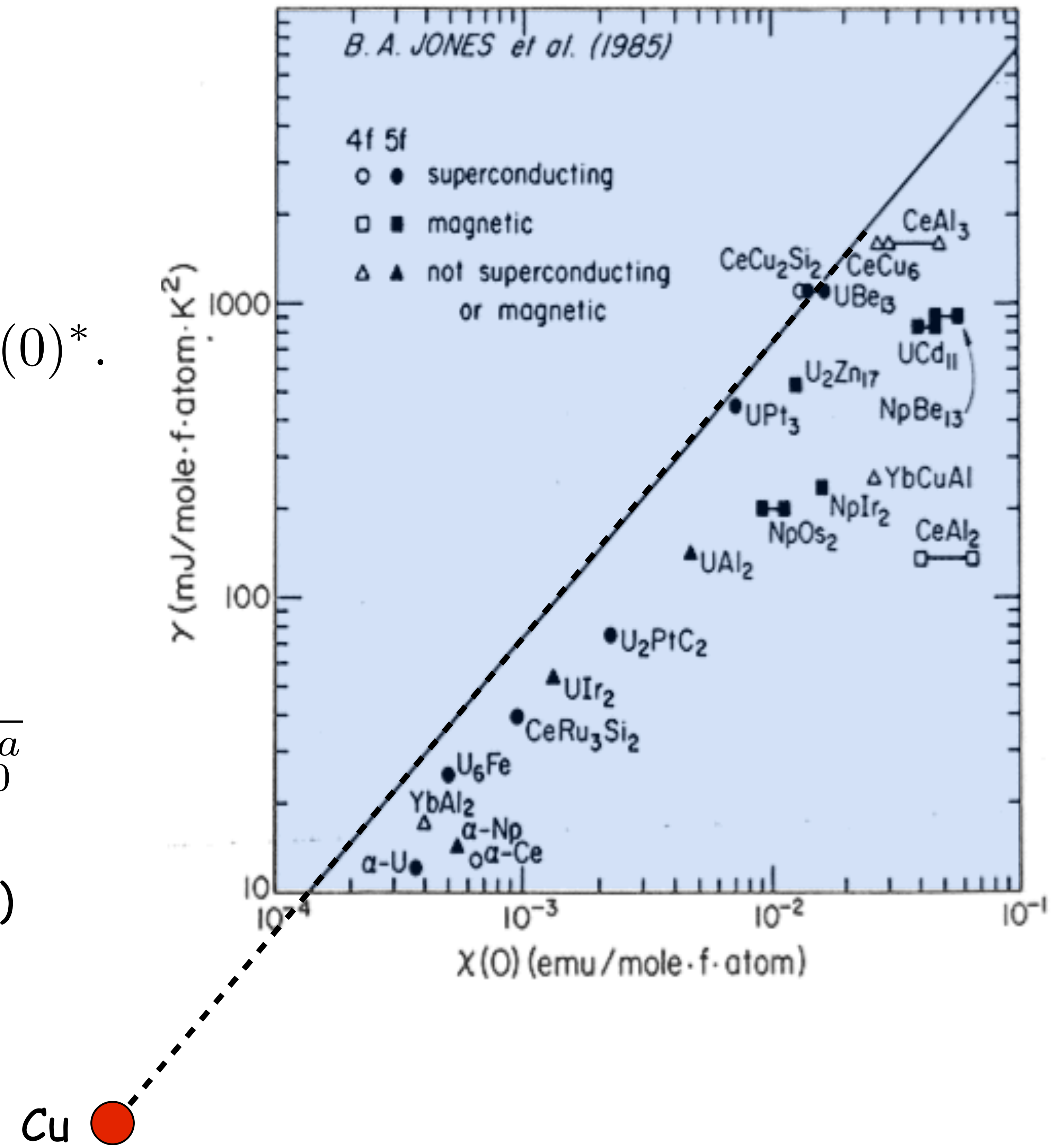
$\gamma Cu \sim 1\ mJ/mol/K^2$,



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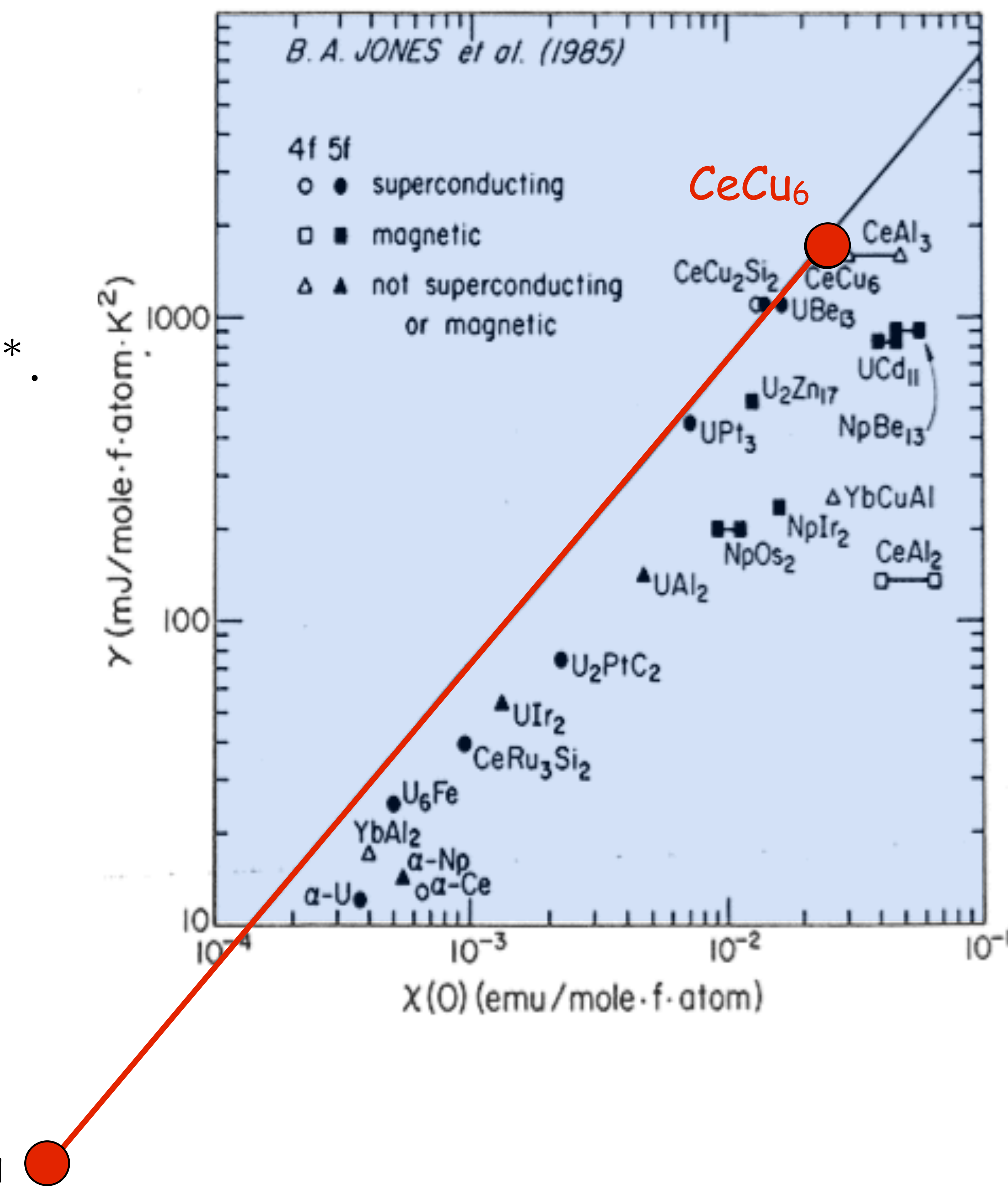

eg Cu vs $CeCu_6$ (copper, spin doped)

$\gamma Cu \sim 1$ mJ/mol/K^2,

$\gamma[CeCu_6] \sim 1000$ mJ/mol/K^2,

$m^*/m_e \sim 1000$

20. Moscow, 1956. Freeman Dyson (front, left), talking with I. Pomeranchuk and Lev Landau.

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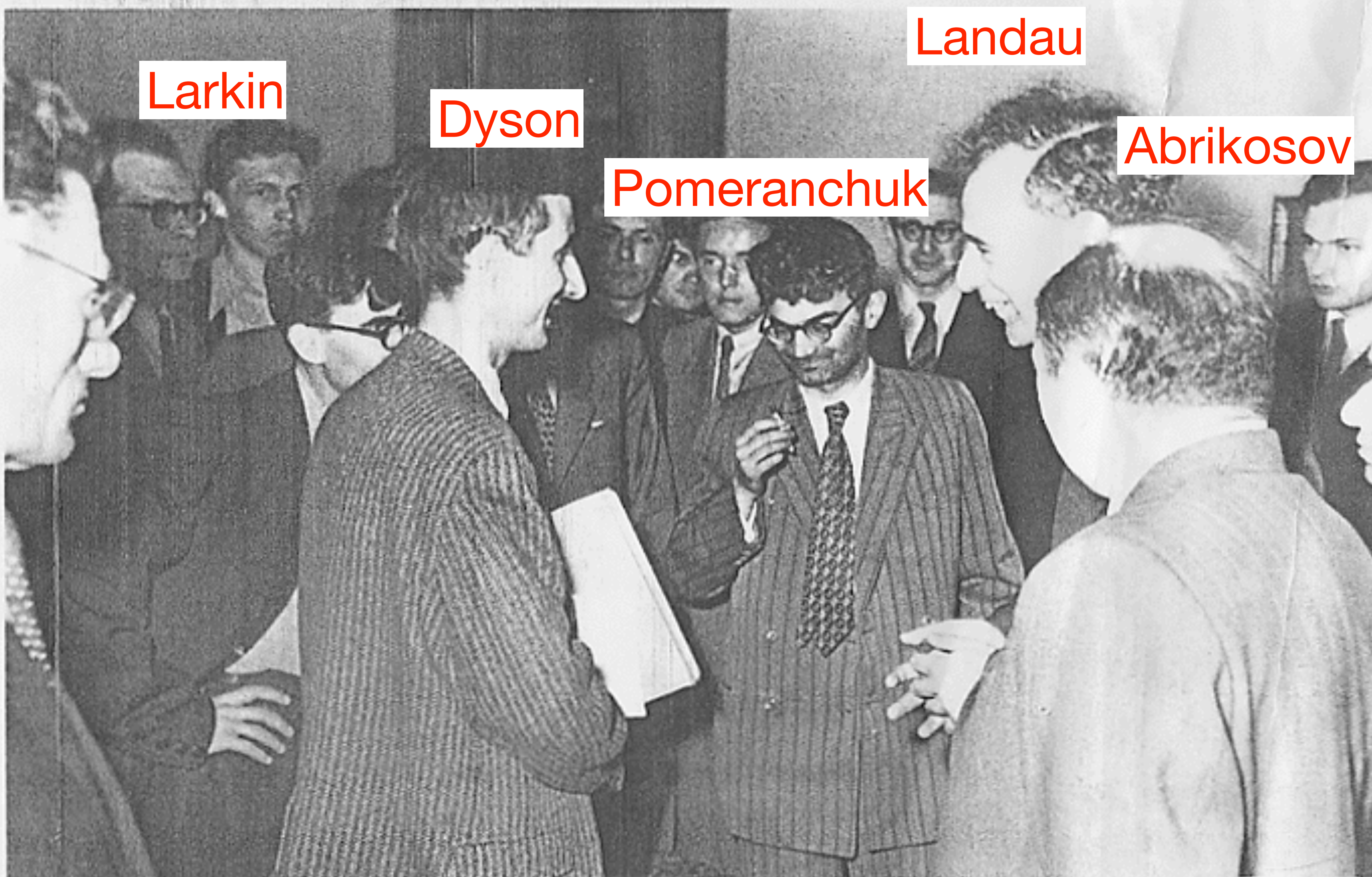

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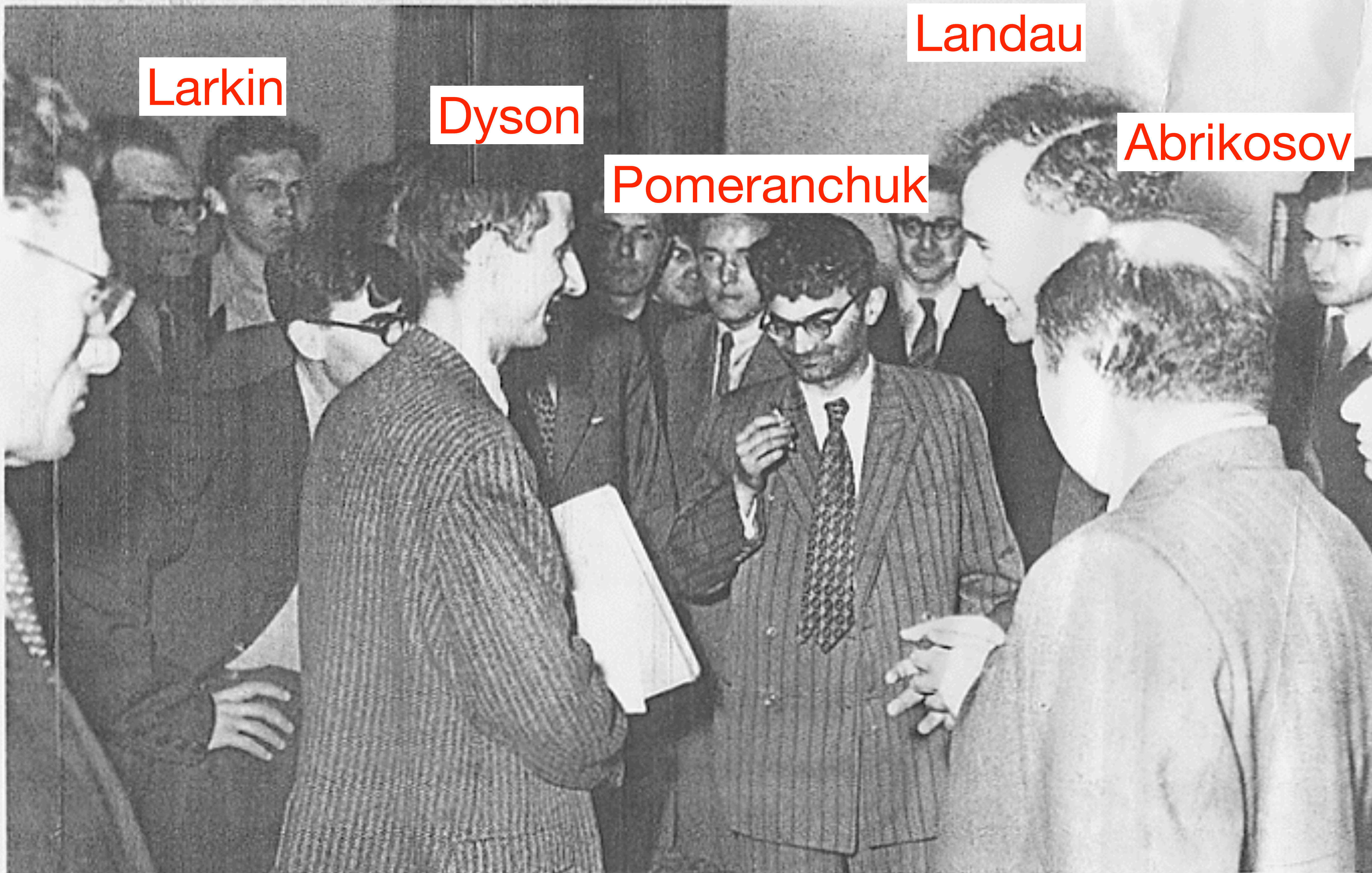
0 Fermi Liquid — Adiabaticity — Landau Fermi Liquid $X \sim U/t$



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$X \sim U/t$

0 Fermi Liquid — Adiabaticity — Landau Fermi Liquid →

X~U/t

Fermi Liquid

0

What happens when
the interaction
becomes too large?

$X \sim U/t$

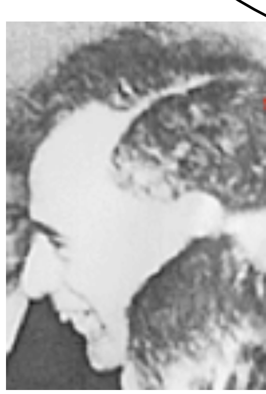
Long range order

Fermi Liquid

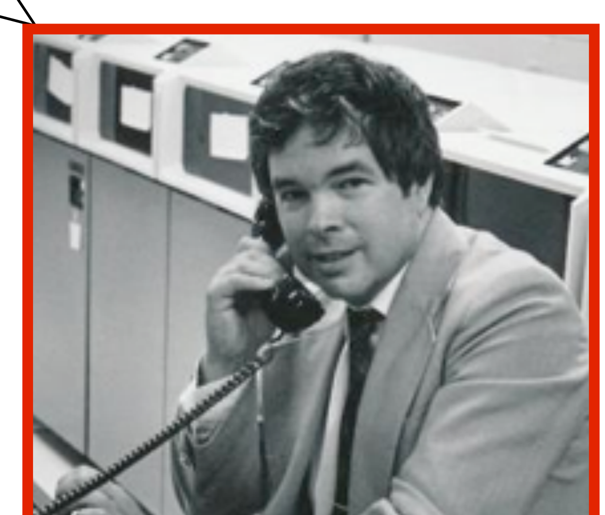
0 X_c

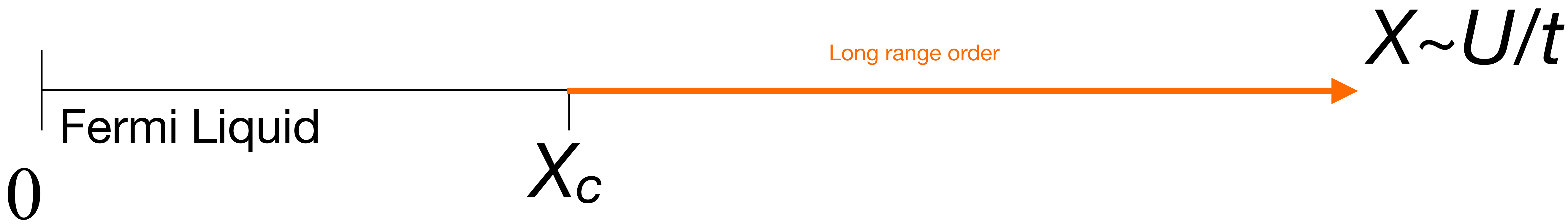
What happens when the interaction becomes too large?

Wigner/ Landau 1934/36



"Electrons order"





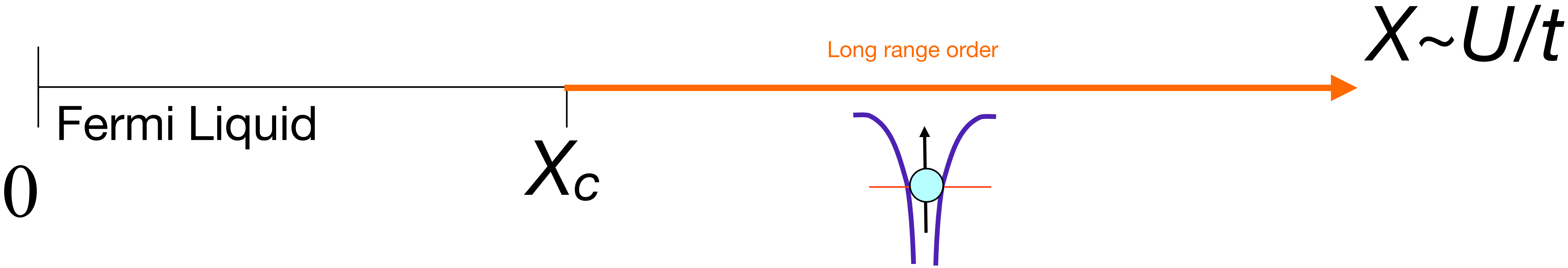
What happens when the interaction becomes too large?

Wigner/ Landau 1934/36

"Electrons order"

Peierls/Mott 1939

"Electrons localize"



What happens when the interaction becomes too large?

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Anderson 1961

"Electrons order"

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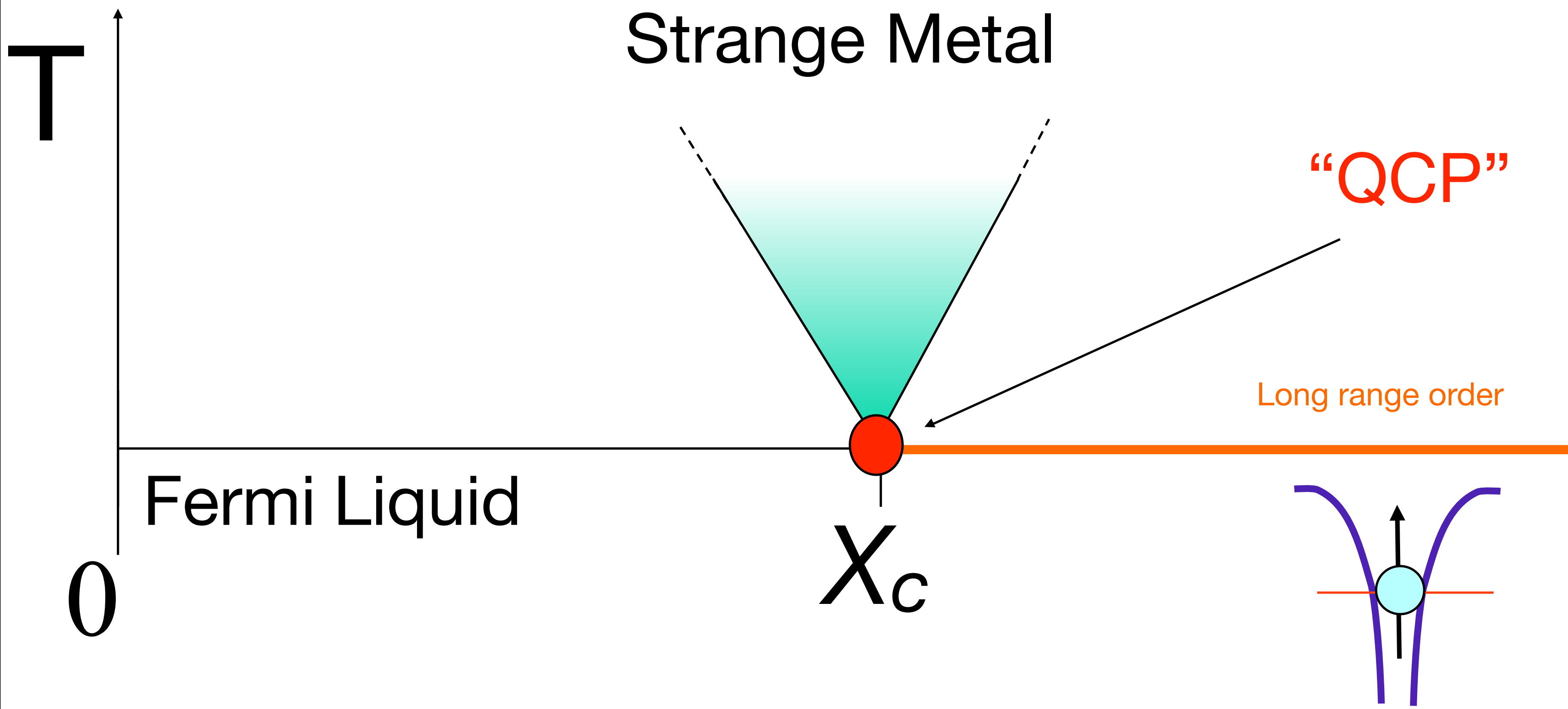
"Moments form"



Hertz, 1976, Doniach, 1977

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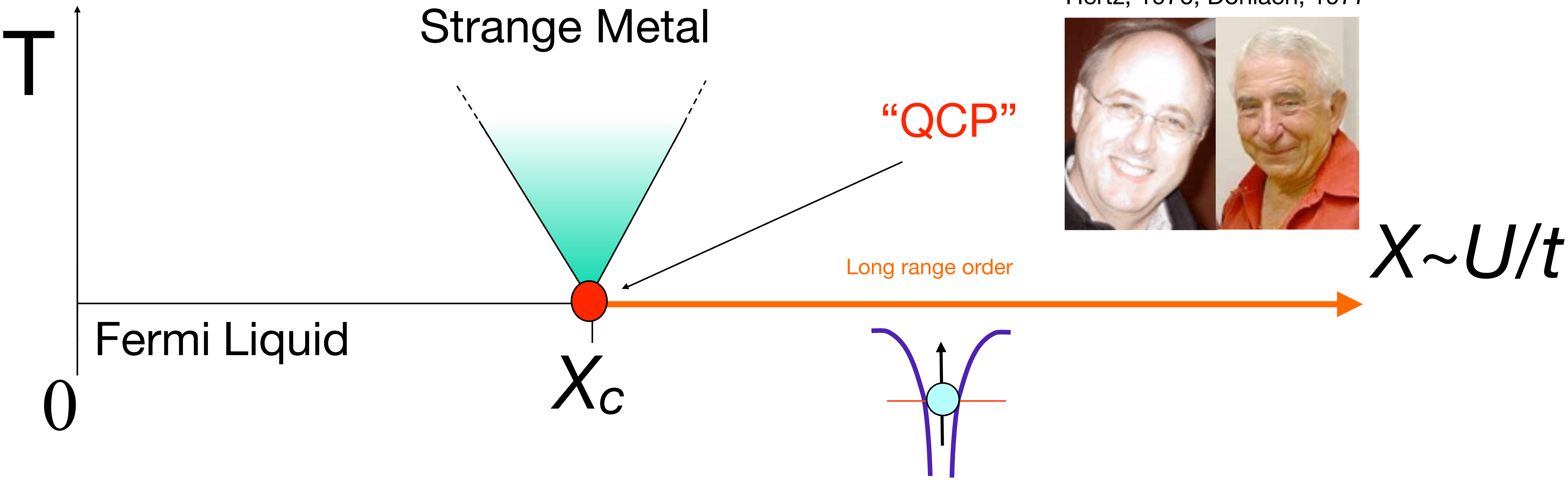

What happens when
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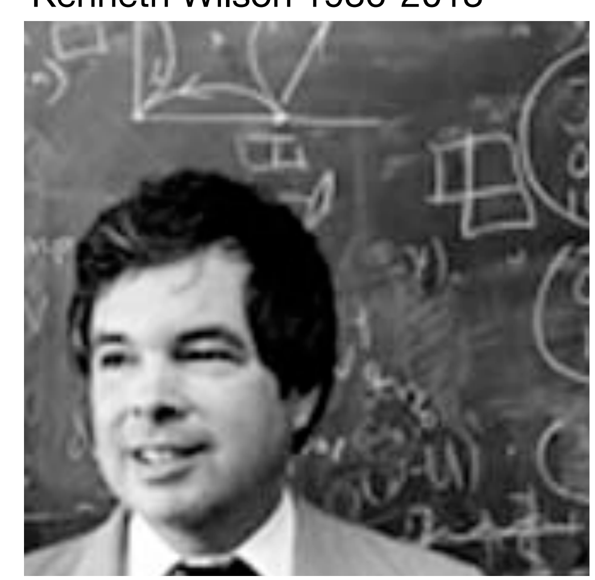
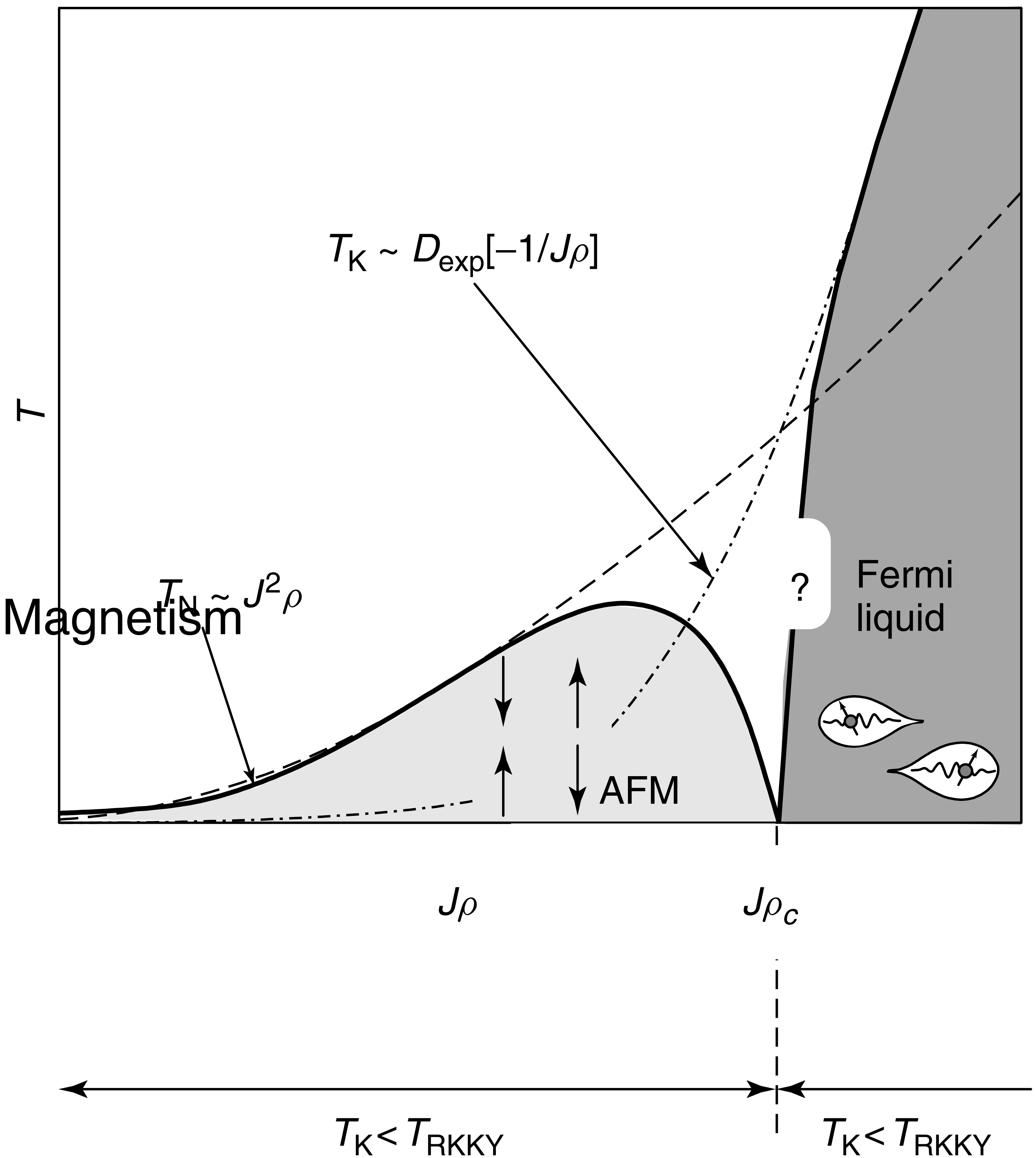
Kenneth Wilson 1936-2013

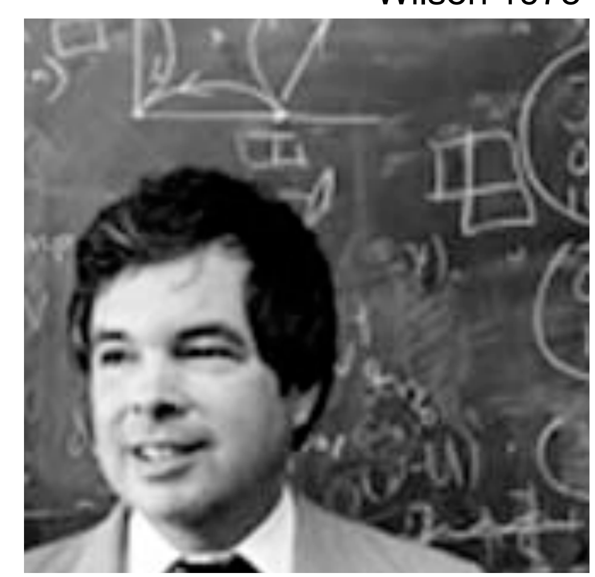


New Fixed Points

Mott, 1973
Doniach 1976



Wilson 1975



New Fixed Points

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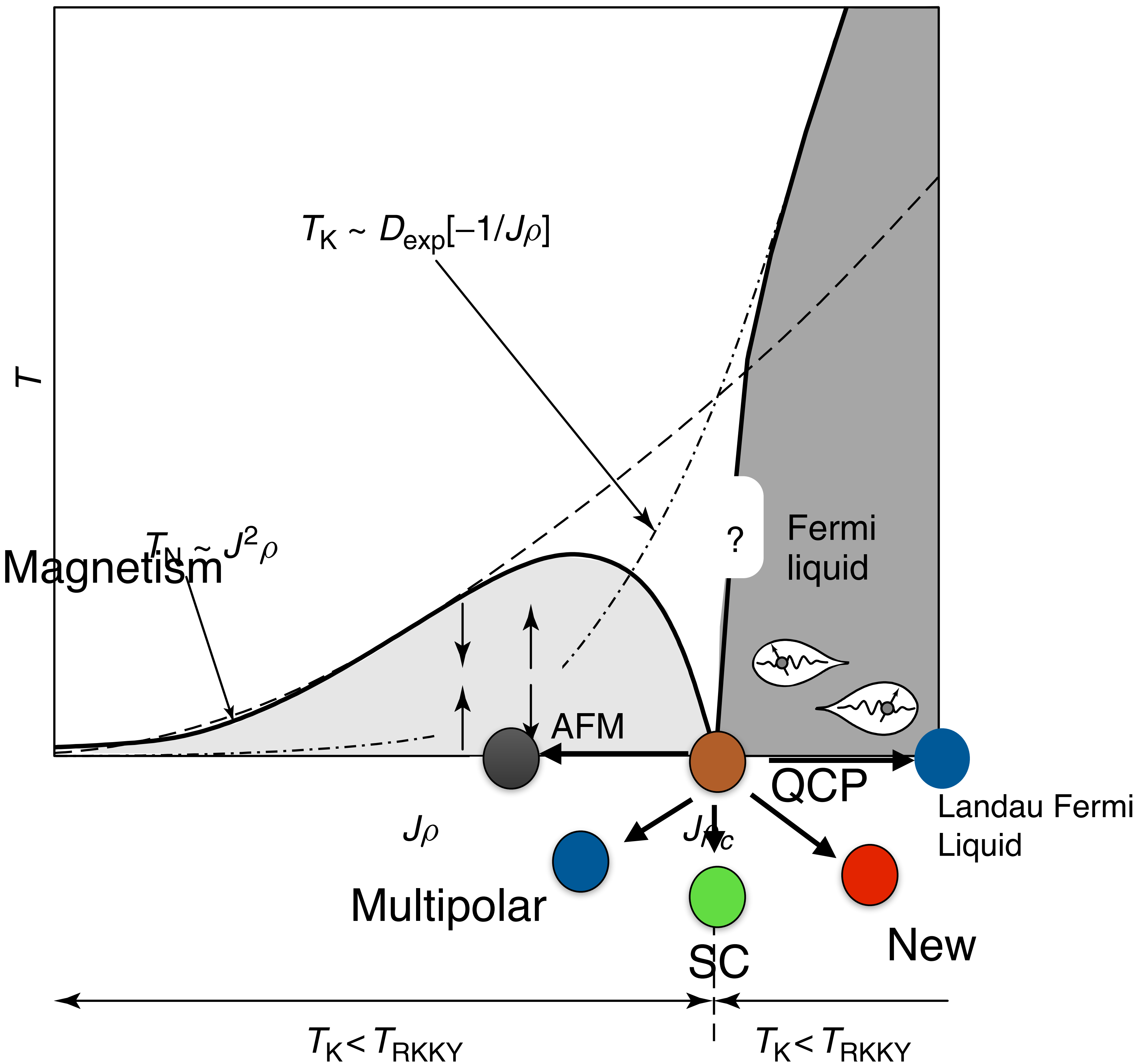
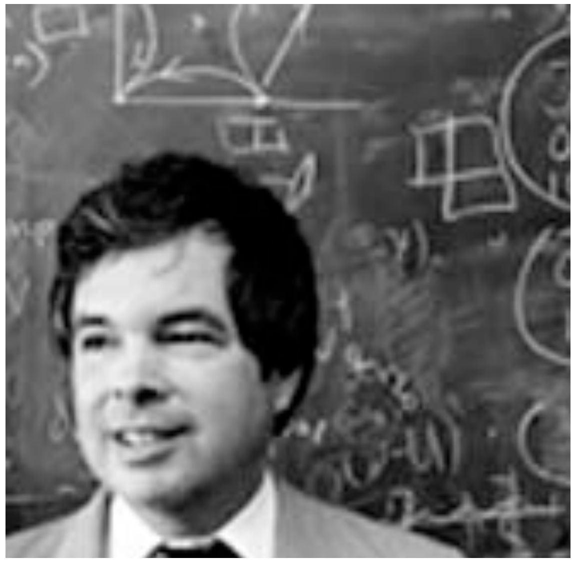
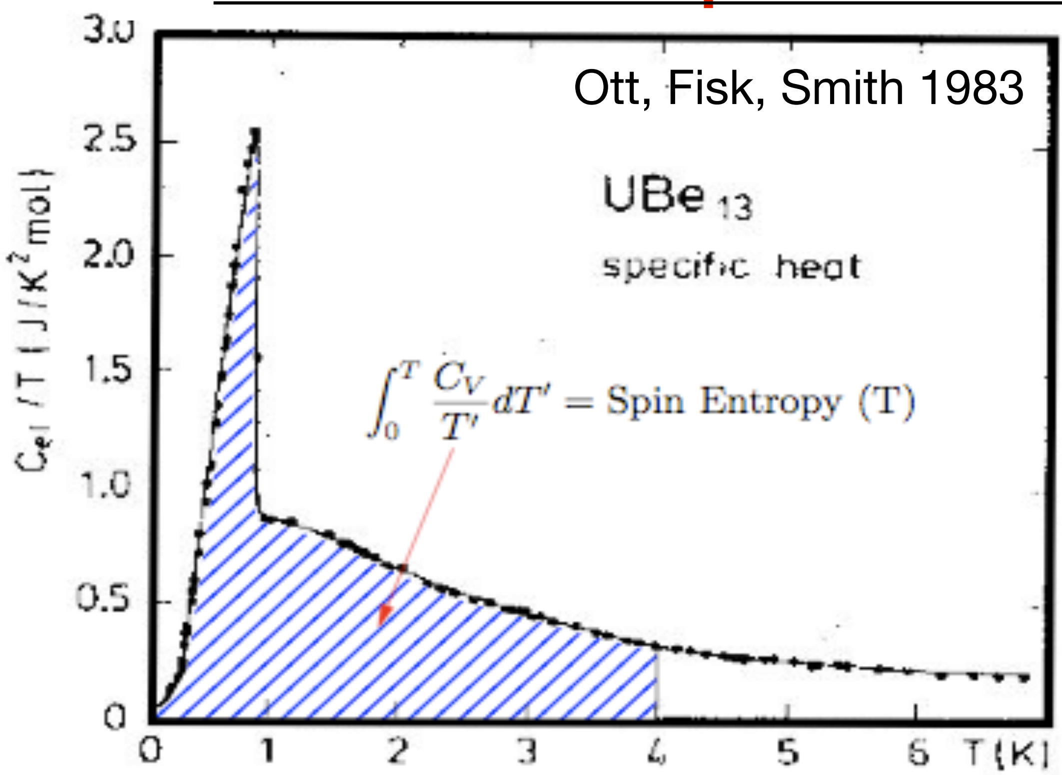


Wilson 1975



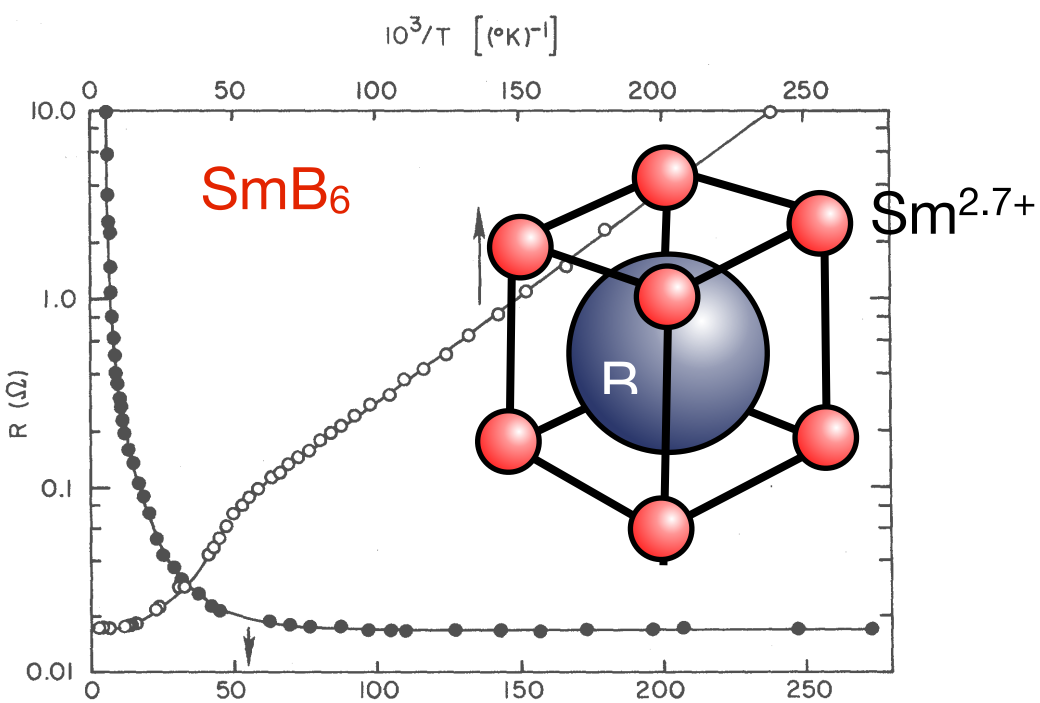
New Fixed Points

→ **New kinds of superconductor**

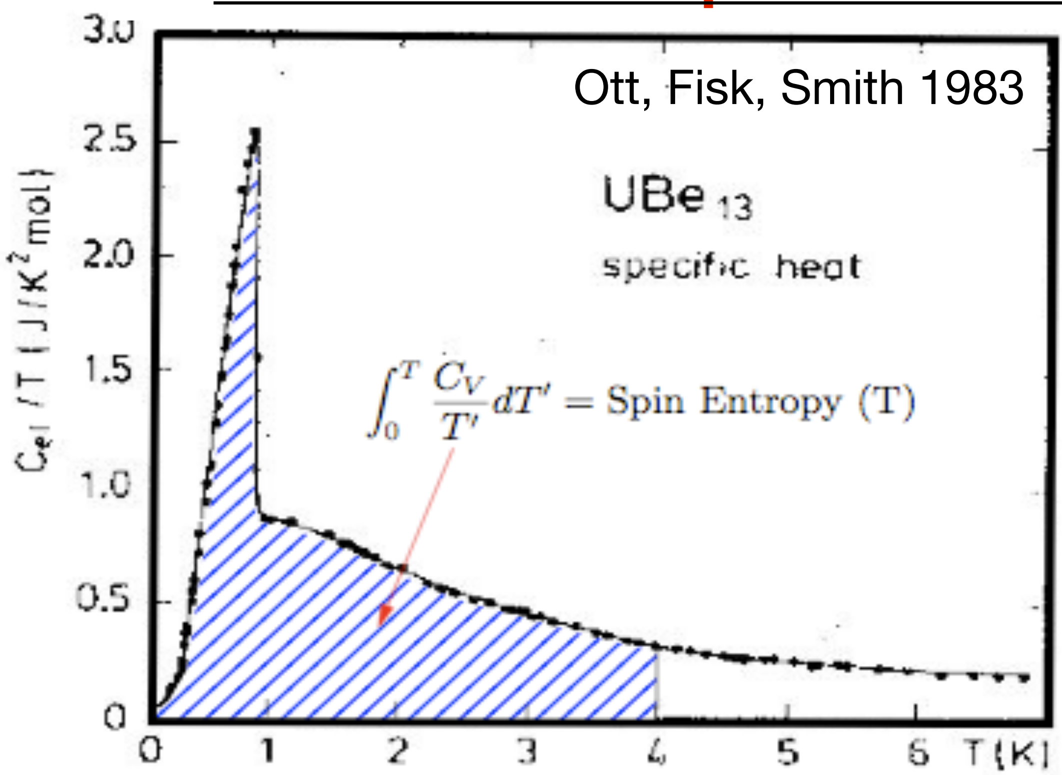


→ New kinds of insulator

Kondo Insulators

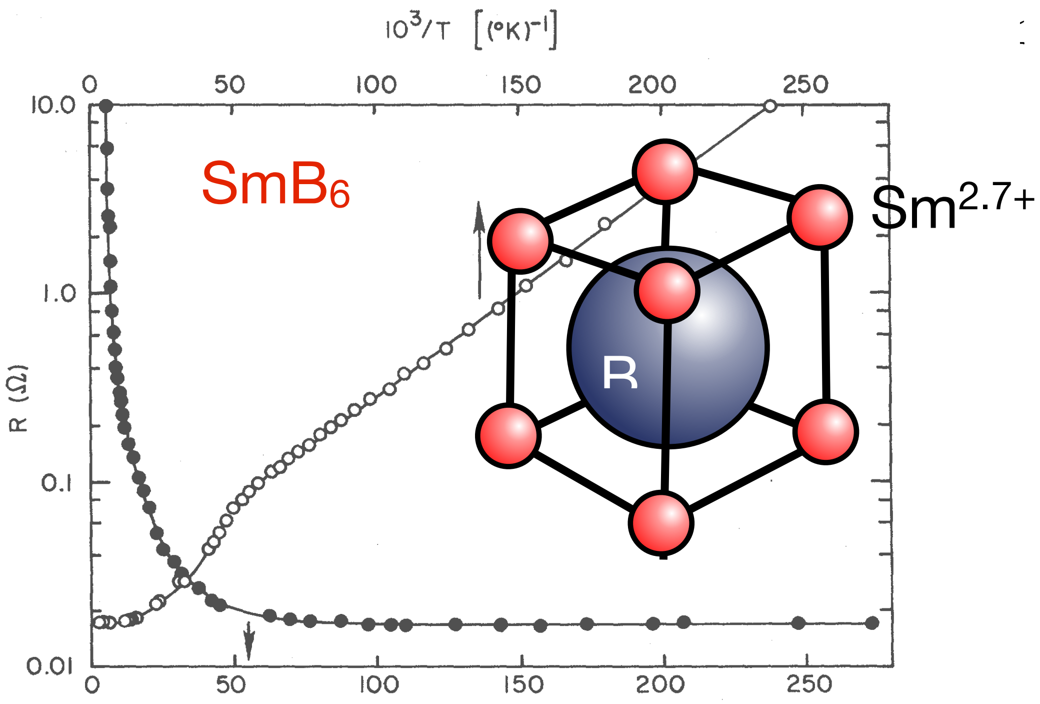


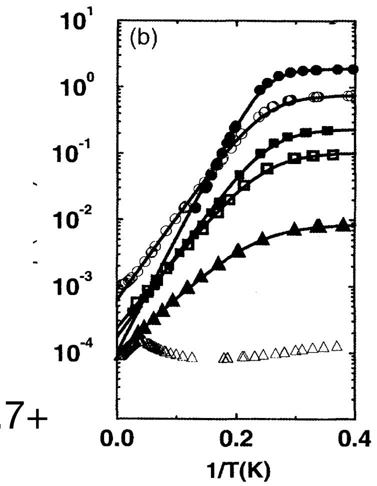
→ New kinds of superconductor



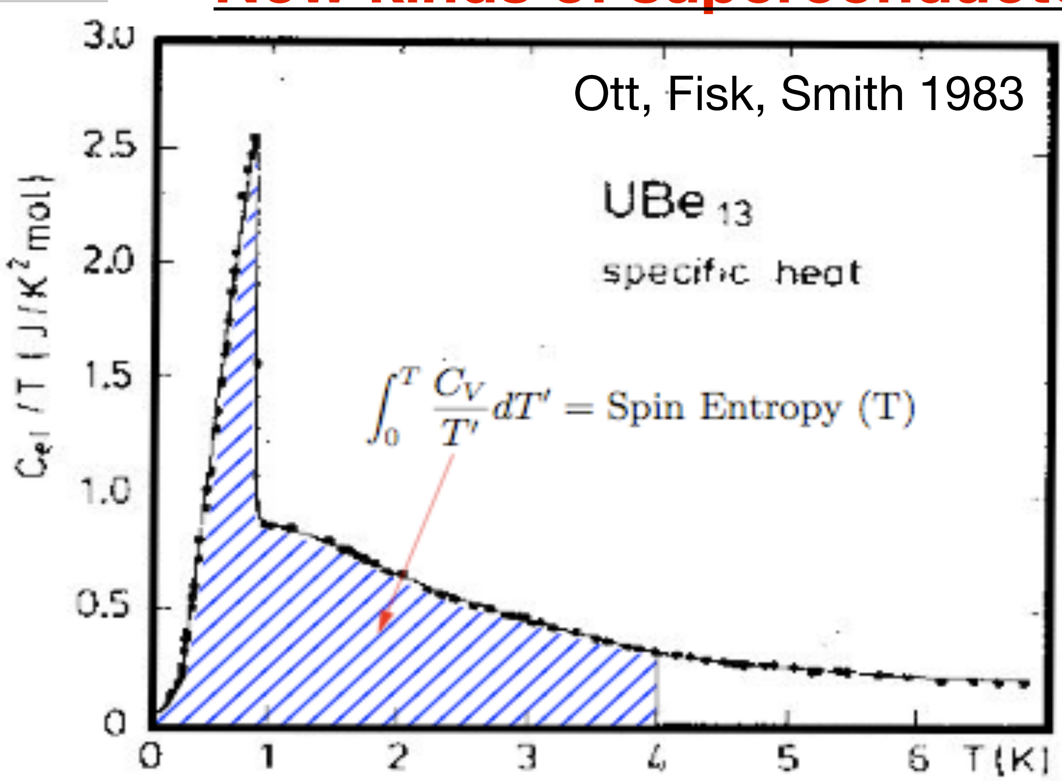
→ New kinds of insulator

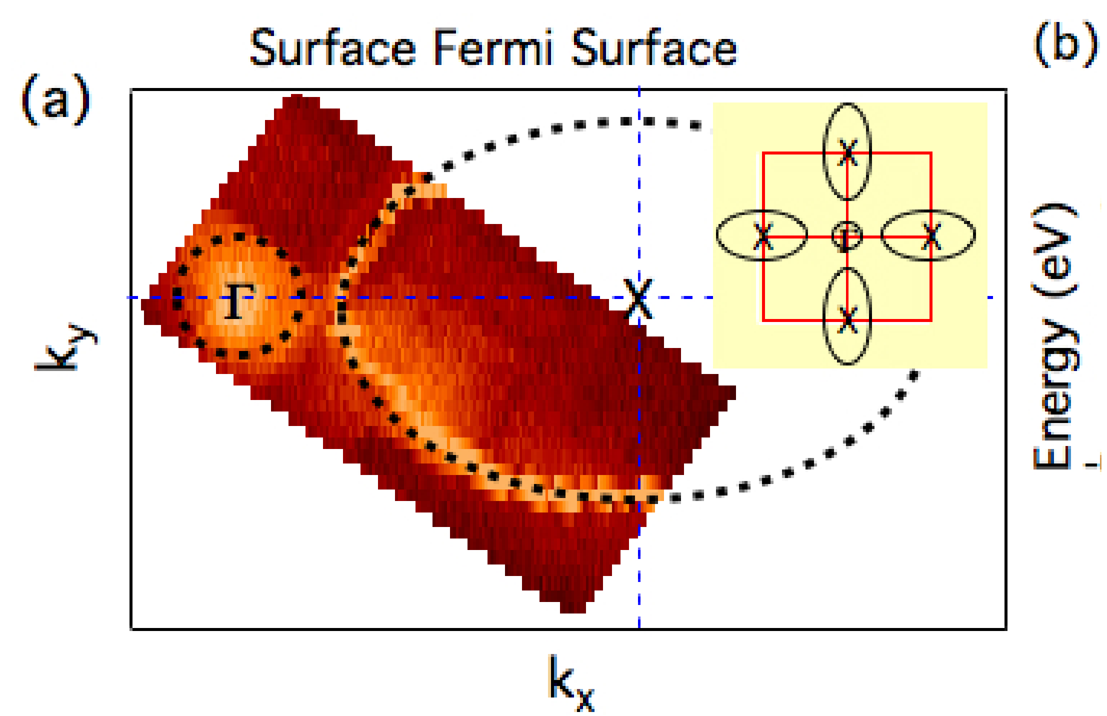
Topological Kondo Insulators





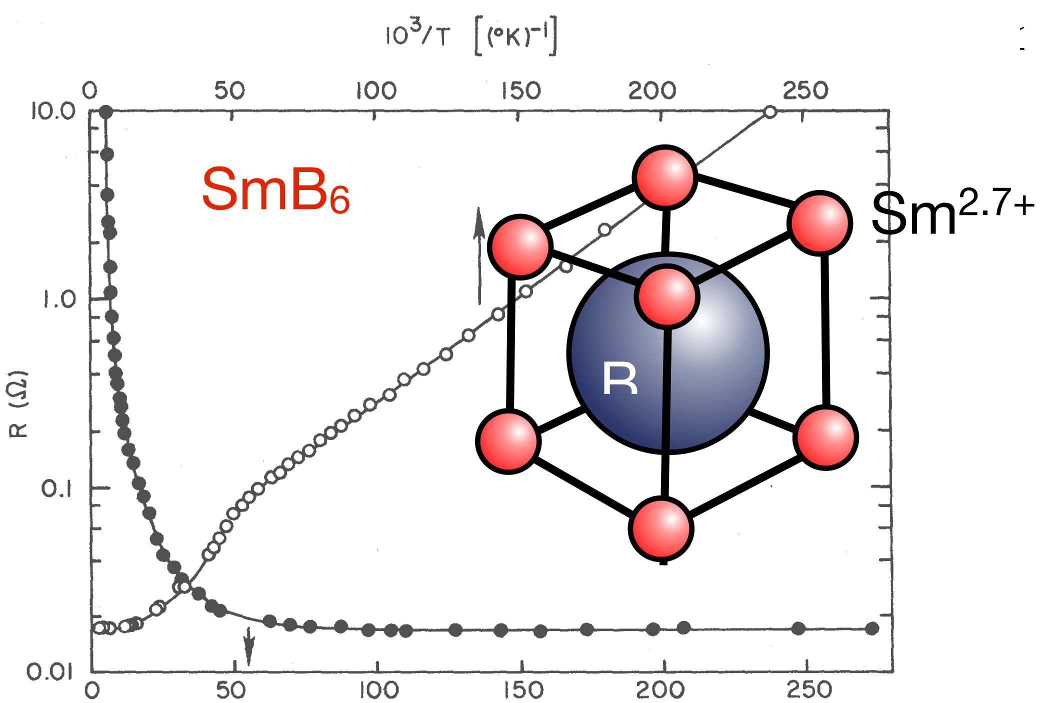
→ New kinds of superconductor

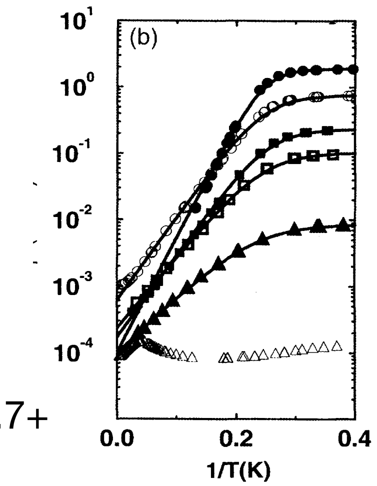




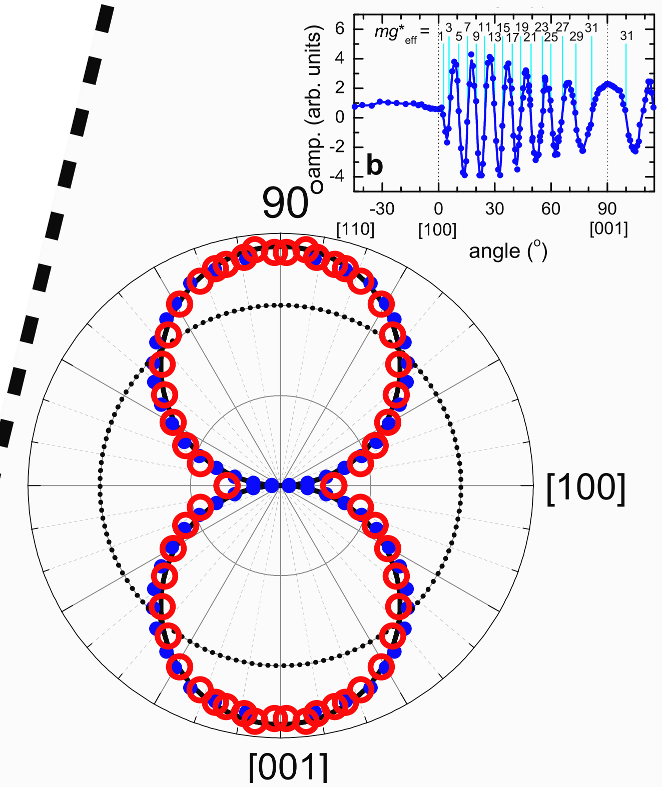
→ New kinds of insulator

Topological Kondo Insulators



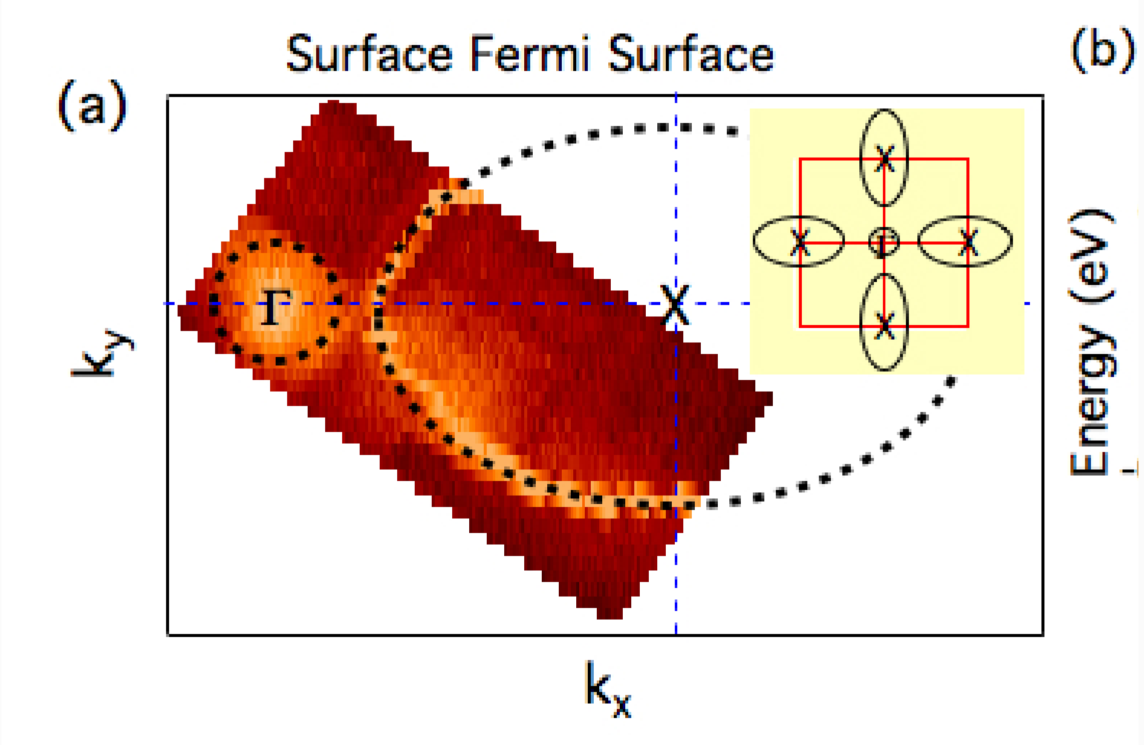


Altarawneh et al., (2012)

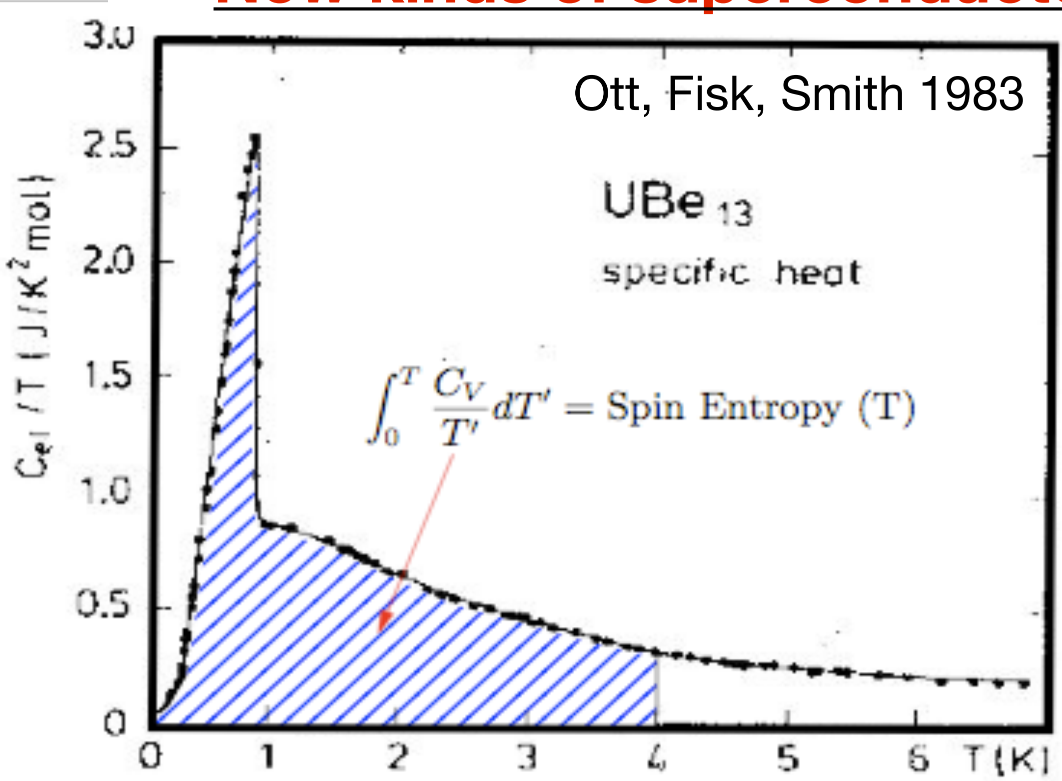


URu_2Si_2

→ New kinds of Electron Order

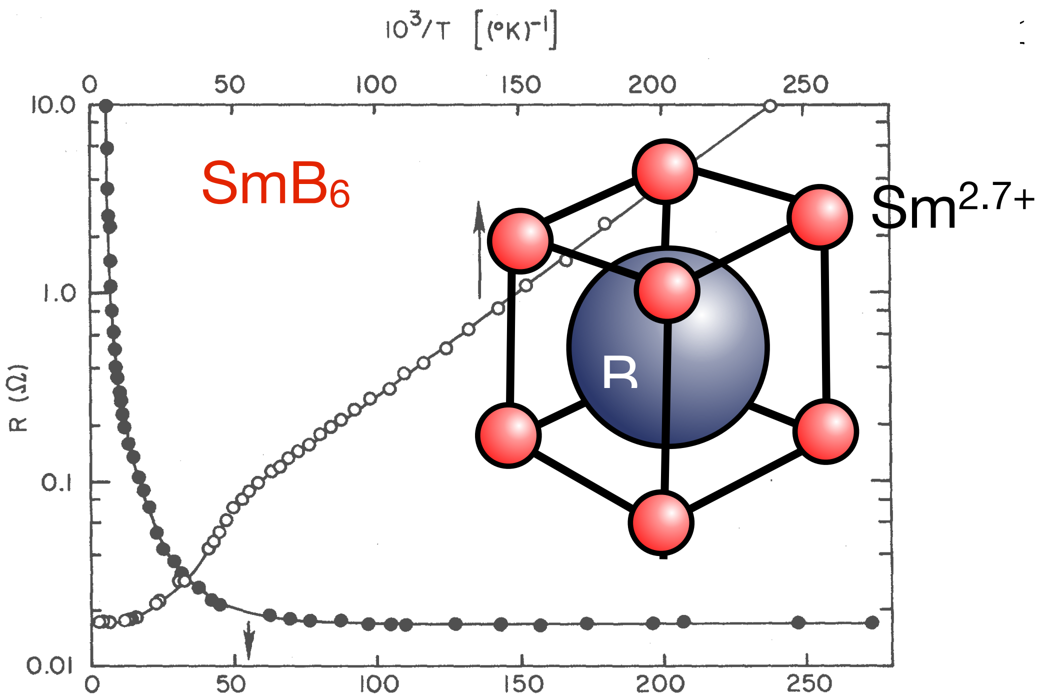


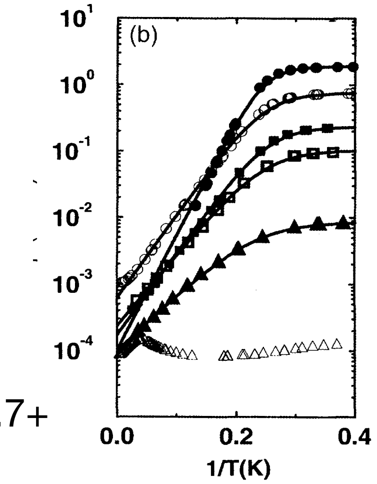
→ New kinds of superconductor



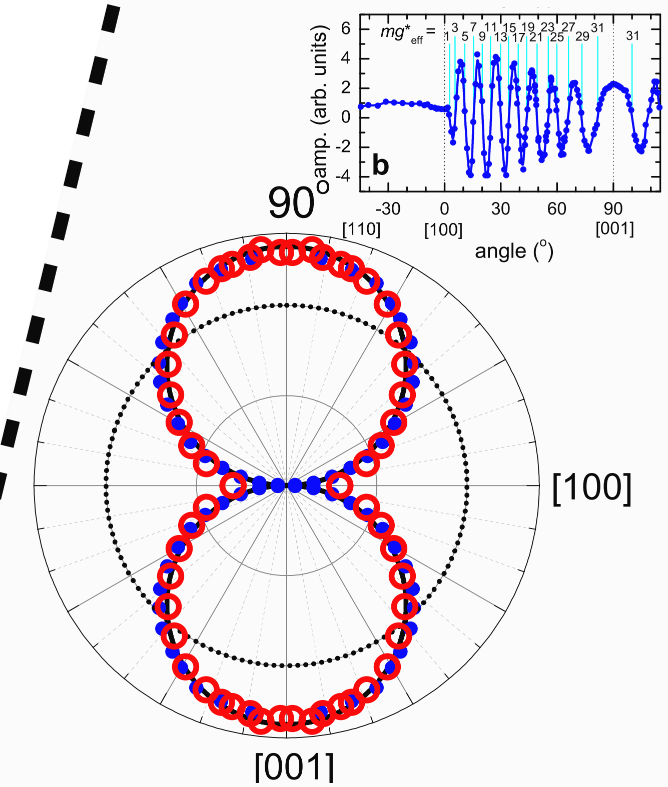
→ New kinds of insulator

Topological Kondo Insulators





Altarawneh et al., (2012)

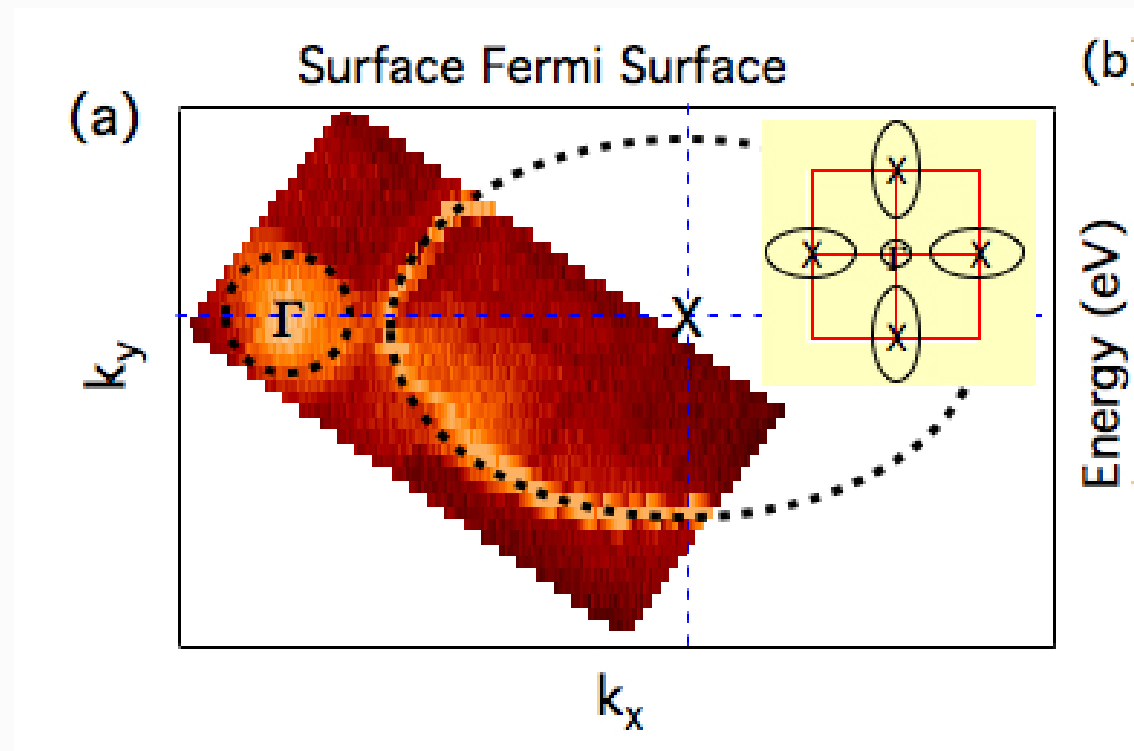


URu_2Si_2

$$\Psi = \begin{pmatrix} \langle \Psi_\uparrow \rangle \\ \langle \Psi_\downarrow \rangle \end{pmatrix}$$

Ising Electrons: *Hastatic order ?*

→ New kinds of Electron Order



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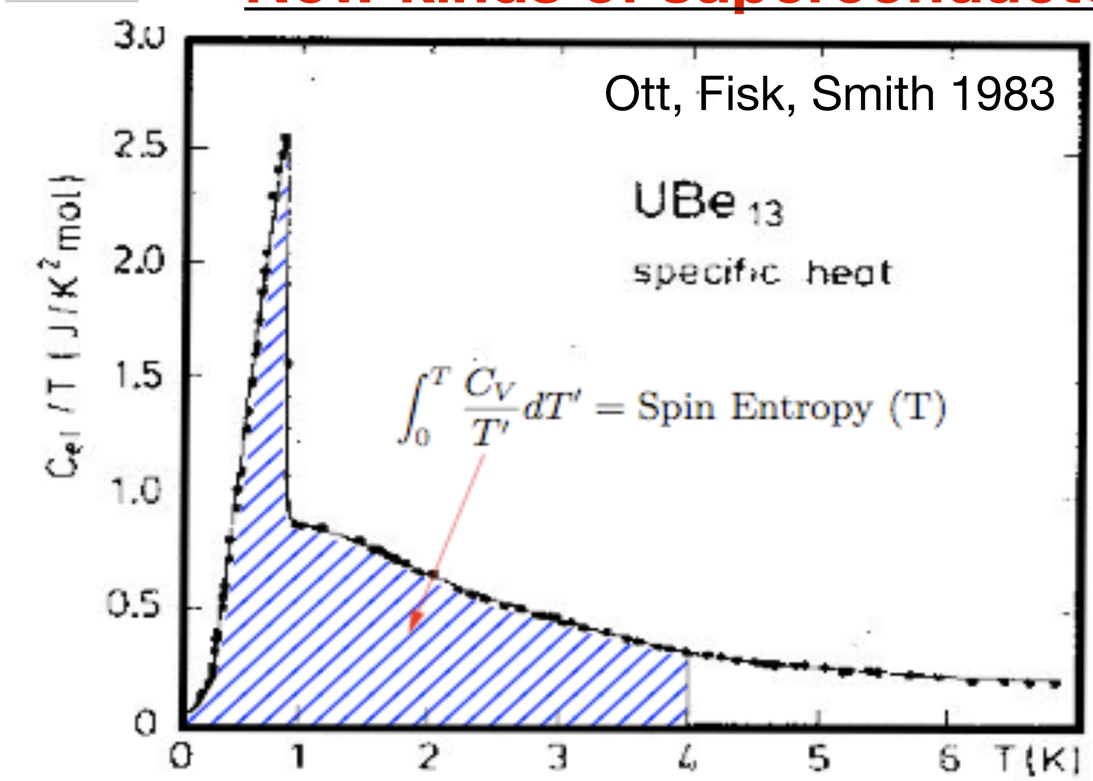
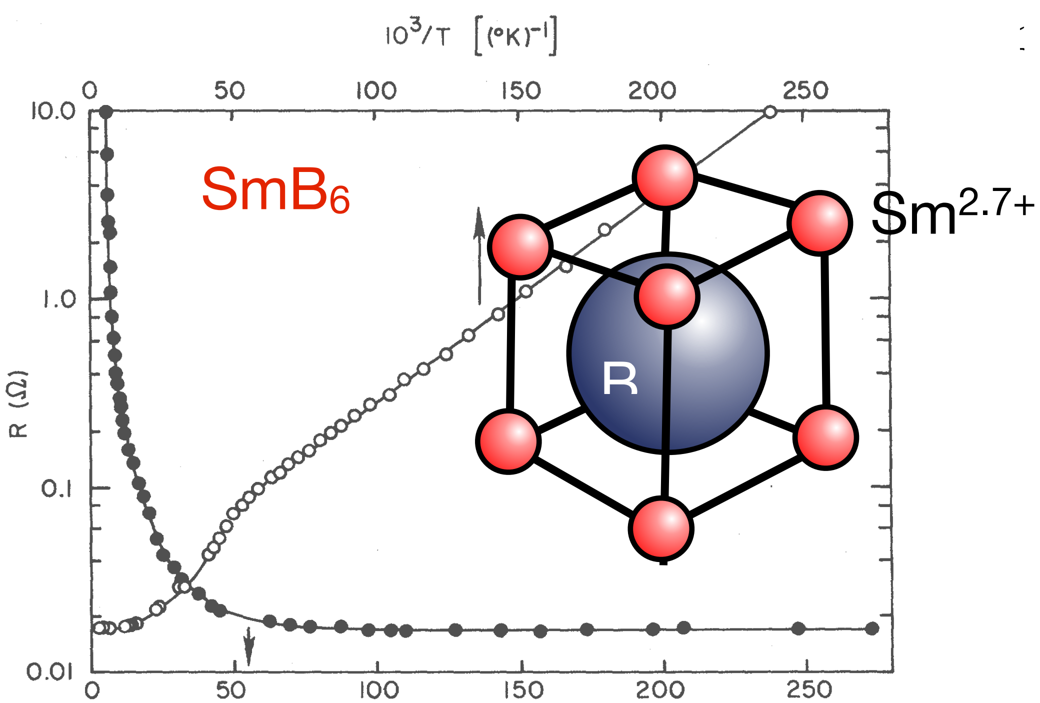
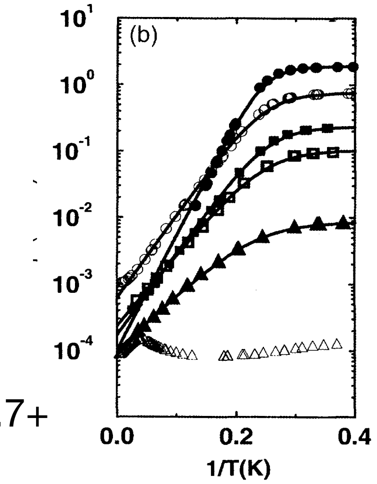


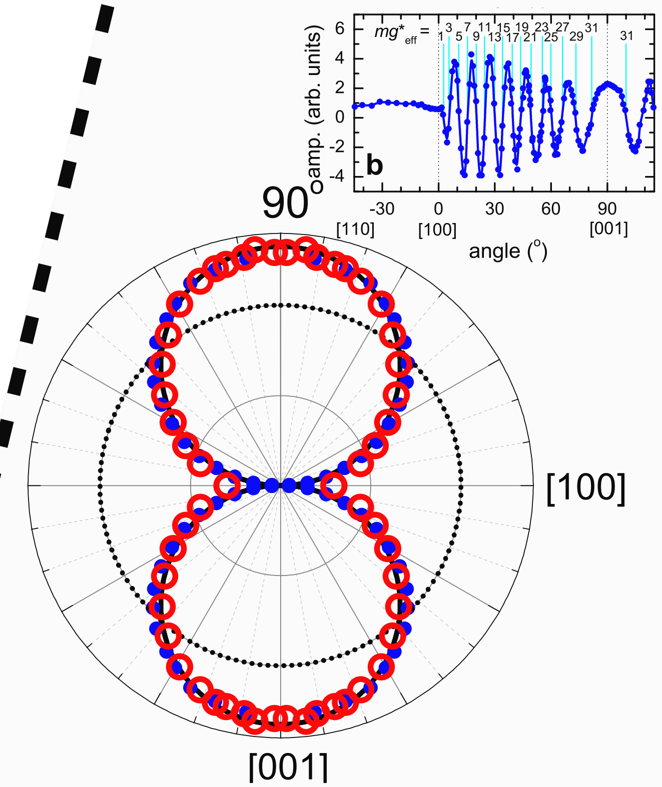
→ New kinds of insulator

Topological Kondo Insulators





Altarawneh et al., (2012)



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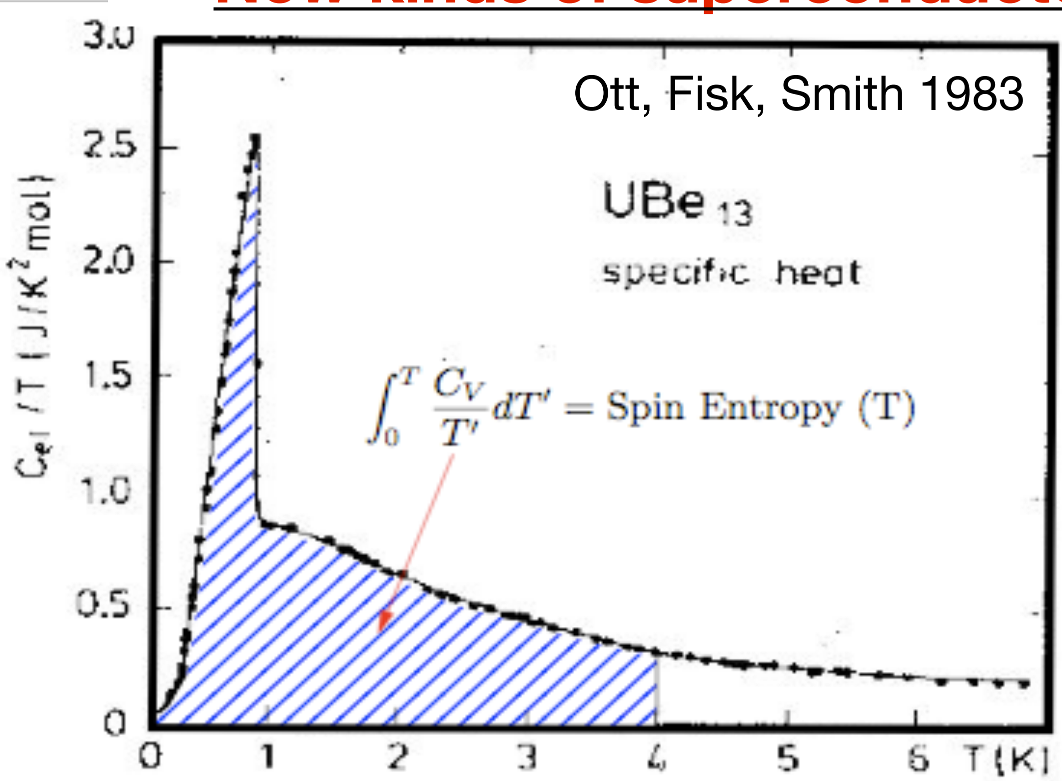
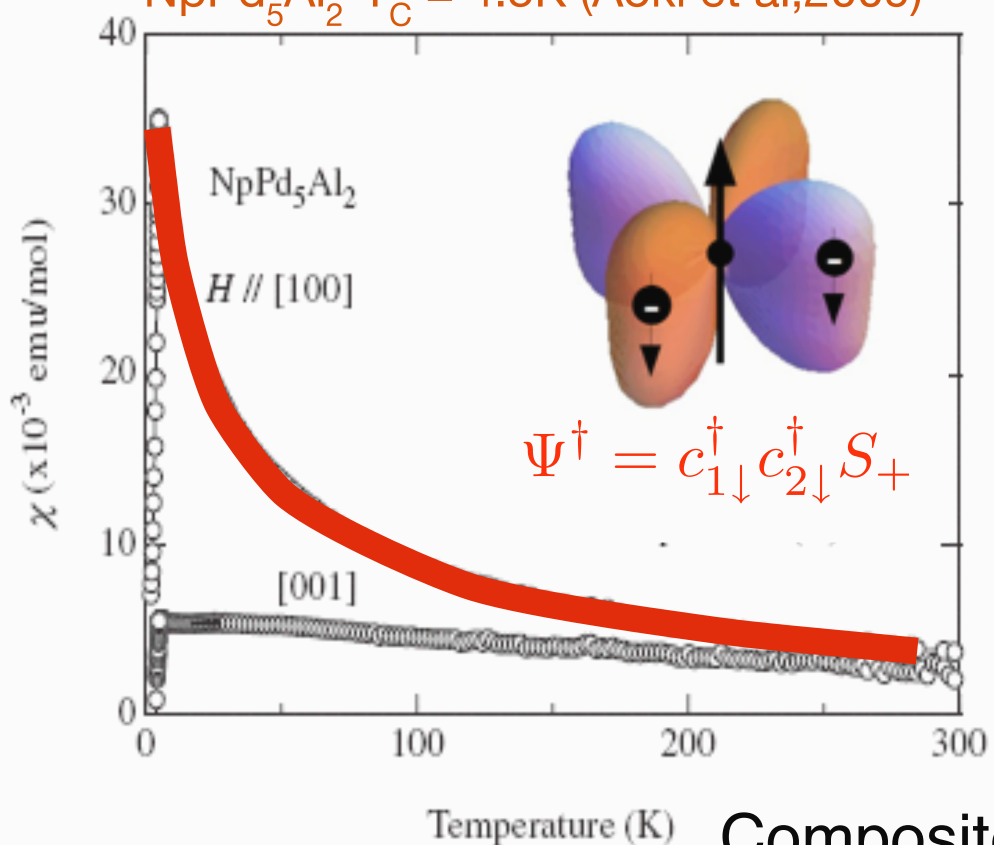


→ New kinds of Electron Order

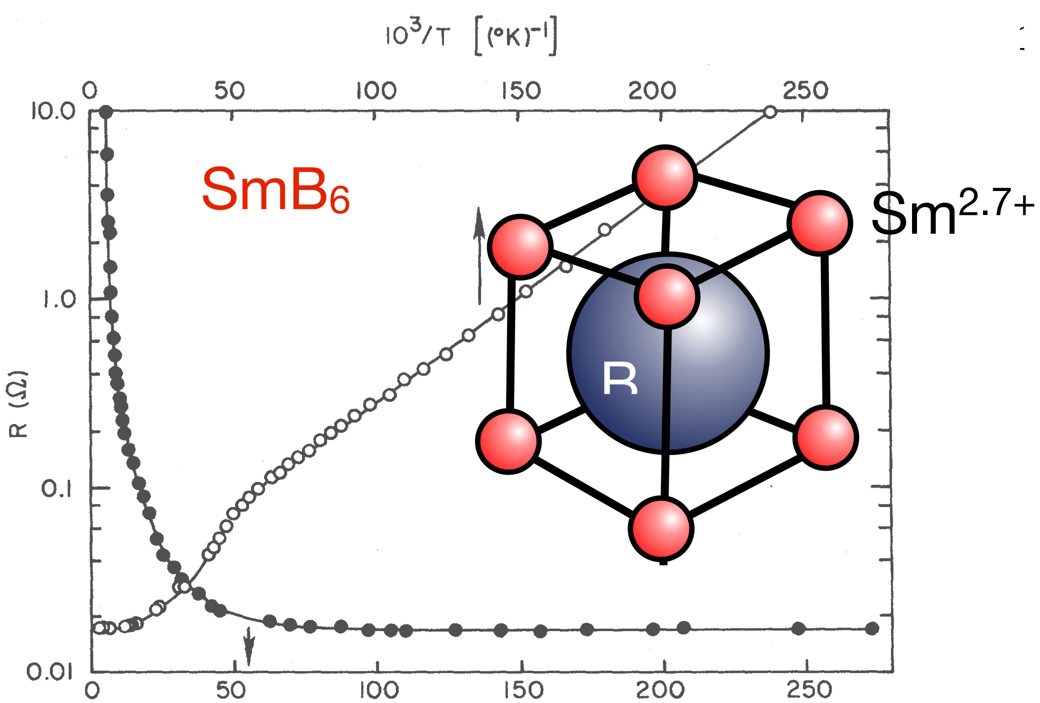
$NpPd_5Al_2$ T_C = 4.5K (Aoki et al,2009)

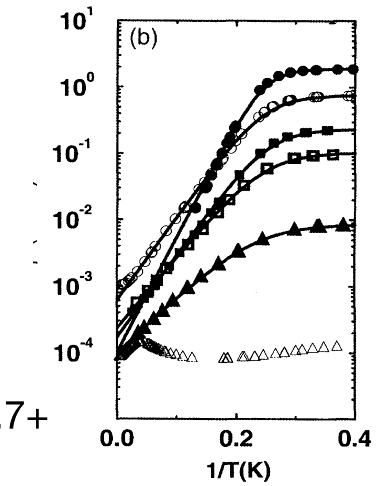


Composite Pairing

→ New kinds of insulator

Topological Kondo Insulators





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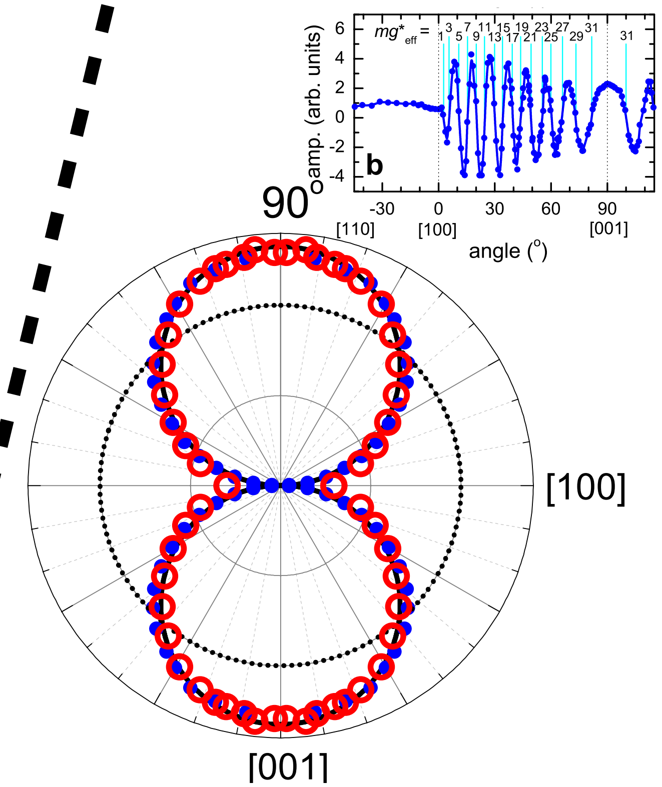
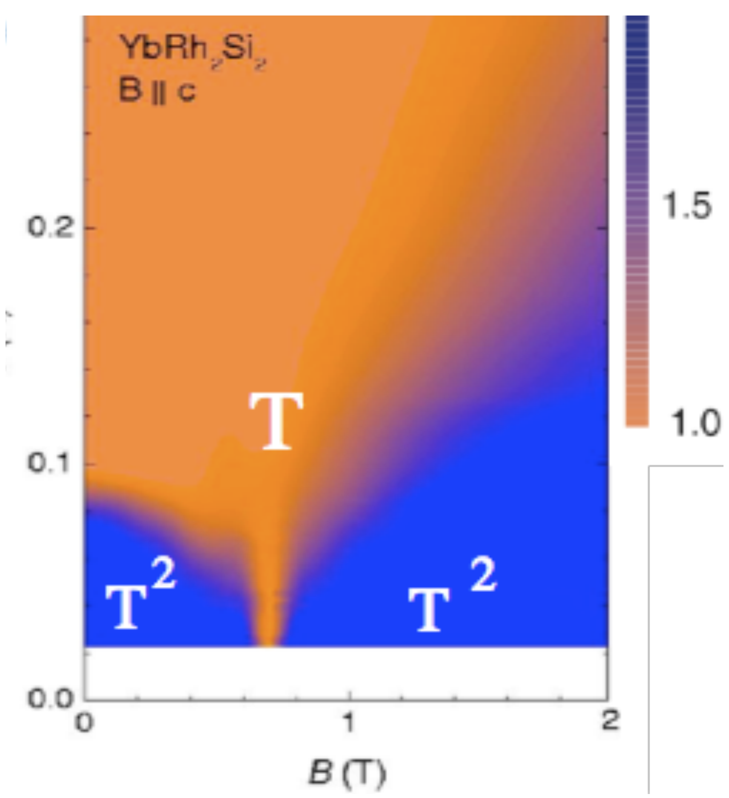


URu_2Si_2

$$\Psi = \begin{pmatrix} \langle \Psi_\uparrow \rangle \\ \langle \Psi_\downarrow \rangle \end{pmatrix}$$

Ising Electrons: *Hastatic order*

→ New kinds of Phase Transition



→ New kinds of Electron Order

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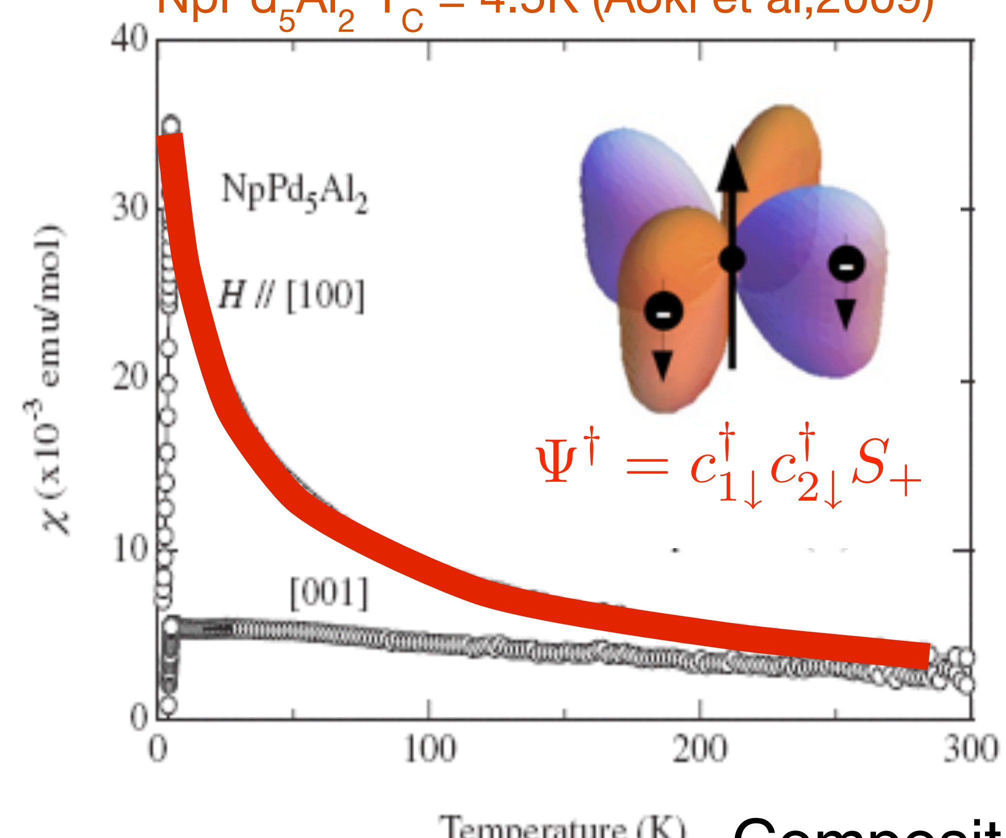
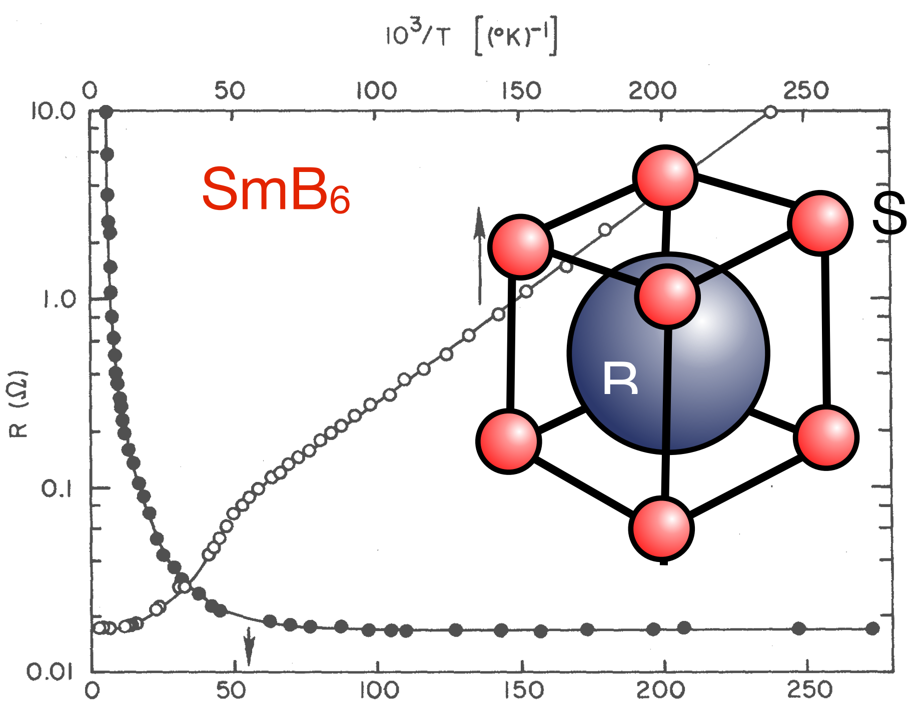


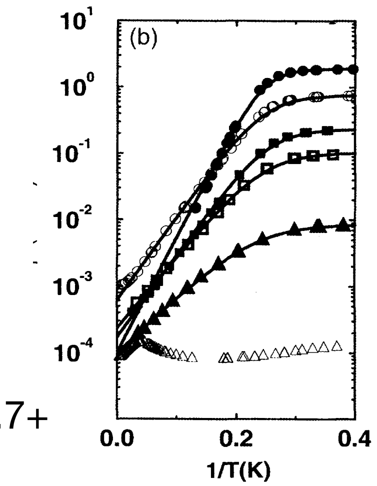
$$\Psi^\dagger = c^\dagger_{1\downarrow} c^\dagger_{2\downarrow} S_+$$

Composite Pairing

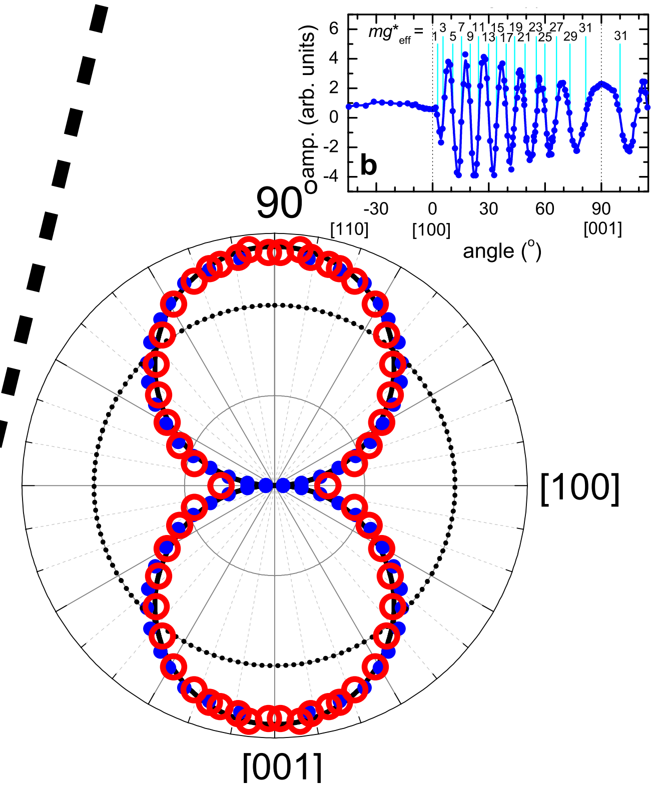
→ New kinds of insulator

Topological Kondo Insulators





Altarawneh et al., (2012)



URu_2Si_2

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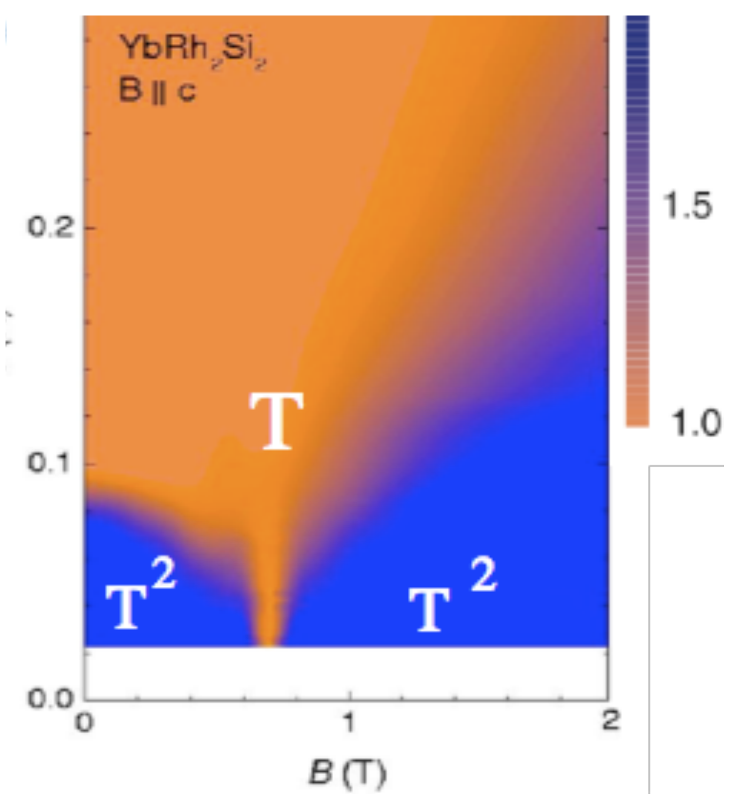
Ising Electrons: *Hastatic order*

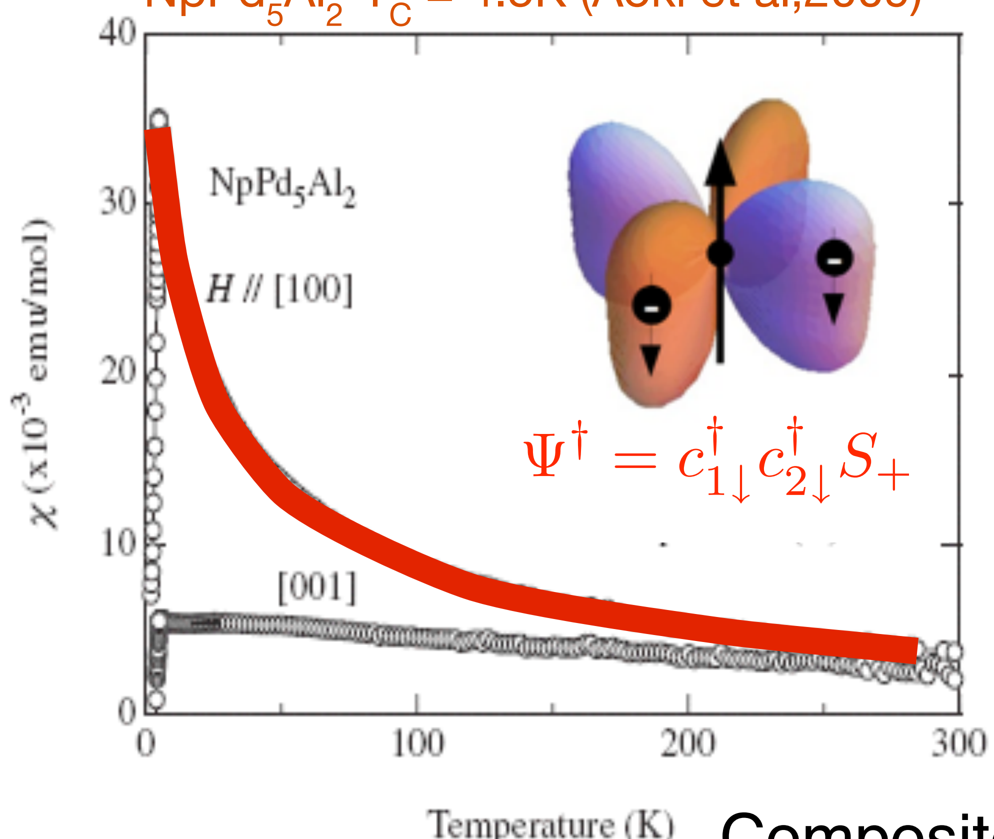
→ New kinds of Electron Order

→ New kinds of Phase Transition



→ Quantum Criticality

$NpPd_5Al_2$ T_C = 4.5K (Aoki et al,2009)



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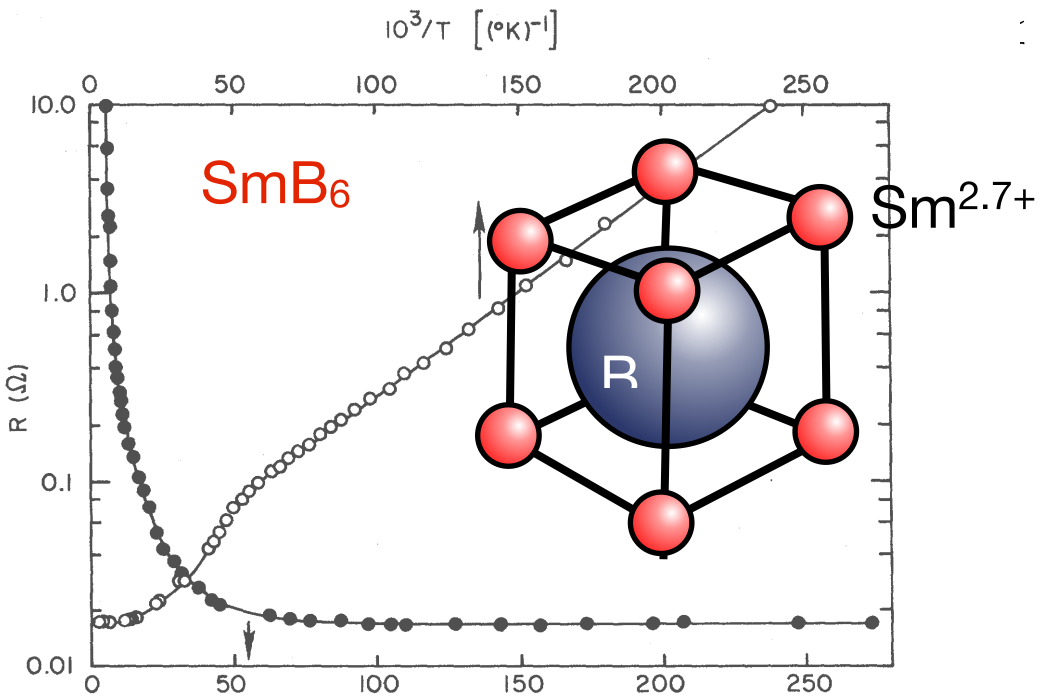
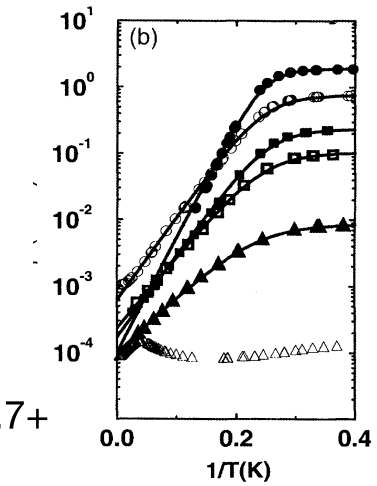
Composite Pairing

→ New kinds of insulator

Topological Kondo Insulators





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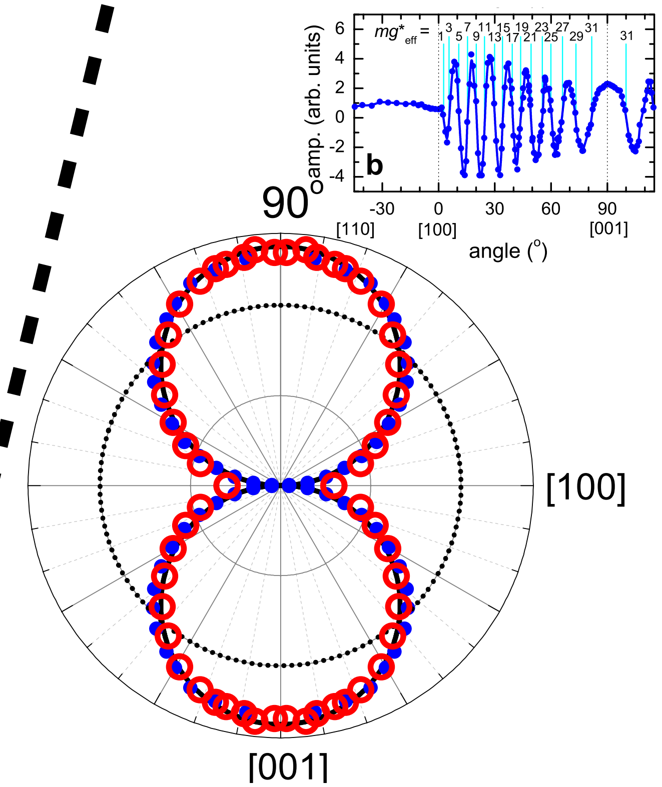

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Ising Electrons: *Hastatic order*

→ New kinds of Electron Order

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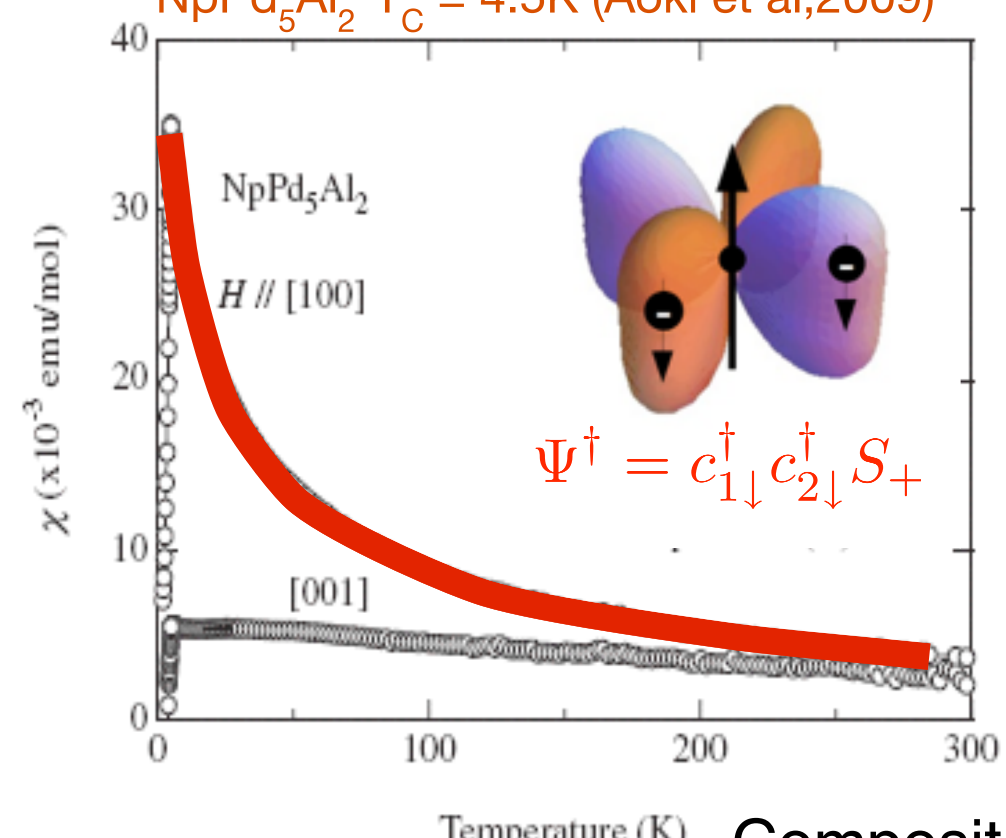

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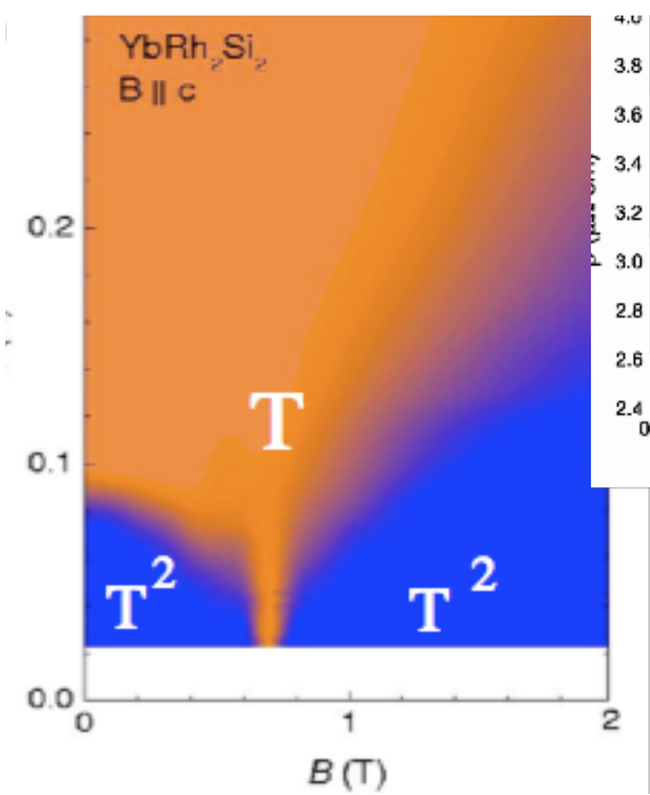
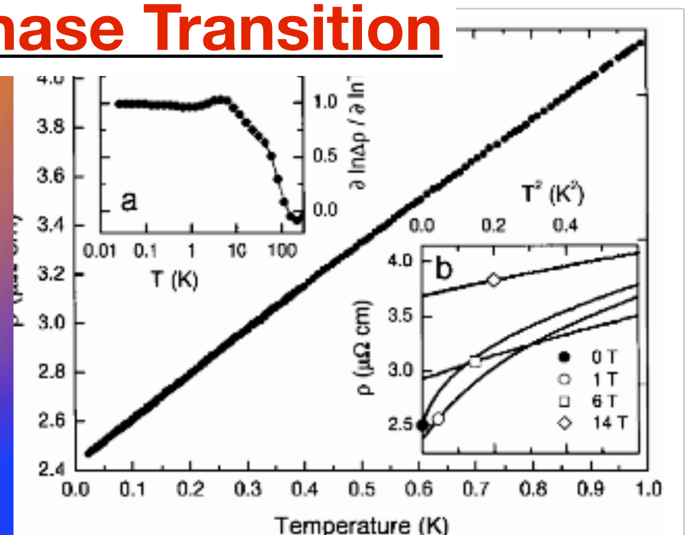




→ Strange Metals

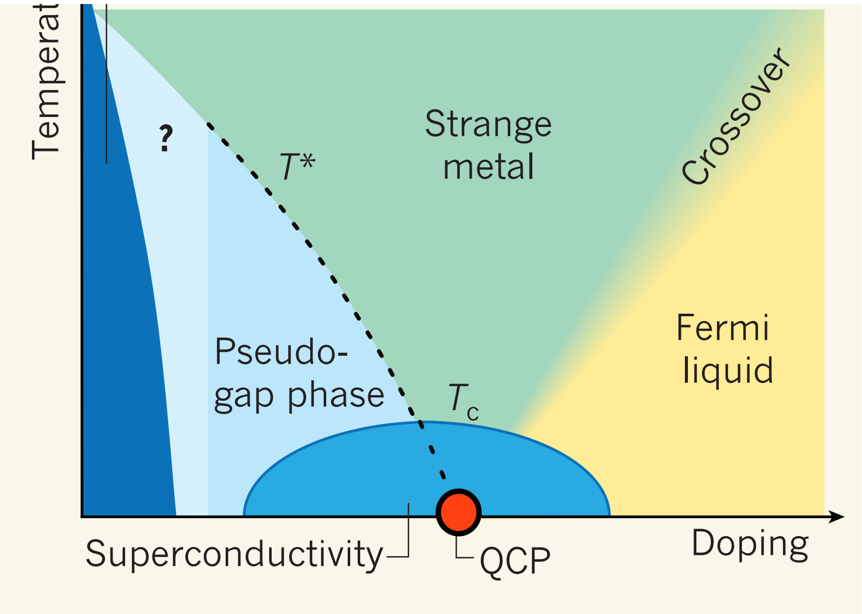
→ Quantum Criticality

Quantum Criticality

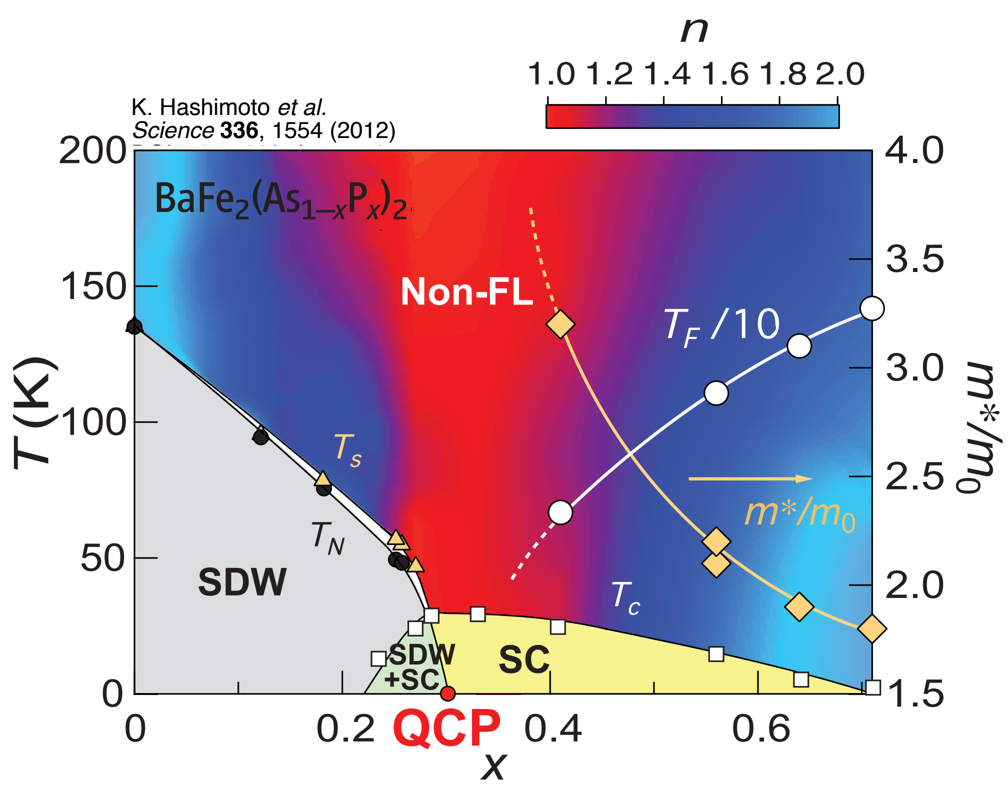
Quantum Criticality and Superconductivity

Quantum Criticality and Superconductivity

3d Cu

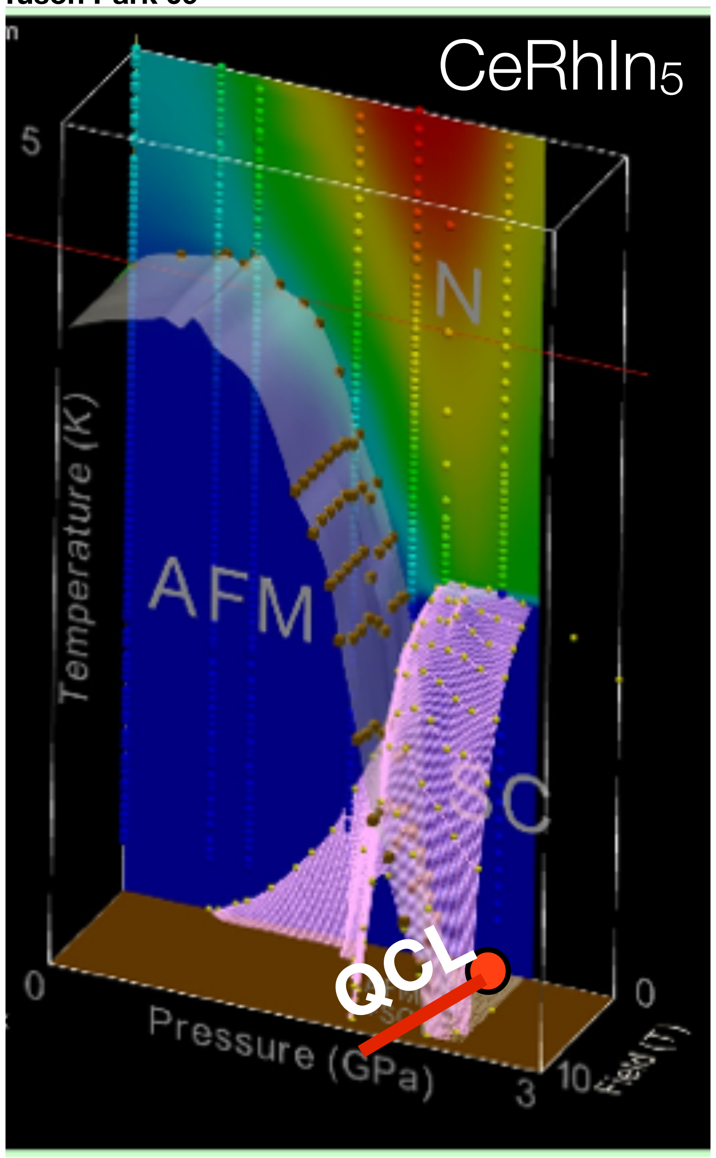


3d Fe



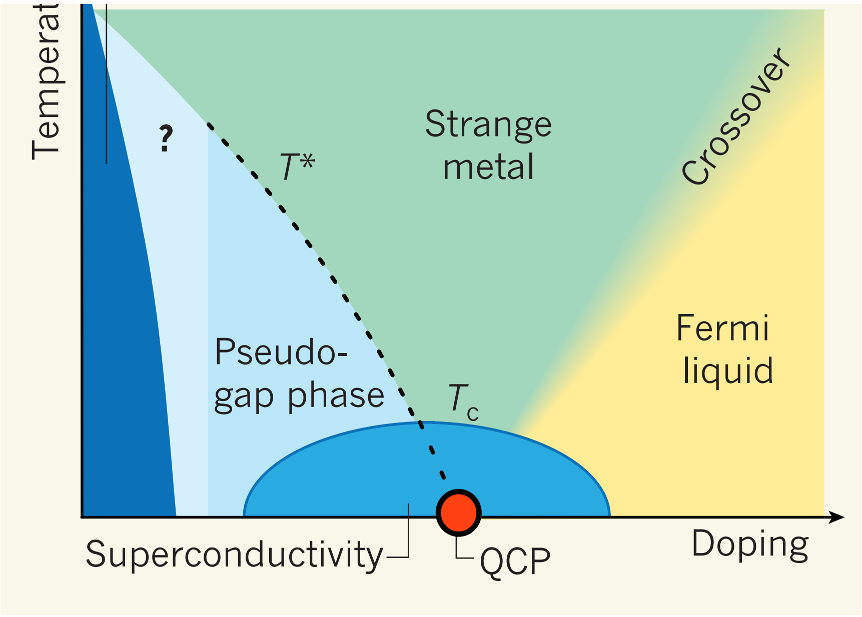
4f Ce

Tuson Park 09

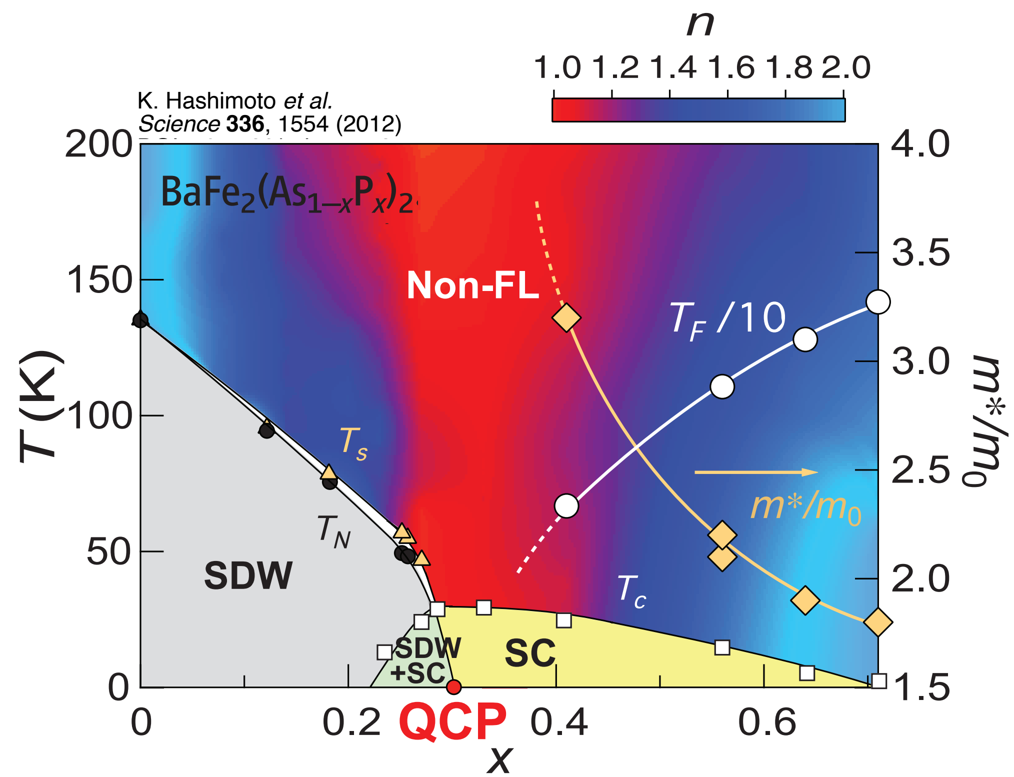


Quantum Criticality and Superconductivity

3d Cu

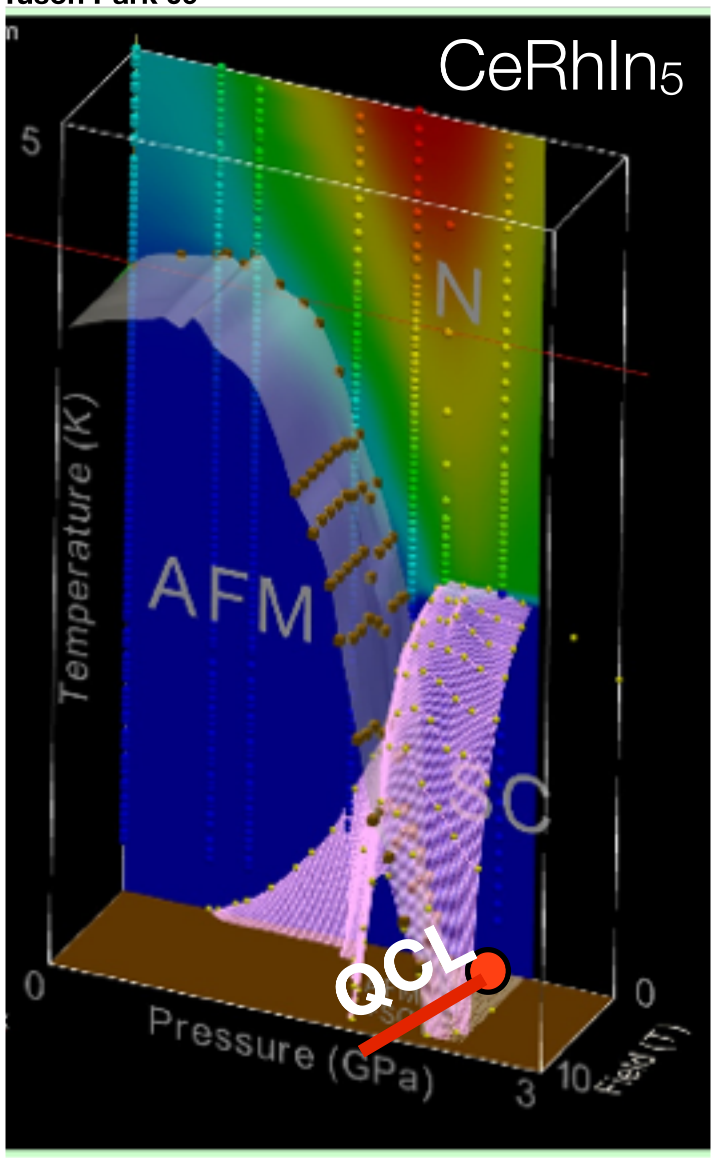


3d Fe



4f Ce

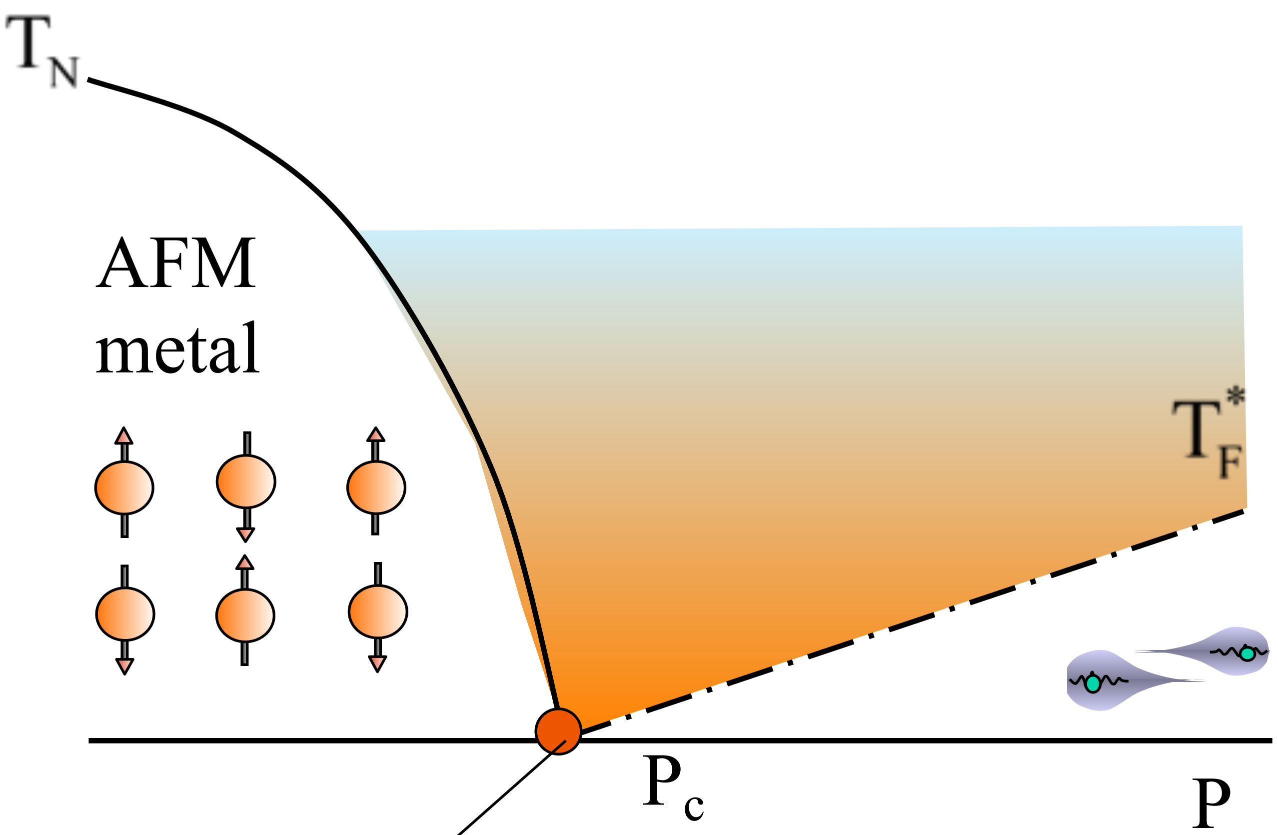
Tuson Park 09



Wanted: a unified conceptual description of magnetism, quantum criticality and superconductivity.

Quantum Criticality: divergent specific heat capacity

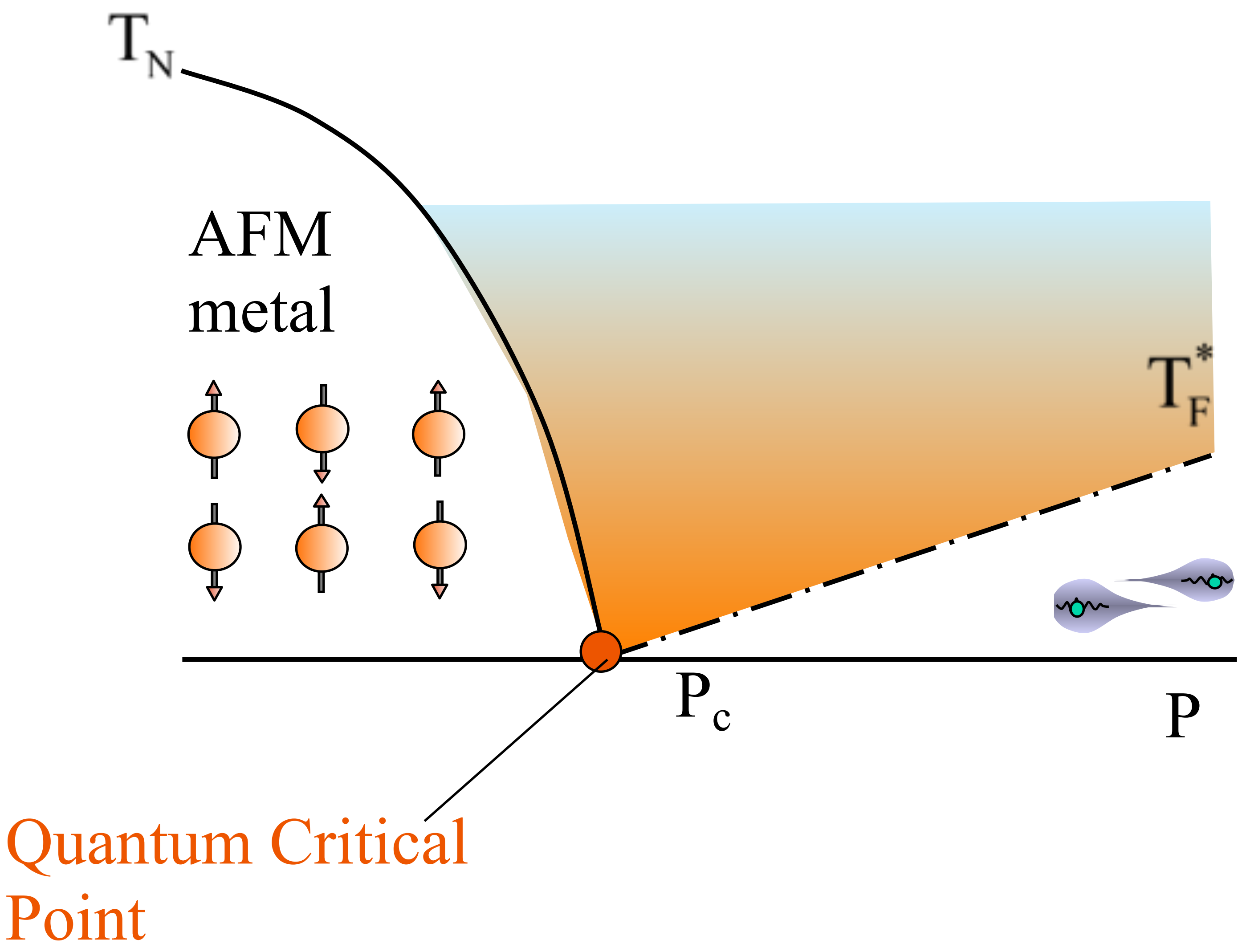
Heavy Fermion Materials



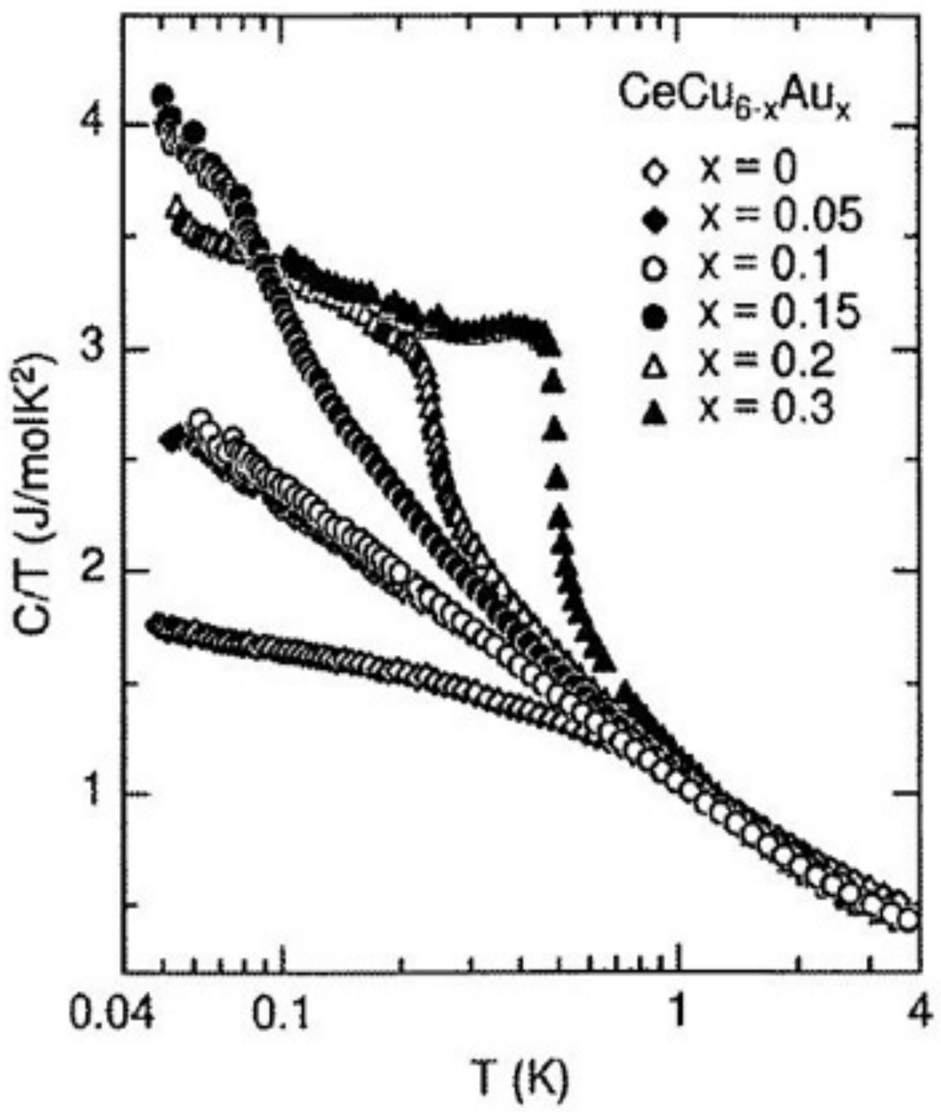
Quantum Critical Point

Quantum Criticality: divergent specific heat capacity

Heavy Fermion Materials

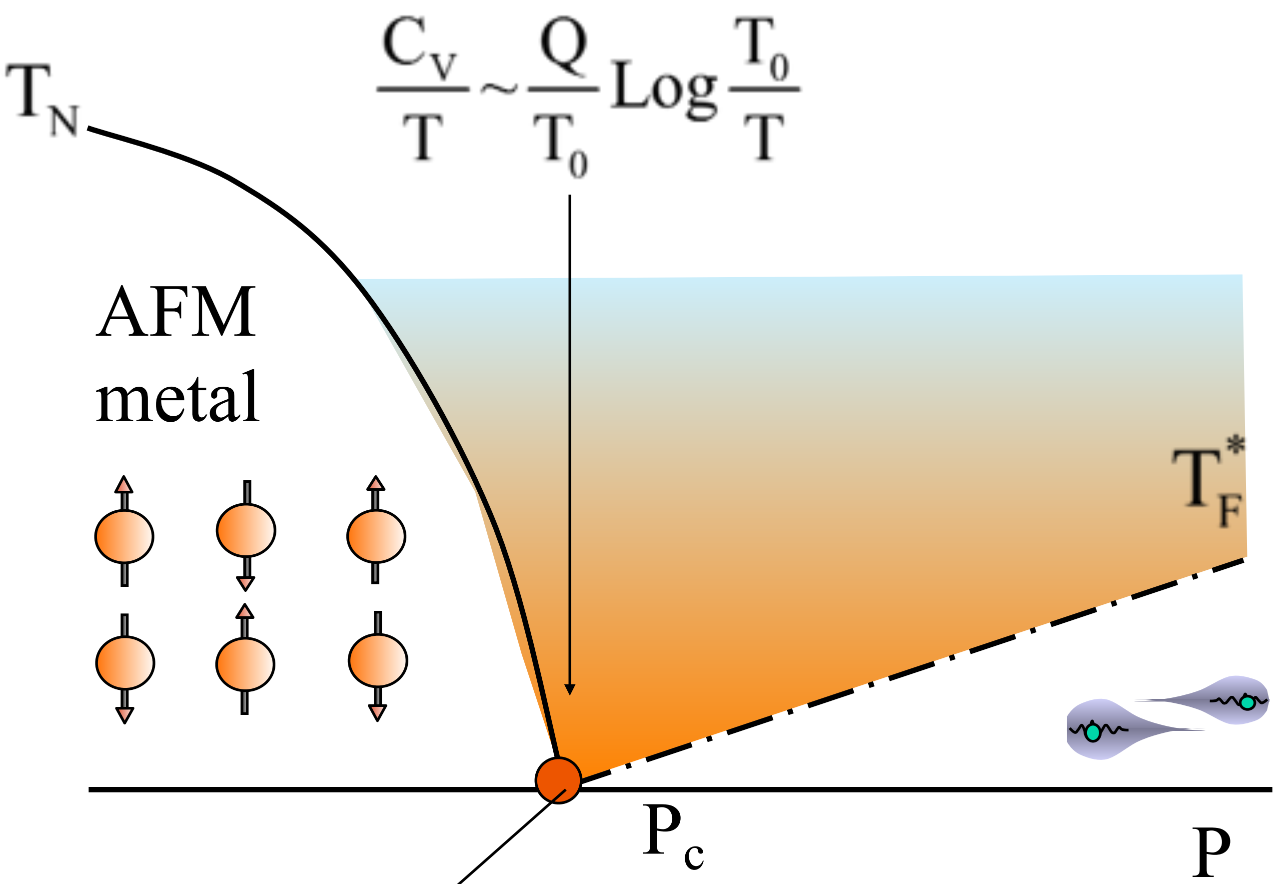


H. Von Lohneyson (1996)



Quantum Criticality:
divergent specific heat capacity

Heavy Fermion
Materials



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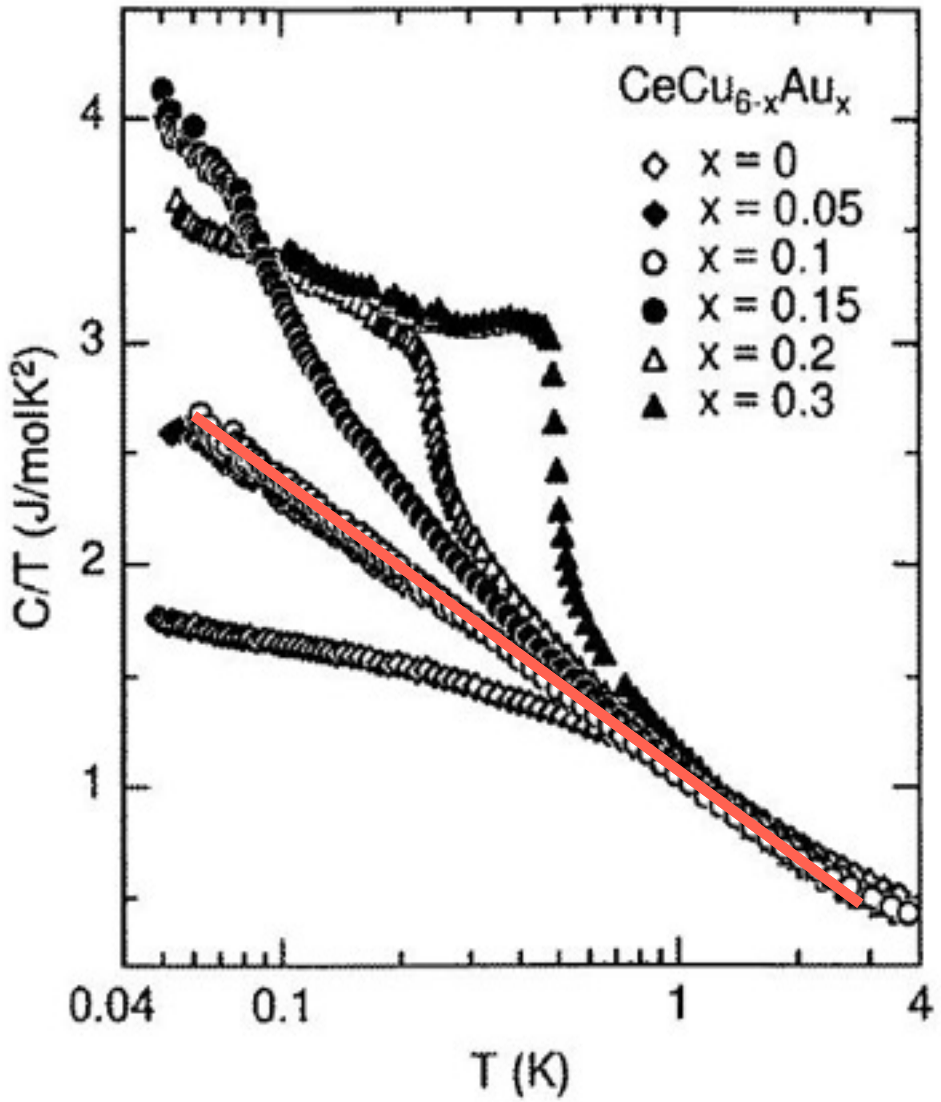
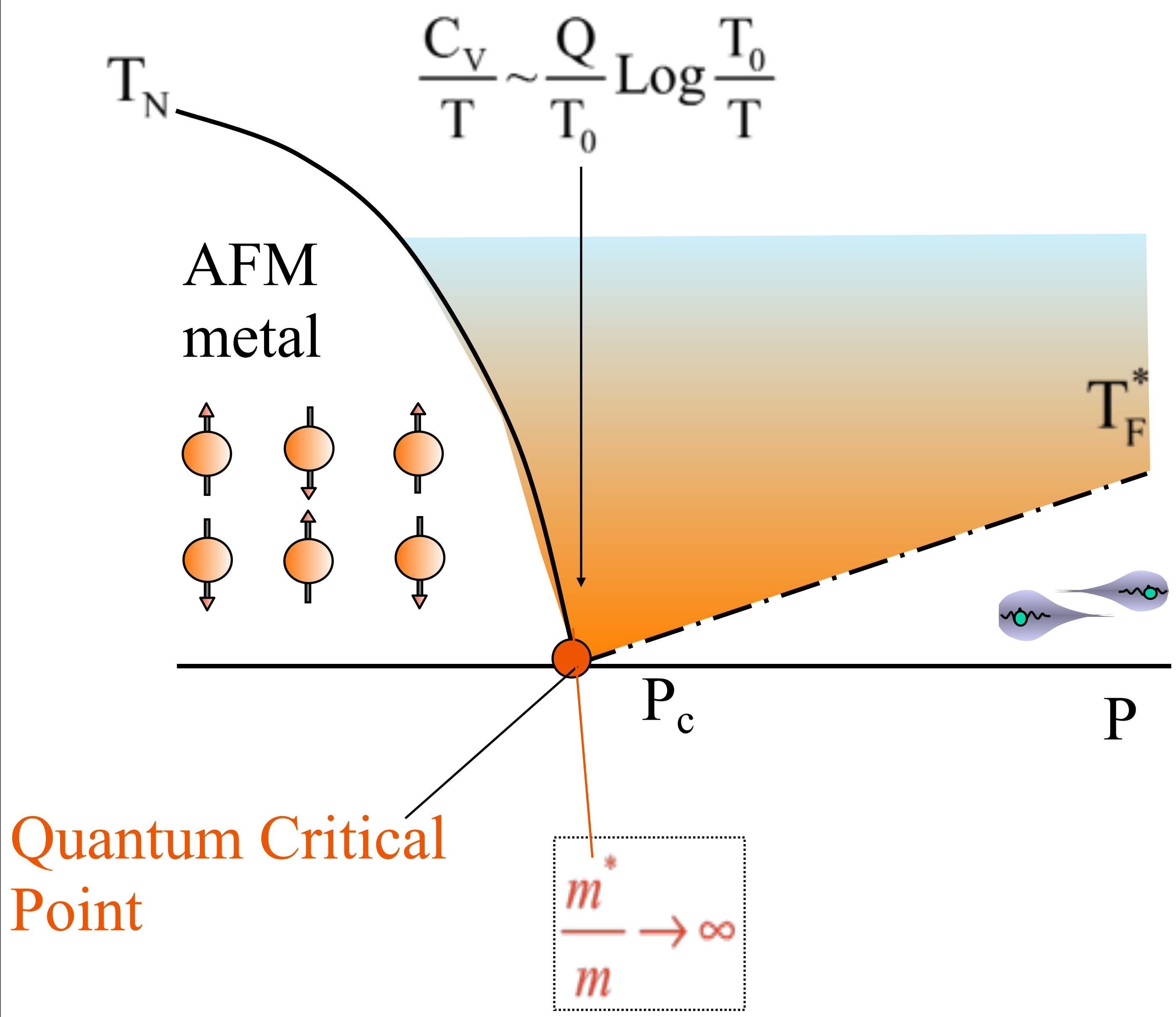


Quantum Criticality: divergent specific heat capacity

Heavy Fermion Materials



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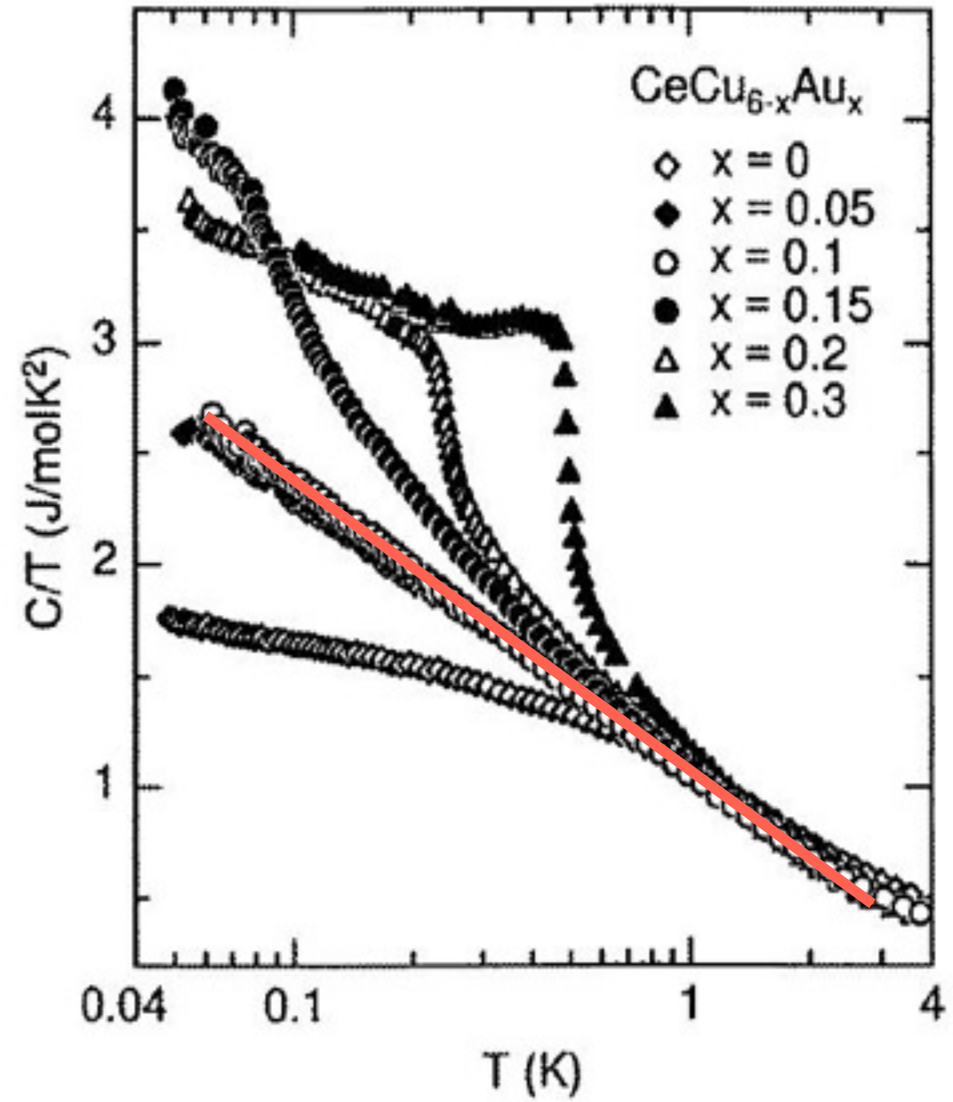
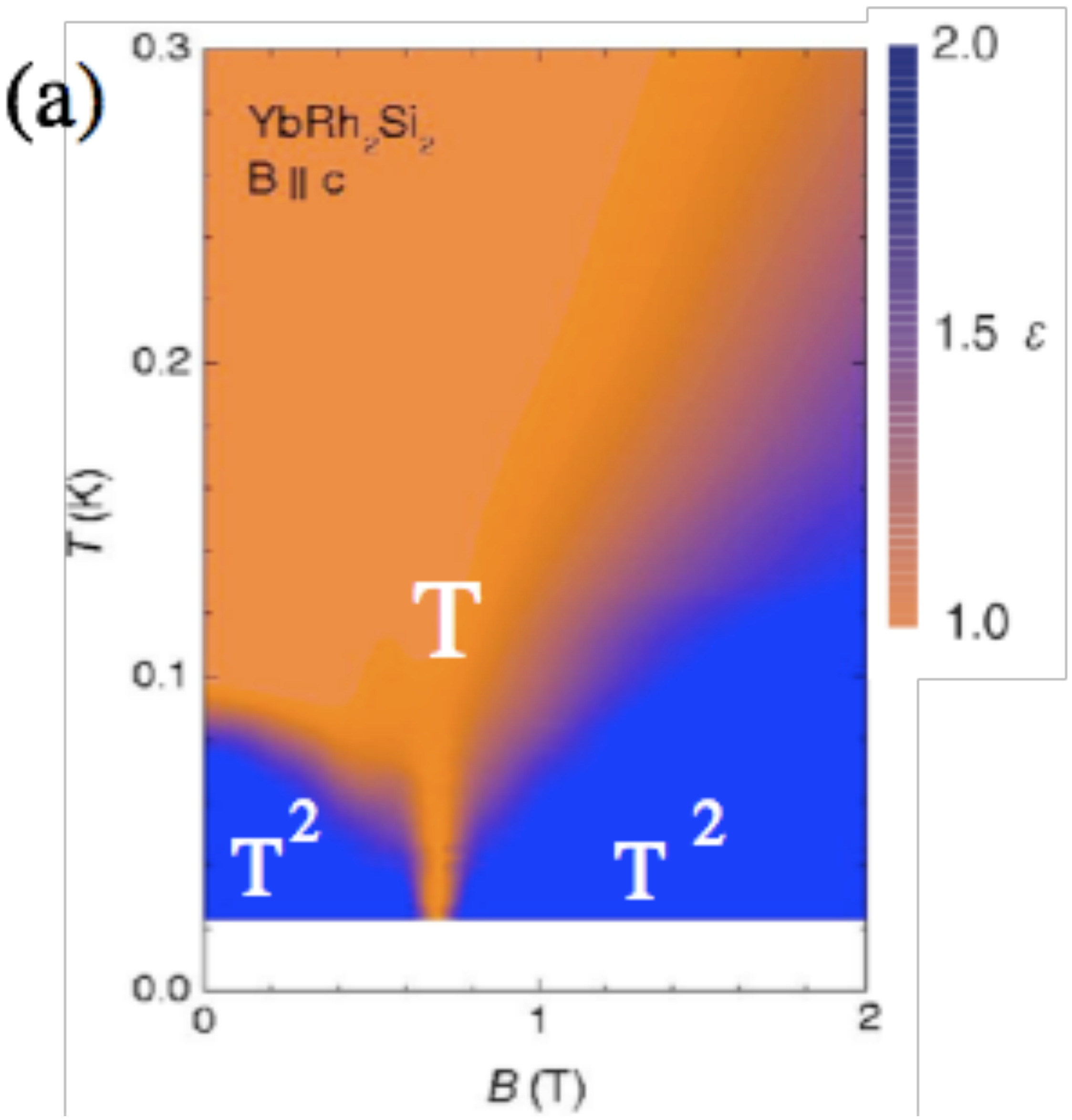


Quantum Criticality:
divergent specific heat capacity
breakdown of Landau Fermi Liquid

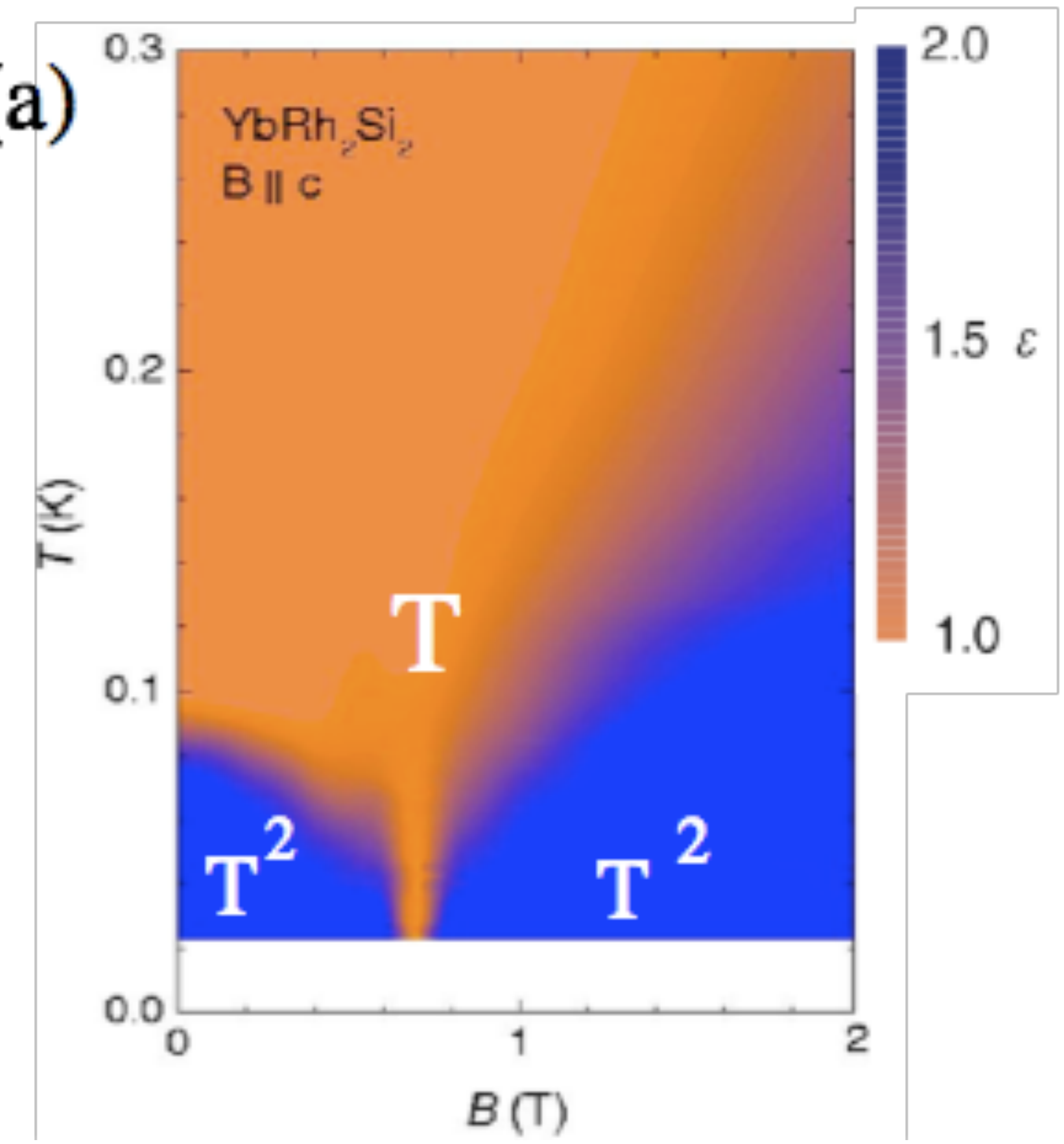


Custers et al, (2003)

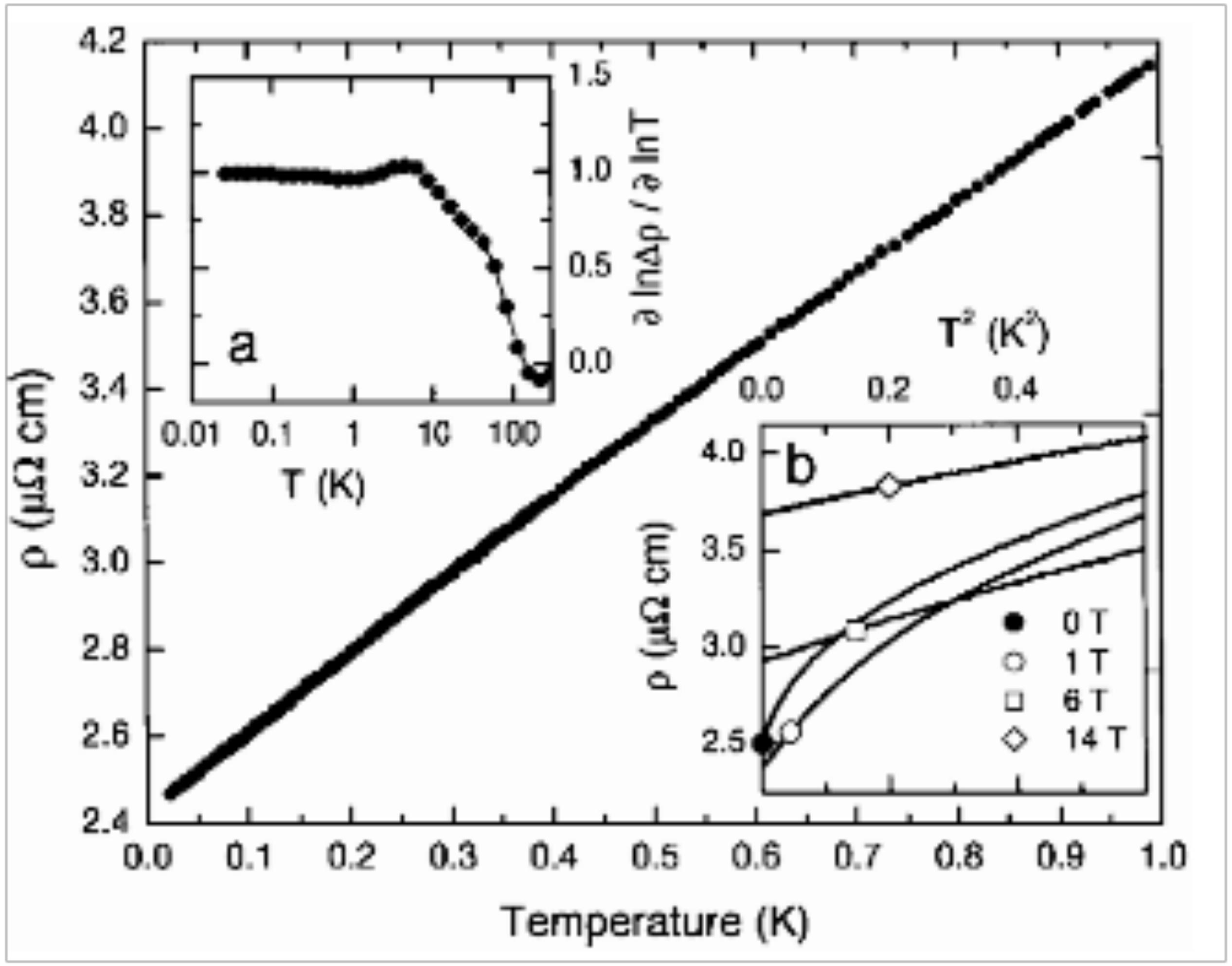
$YbRh_2Si_2$: Field tuned quantum criticality.

Quantum Criticality:
divergent specific heat capacity
breakdown of Landau Fermi Liquid

(a)



(b)



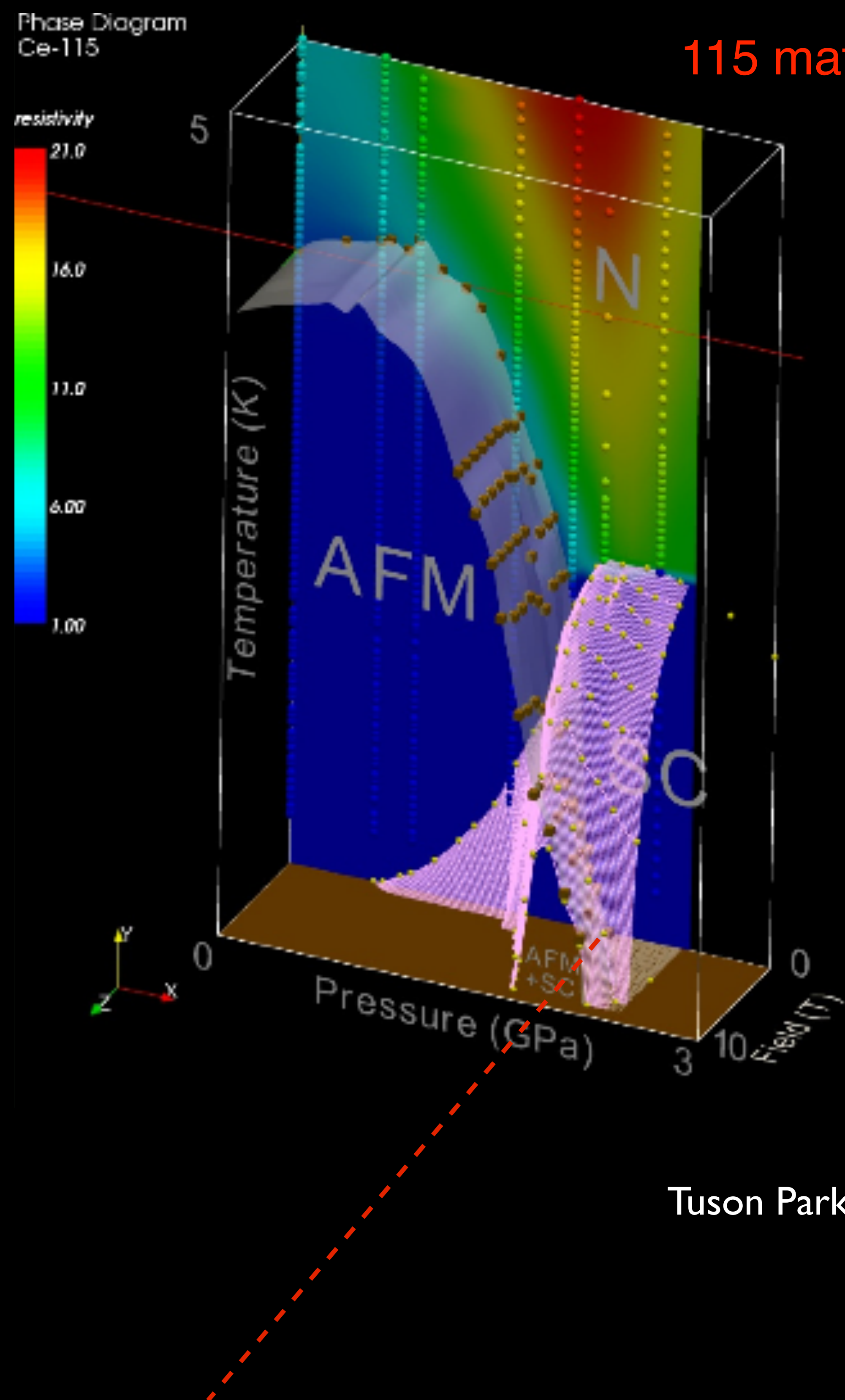
Custers et al, (2003)

YbRh$_2$Si$_2$: Field tuned quantum criticality.

Reconstruction of the Fermi Surface and mass divergence

CeRhIn$_5$

115 material



Tuson Park, (2007).

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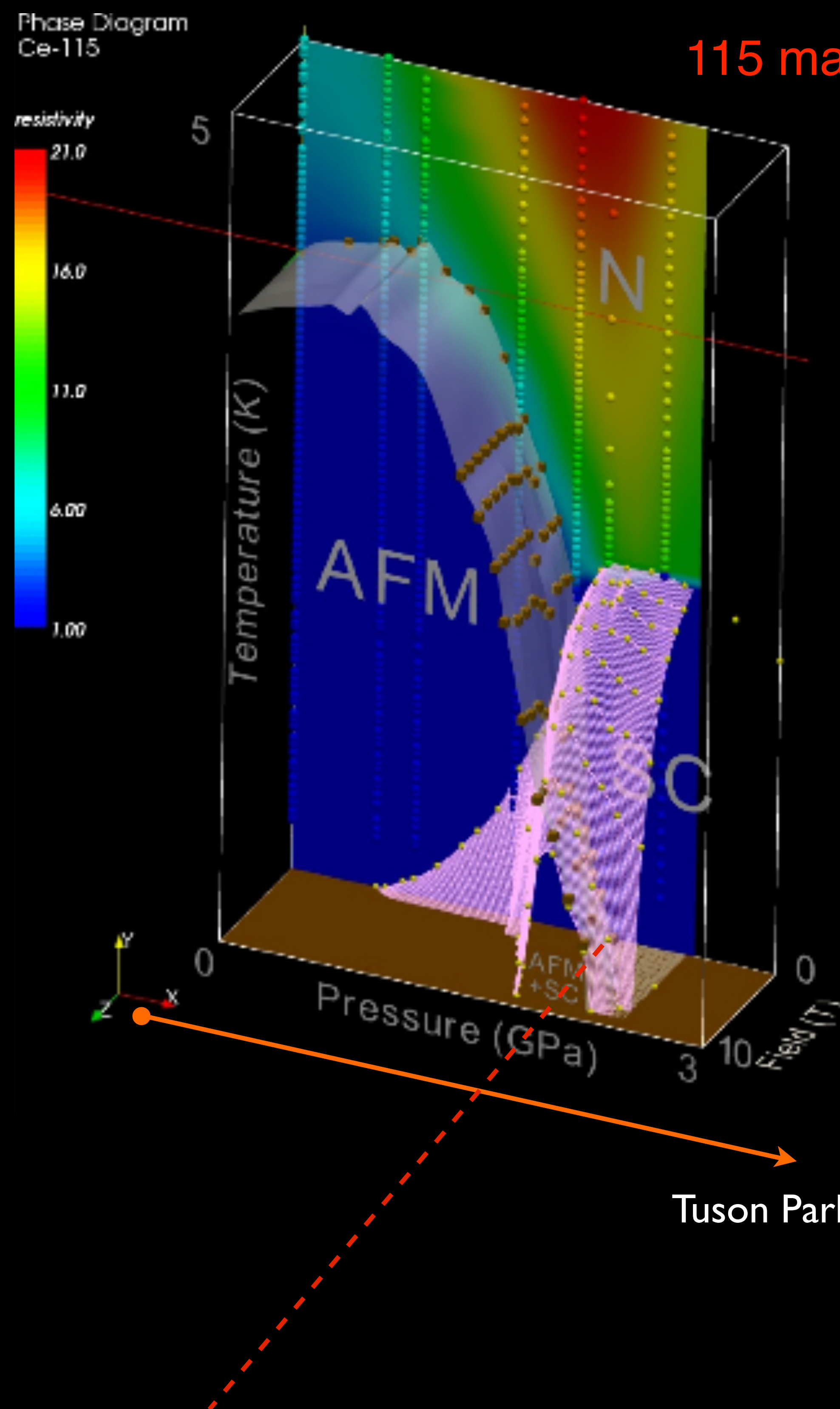

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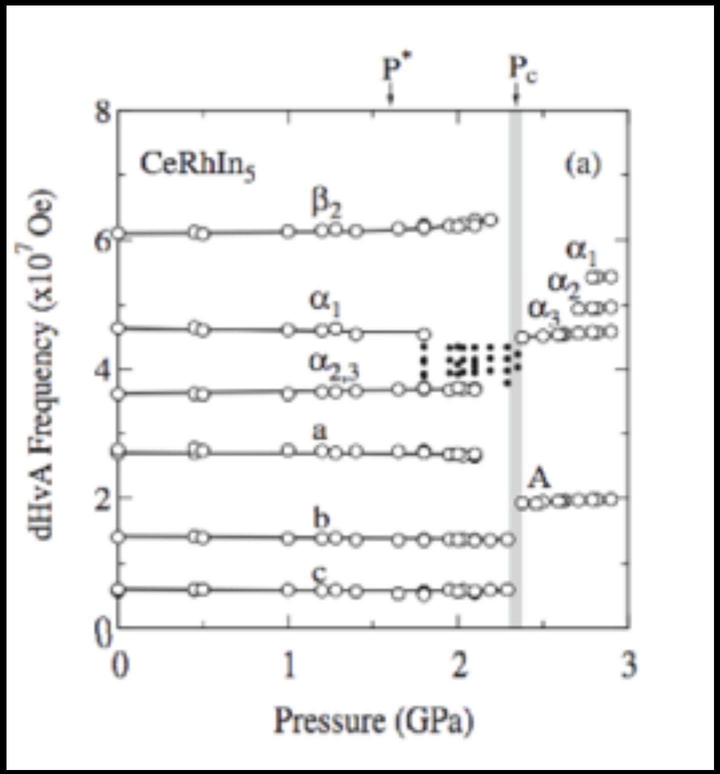

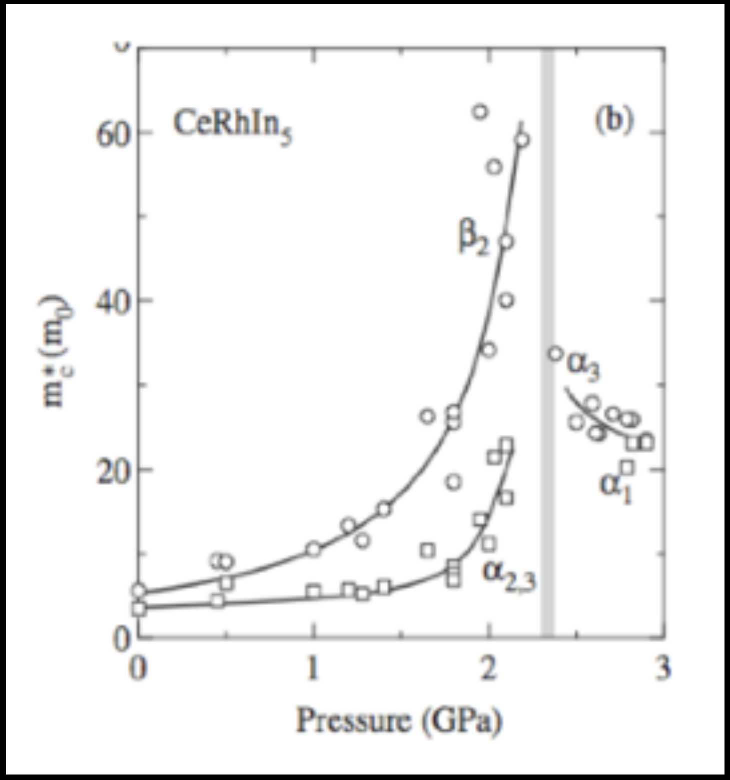

Shimuzu et al (2006)

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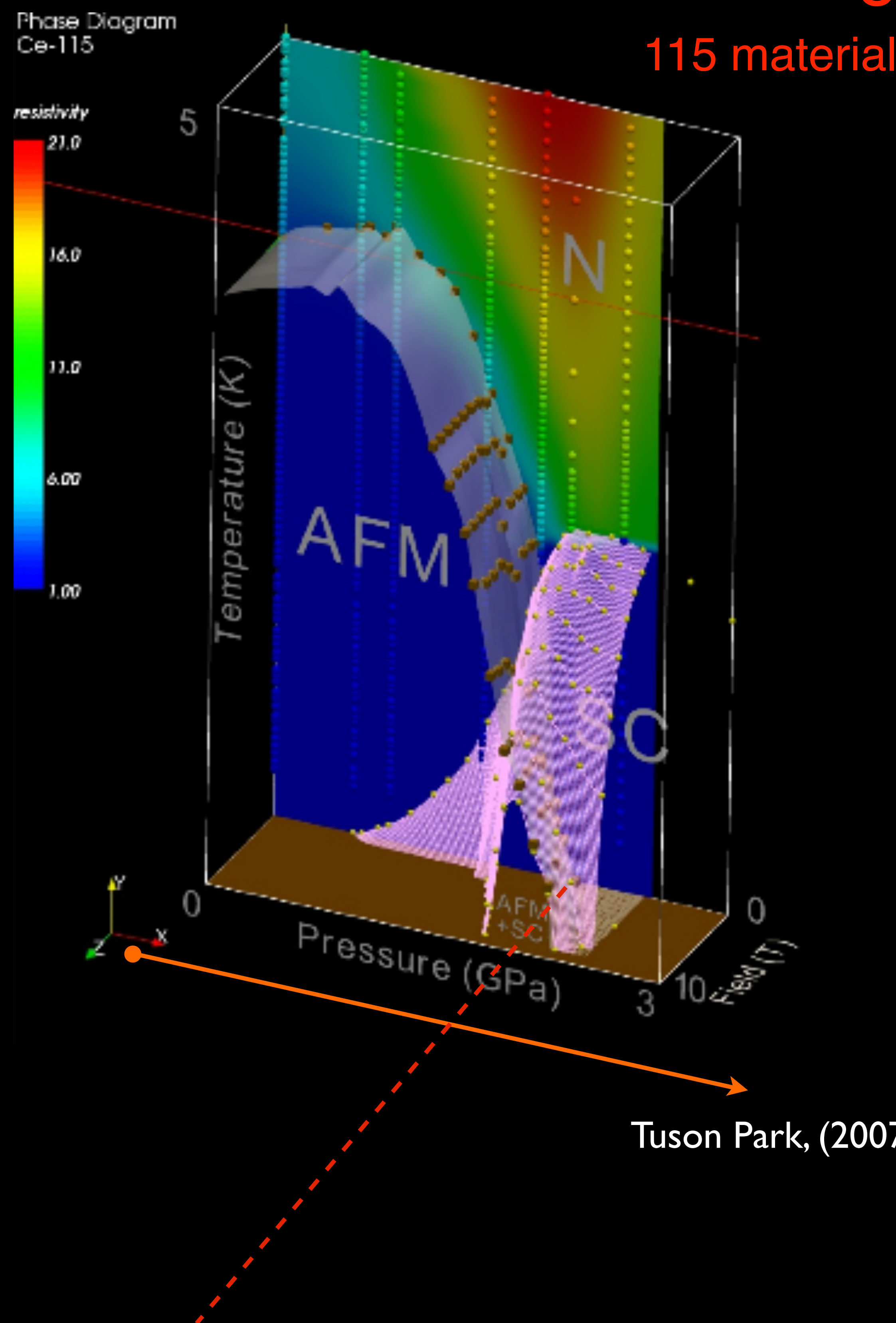

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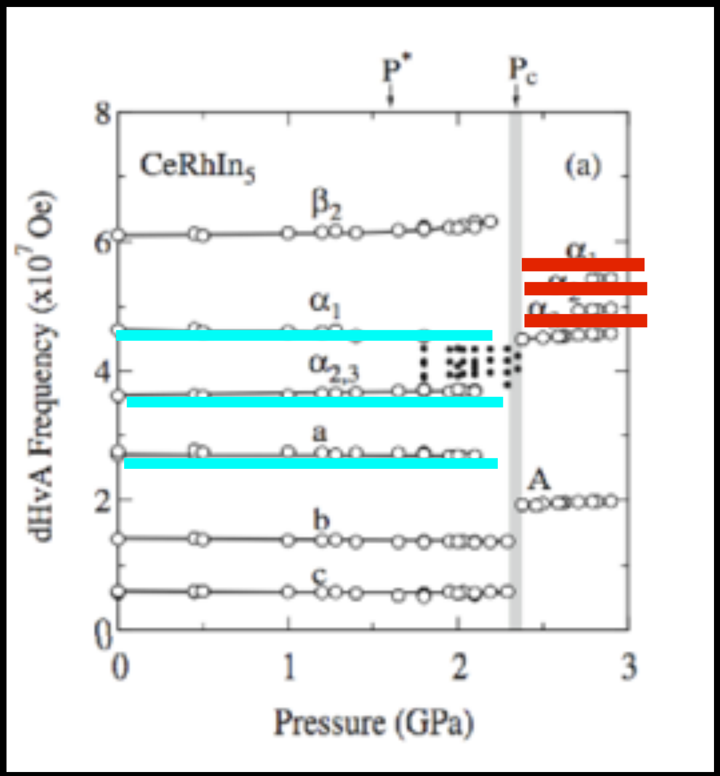

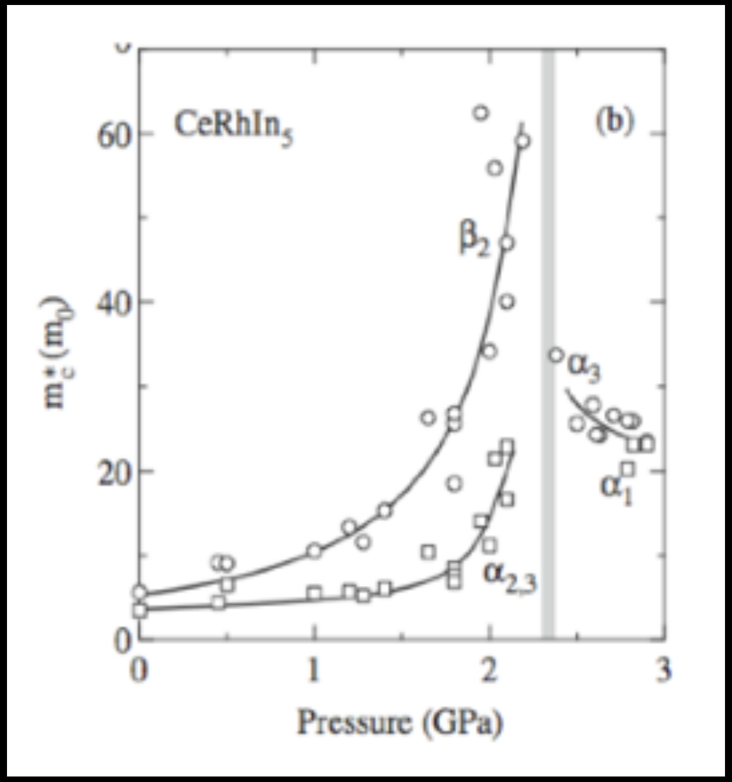

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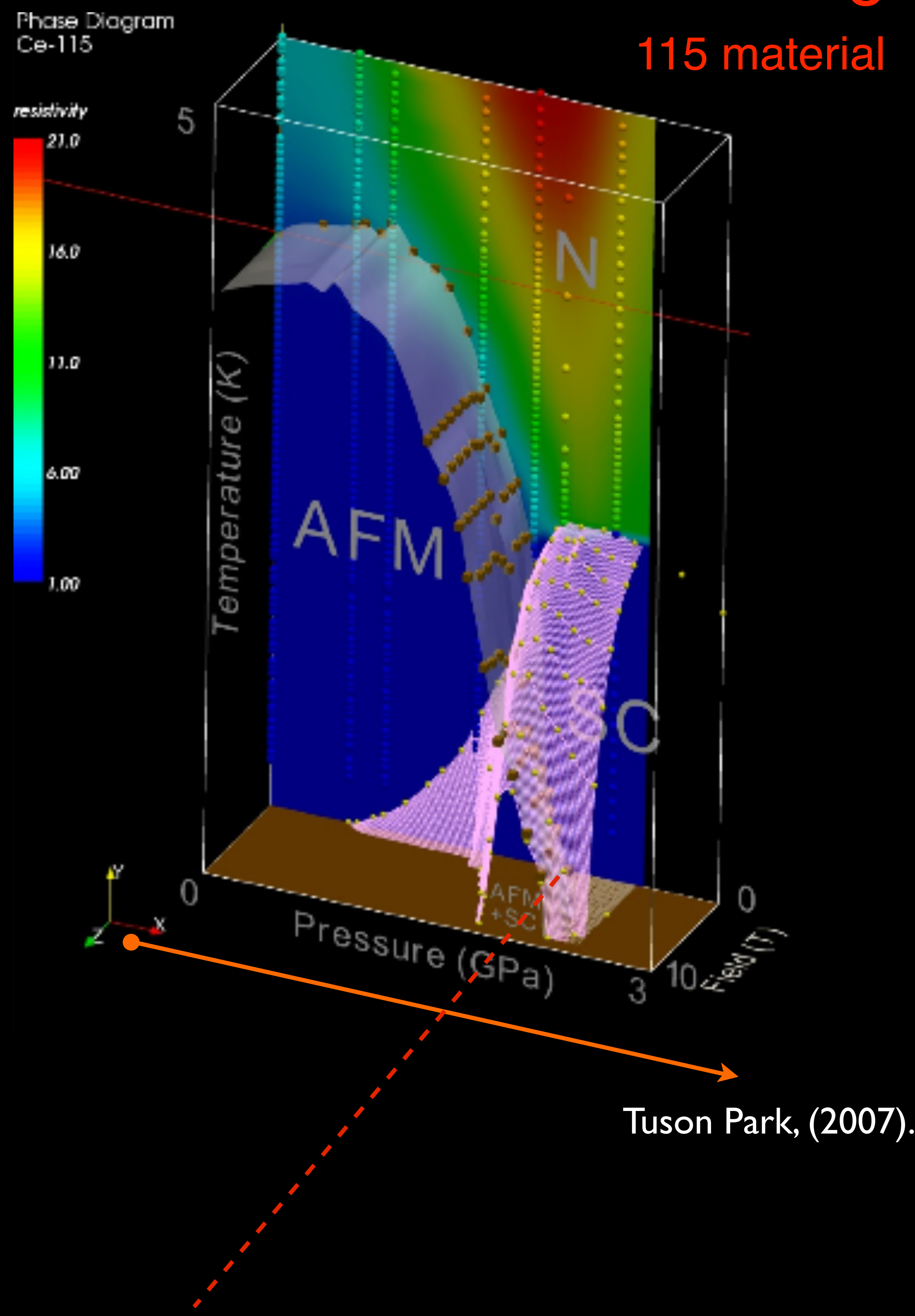

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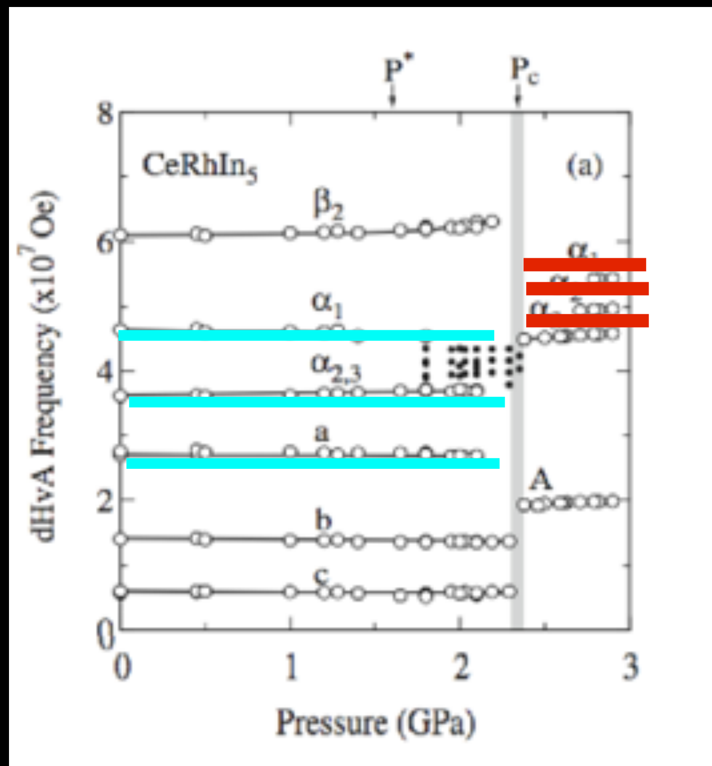

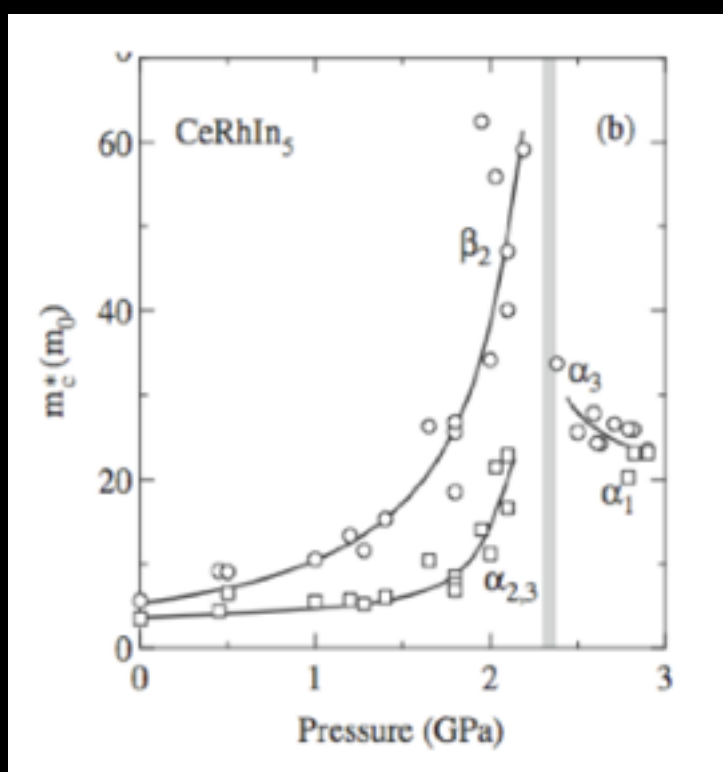

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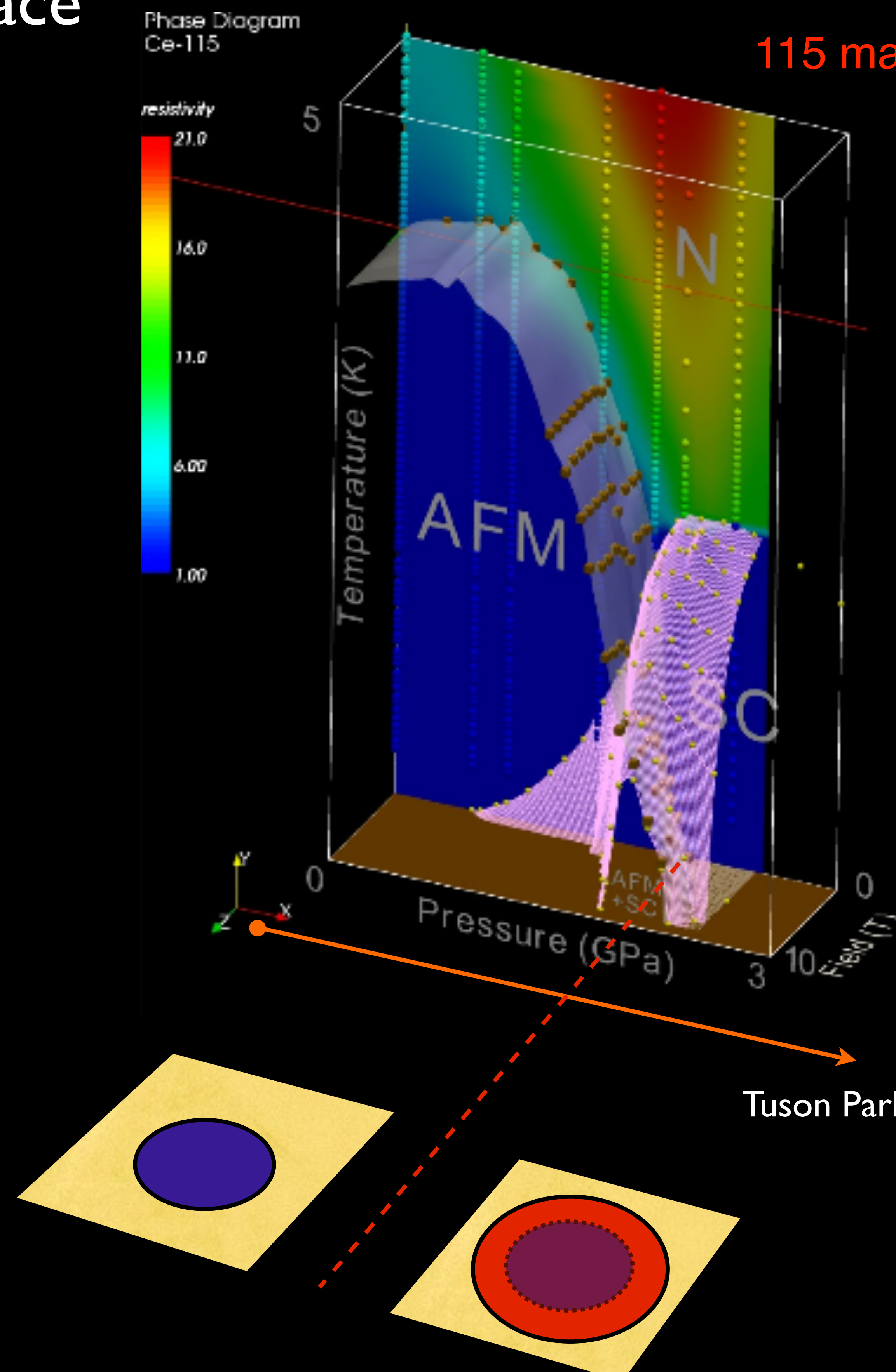
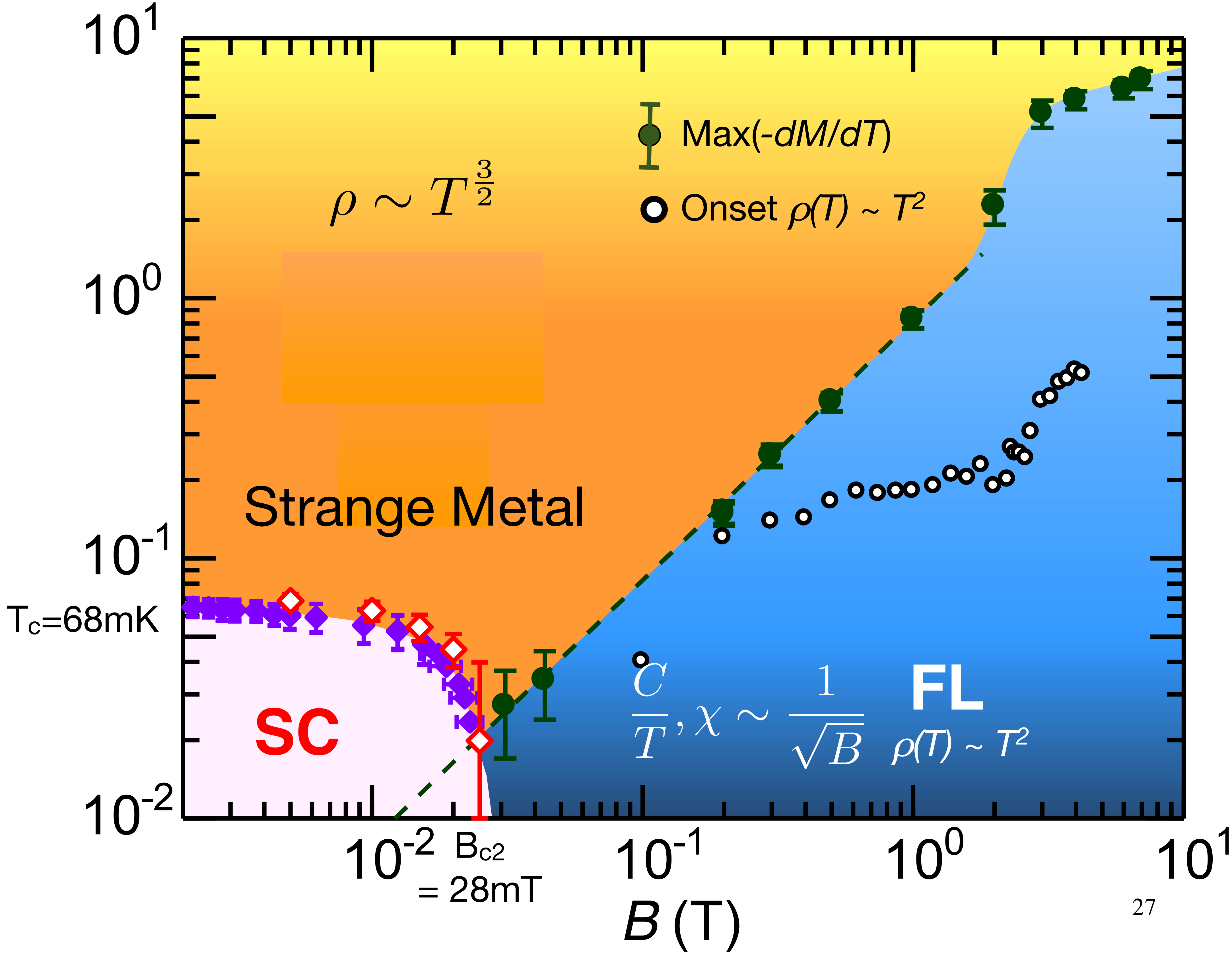


Tuson Park, (2007).

$YbAlB_4$

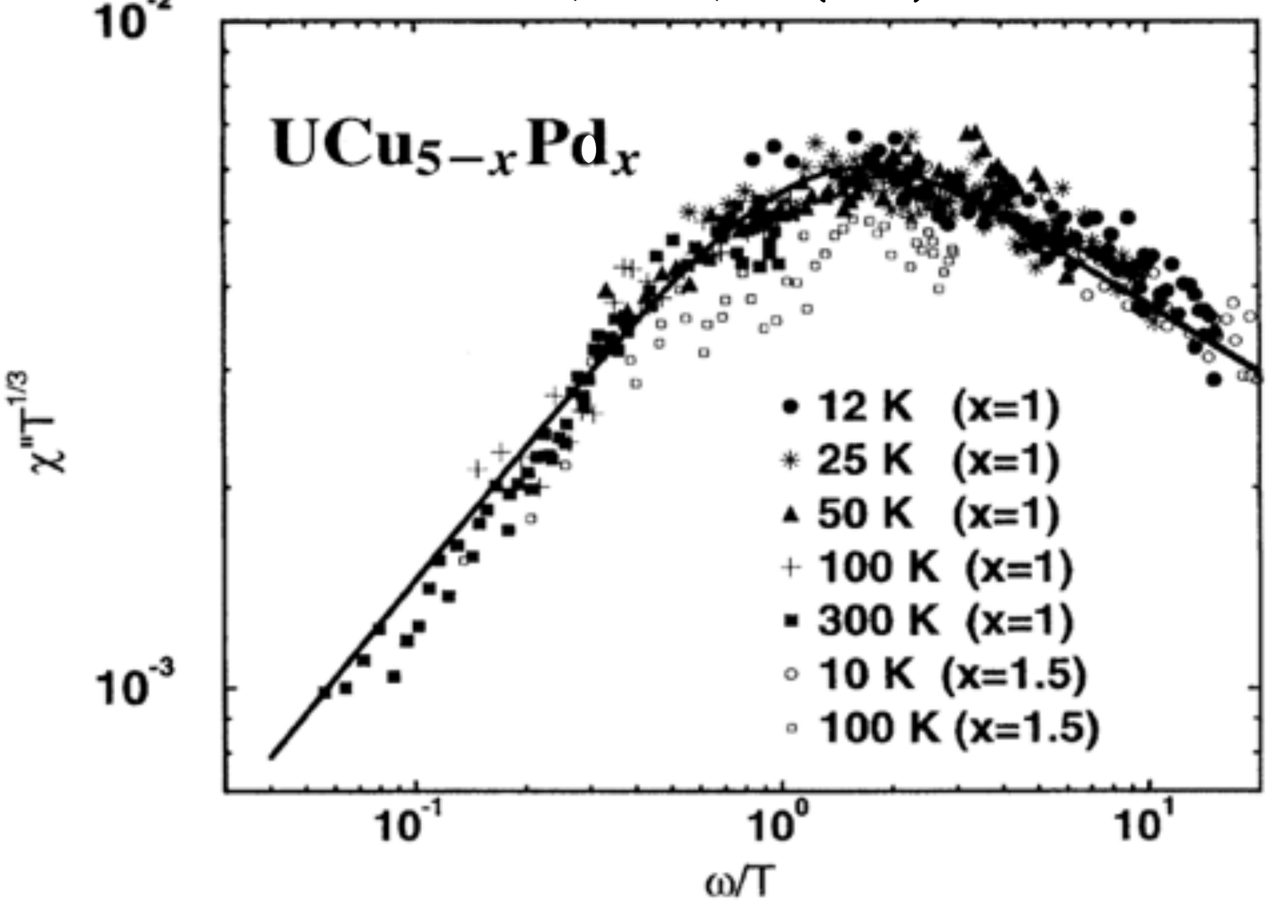


10^1
10^0
10^-1
10^-2
Tc=68mK
Max(-dM/dT)
Onset ρ(T) ~ T²
ρ ~ T^{3/2}
Strange Metal
SC
C/T, χ ~ 1/√B
FL
ρ(T) ~ T²
10^-2
Bc2
= 28mT
10^-1
10^0
10^1
B (T)

E/T Scaling:

Meigan Aronson

M. C. Aronson et al, PRL 75, 725 (1995).



Rutgers Center for Materials Theory

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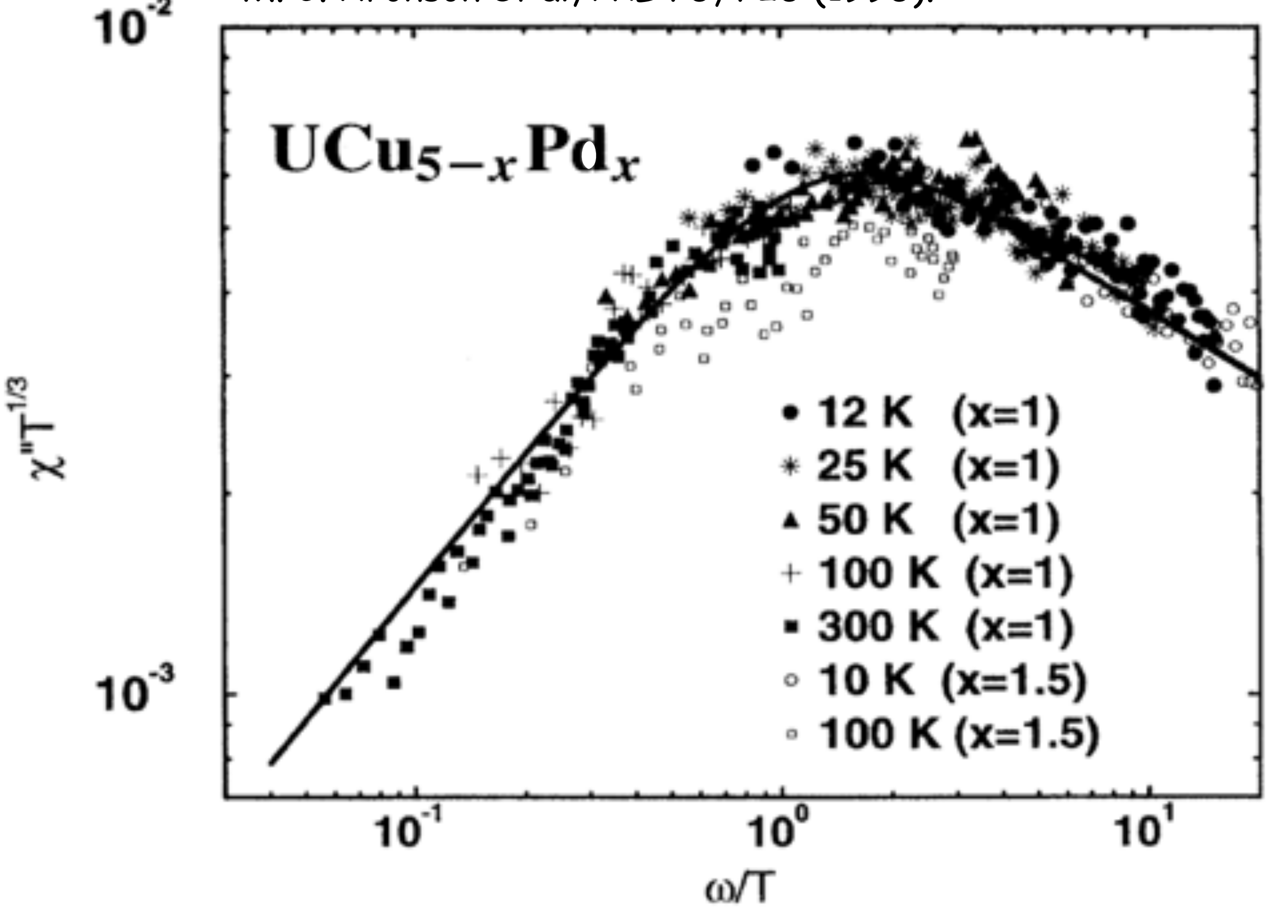

$$\chi''(E) = \frac{1}{E^{1-\alpha}} G\left(\frac{E}{T}\right)$$

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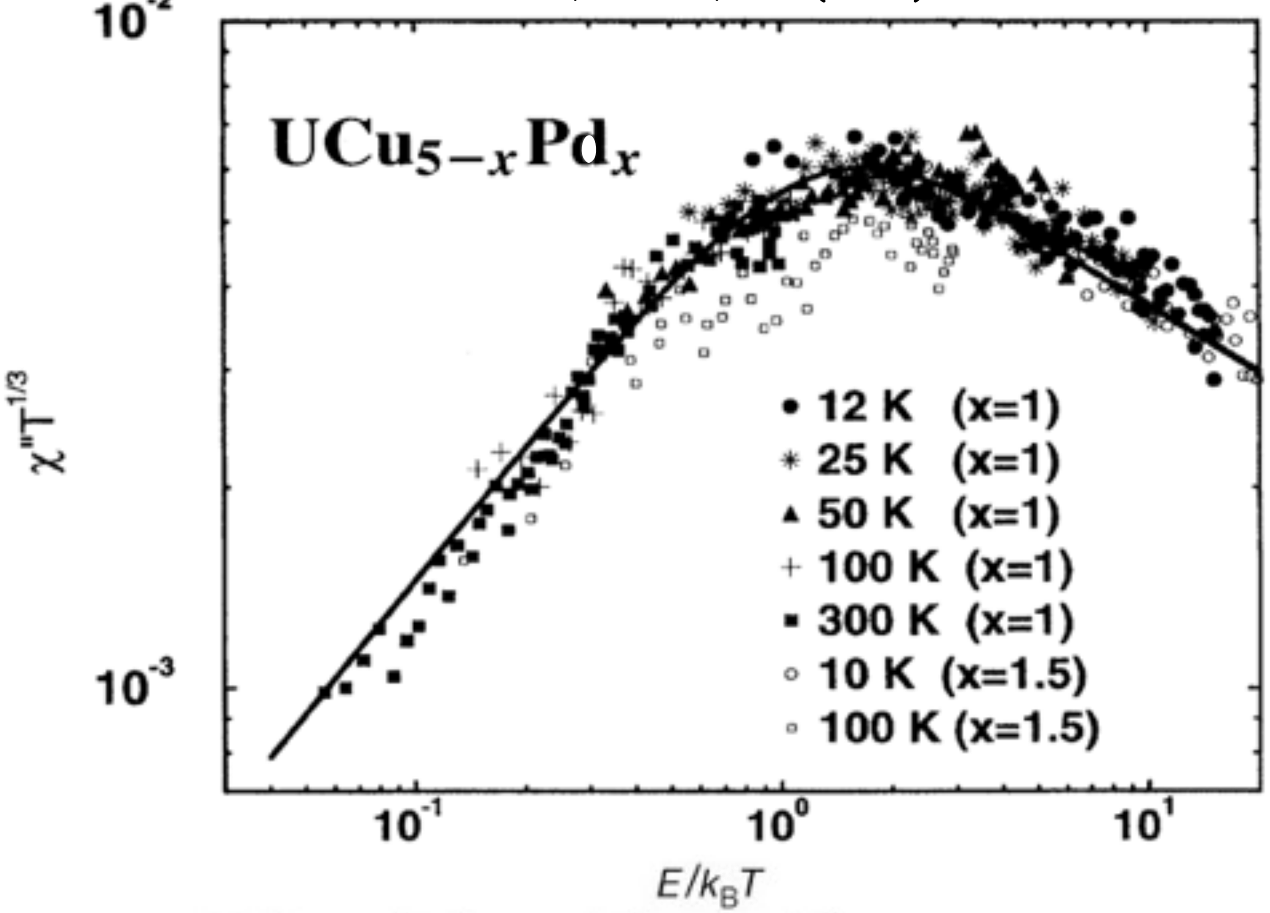

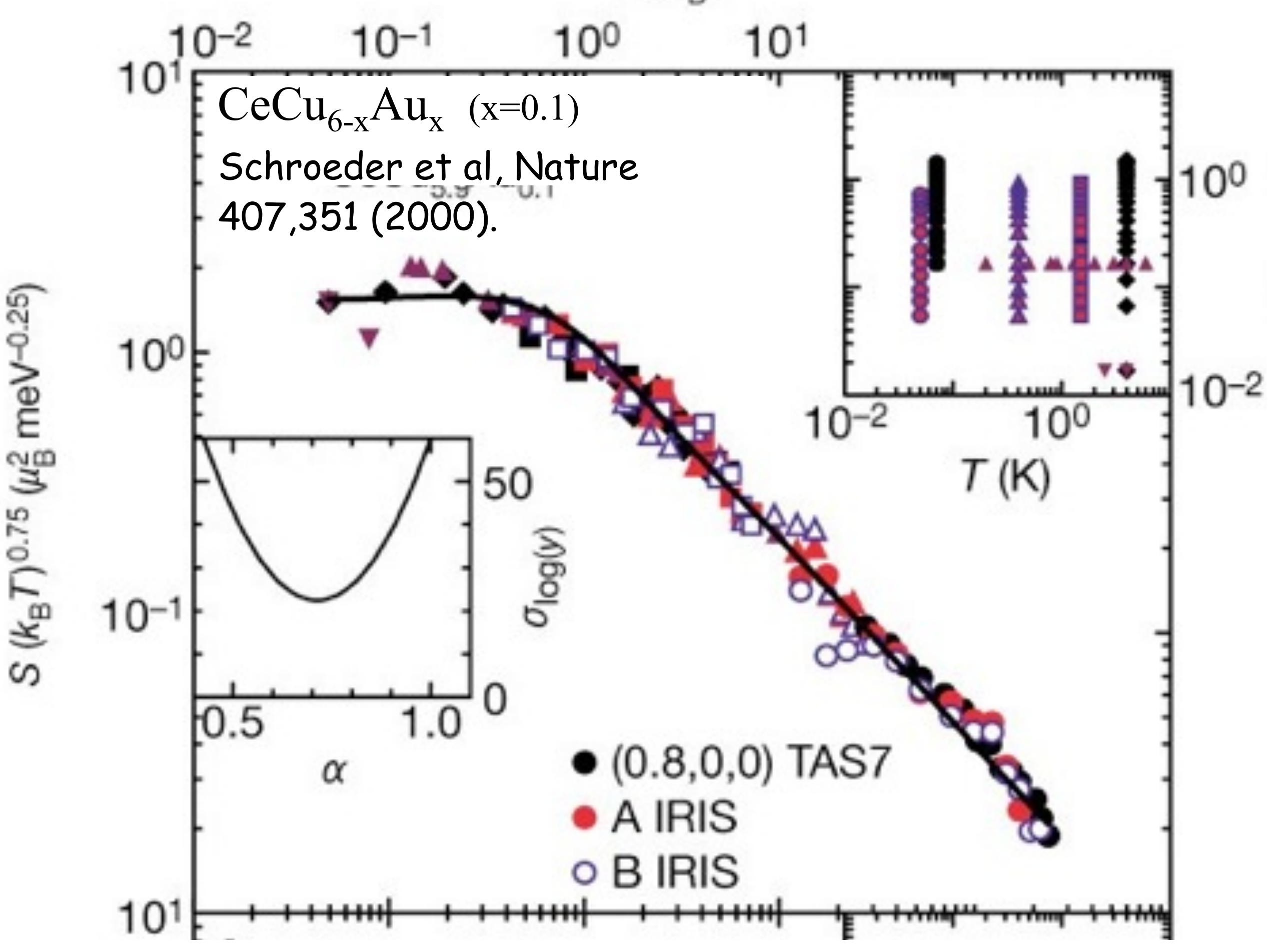

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Physics Below the upper Critical Dimension.

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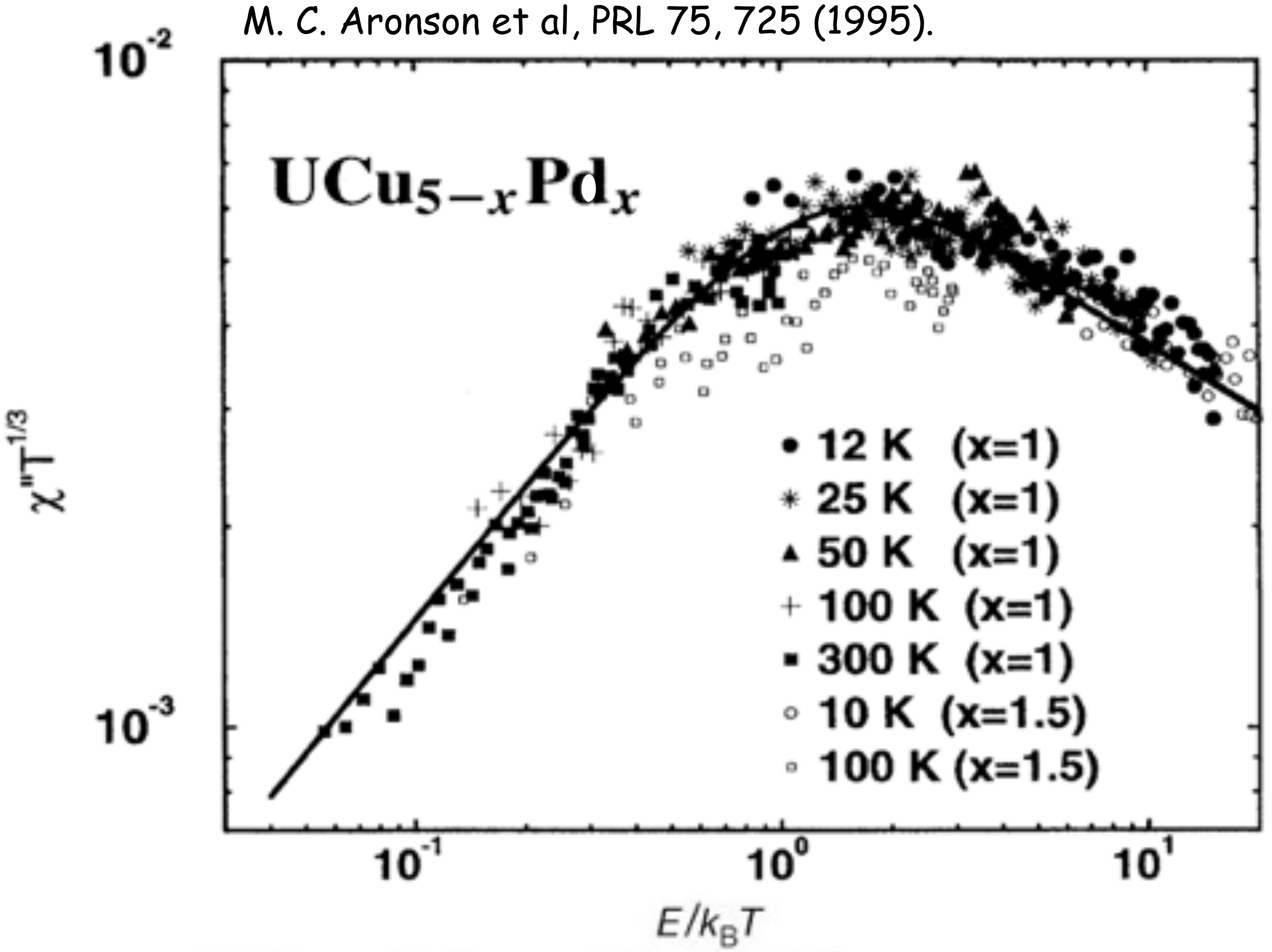
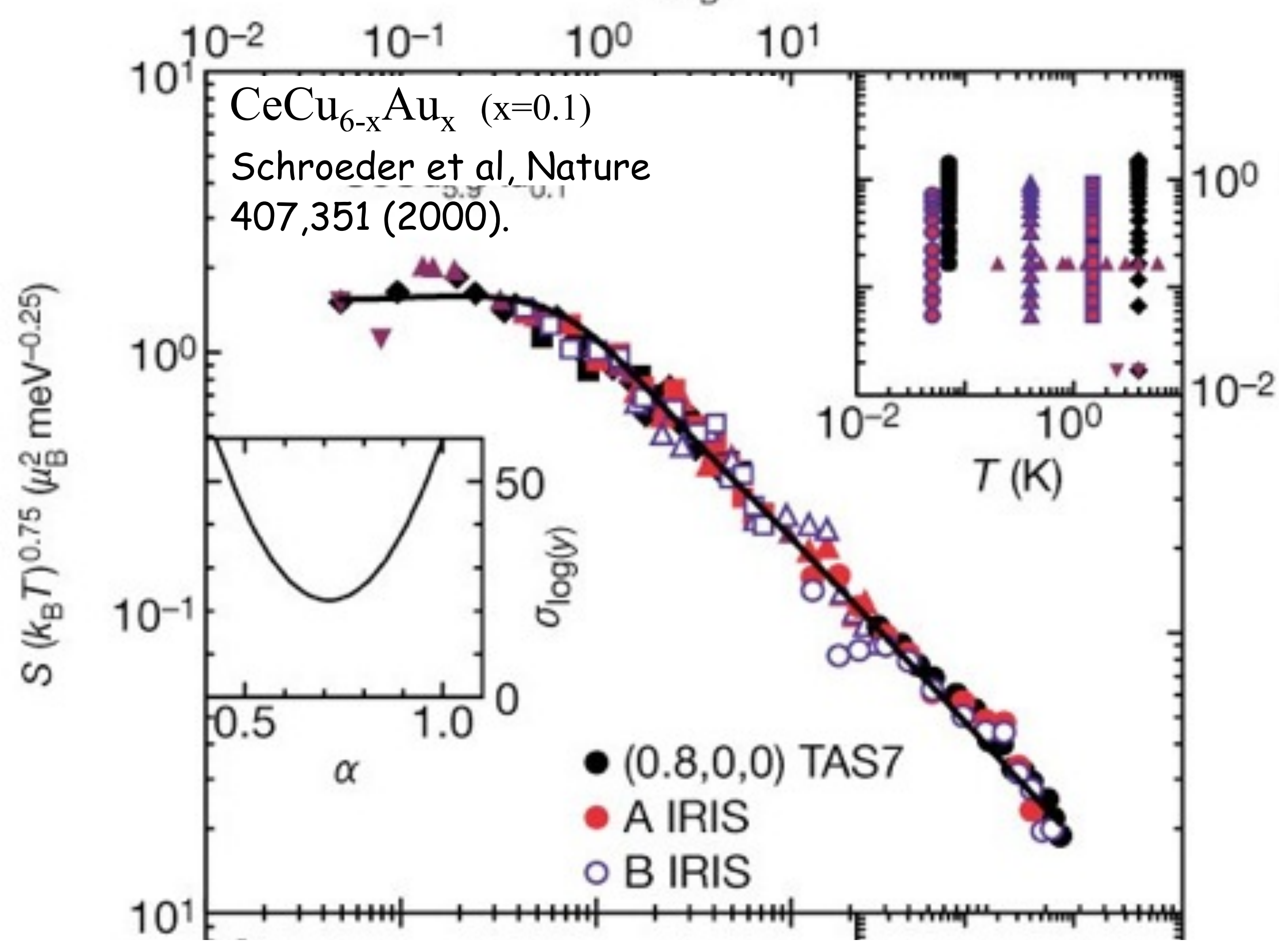
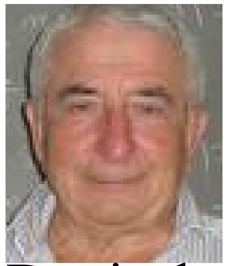




Standard Model: Quantum SDW?


Moriya


Doniach

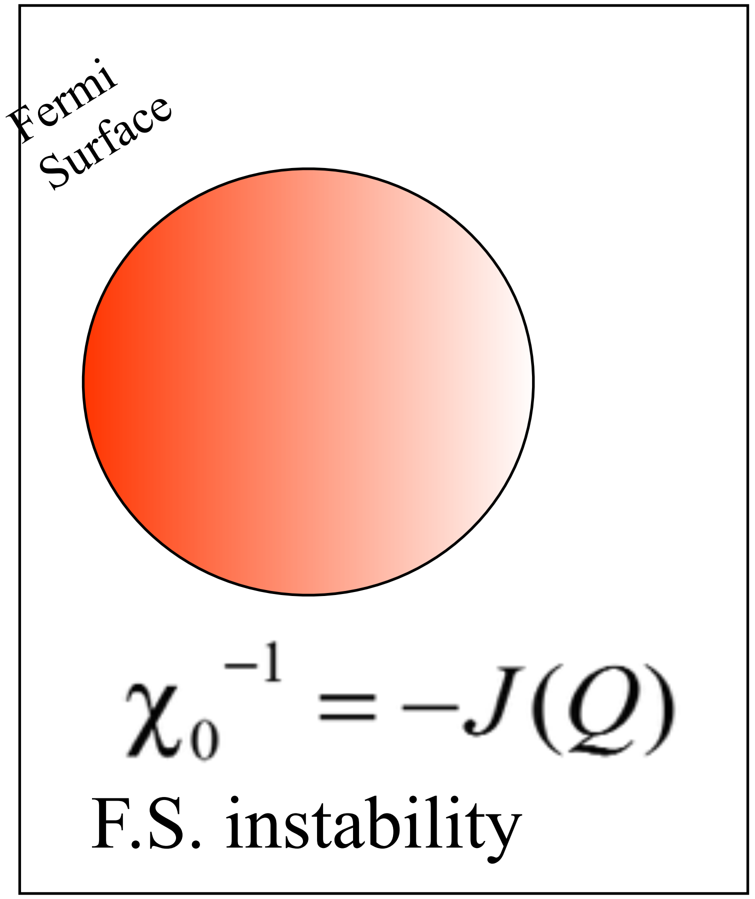

Schrieffer


Hertz


Millis

- Moriya, Doniach, Schrieffer (60s)
- Hertz (76)
- Millis (93)

$$d_{eff} = d + z$$



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Moriya Doniach Schrieffer Hertz Millis

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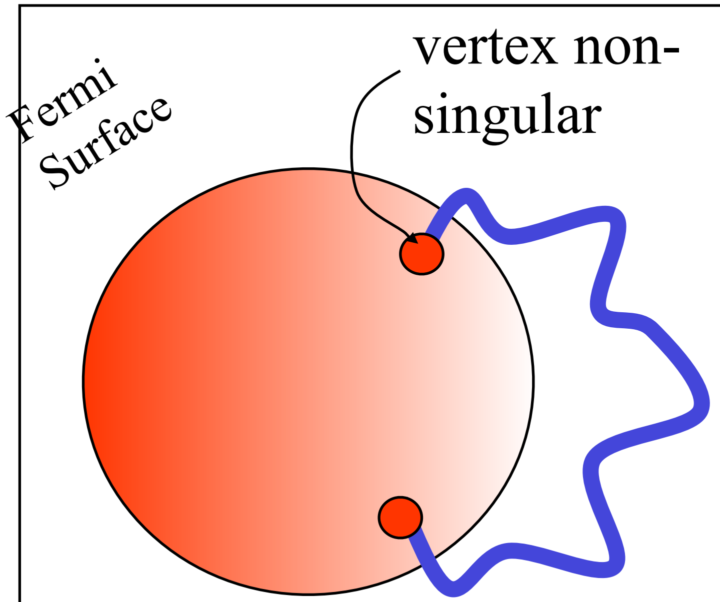

$$\chi_0^{-1} = -J(Q)$$

F.S. instability

$$\chi^{-1}(q,\omega) \propto (\xi^{-2} + (q-Q)^2 - i\omega / \Gamma)$$

Rutgers
Center for Materials Theory

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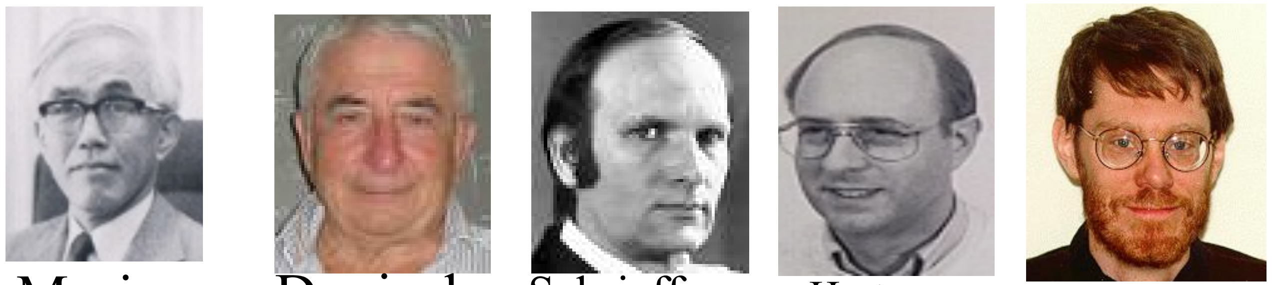

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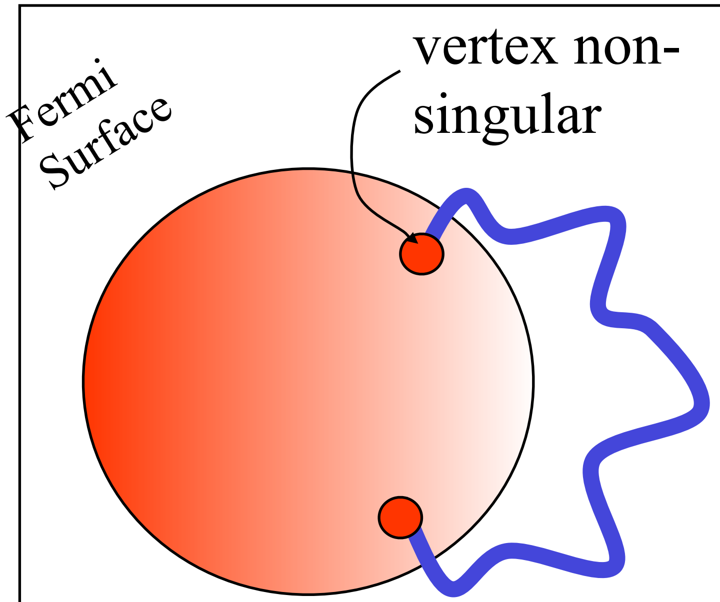

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Rutgers
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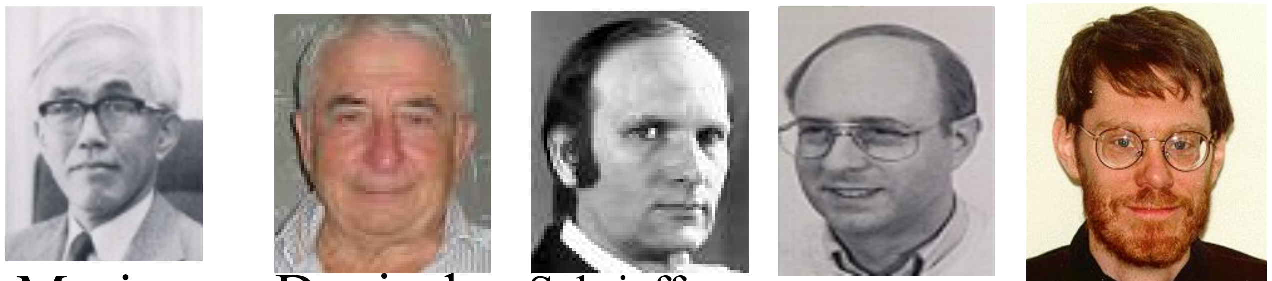

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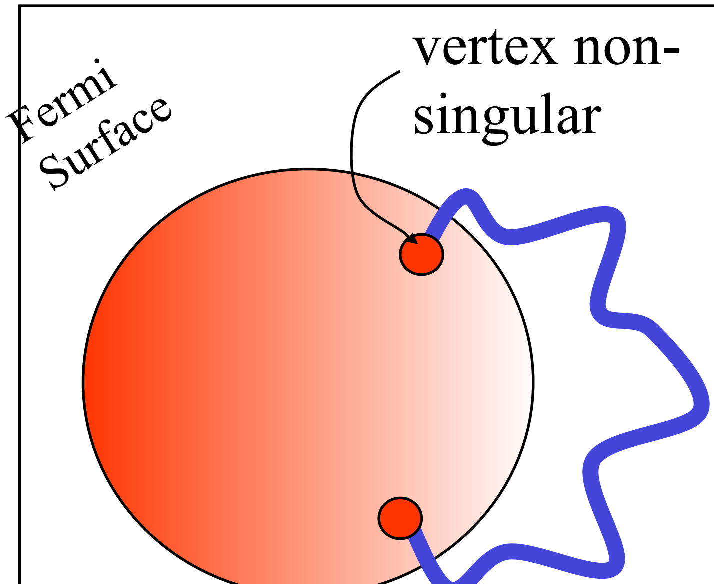

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Time counts as z =2 scaling dimensions

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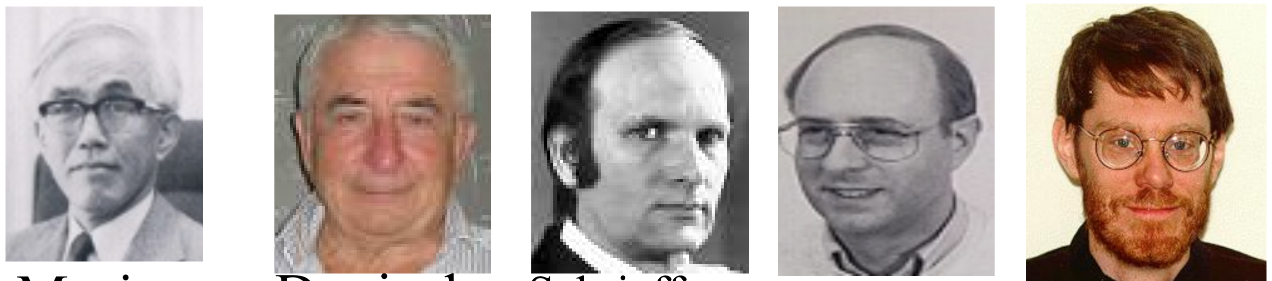

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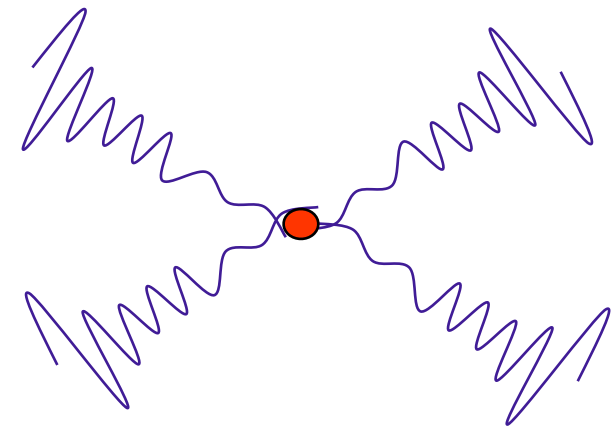

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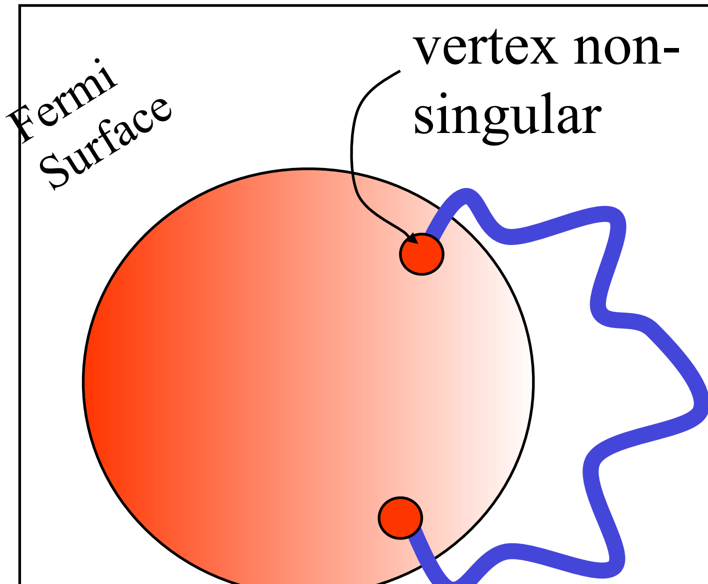
$$d_{eff} = d + z$$

If $d + z = d + 2 > 4$:
ϕ^4 terms "irrelevent"
Critical modes are <u>Gaussian</u>.
T is not the only energy scale.



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Rutgers Center for Materials Theory

New Ideas

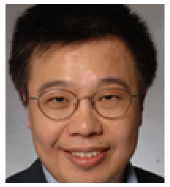
New Ideas

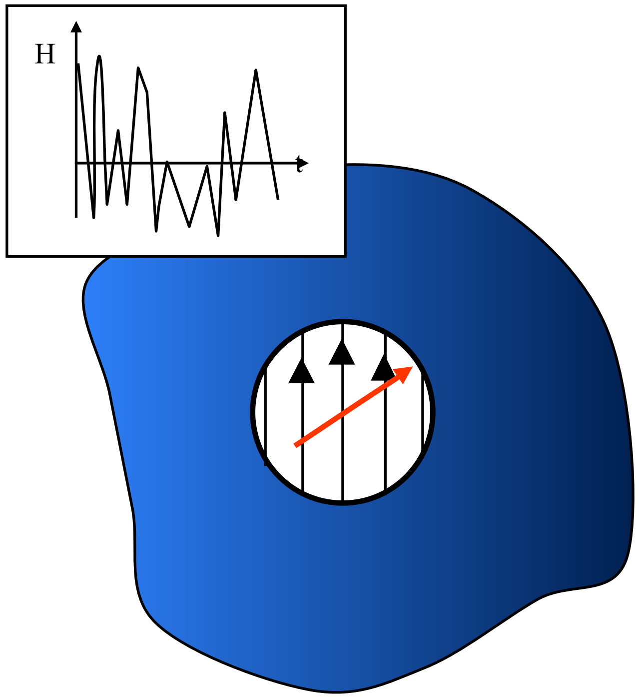
- **Local quantum criticality**

(Si, Ingersent, Smith, Rabello, Nature 2001):
Spin is the critical mode,
Fluctuations critical in time.

Requires a two dimensional spin fluid

Si, Ingersent





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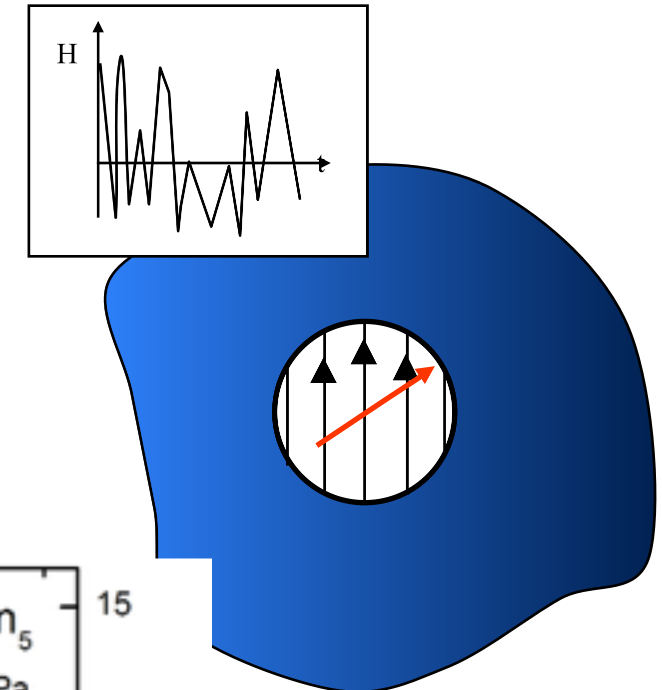

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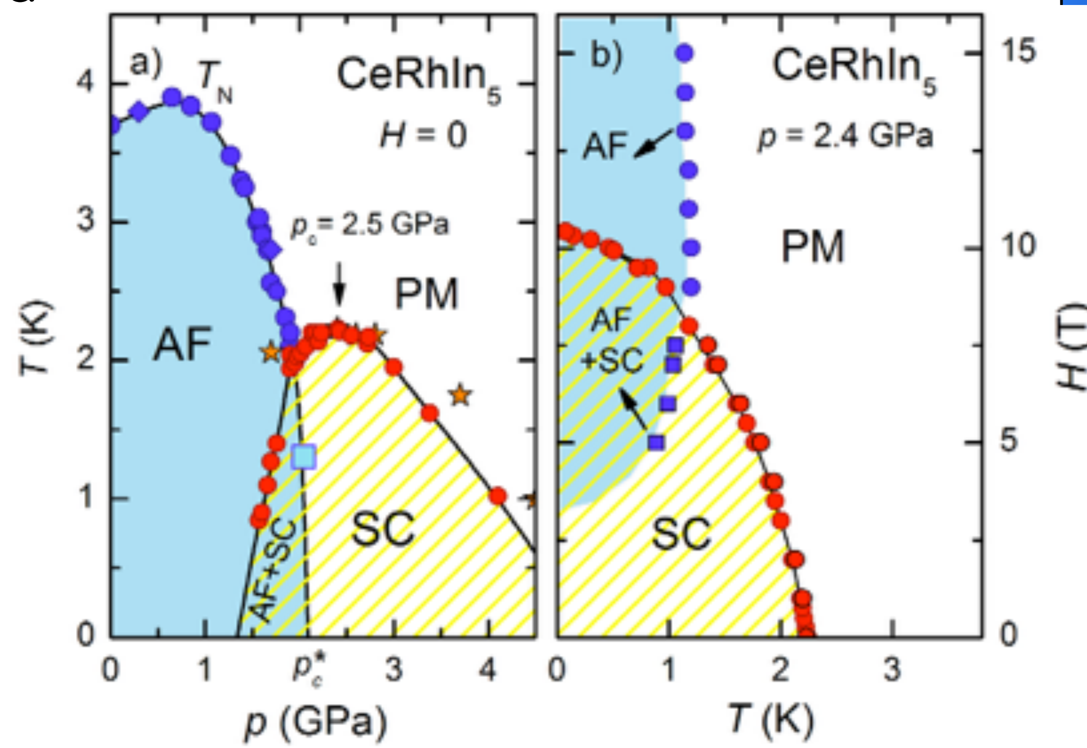
- **Two fluid scenario.**




D. Pines Z. Fisk S. Nakatsuji Y. Yang



Nature (2008), PRL (2004)

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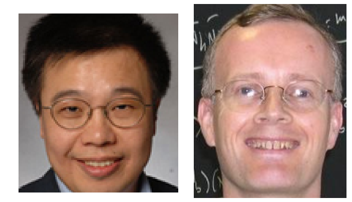

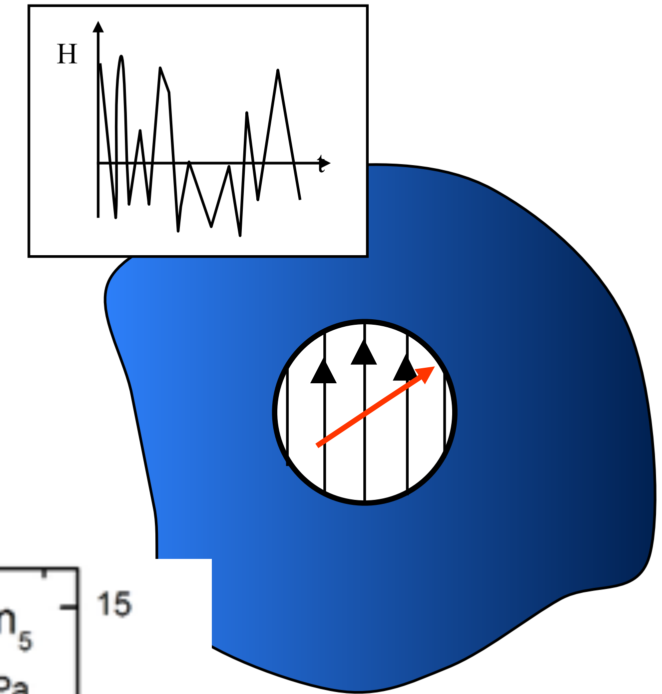

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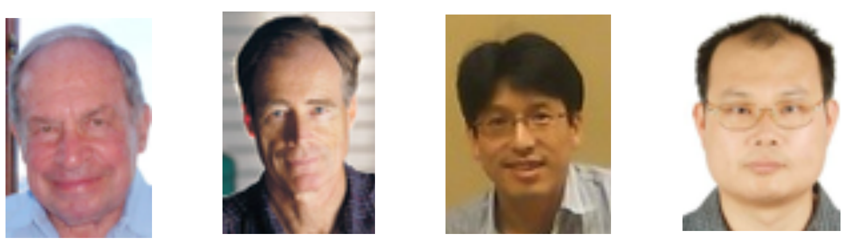

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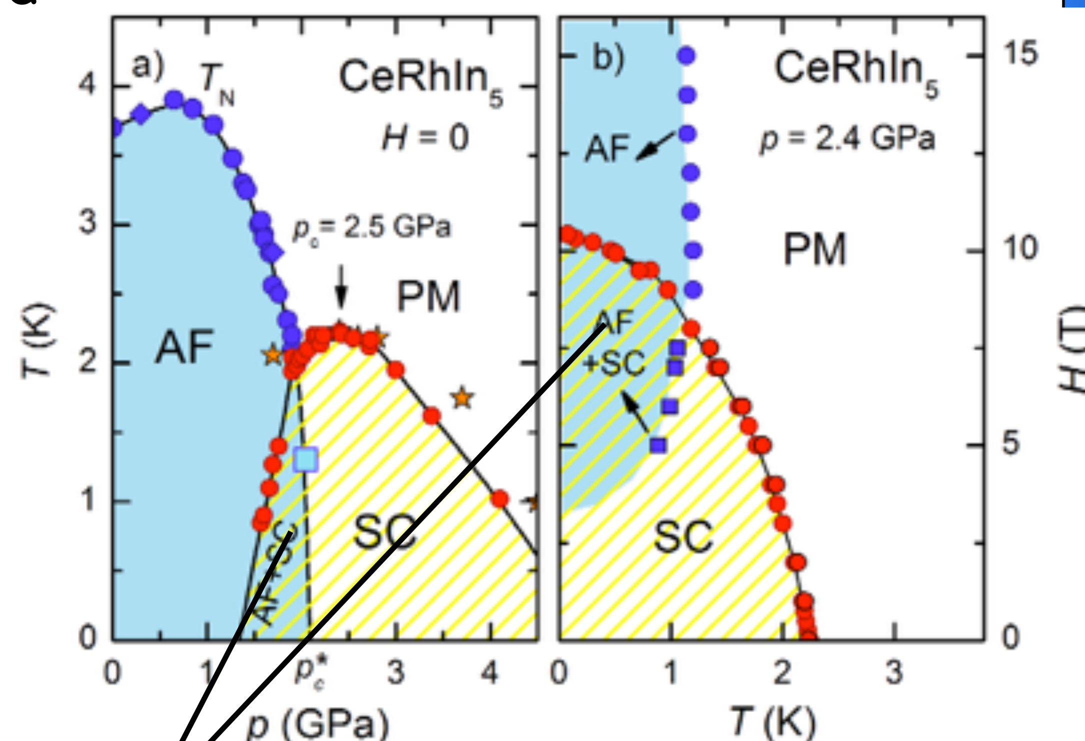

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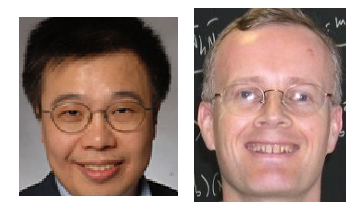

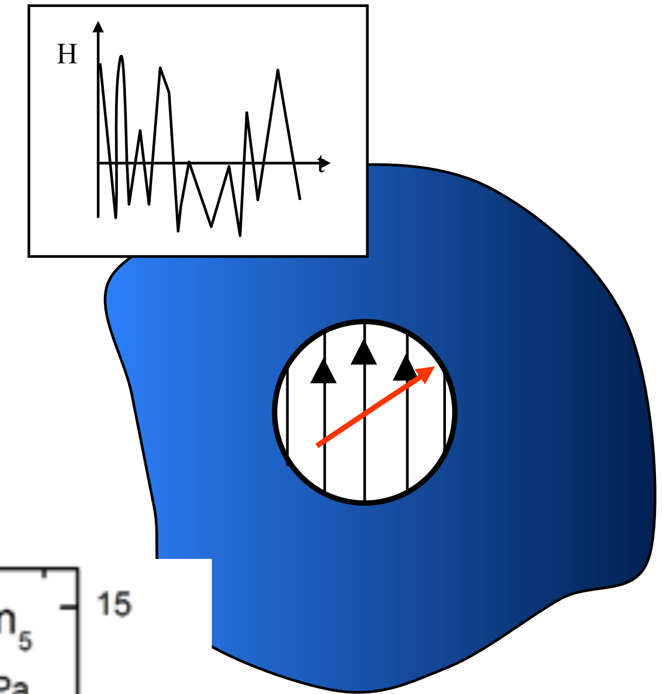

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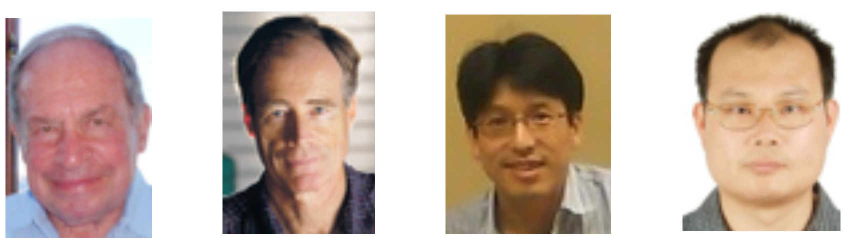

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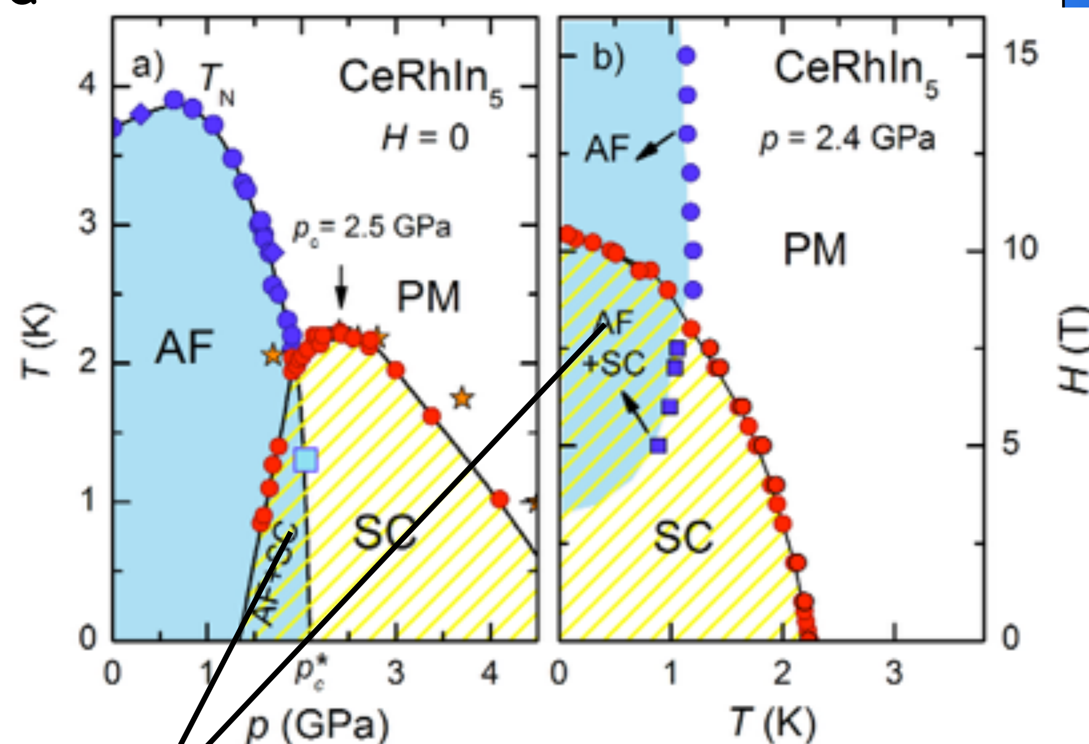

Description of unconventional QCP requires new formalism.

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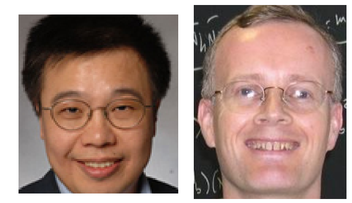

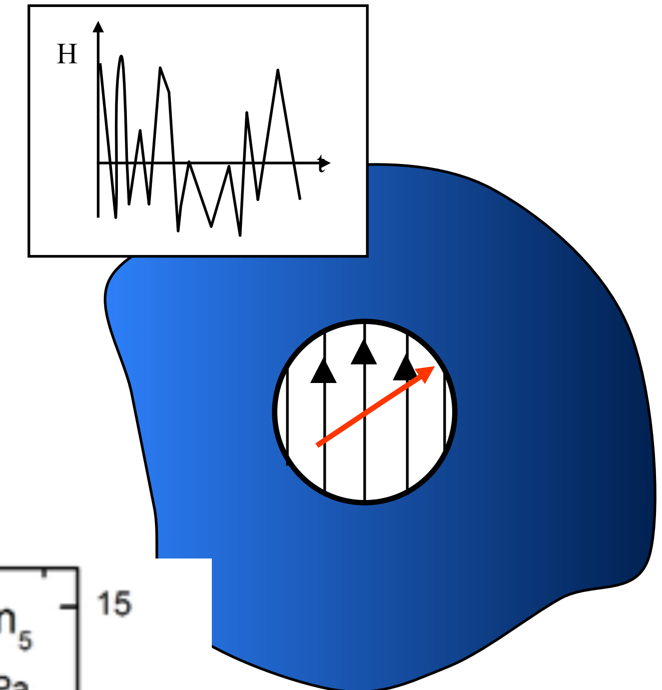

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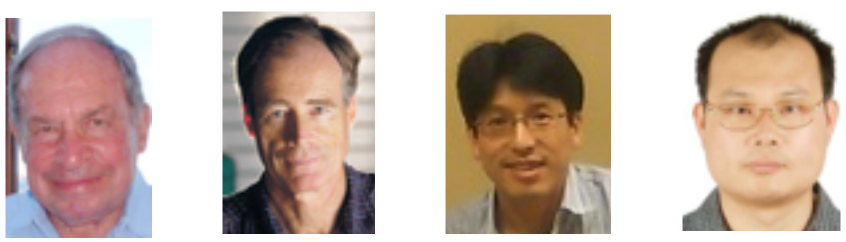

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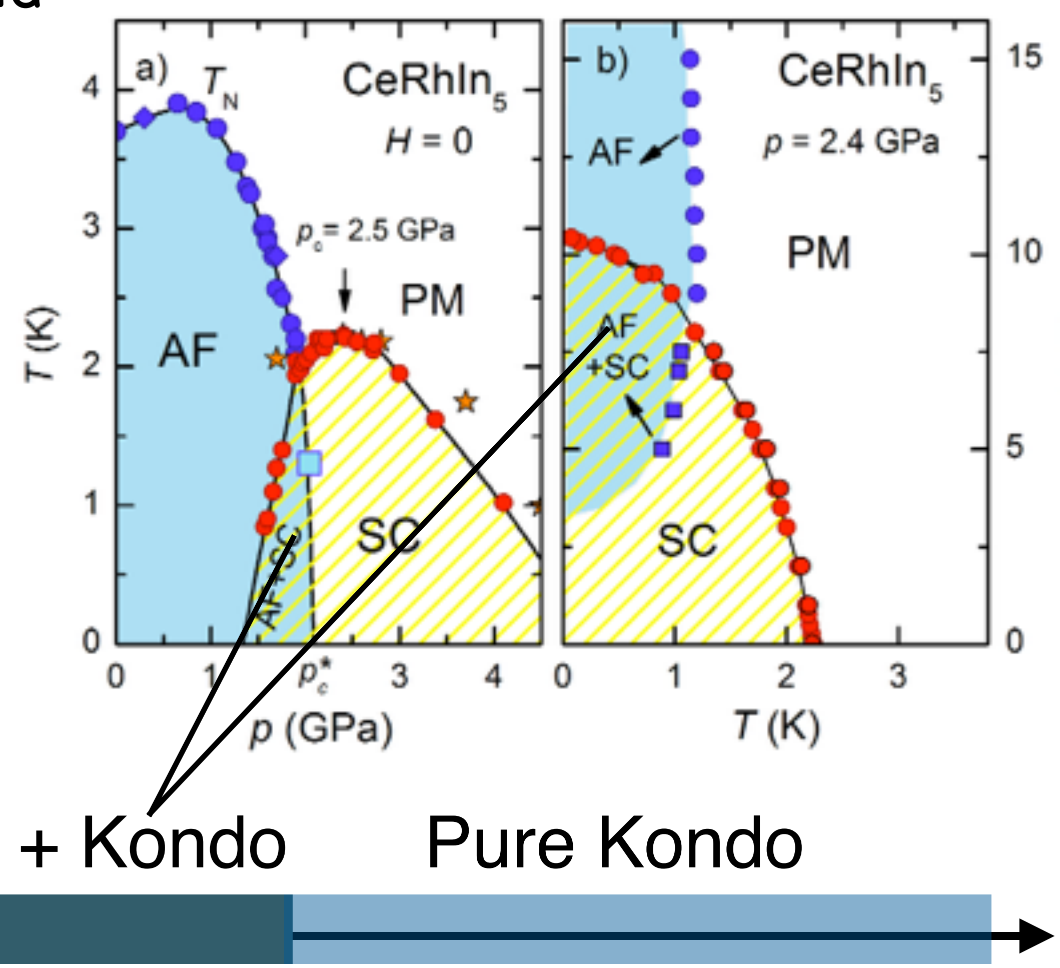

- **Supersymmetry?**

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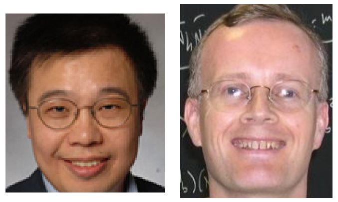

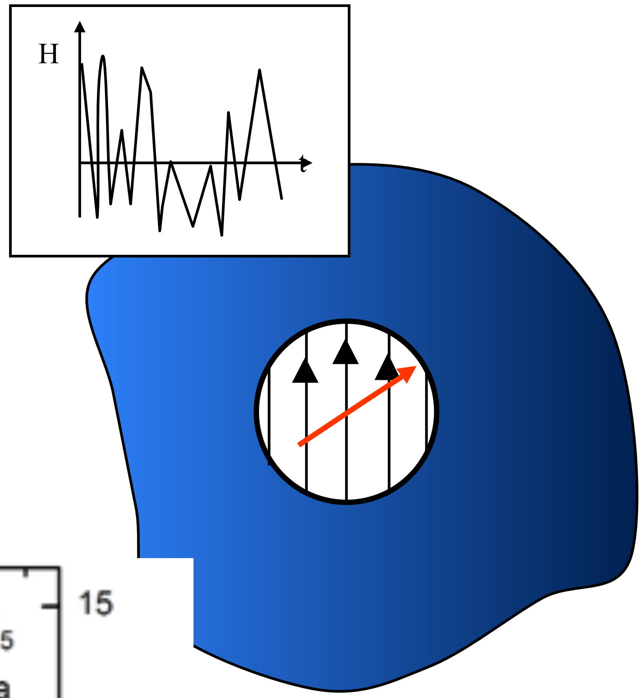

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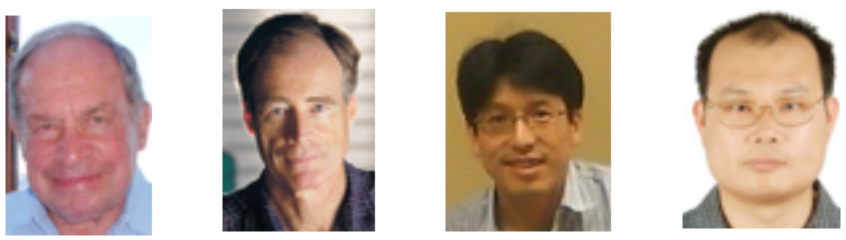

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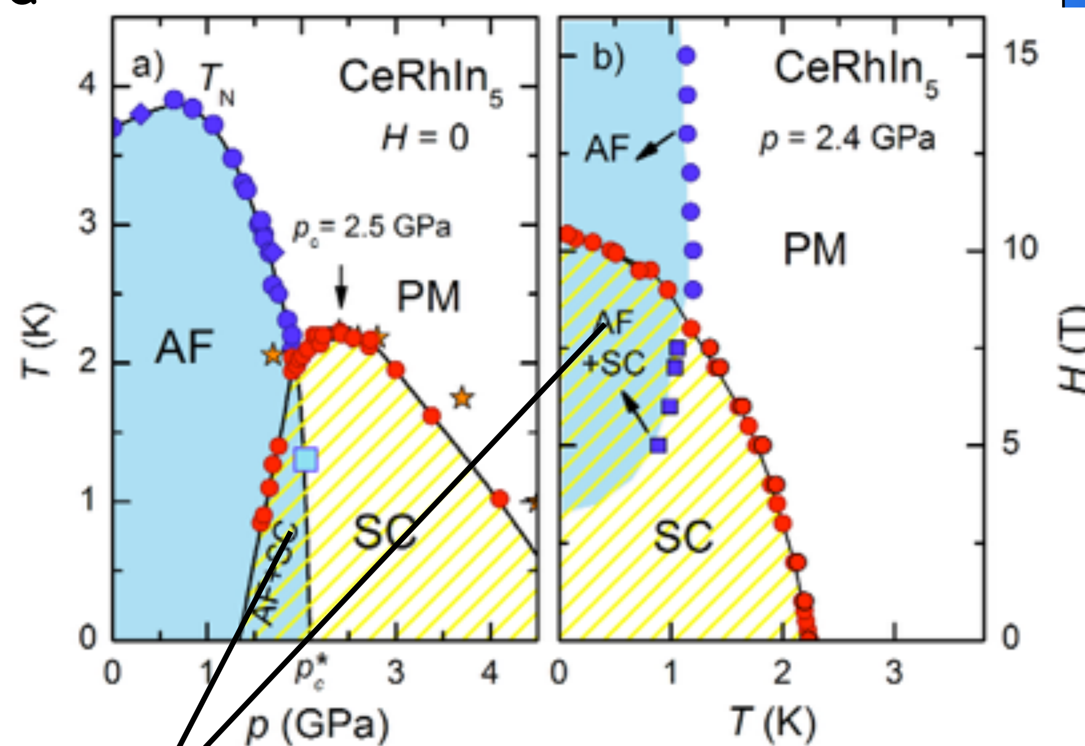

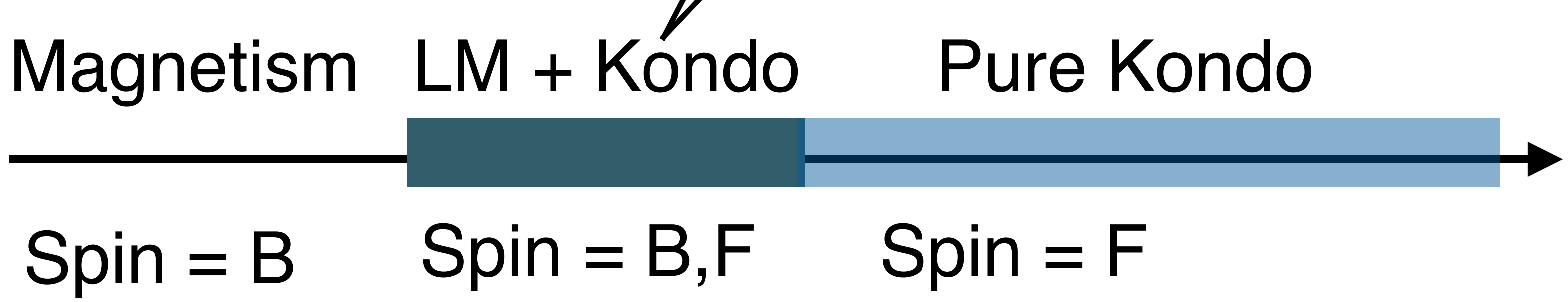

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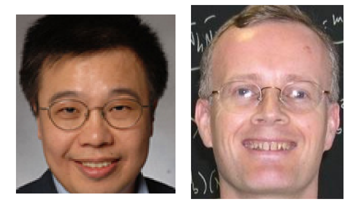

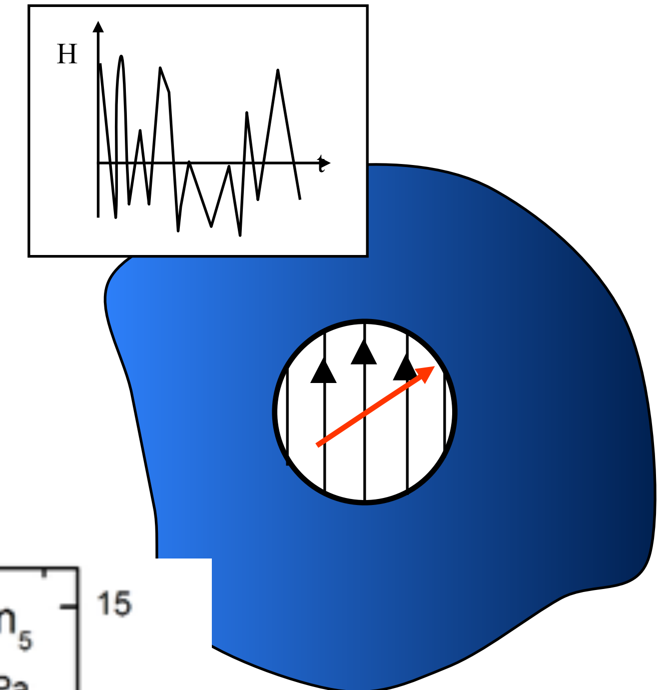

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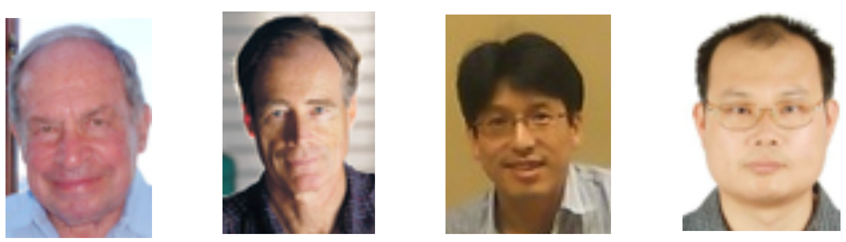

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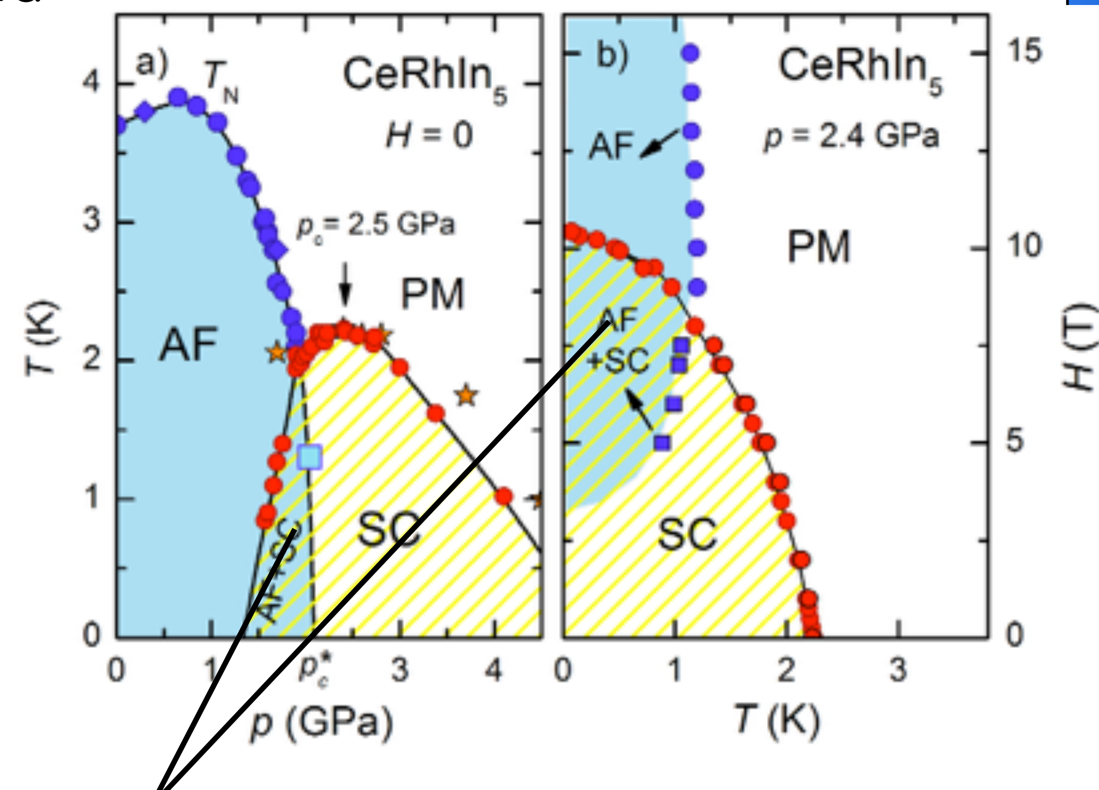

Magnetism LM + Kondo Pure Kondo

Spin = B Spin = B,F Spin = F

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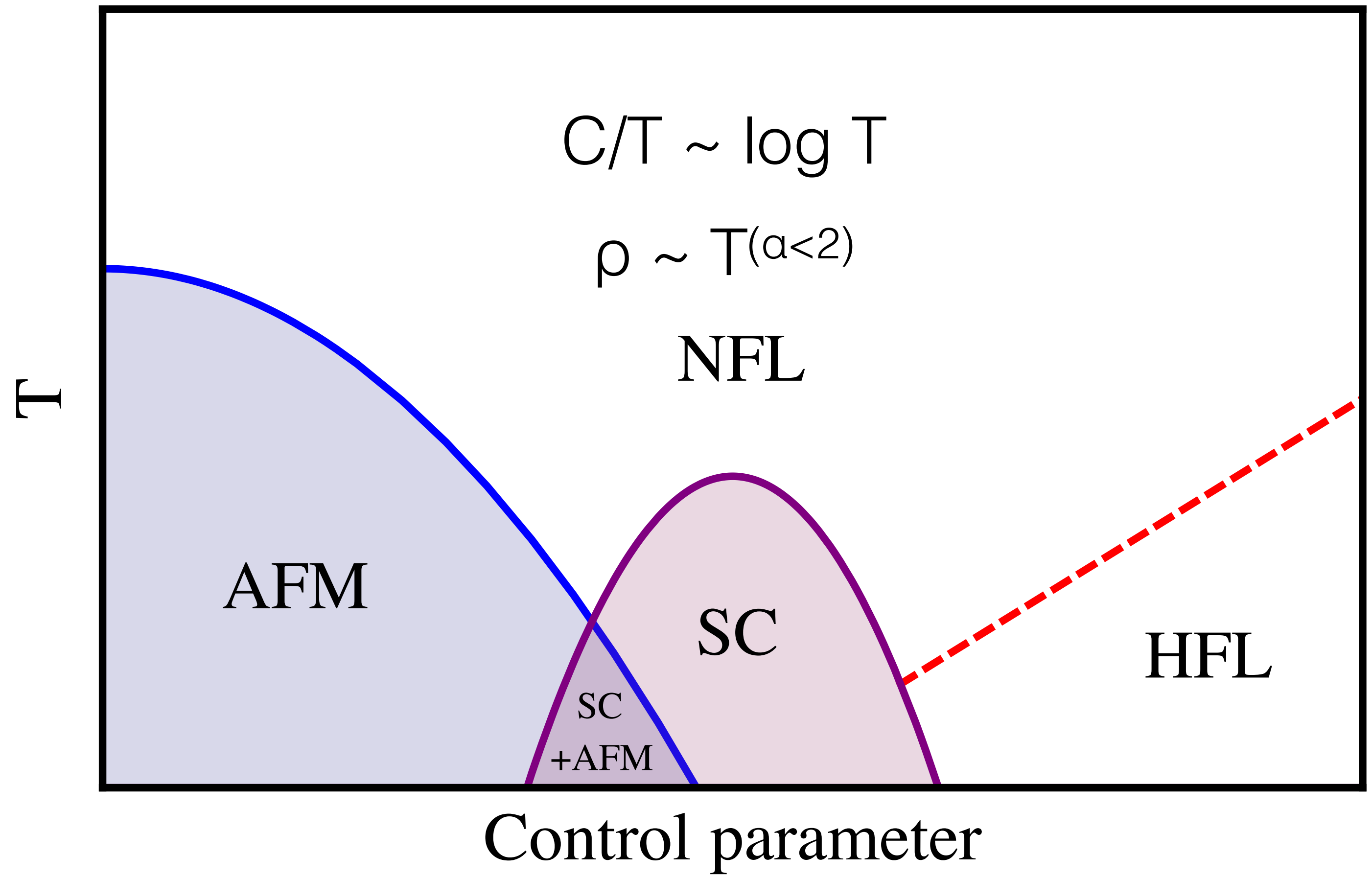
Strange Metal = Unbroken Susy?

Heavy Fermion Systems: Why Supersymmetric Spins?



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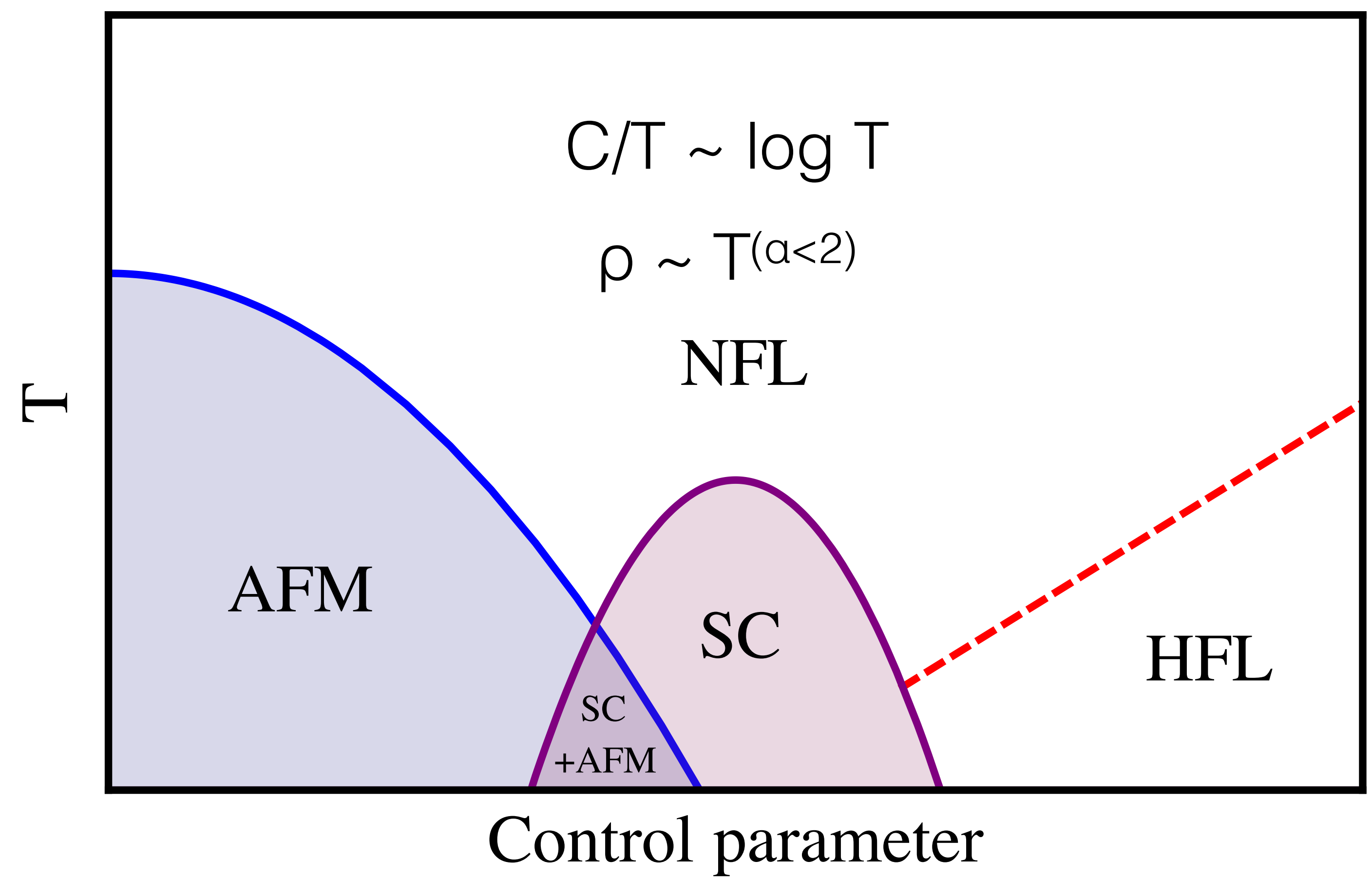


Bosonic

$\mathbf{S}_B = b^\dagger_\alpha \mathbf{\Gamma}_{\alpha\beta} b_\beta$

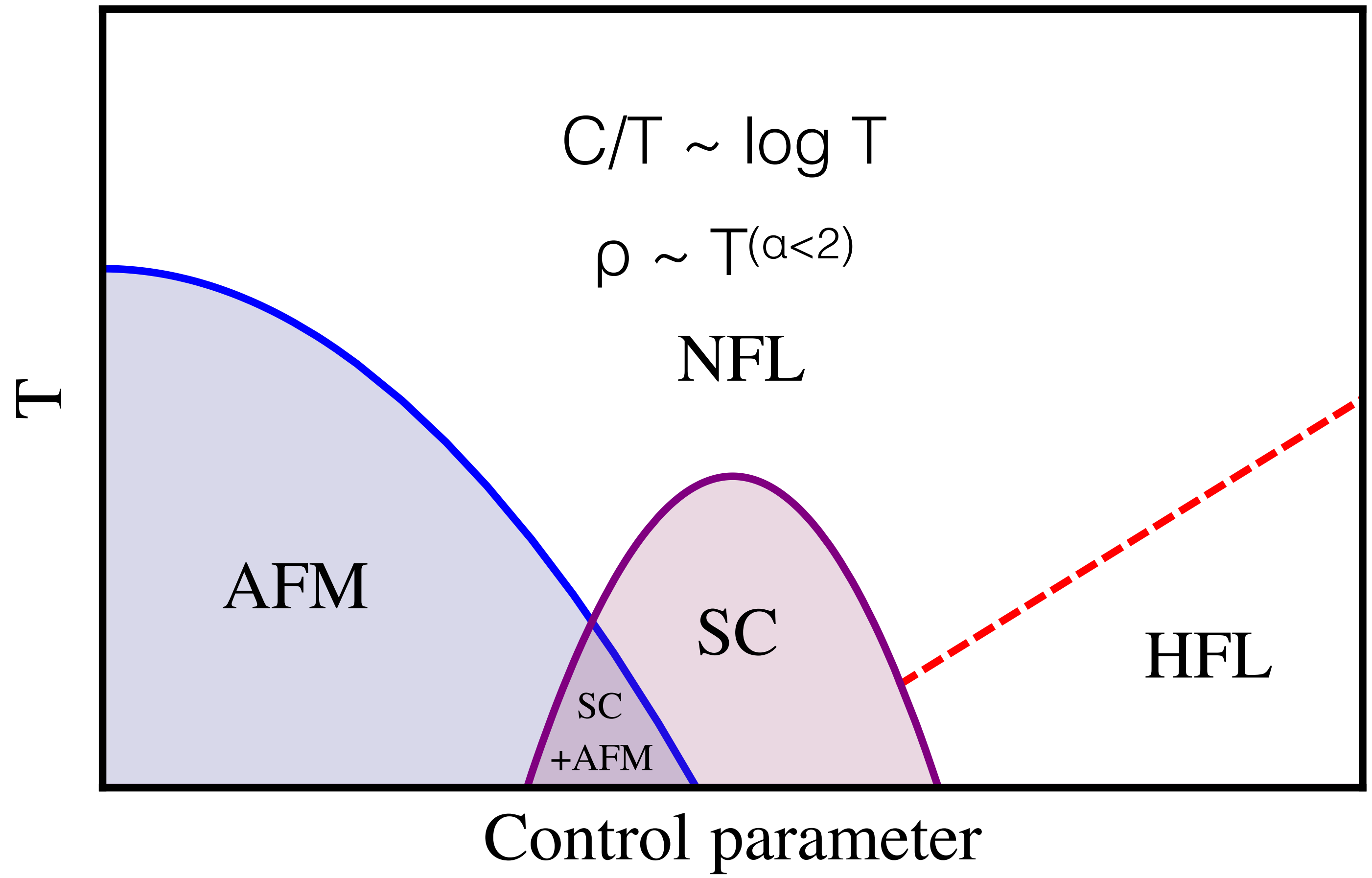
$|\Psi\rangle = P_G |\Psi_B\rangle$

Fermionic

$\mathbf{S}_F = f^\dagger_\alpha \mathbf{\Gamma}_{\alpha\beta} f_\beta$

$|\Psi\rangle = P_G |\Psi_F\rangle$

Heavy Fermion Systems: Why Supersymmetric Spins?



Bosonic

$$\mathbf{S}_B = b^\dagger_\alpha \mathbf{\Gamma}_{\alpha\beta} b_\beta$$

$$|\Psi\rangle = P_G |\Psi_B\rangle$$

Fermionic

$$\mathbf{S}_F = f^\dagger_\alpha \mathbf{\Gamma}_{\alpha\beta} f_\beta$$

$$|\Psi\rangle = P_G |\Psi_F\rangle$$

How to describe the generic HF phase diagram in its entirety?

Supersymmetric Spin

$$\mathbf{S} = f^\dagger_\alpha \mathbf{\Gamma}_{\alpha\beta} f_\beta + b^\dagger_\alpha \mathbf{\Gamma}_{\alpha\beta} b_\beta$$

$$|\Psi\rangle = P_G |\Psi_B\rangle \otimes |\Psi_F\rangle$$

Gan, Coleman and Andrei, 1992
Coleman, Pepin, Tsvelik, 2000

Symmetries of the SUSY-SP(N) Spin

$$\mathbf{S} = f^{\dagger}_{\alpha}\mathbf{\Gamma}_{\alpha\beta} f_{\beta} + b^{\dagger}_{\alpha}\mathbf{\Gamma}_{\alpha\beta} b_{\beta},$$

SP(N) generators

Spin commutes with the following operator bilinears:

$$\Psi_0 = n_b + N/2, \qquad n_b = b^{\dagger}_{\alpha} b_{\alpha}$$

$$\Psi_1 = \frac{\psi^{\dagger}+\psi}{2}, \qquad \psi = \tilde{\alpha} f_{\alpha} f_{-\alpha}$$

$$\Psi_2 = \frac{\psi^{\dagger}-\psi}{2i}, \qquad \psi^{\dagger} = \tilde{\alpha} f^{\dagger}_{-\alpha} f^{\dagger}_{\alpha}$$

$$\Psi_3 = n_f - N/2, \qquad n_f = f^{\dagger}_{\alpha} f_{\alpha}$$

$$X_1 = \theta + \eta, \qquad \theta = b^{\dagger}_{\alpha} f_{\alpha}$$

$$X_2 = \theta - \eta, \qquad \eta = \tilde{\alpha} f_{\alpha} b_{-\alpha}$$

“Super-Algebra”: SU(2|1)

Ψ_0 — **U(1)**

$$[\Psi_i, \Psi_j] = 2i\epsilon_{ijk}\Psi_k \qquad i,j,k = \{1,2,3\}$$ — **SU(2)**

(**Even**: U(1) and SU(2))

$$\{X_i, X^{\dagger}_j\} = 2(\Psi_0 \delta_{ij} + \Psi_3 a_{ij}),$$

$$\{X_i, X_j\} = 0, \qquad i,j = \{1,2\}$$

— **Odd**

Results

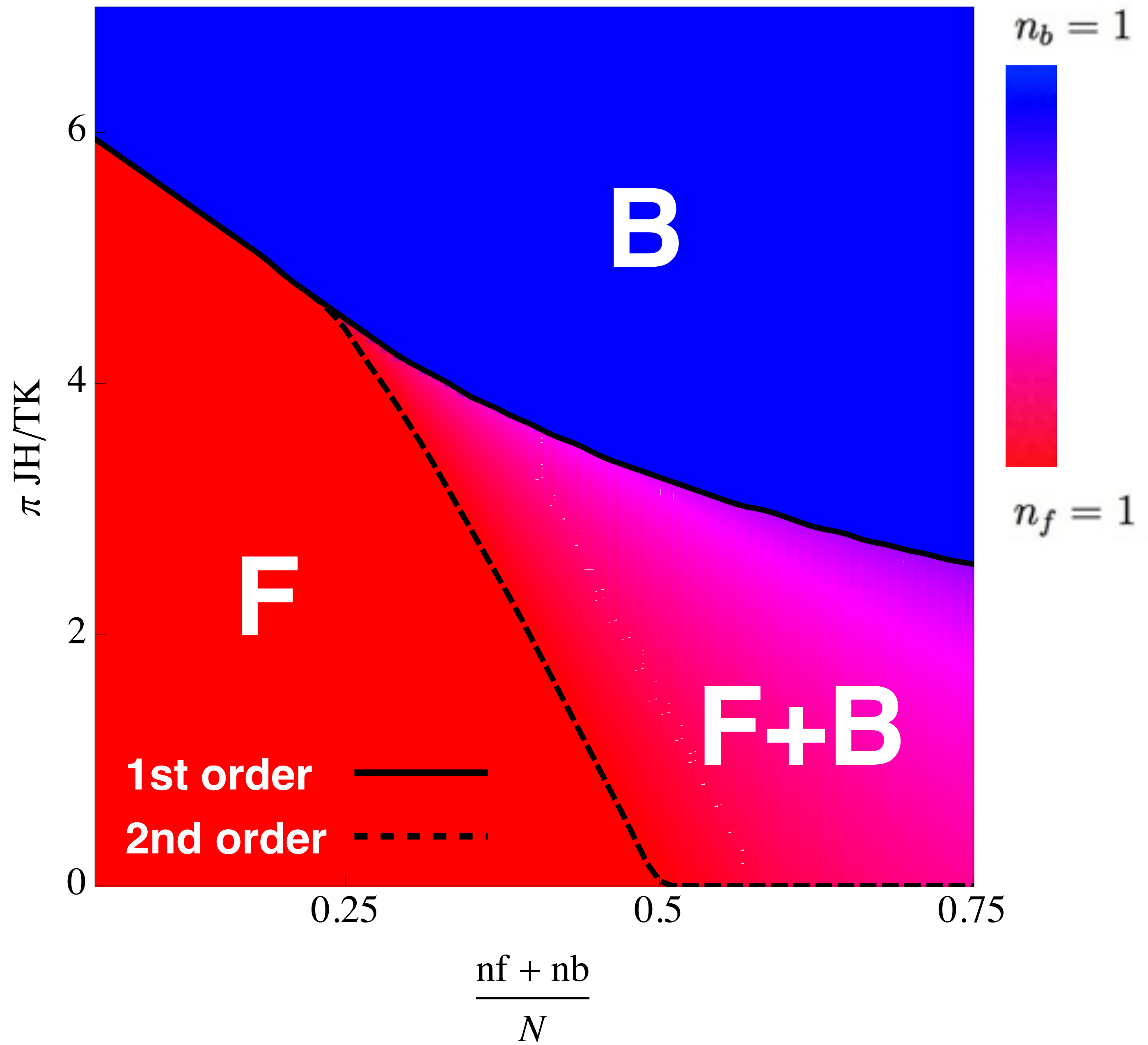
Within a static mean field solution the free energy have the following closed form:

$$F = -2\sin(\pi n_f) - \frac{\pi J_H}{T_K}(q_0 - n_f)(q_0 - n_f + 1)$$

$$n_f + n_b = q_0$$

The energy will be minimized by different representations in different areas of the phase diagram

- ✦ **F+B Phase → Coexistence;**
- ✦ **2nd order transition F → F+B;**
- ✦ **Fermionic modes go soft;**
- ✦ **Unusual critical behavior;**



Open Challenges.

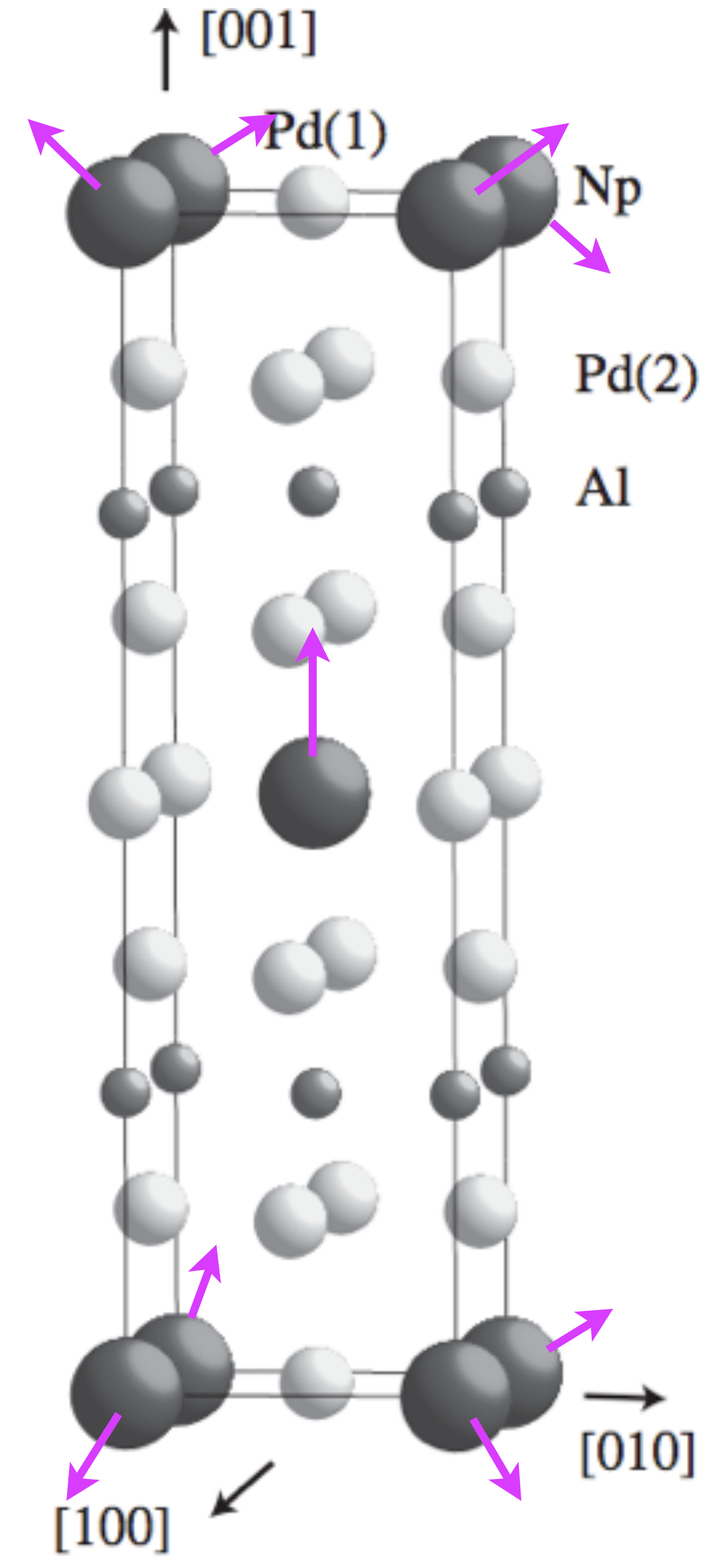
- QCPs: Explanation of universality of

$C/T \sim \mathrm{Log}(T_0/T)$, $\rho \sim T^{1+\alpha}$?

- Co-existence heavy fermions & LM AFM = Two fluid behavior? [Supersymmetry? B/F]

Heavy Fermion Superconductivity
The Nature of Magnetic Pairing.

The remarkable case of $NpPd_5Al_2$



4.5K Heavy Fermion S.C

$NpAl_2Pd_5$

Aoki et al 2007

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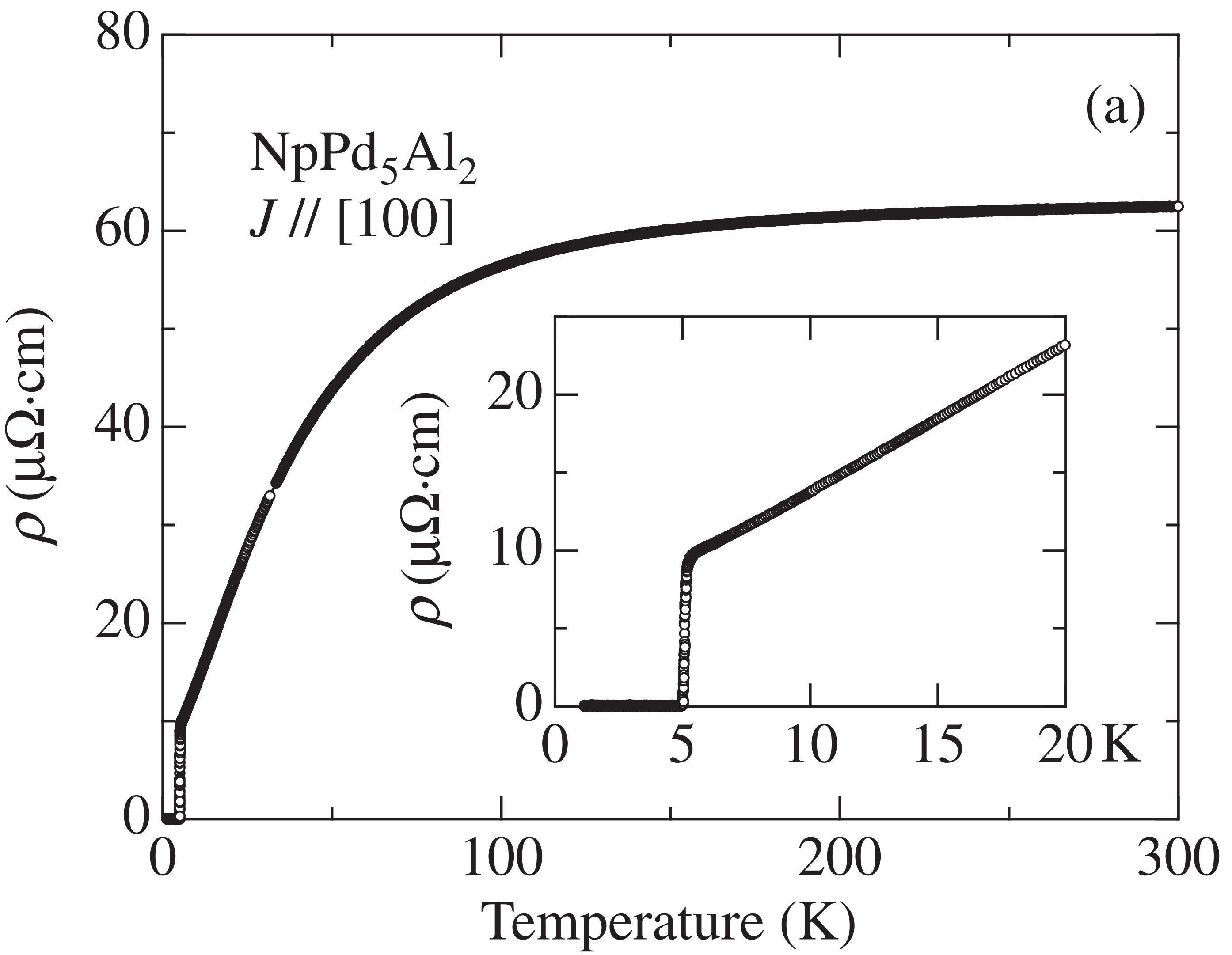

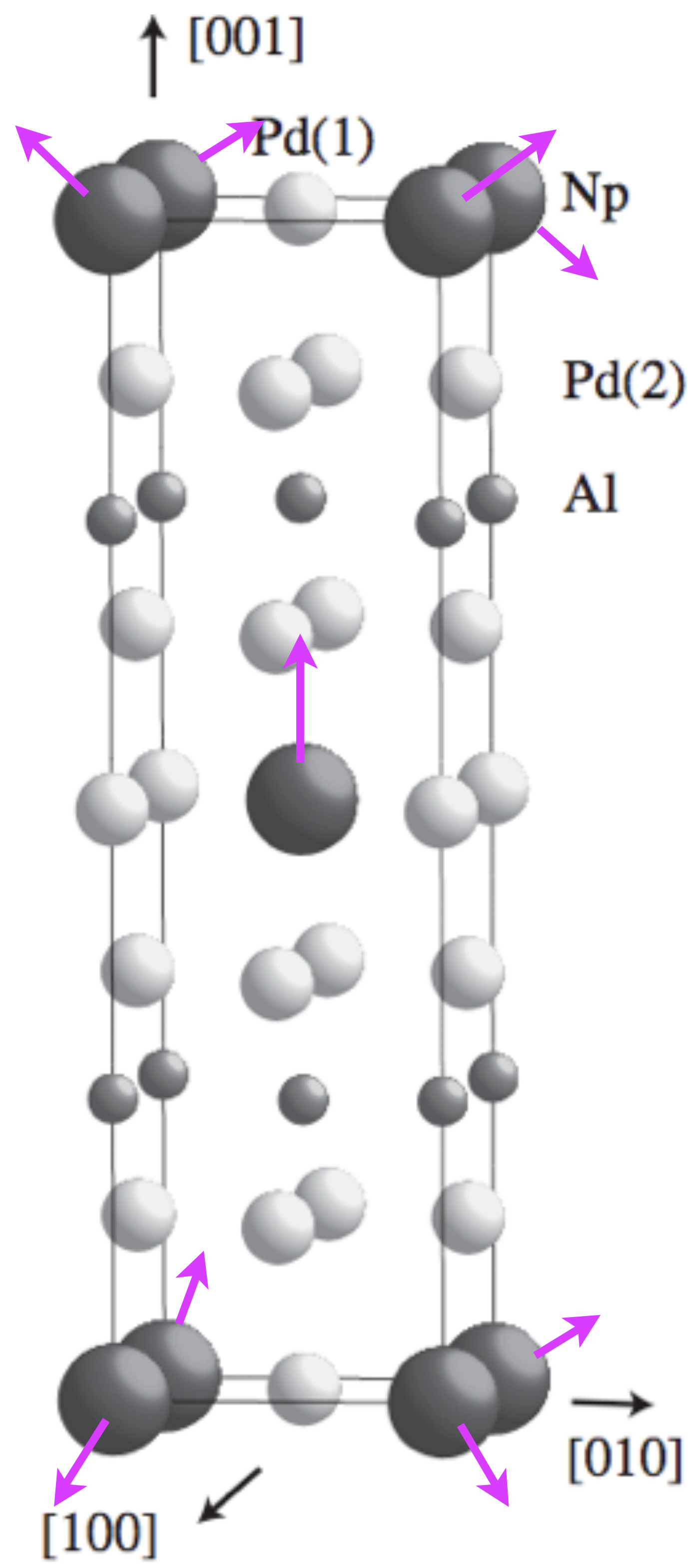

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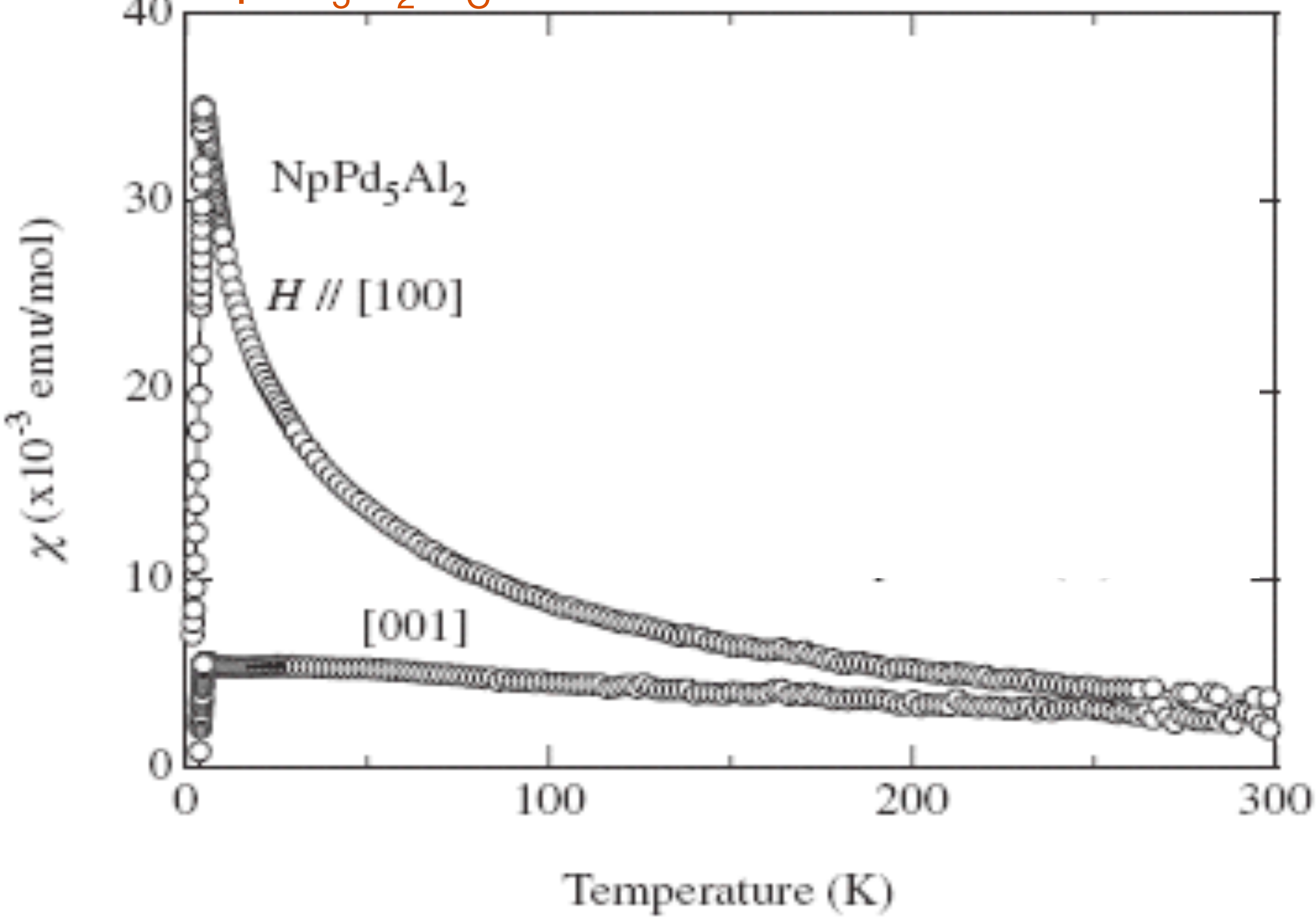
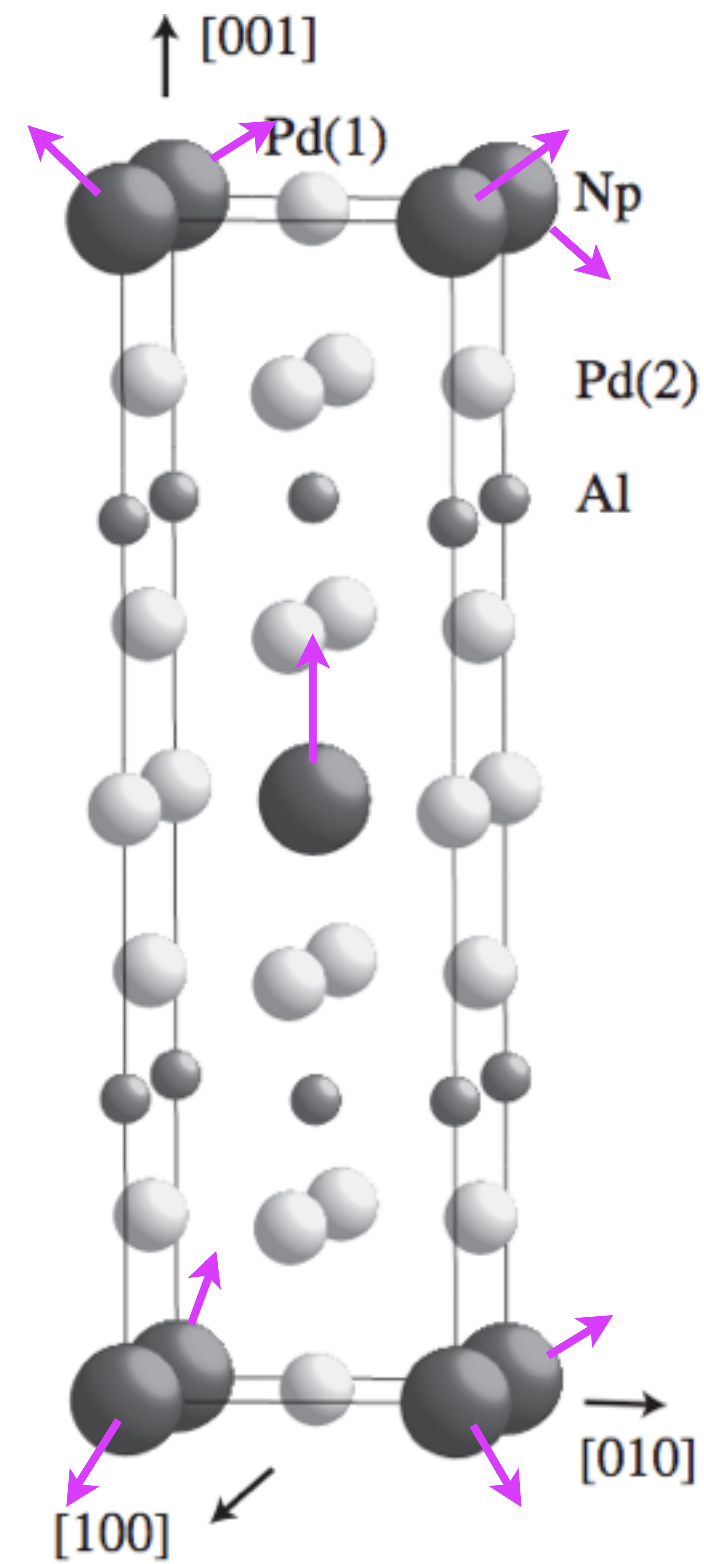
$NpPd_5Al_2$ T_C = 4.5K





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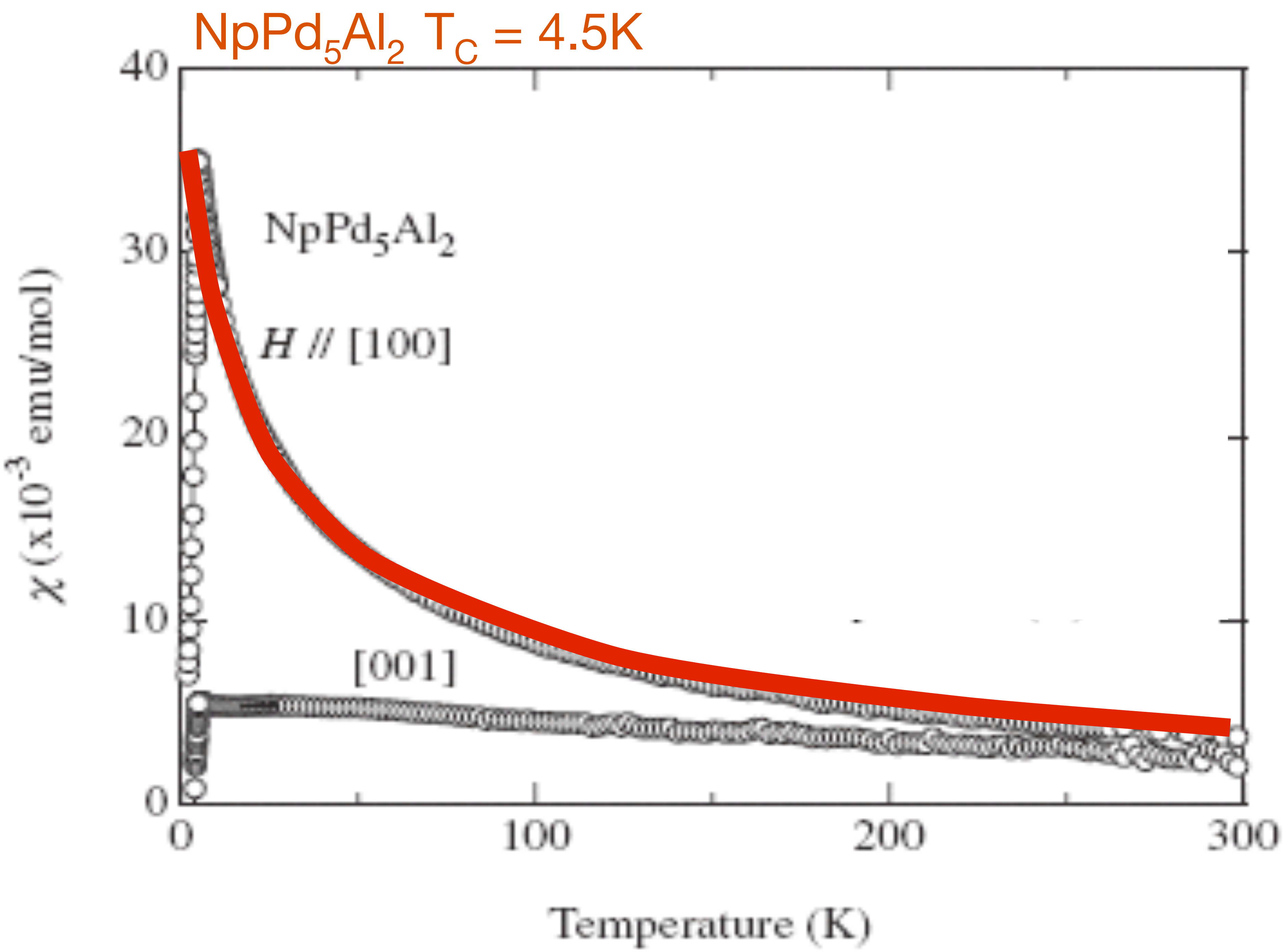

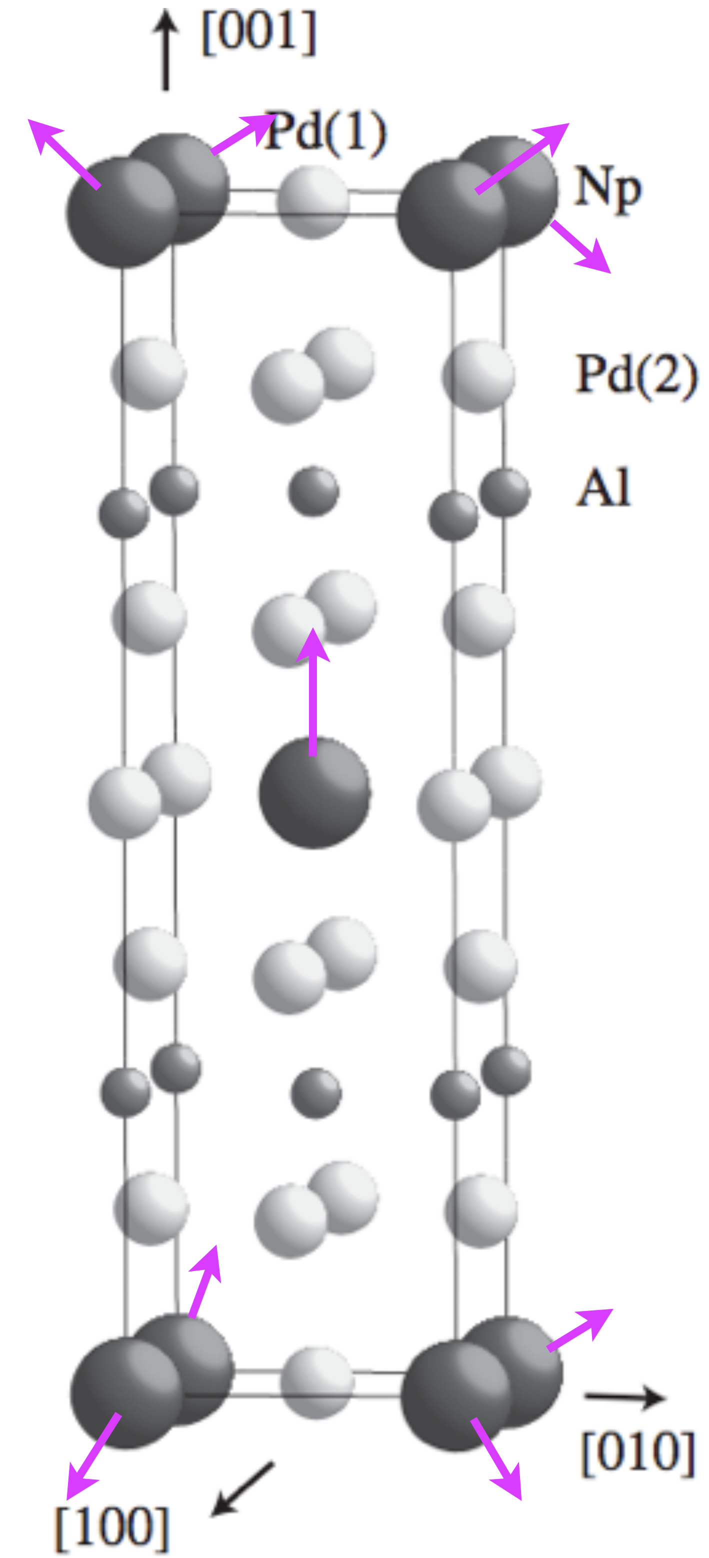

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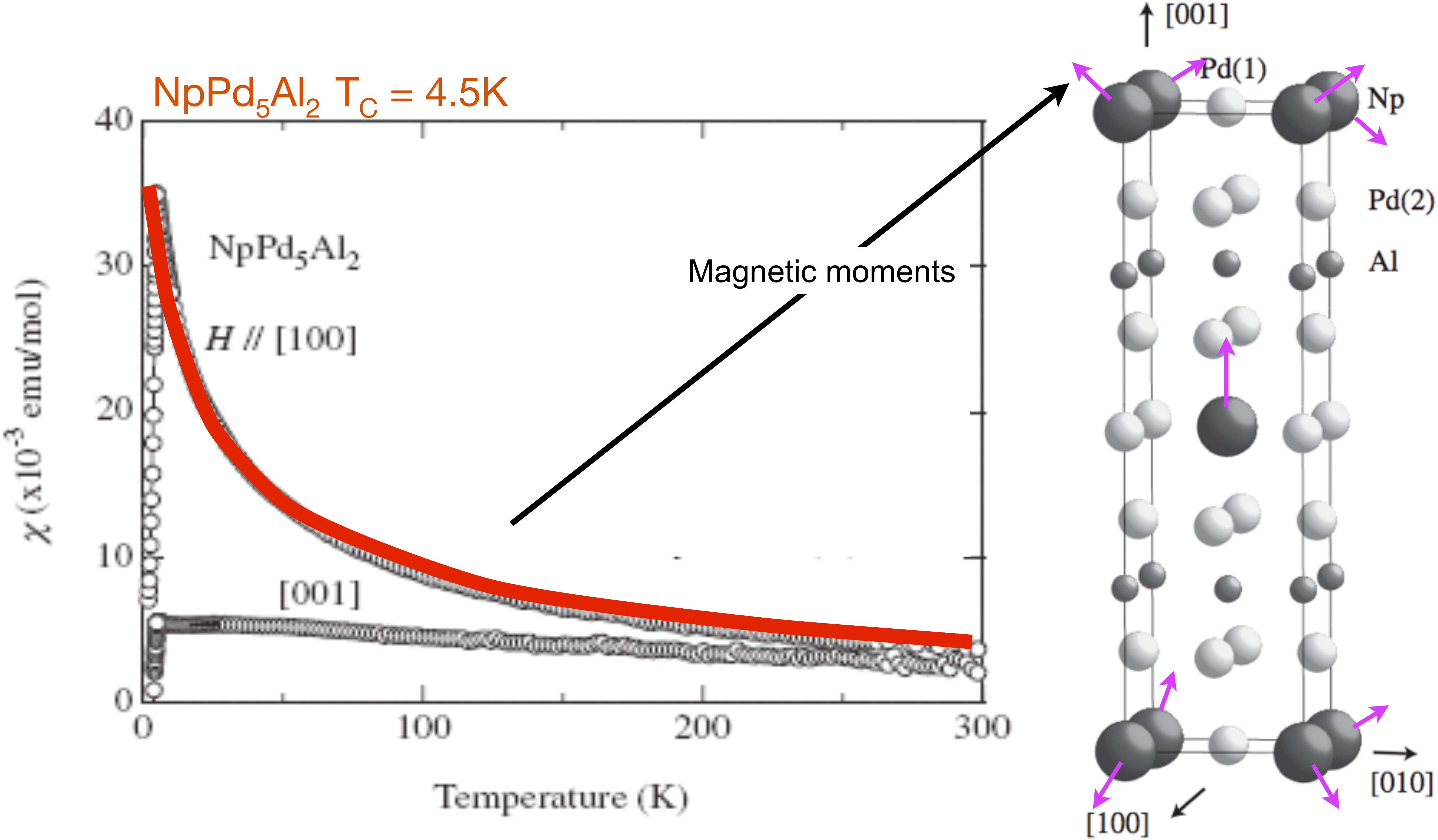

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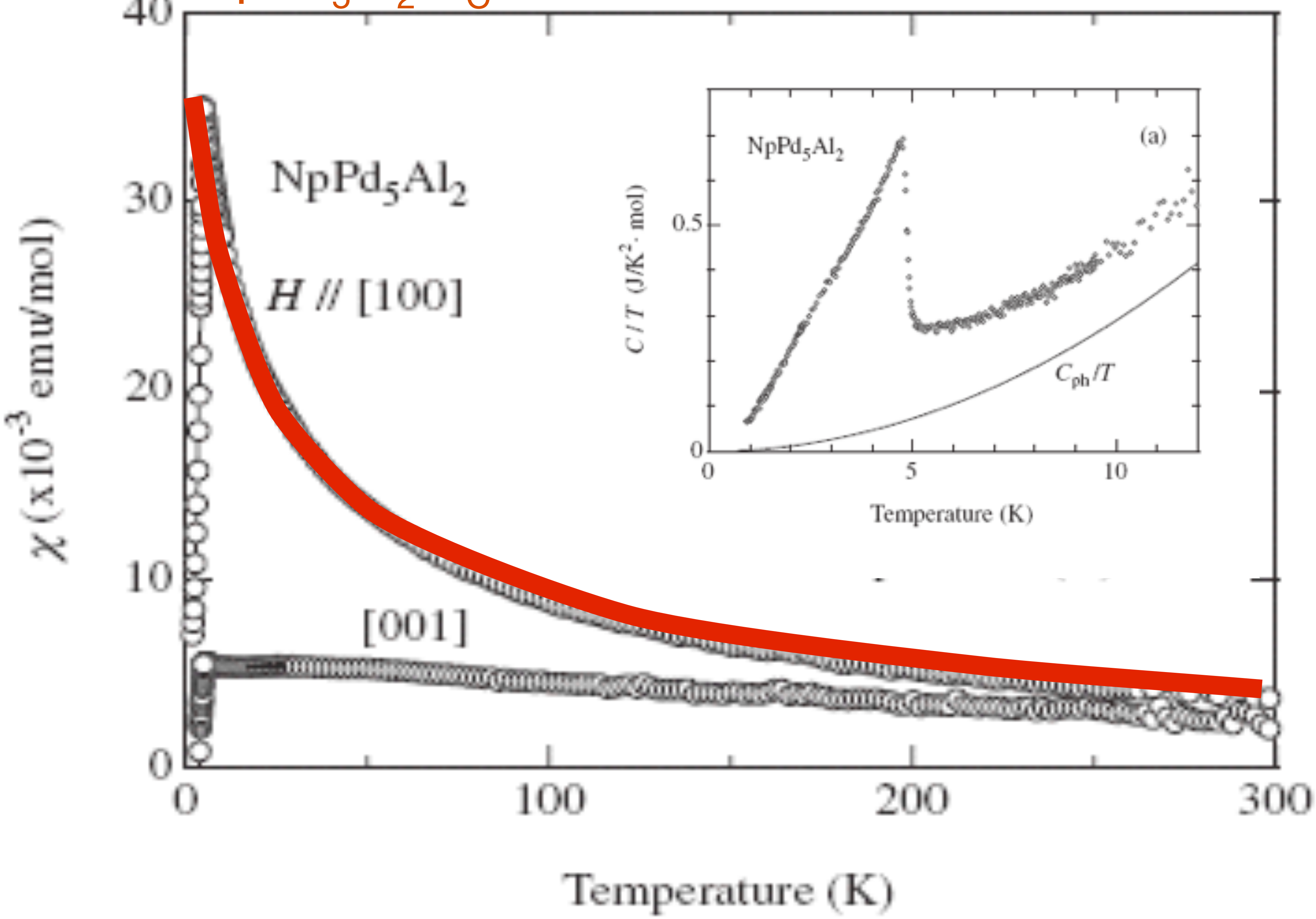

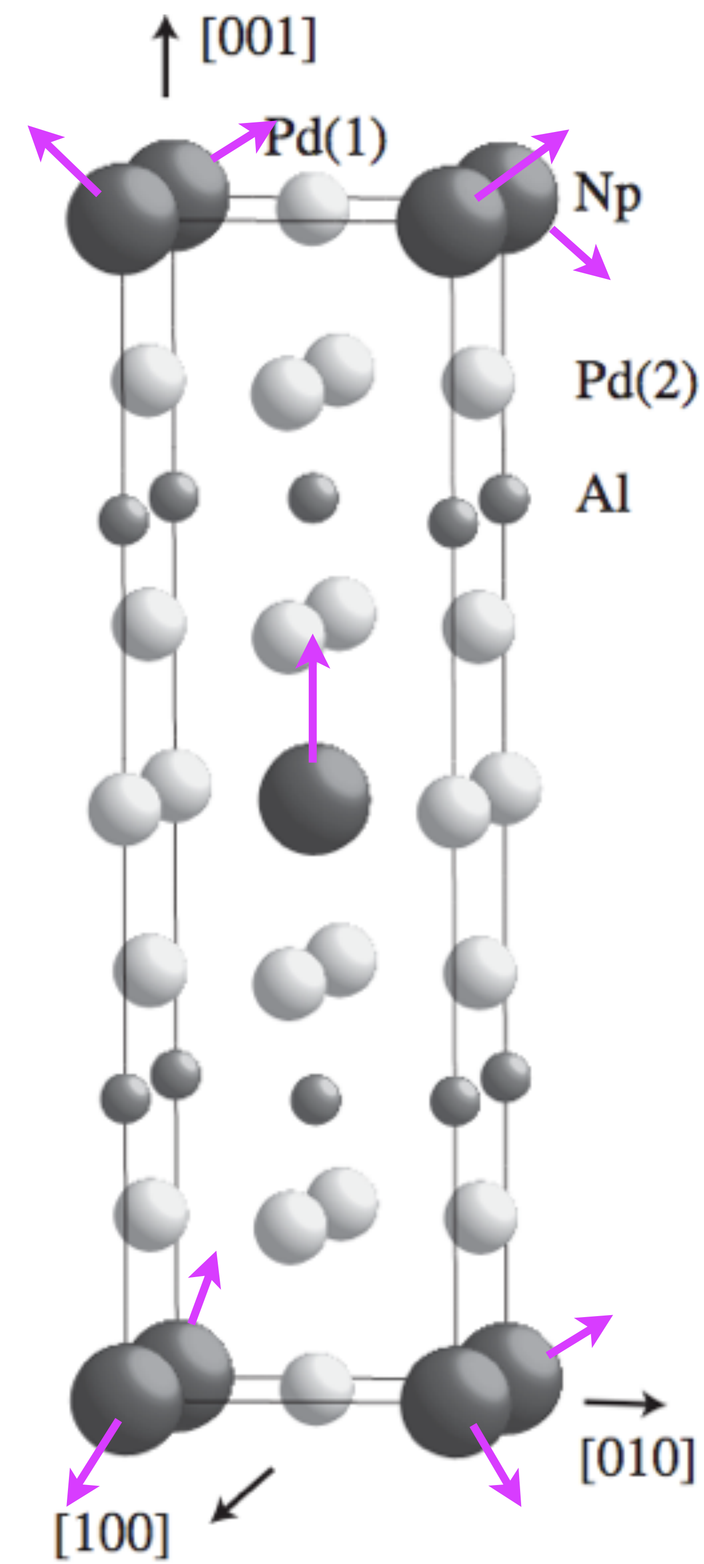

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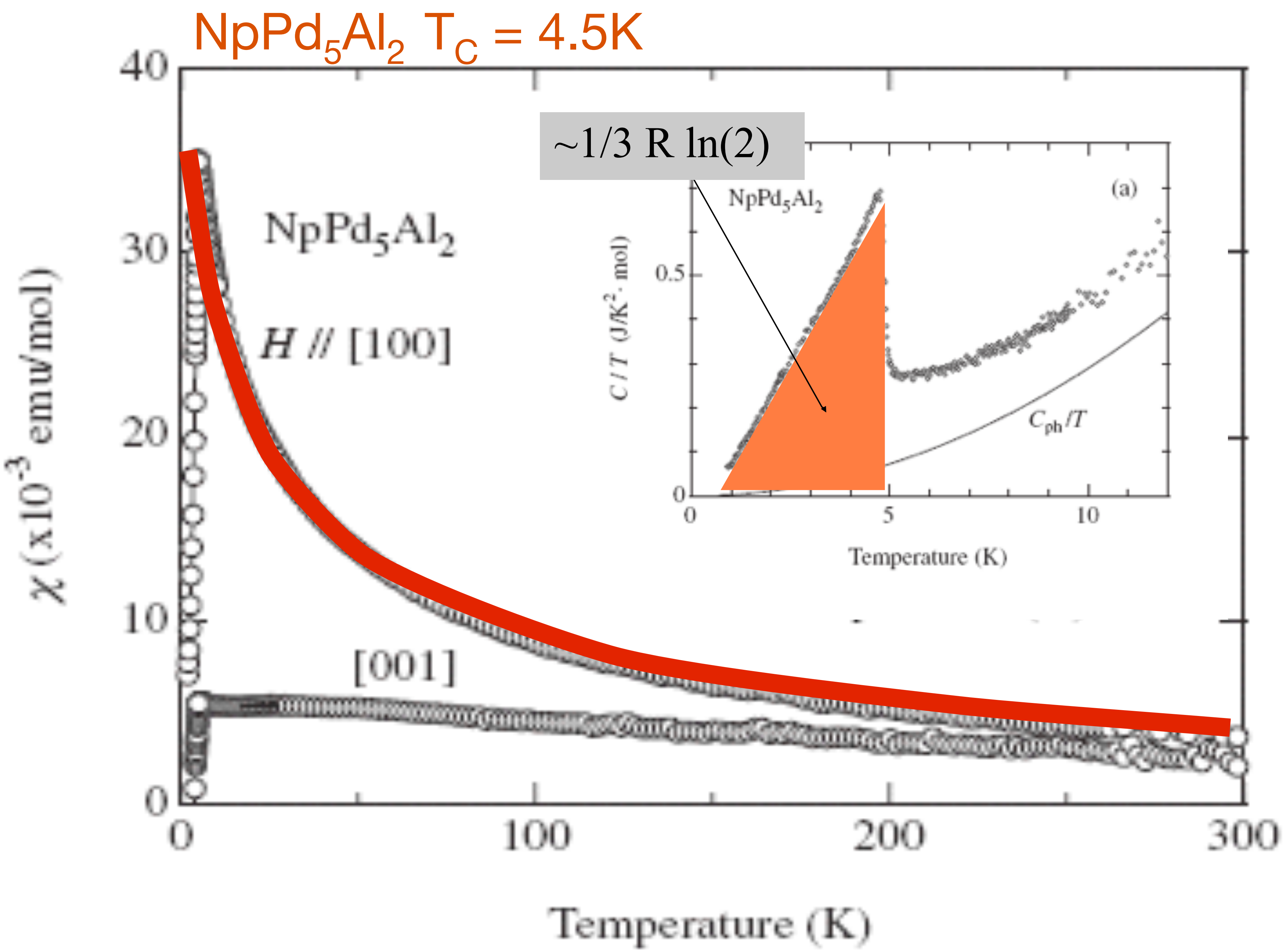

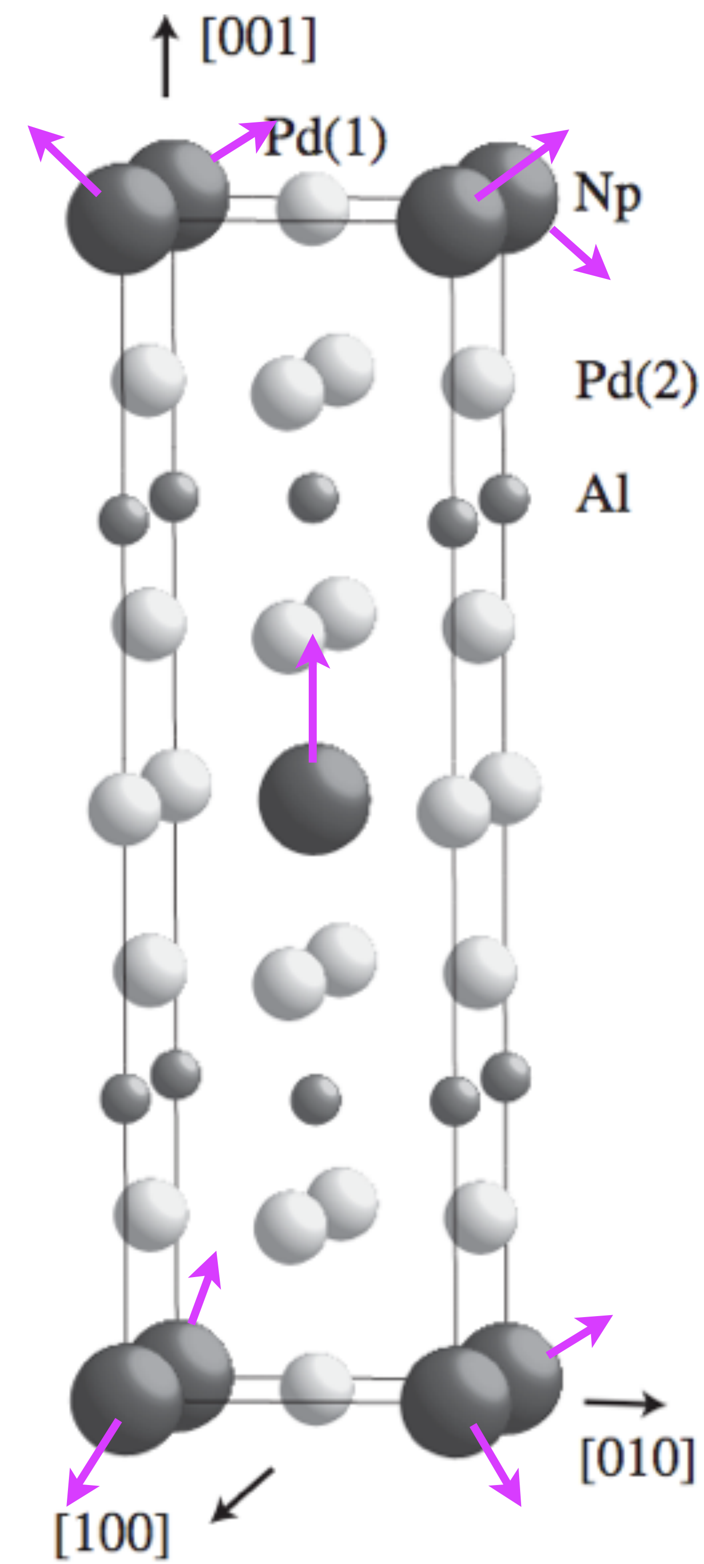

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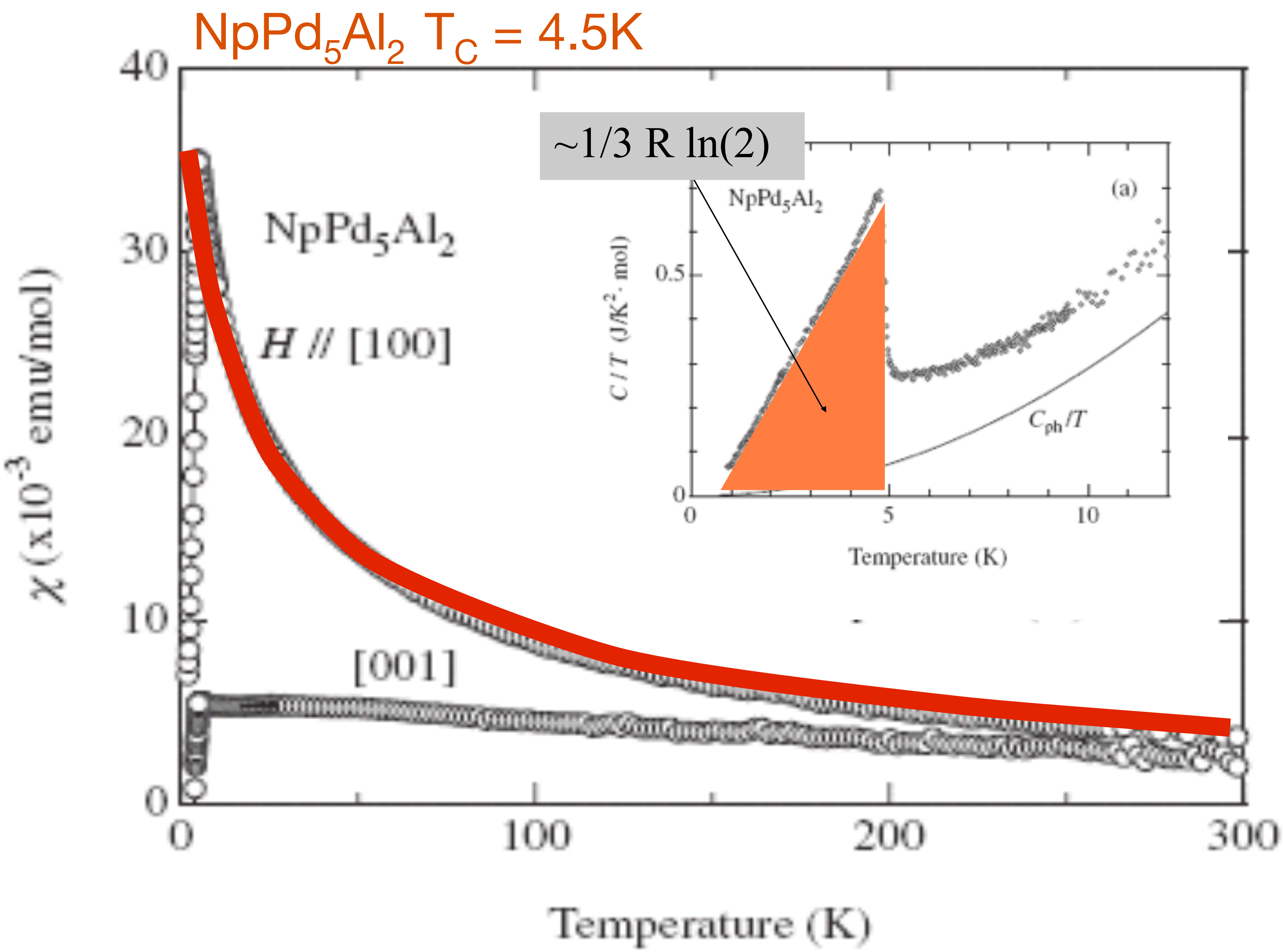

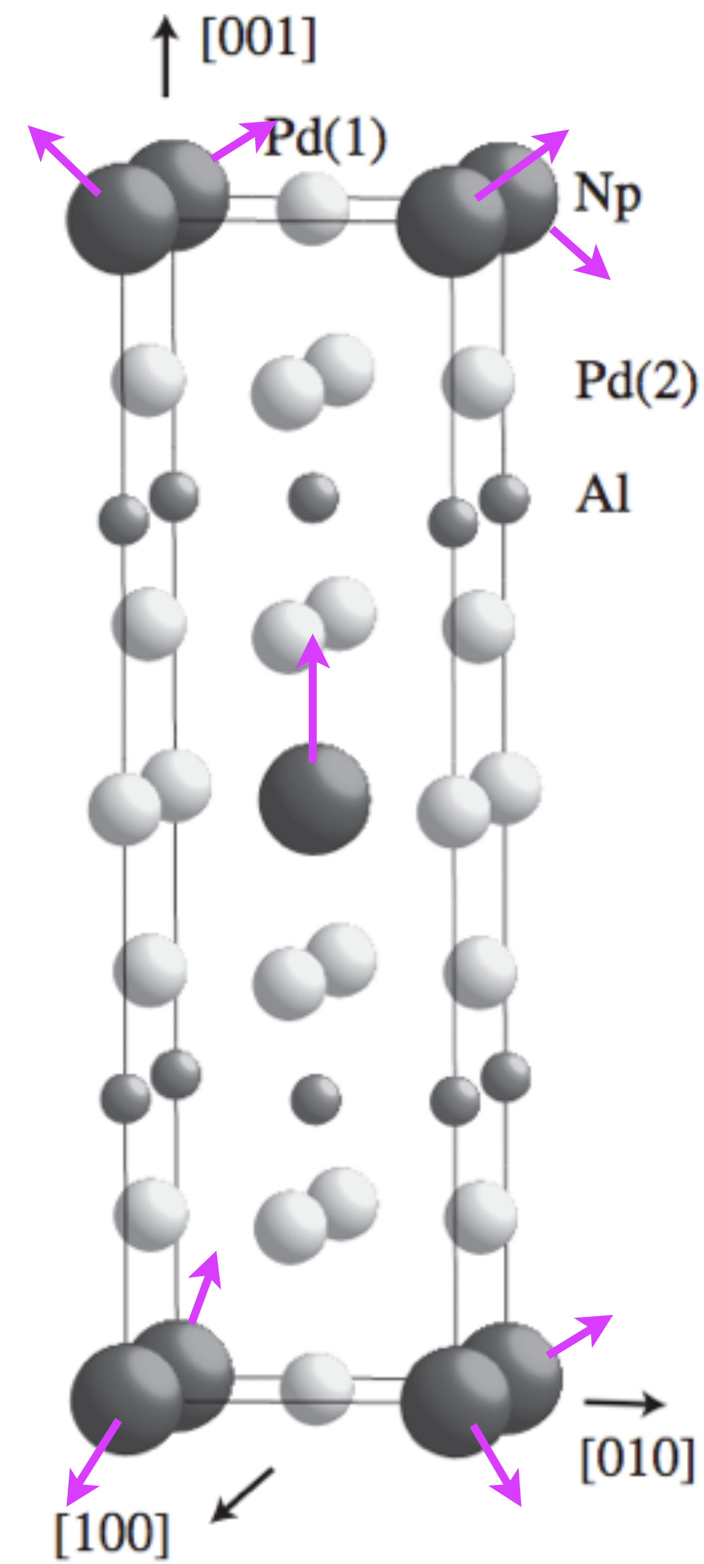

How does the spin form the condensate?

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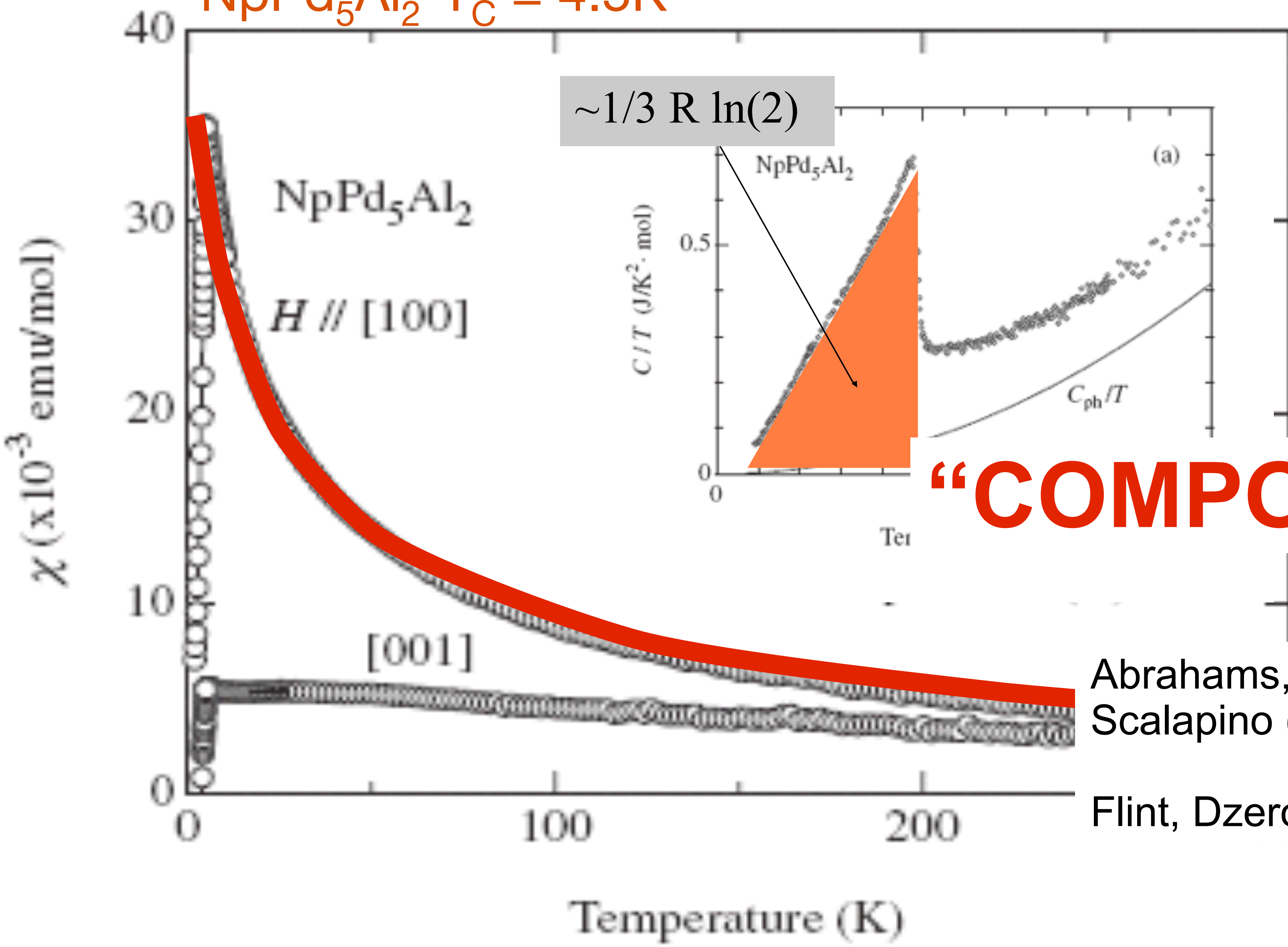
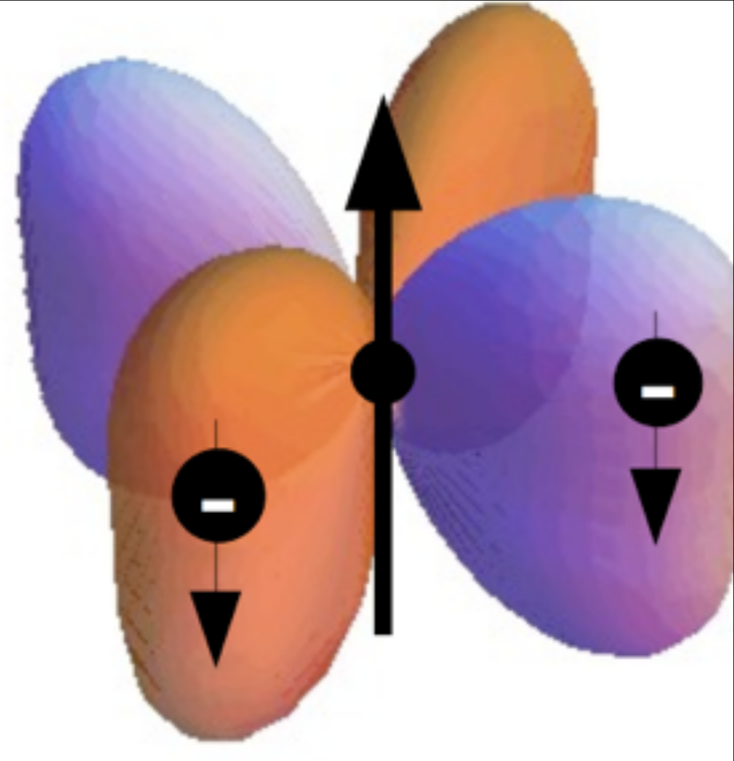




Abrahams, Balatsky, Schrieffer and Scalapino (1994)

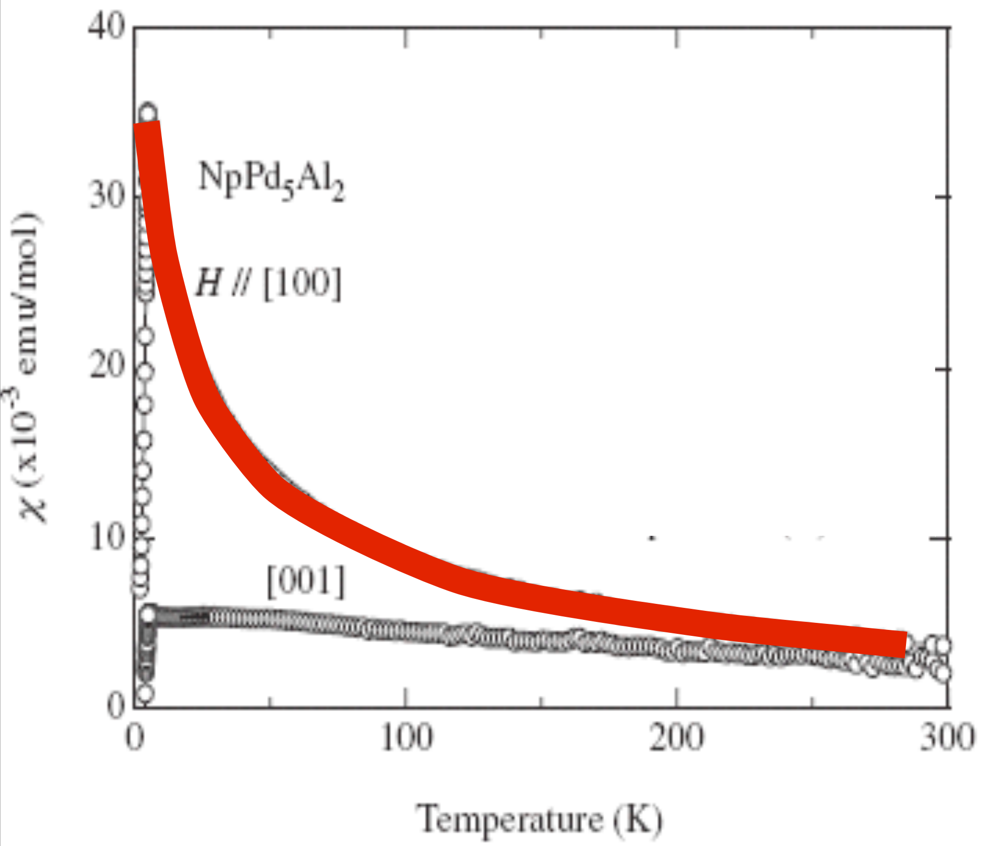
Flint, Dzero, Coleman (2008)

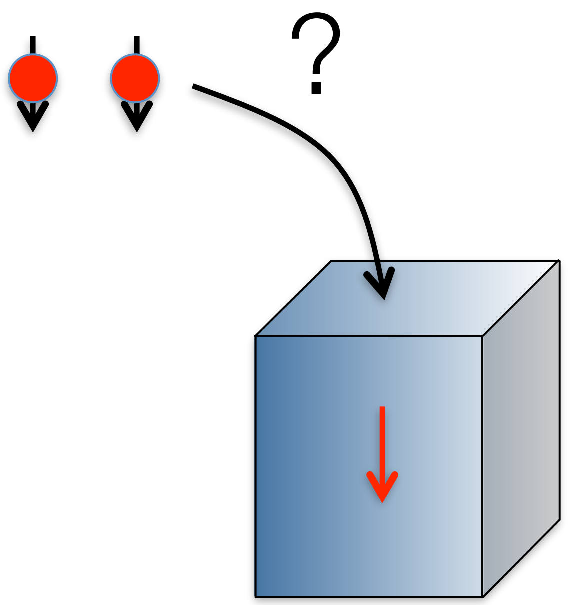
How does the spin form the condensate?

4.5K Heavy Fermion S.C
$NpAl_2Pd_5$
Aoki et al 2007

Composite pairing

$NpPd_5Al_2$ $T_C = 4.5K$





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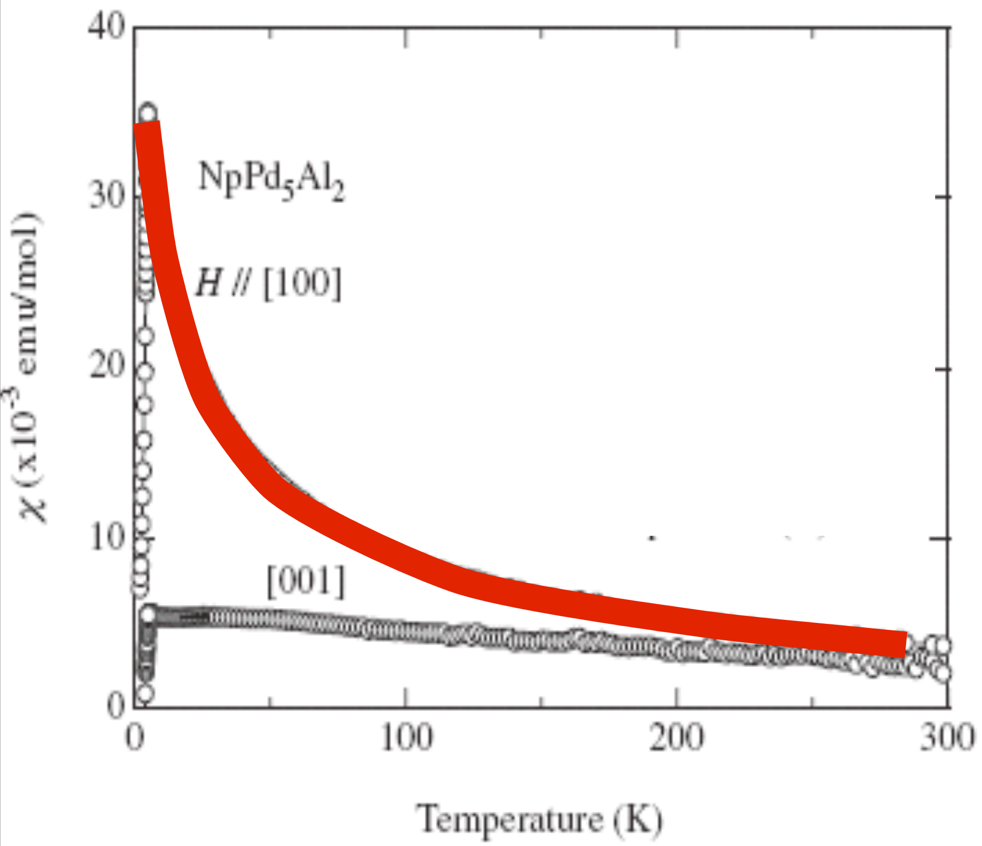

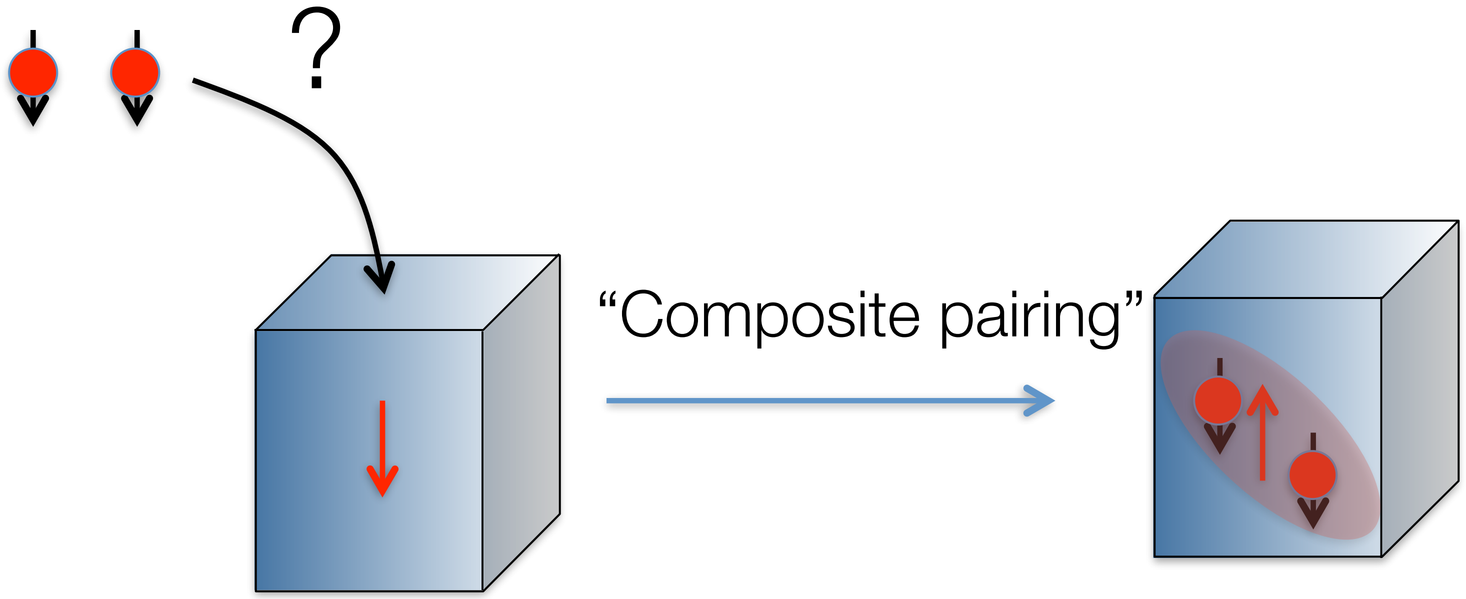

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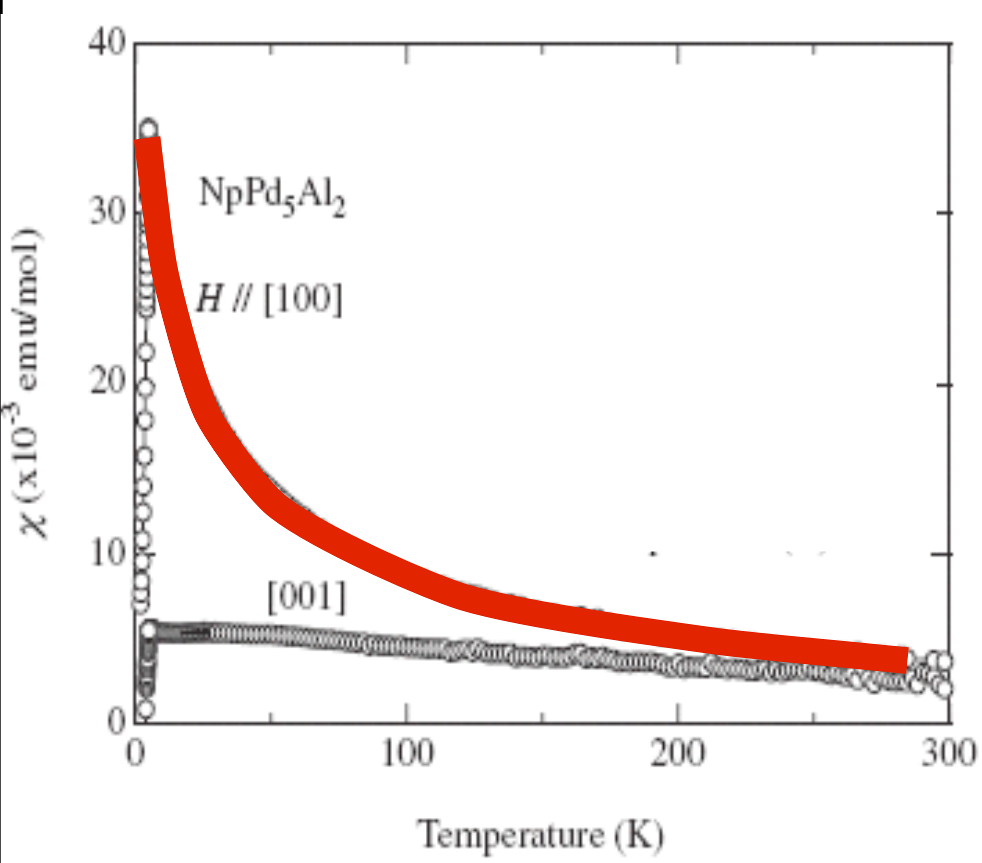

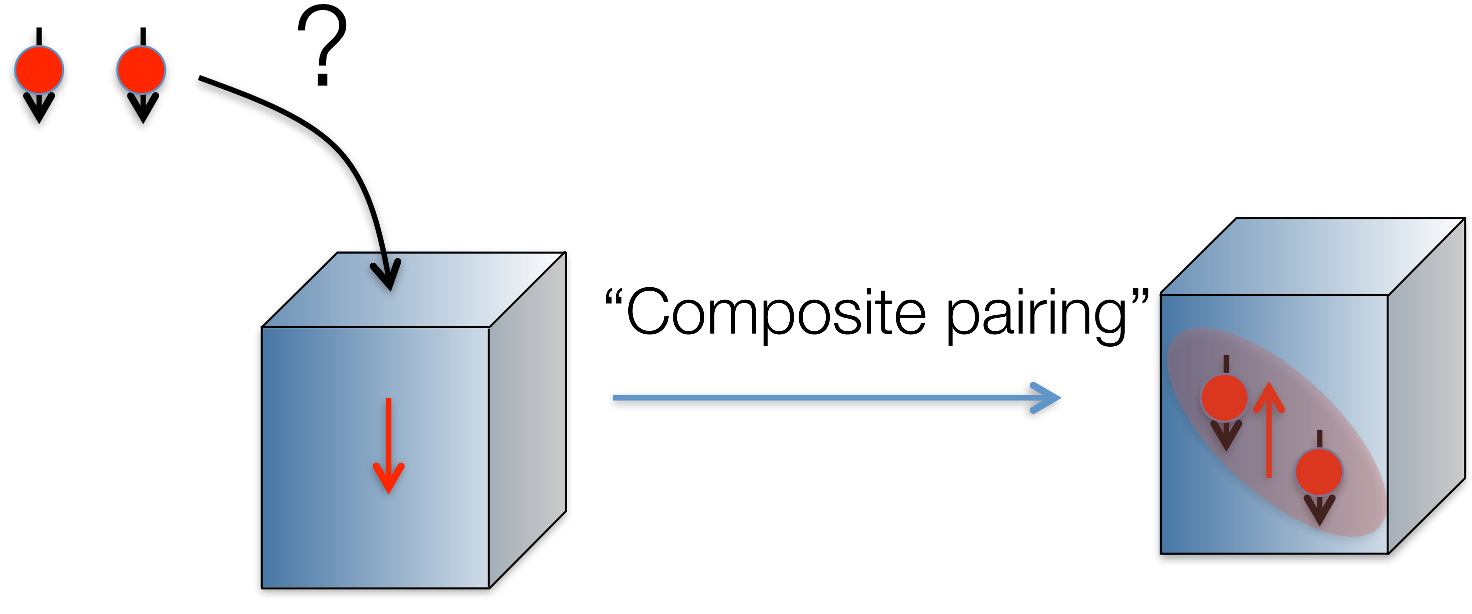

Heavy Cooper pair = (pair x spinflip)

Composite pairing

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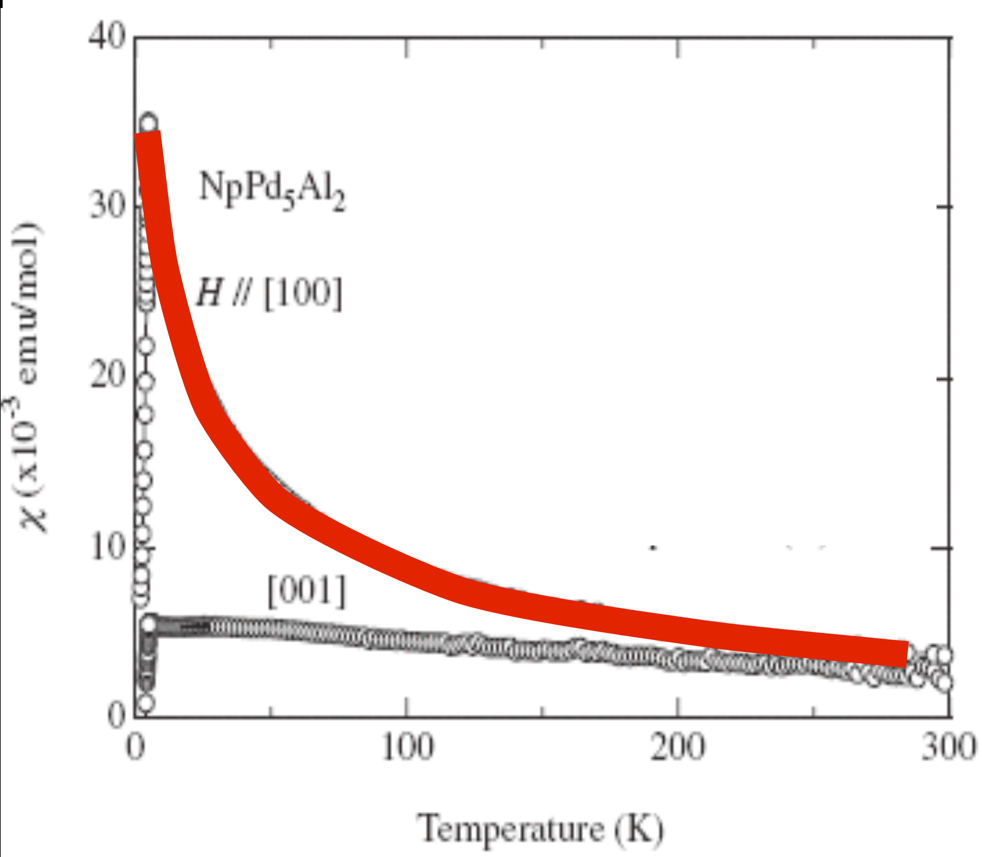

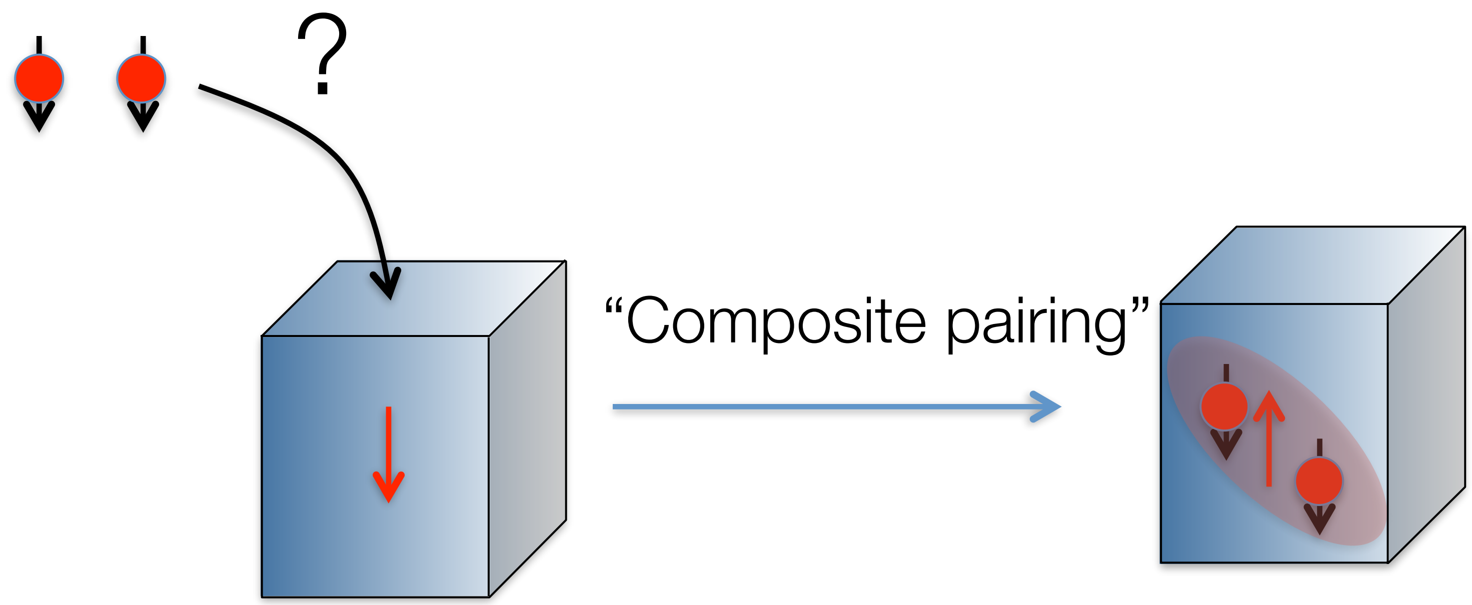

Heavy Cooper pair = (pair x spinflip)

$$\Psi^\dagger = c^\dagger_{1\downarrow} c^\dagger_{2\downarrow} S_+$$

Composite pairing

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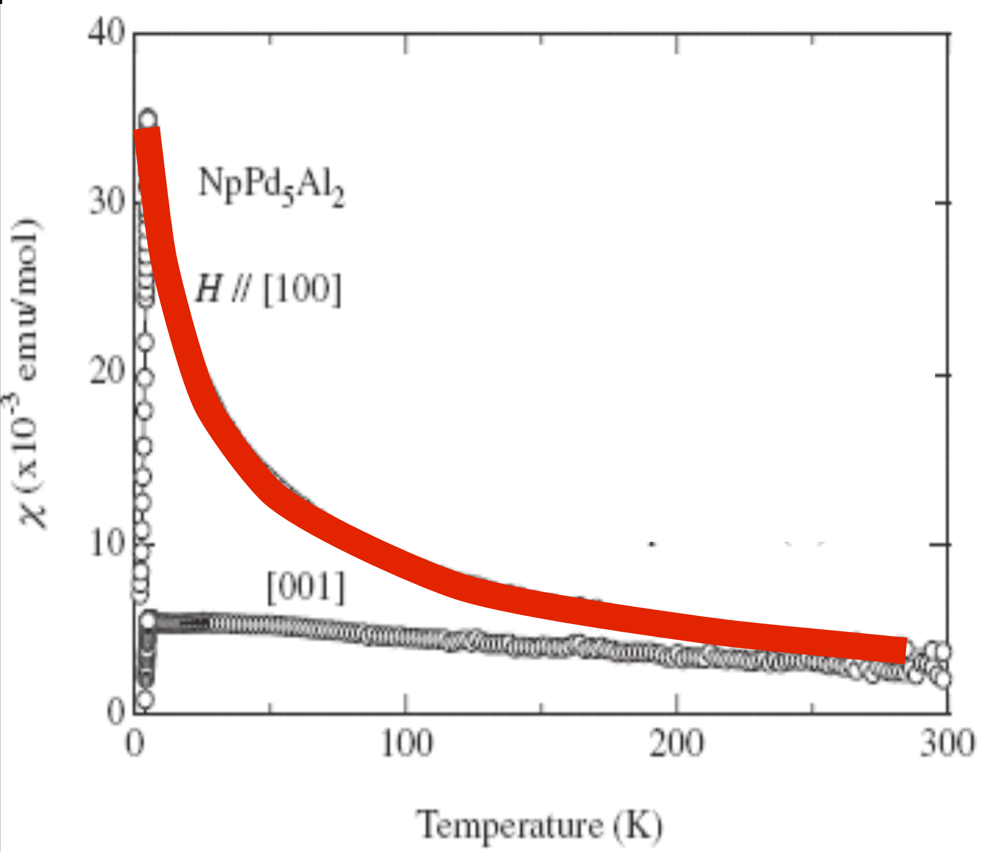

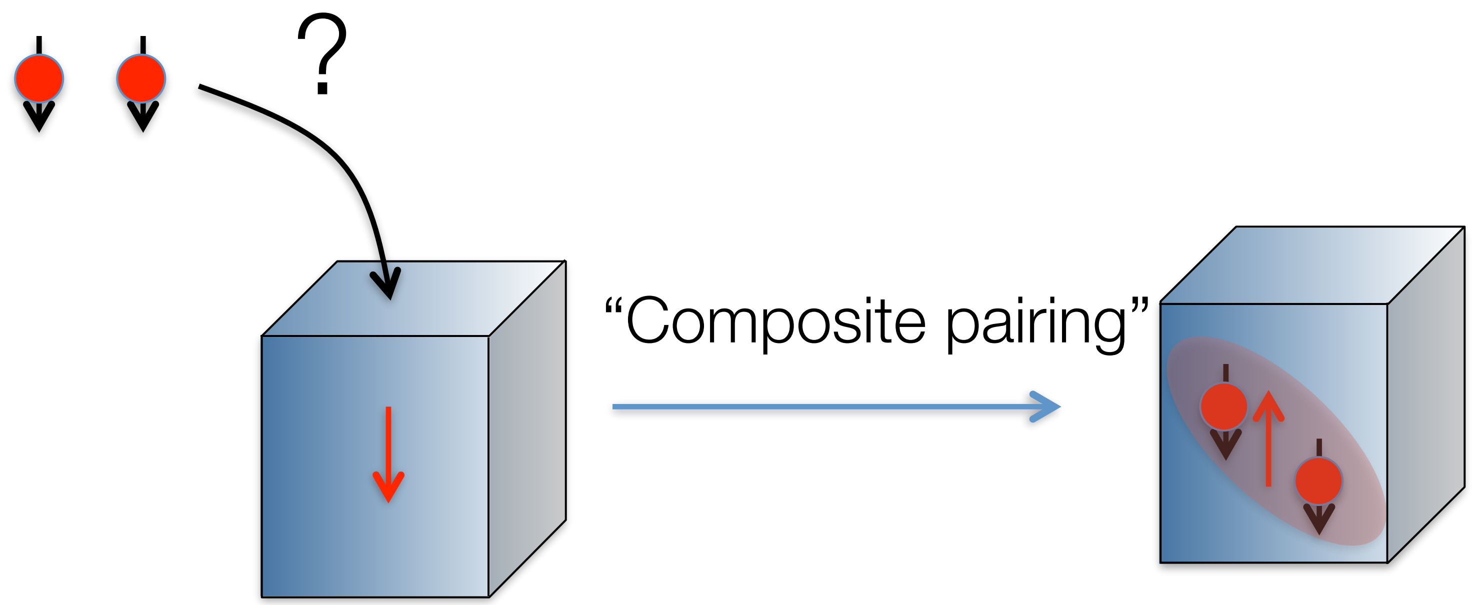

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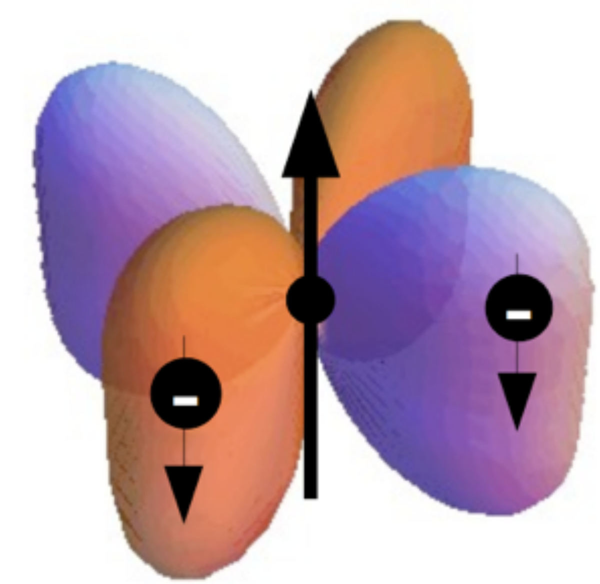


Abrahams, Balatsky, Scalapino, Schrieffer 1995

A solvable model of composite pairing.

PC, Tsvelik, Kee, Andrei PRB 60, 3605 (1999).

Flint, Dzero, PC, Nature Physics 4, 643 (2008).

Flint, PC, PRL, 105, 246404 (2010).

Flint, Nevidomskyy, PC, PRB 84, 064514 (2011).

$$H = \sum_k \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{1}{N} \sum_{\mathbf{k},\mathbf{k}'} \left(J_1 \psi^\dagger_{1a}(j) \psi_{1b}(j) + J_2 \psi^\dagger_{2a}(j) \psi_{2b}(j) \right) S^{ba}(j)$$

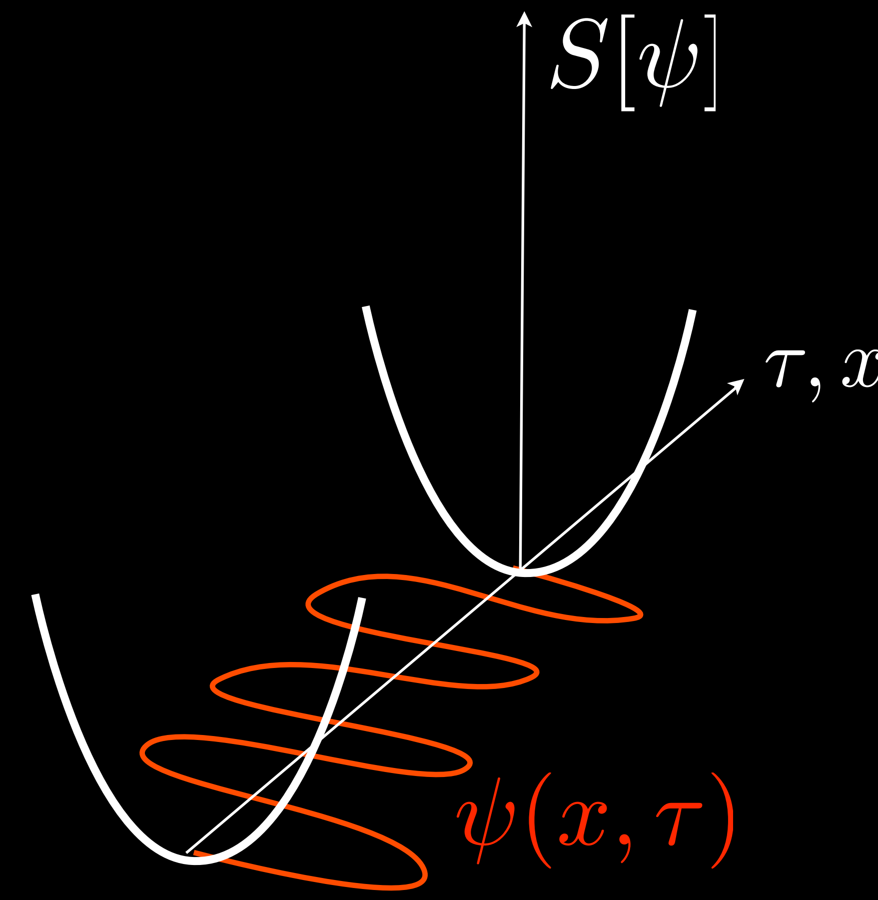
Single FS, two channels.

$$\psi_\Gamma(j) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \gamma_{\Gamma\mathbf{k}} \quad c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}_j}$$

cf Cox, Pang, Jarell (96)
PC, Kee, Andrei, Tsvelik (98)

Path Integral

$$Z = \int_{\text{Fields}} e^{-\quad S[\psi]}$$



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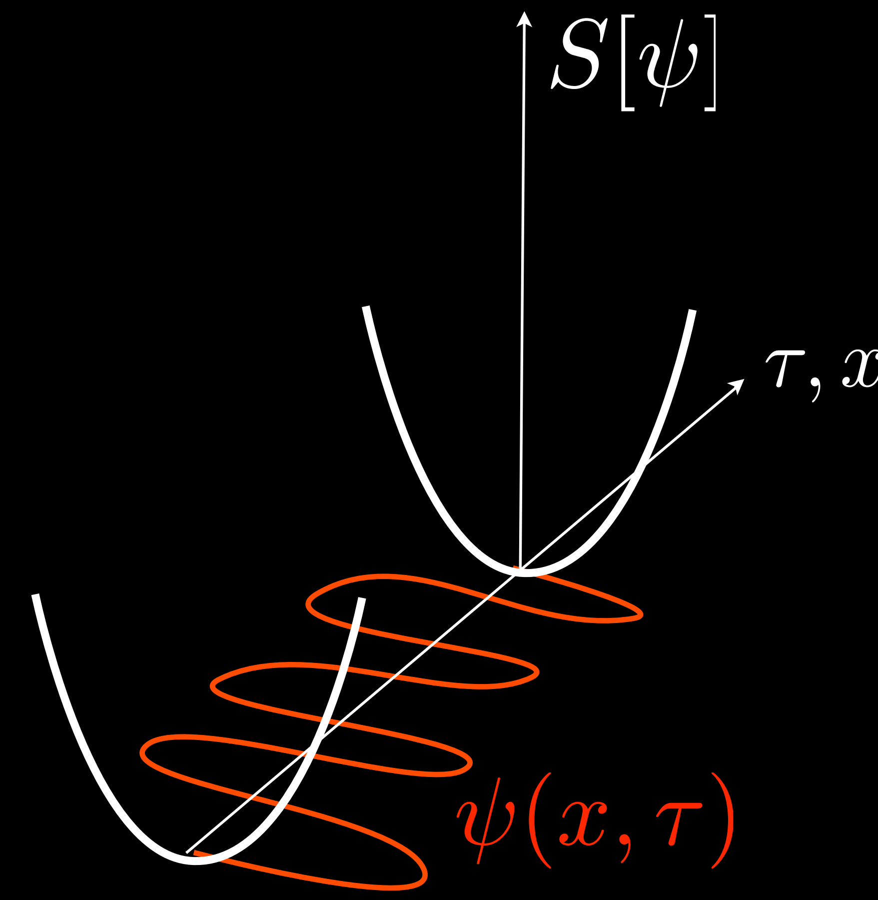

Wild quantum fluctuations!

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How can we tame the wild Quantum fluctuations?

Path Integral

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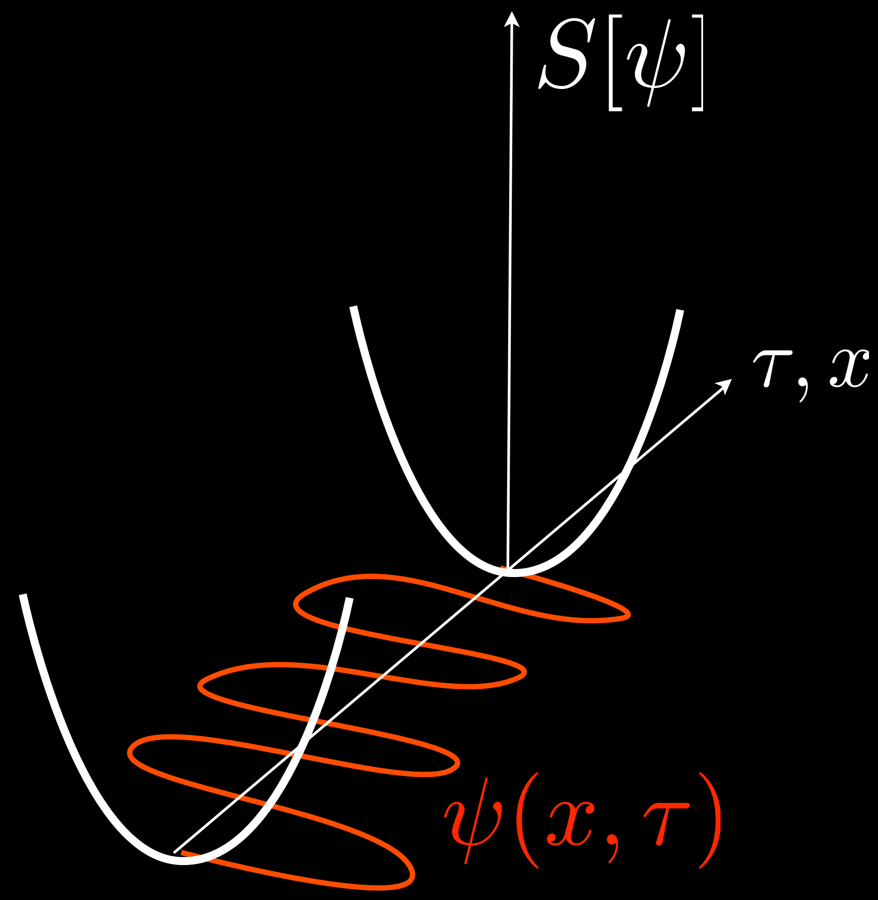

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Large *N* expansion.

Path Integral

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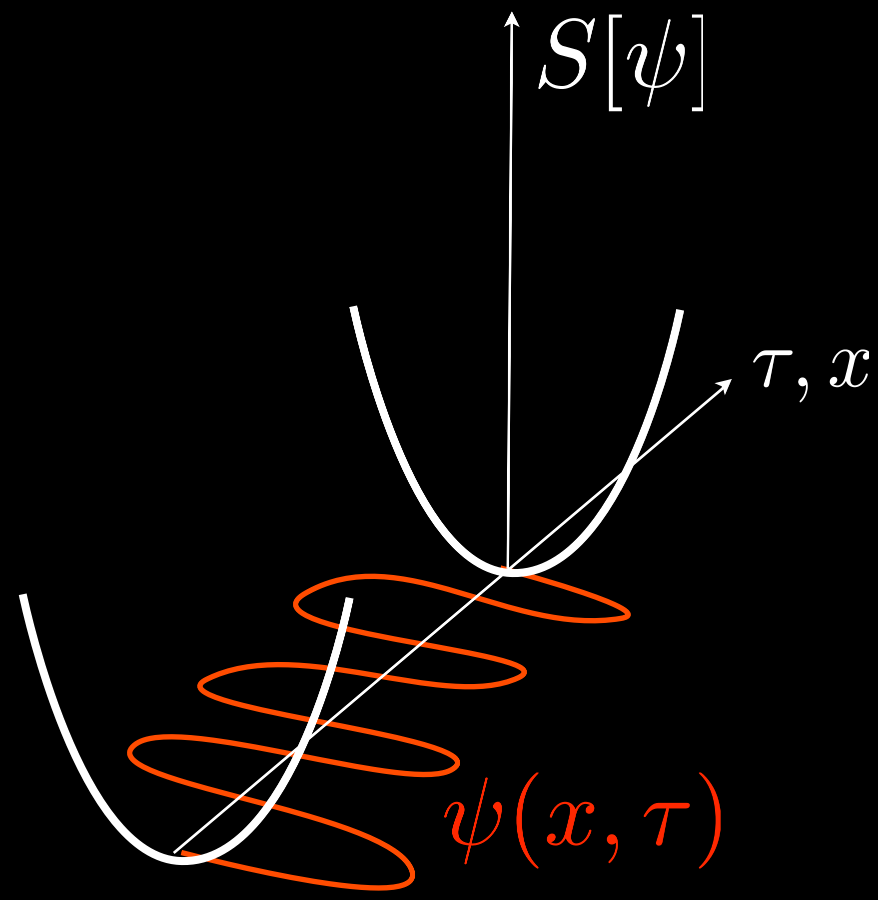

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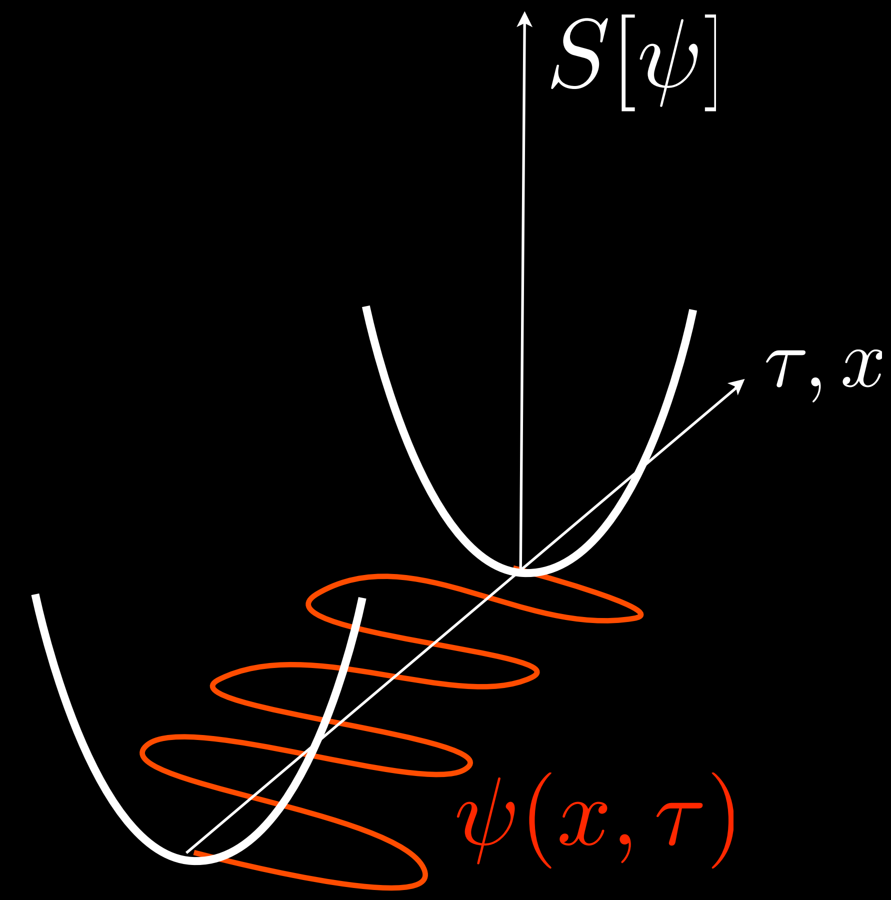

$$\sigma \in \left(-\frac{1}{2}, \frac{1}{2}\right) \longrightarrow \left(-\frac{N}{2}, \frac{N}{2}\right)$$

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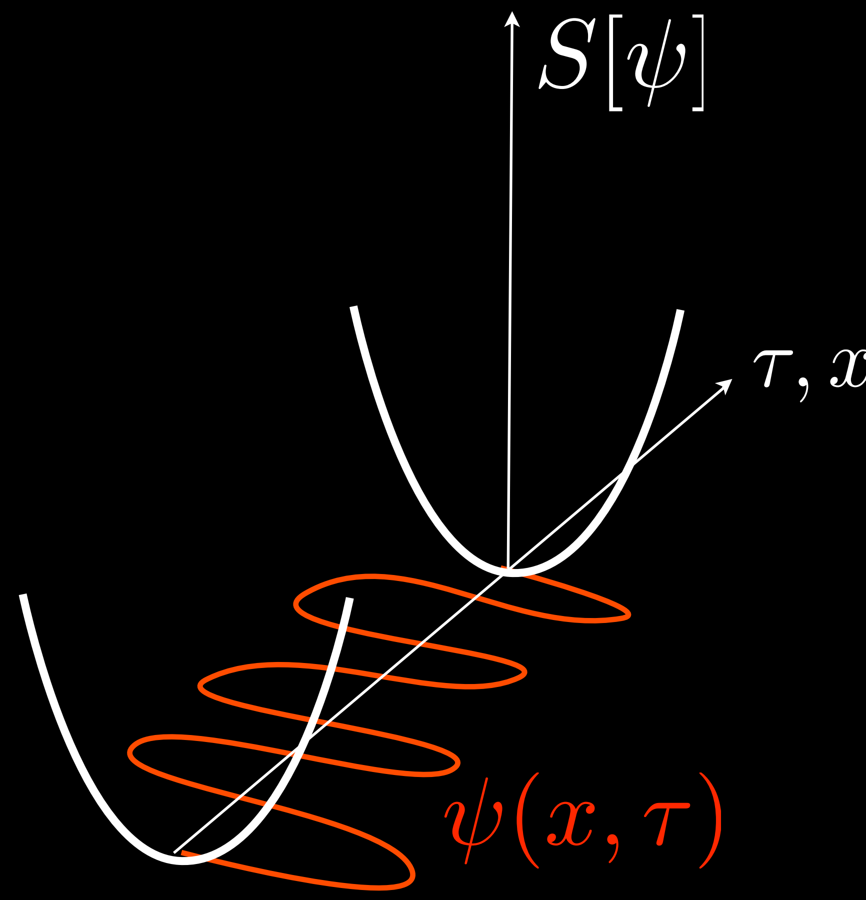
Large *N* expansion.

$$Z = \int_{\text{Fields}} e^{-NS[\psi]}$$



$$\sigma \in (-\frac{1}{2}, \frac{1}{2}) \longrightarrow (-\frac{N}{2}, \frac{N}{2})$$

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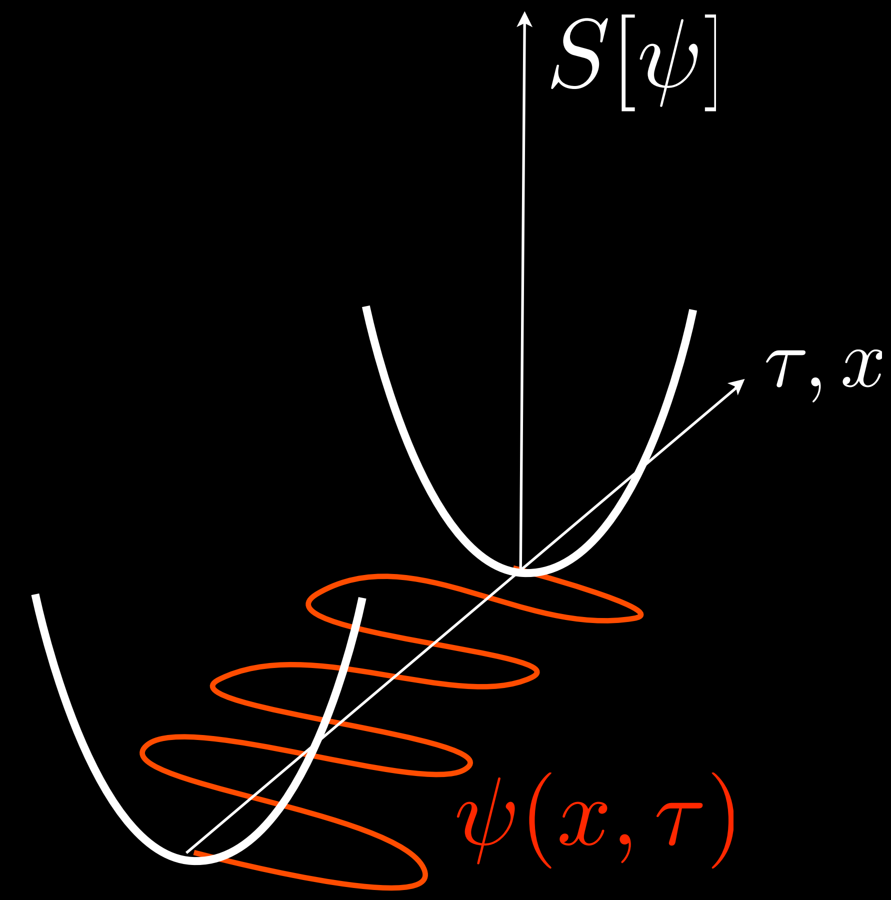
Large *N* expansion.

$$Z = \int_{\rm Fields} e^{-\frac{S[\psi]}{1/N}}$$



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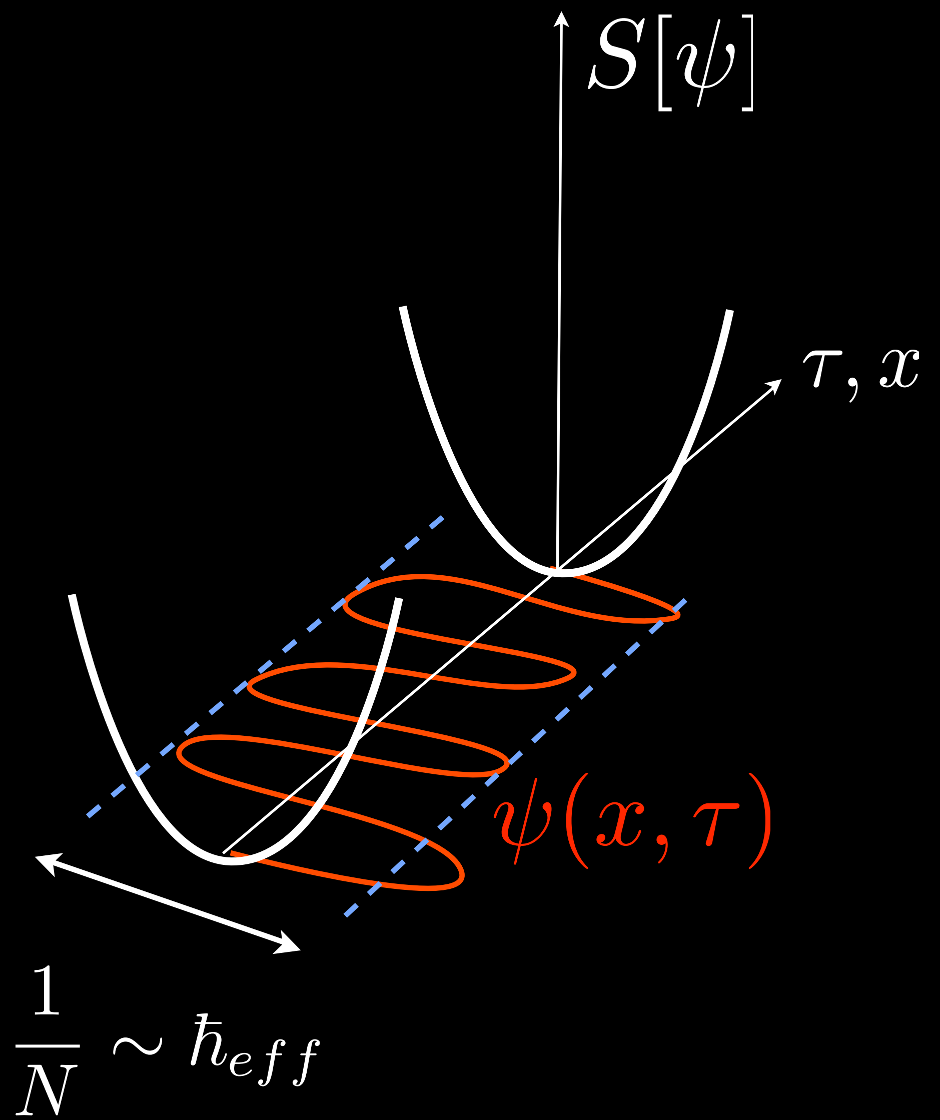

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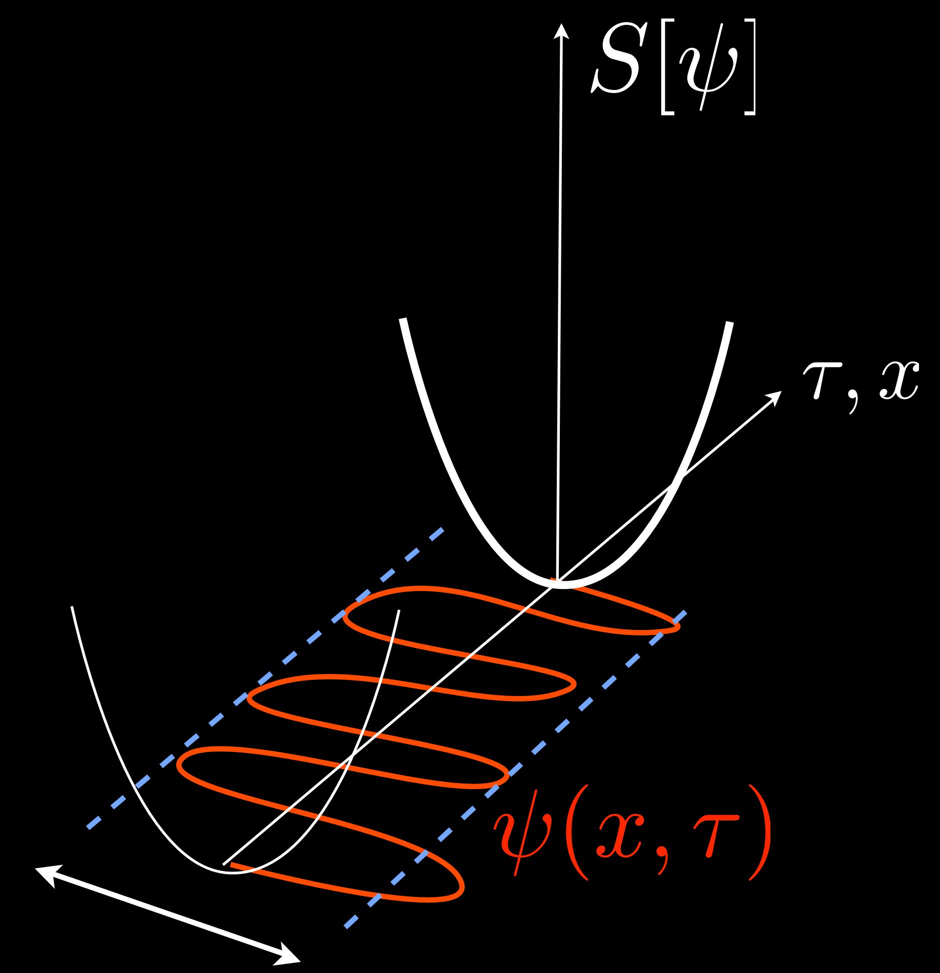
$$N \longrightarrow \infty$$



$$\sigma \in \left(-\frac{1}{2}, \frac{1}{2}\right) \longrightarrow \left(-\frac{N}{2}, \frac{N}{2}\right)$$

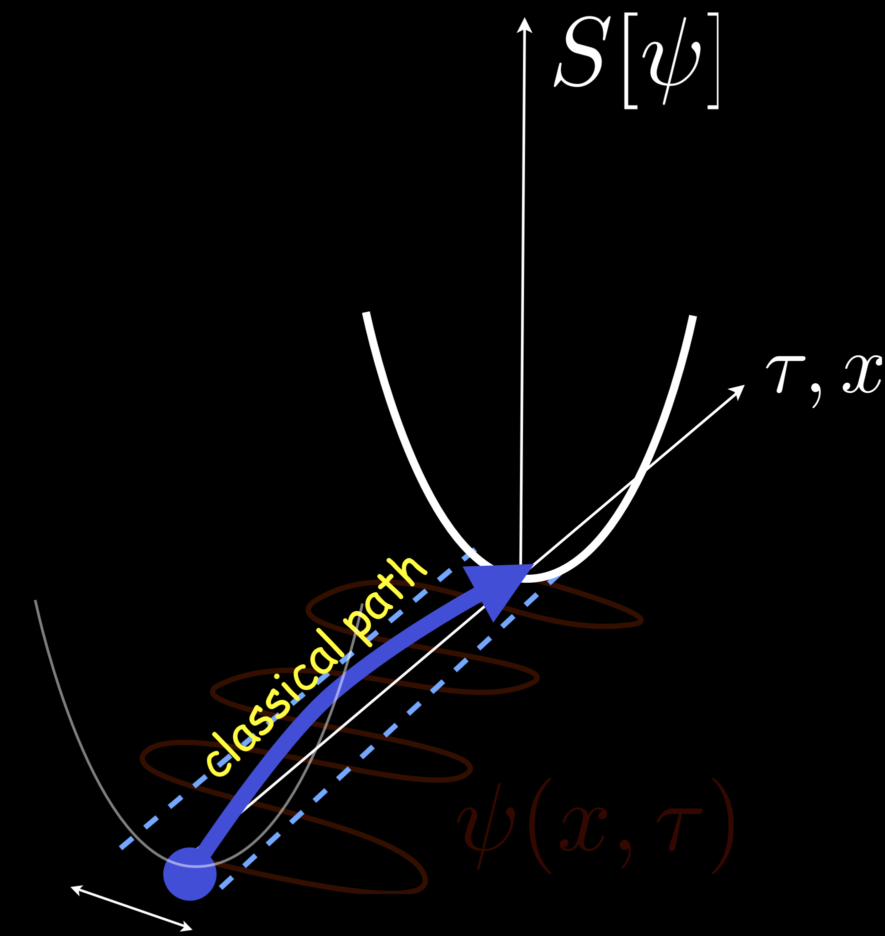
$$H = \sum_k \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{1}{N} \sum_{\mathbf{k},\mathbf{k}'} \left(J_1 \psi^\dagger_{1a}(j)\psi_{1b}(j) + J_2 \psi^\dagger_{2a}(j)\psi_{2b}(j) \right) S^{ba}(j)$$

Single FS, two channels.

$$\psi_\Gamma(j) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \gamma_{\Gamma\mathbf{k}} \; c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}_j}$$

cf Cox, Pang, Jarell (96)
PC, Kee, Andrei, Tsvelik (98)

$$N \longrightarrow \infty$$



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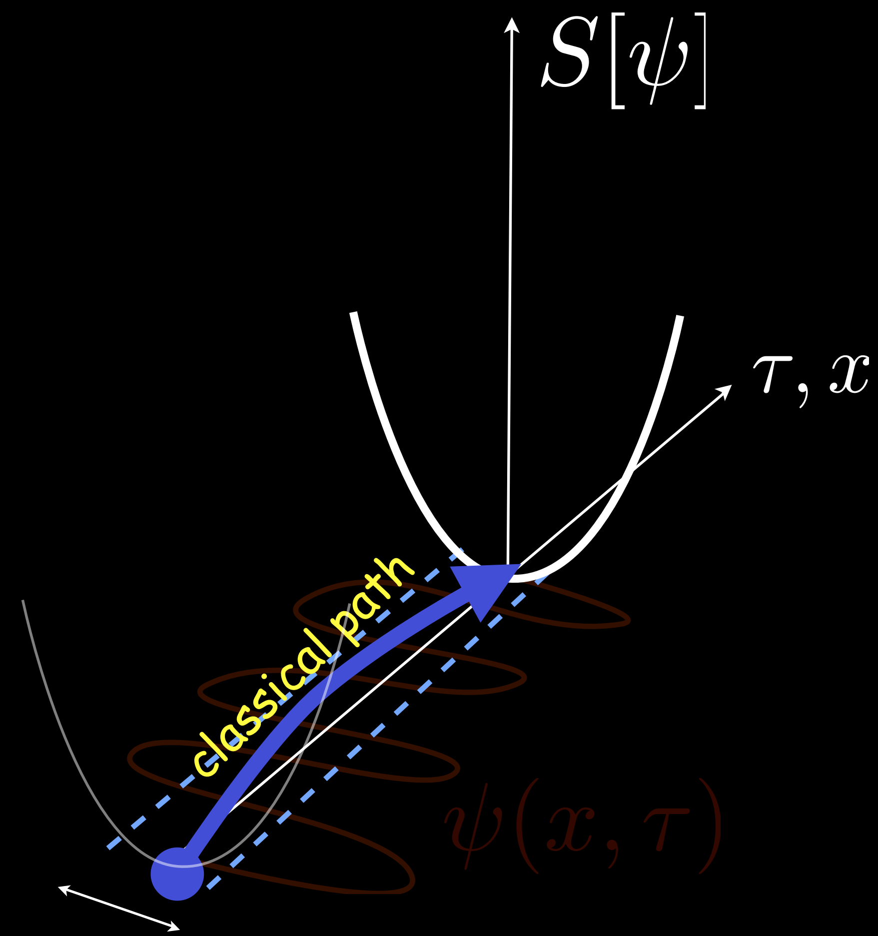

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Rutgers NHETC.

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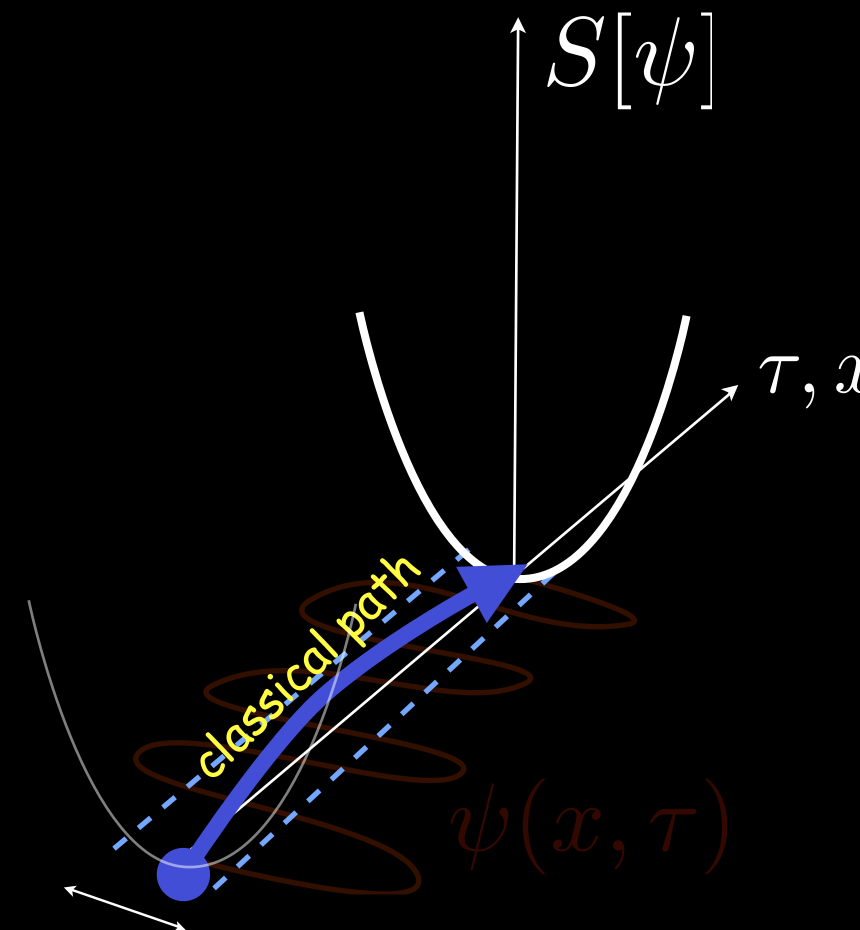

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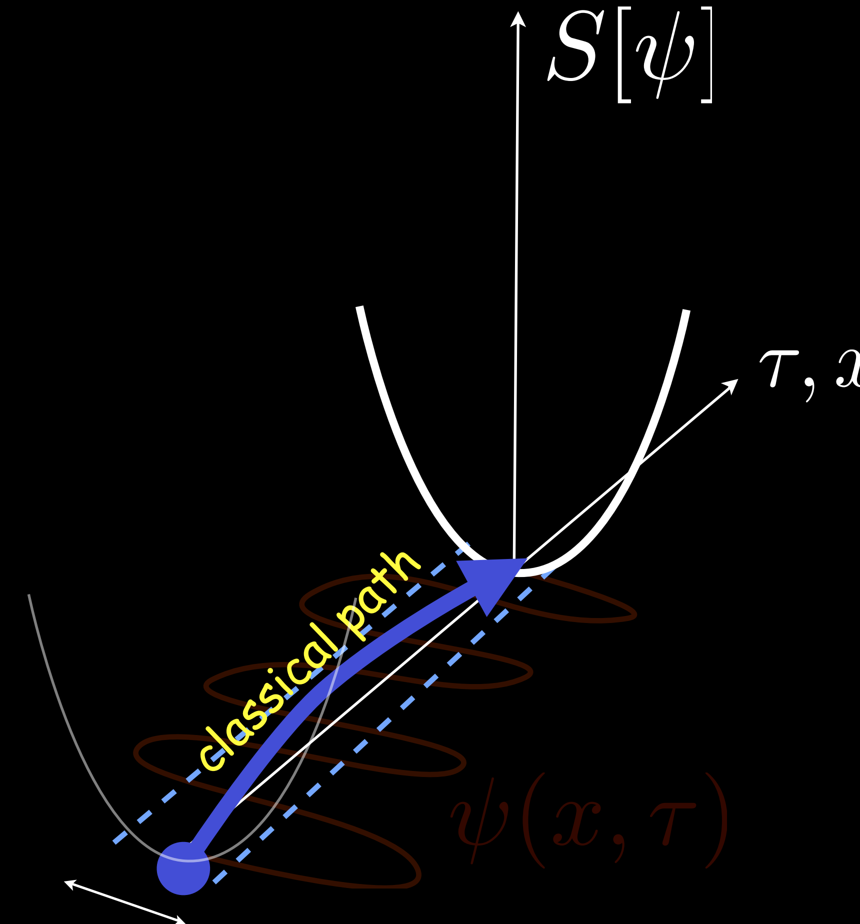

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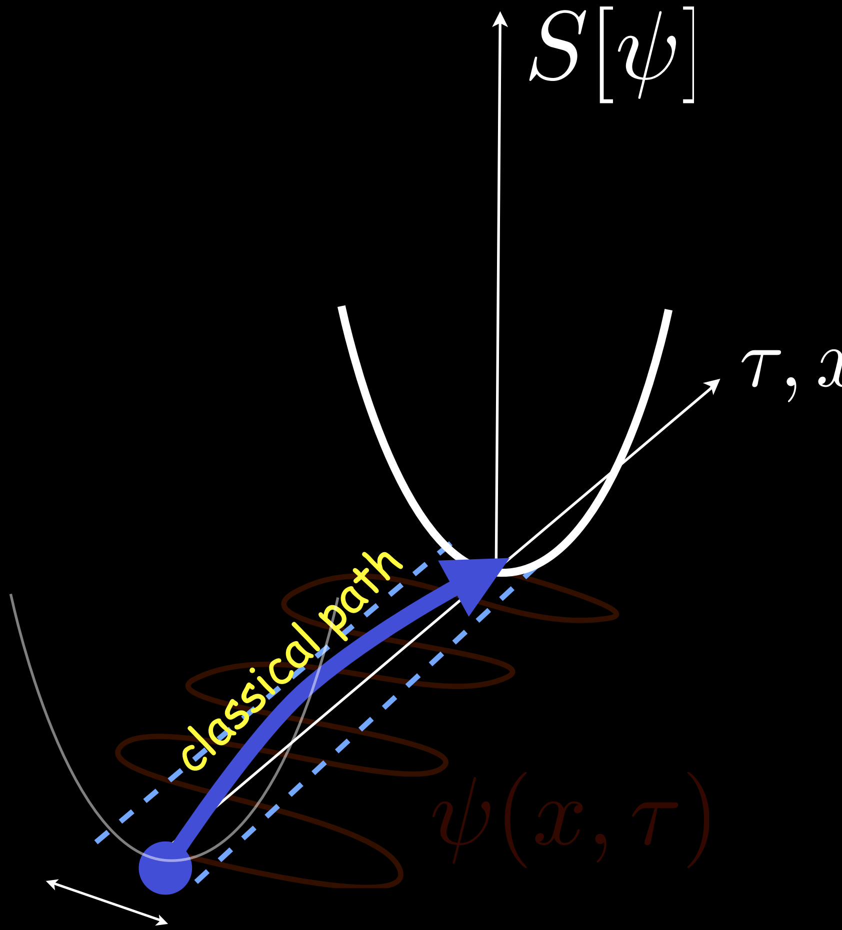

SU(N): *Mesons* $\bar{q}q$ *Baryons* $q_1 q_2 \dots q_N$

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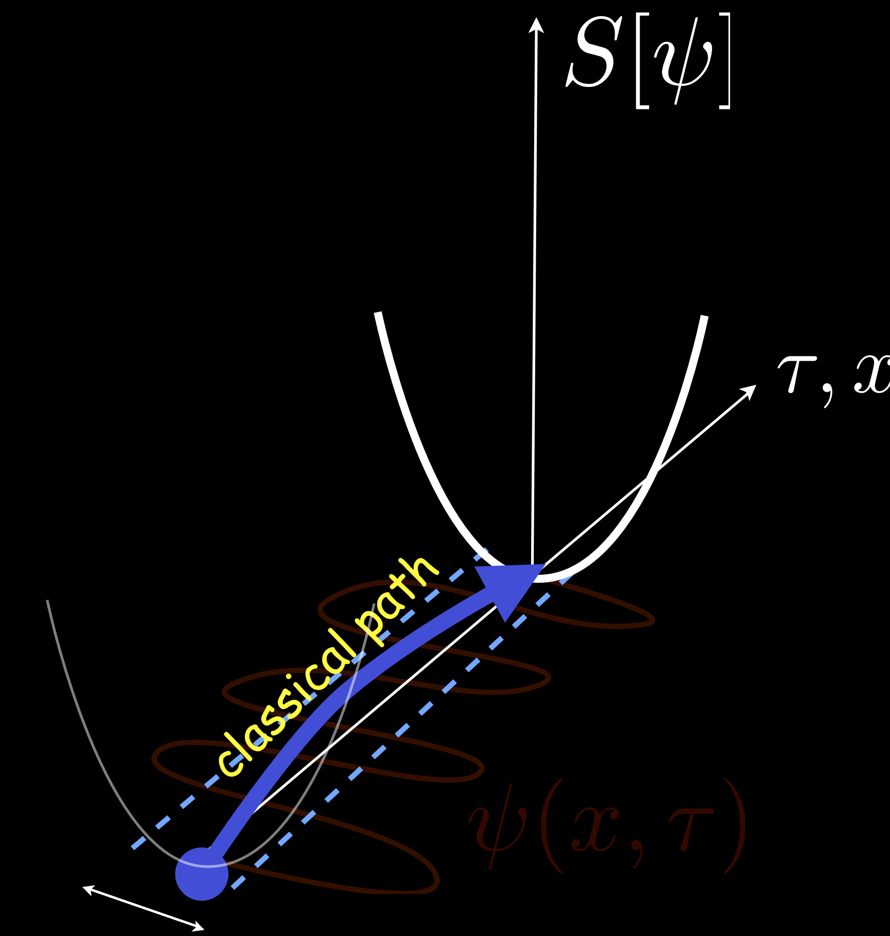

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	$\bar{q}q$	$q_1 q_2 \cdots q_N$
		Cooper pairs
SP(N):	$\bar{q}q$	$q_a q_{-a}$

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$N \rightarrow \infty$

"Symplectic Large N" R. Flint and PC '08

$$S^{ba} = f_b^\dagger f_a - \mathrm{sgn}(a)\mathrm{sgn}(b) f^\dagger_{-b} f_{-a}$$

	Mesons	Baryons
SU(N):	$\bar{q}q$	$q_1 q_2 \ldots q_N$
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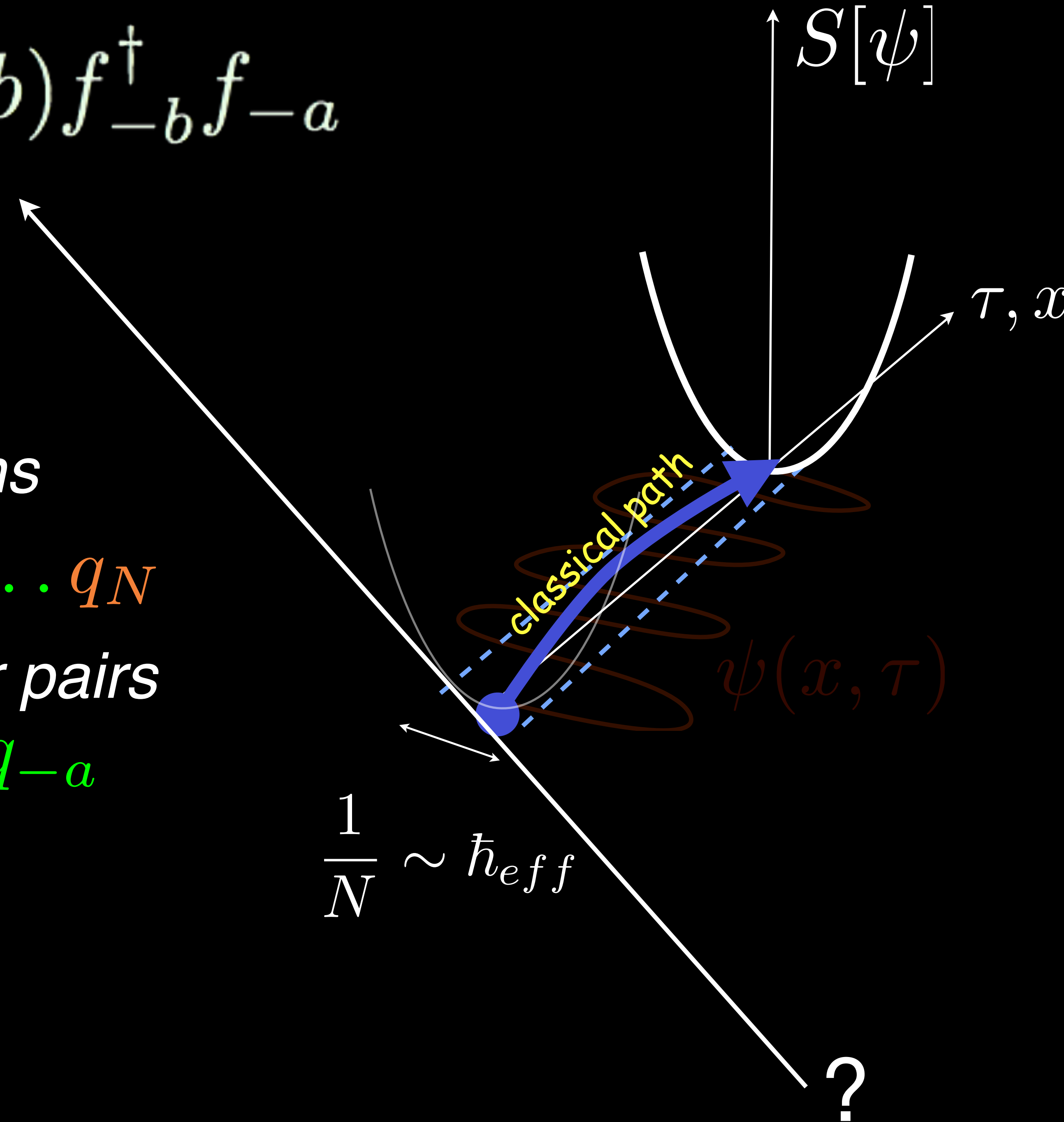

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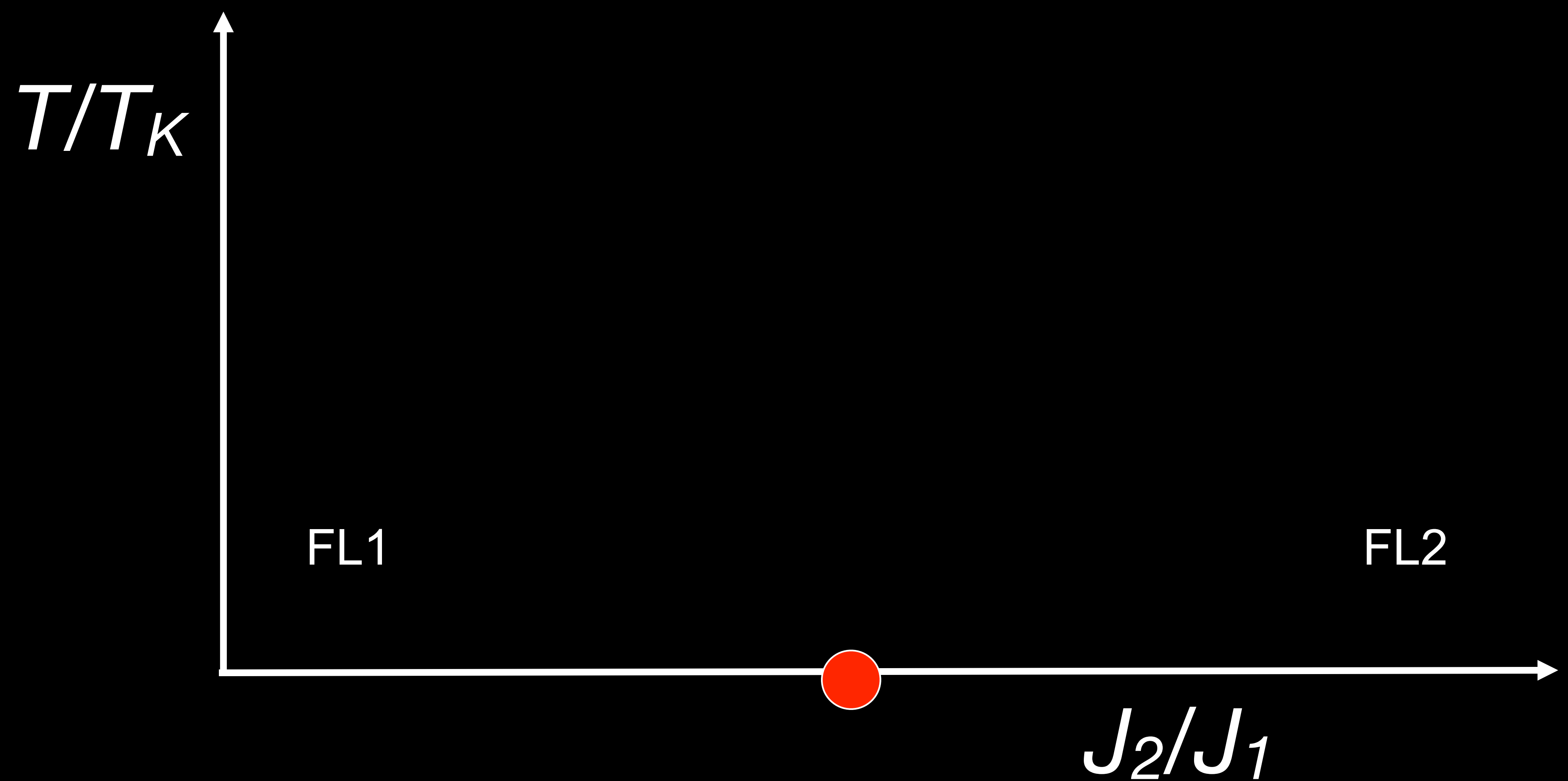
Impurity: quantum critical point for $J_1 = J_2$

Nozieres and Blandin 1980



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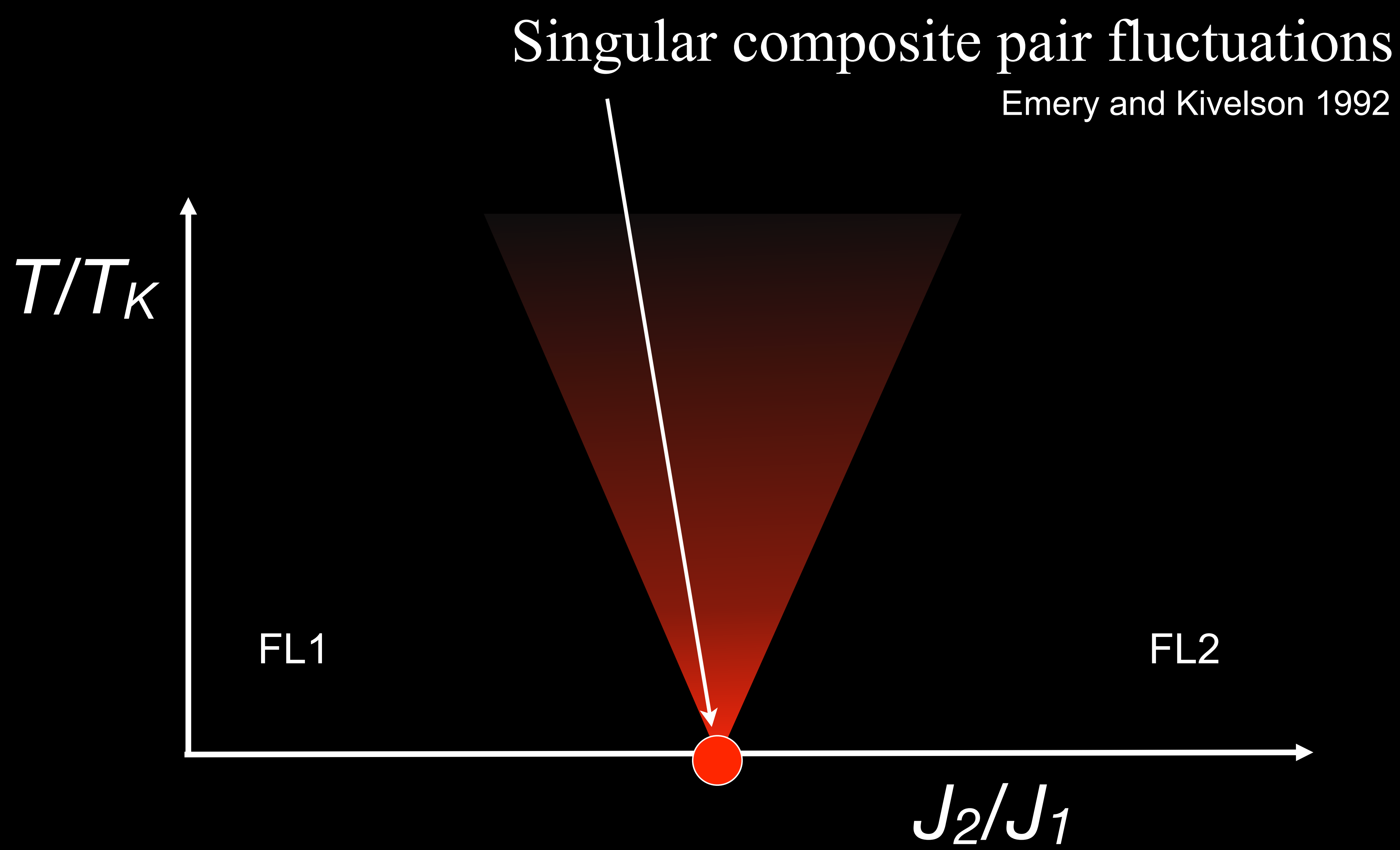

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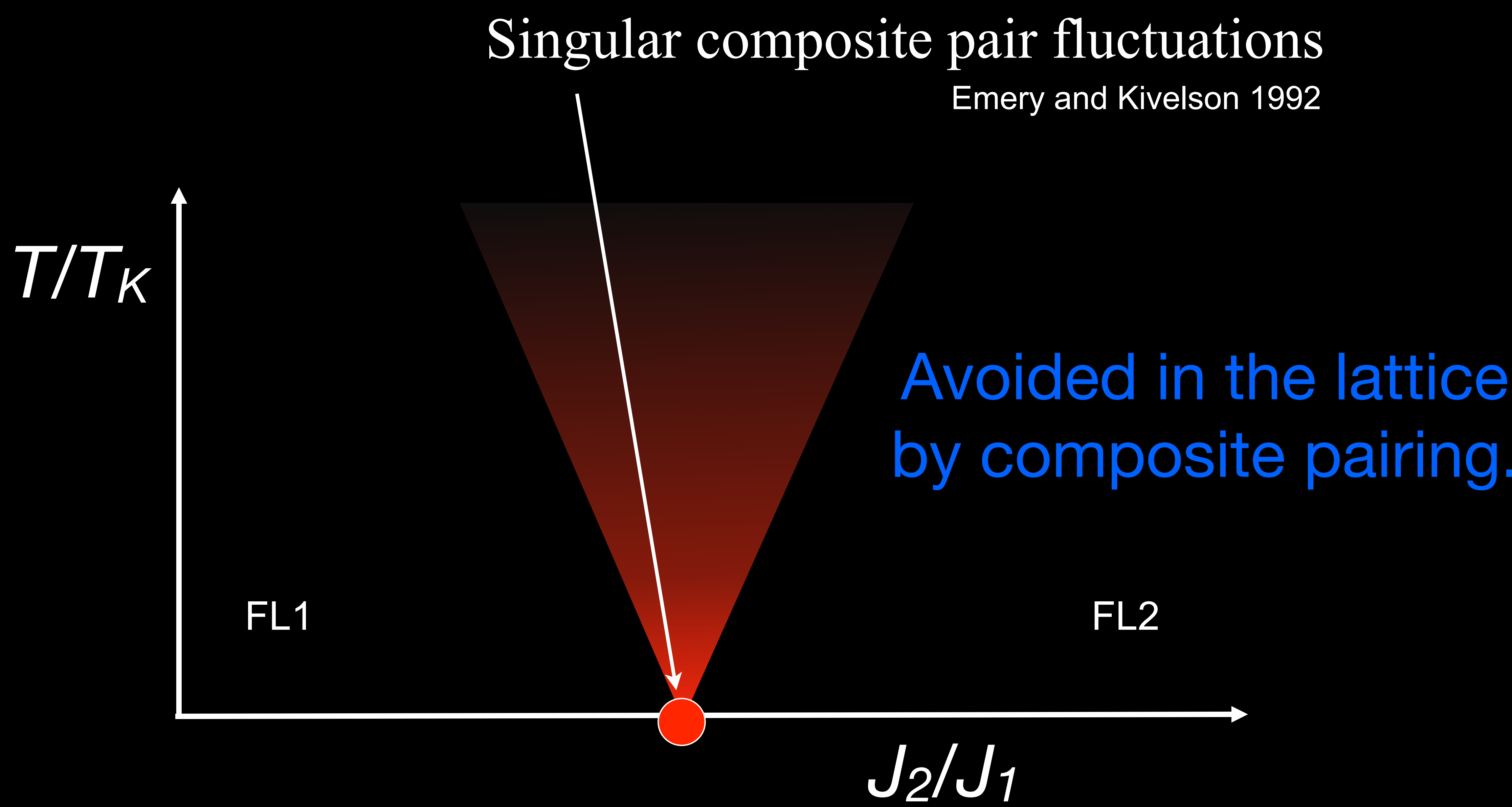

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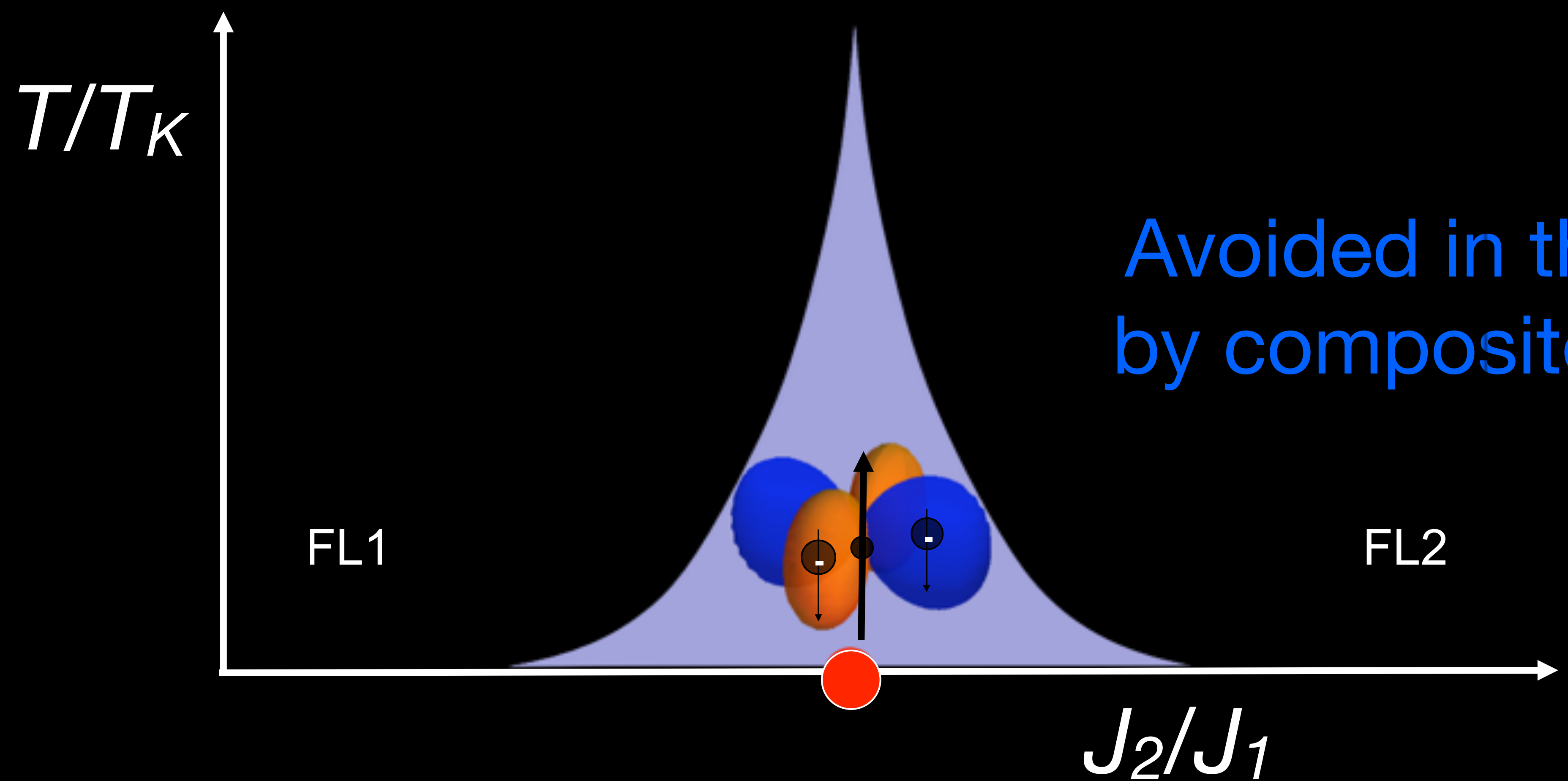
Singular composite pair fluctuations

Emery and Kivelson 1992



Avoided in the lattice by composite pairing.

$$H = \sum_k \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{1}{N}\sum_{\mathbf{k},\mathbf{k}'}\left(J_1 \psi^\dagger_{1a}(j)\psi_{1b}(j) + J_2 \psi^\dagger_{2a}(j)\psi_{2b}(j)\right) S^{ba}(j)$$

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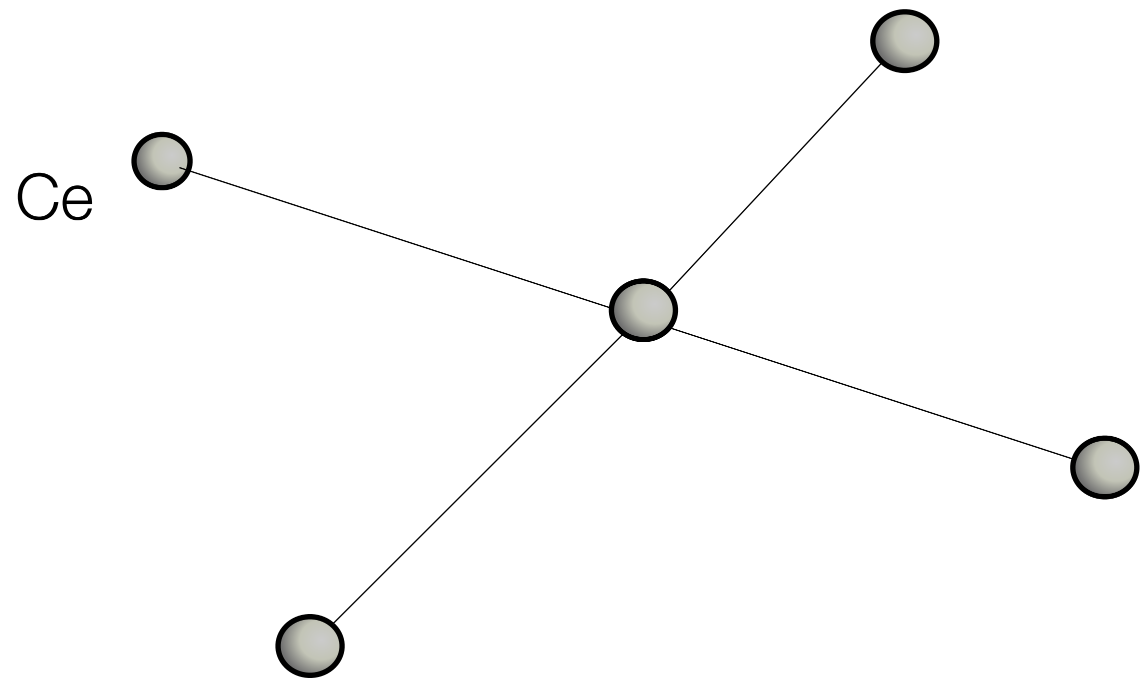
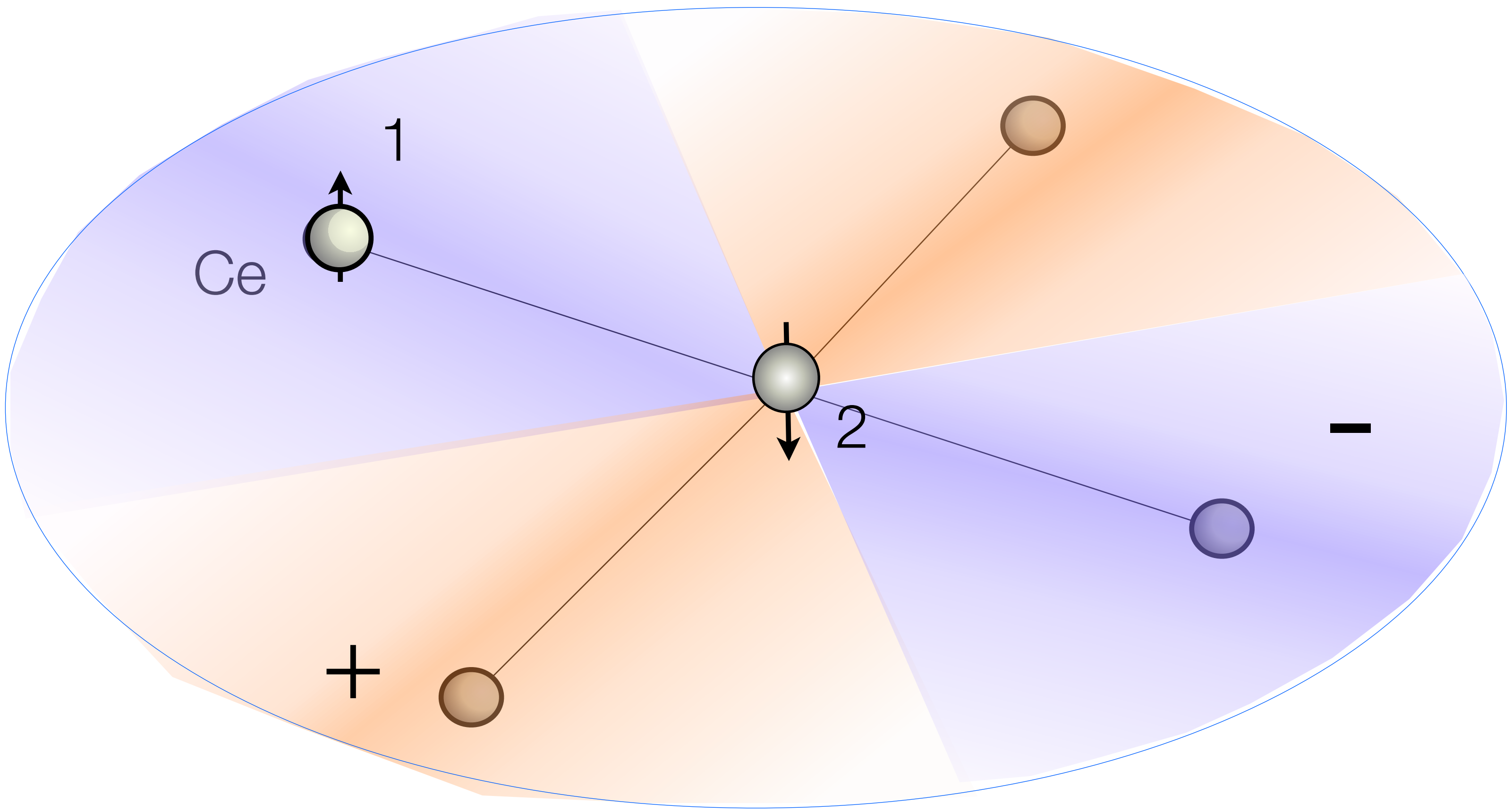
Real-space structure of pair



Real-space structure of pair

Magnetic pair: intercell

$$\Psi_M^\dagger = \Delta_d(1-2) f_\uparrow^\dagger(1) f_\downarrow^\dagger(2)$$



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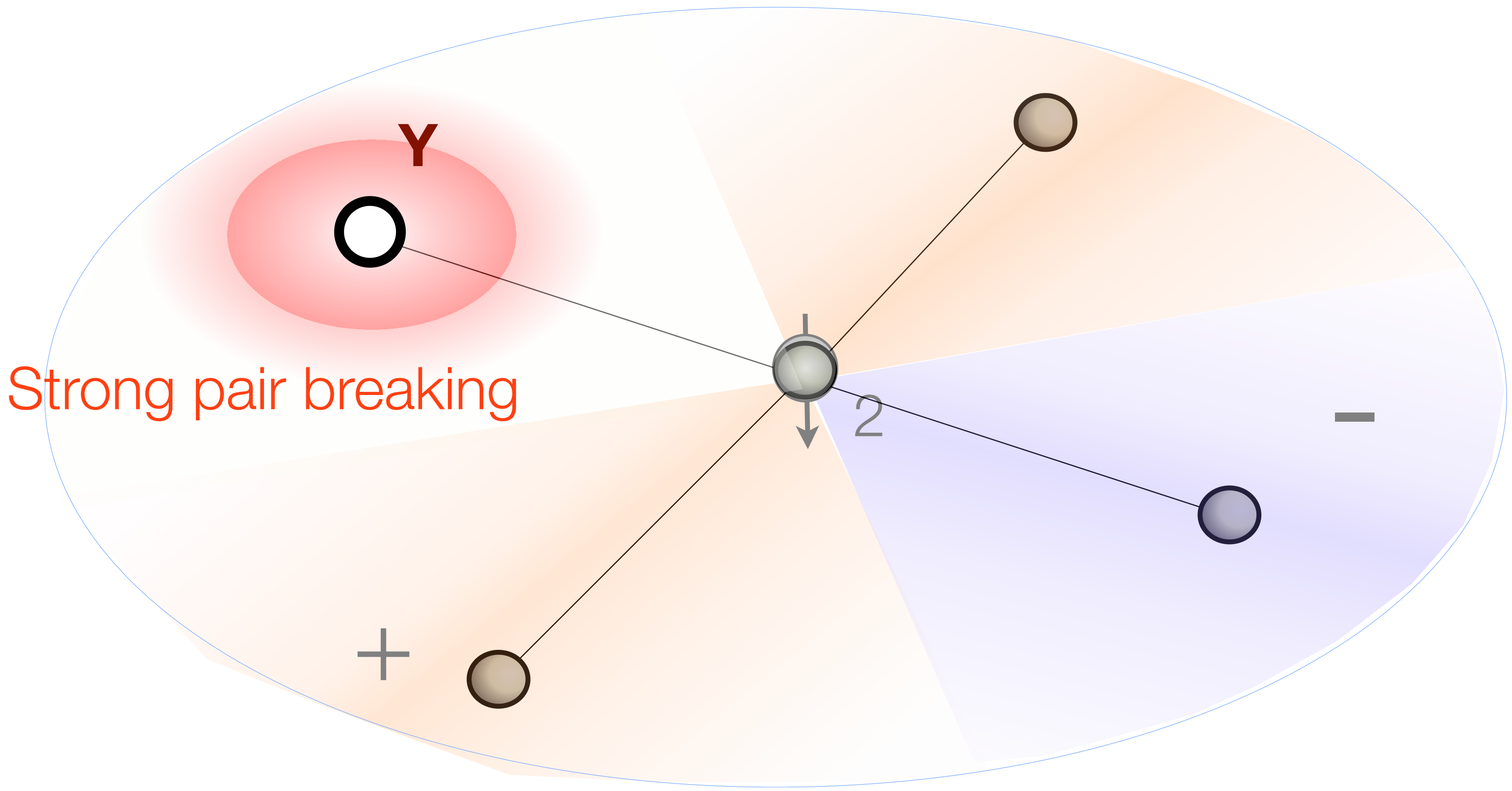

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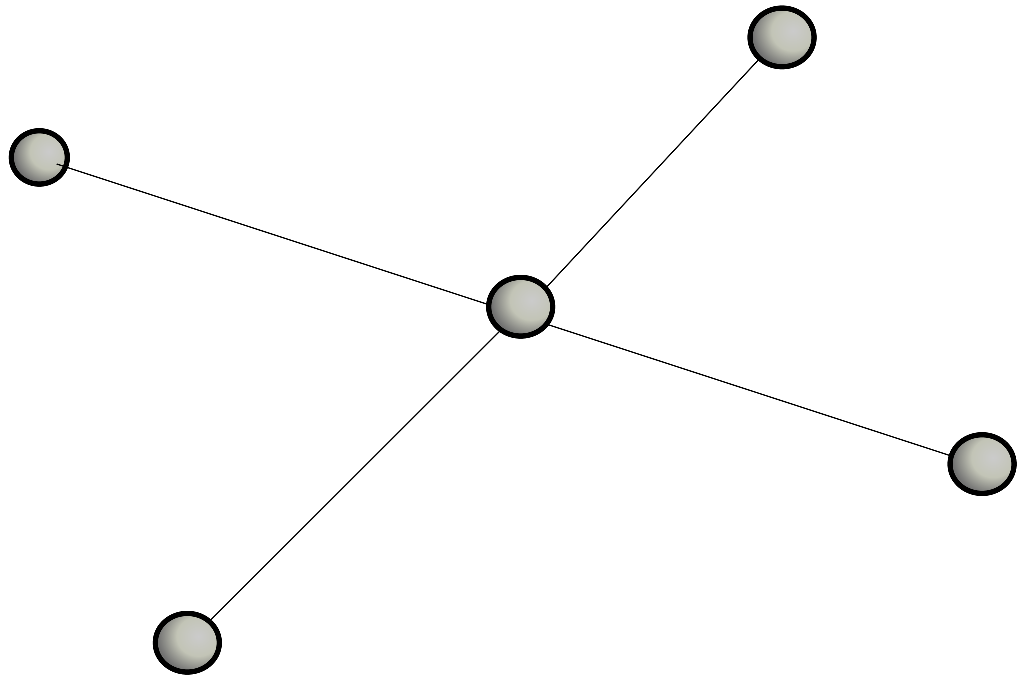

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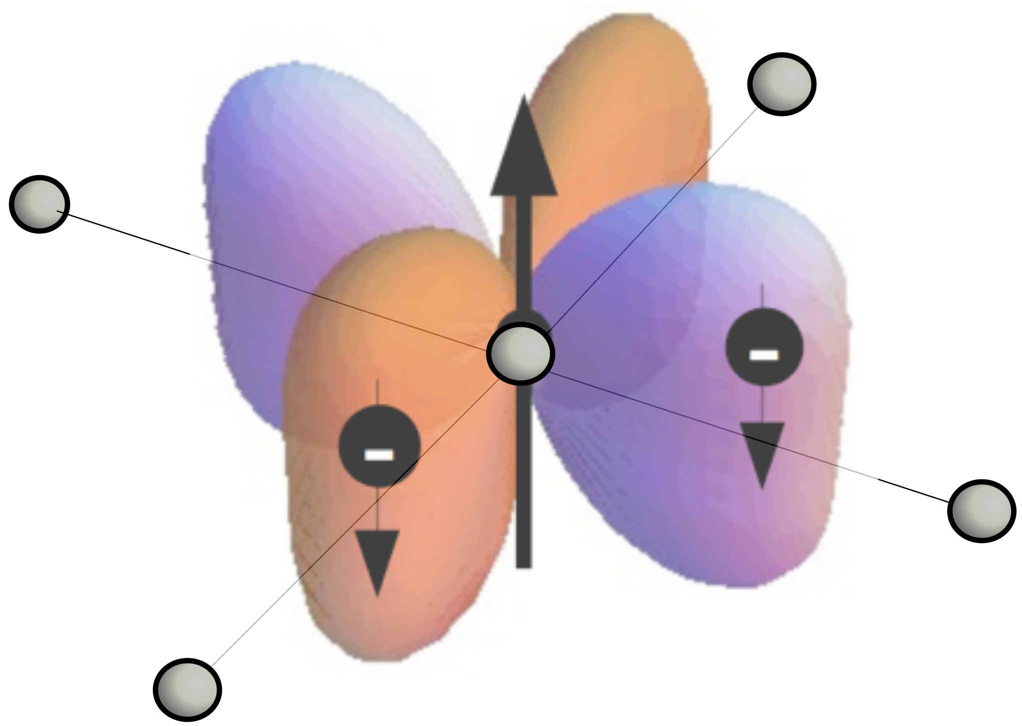

Composite pair

$$\Psi_C^\dagger = c_{1\downarrow}^\dagger c_{2\downarrow}^\dagger S_+$$

Abrahams, Balatsky, Scalapino, Schrieffer 1995

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Flint, Dzero, Coleman , Nat. Phys, (2008)

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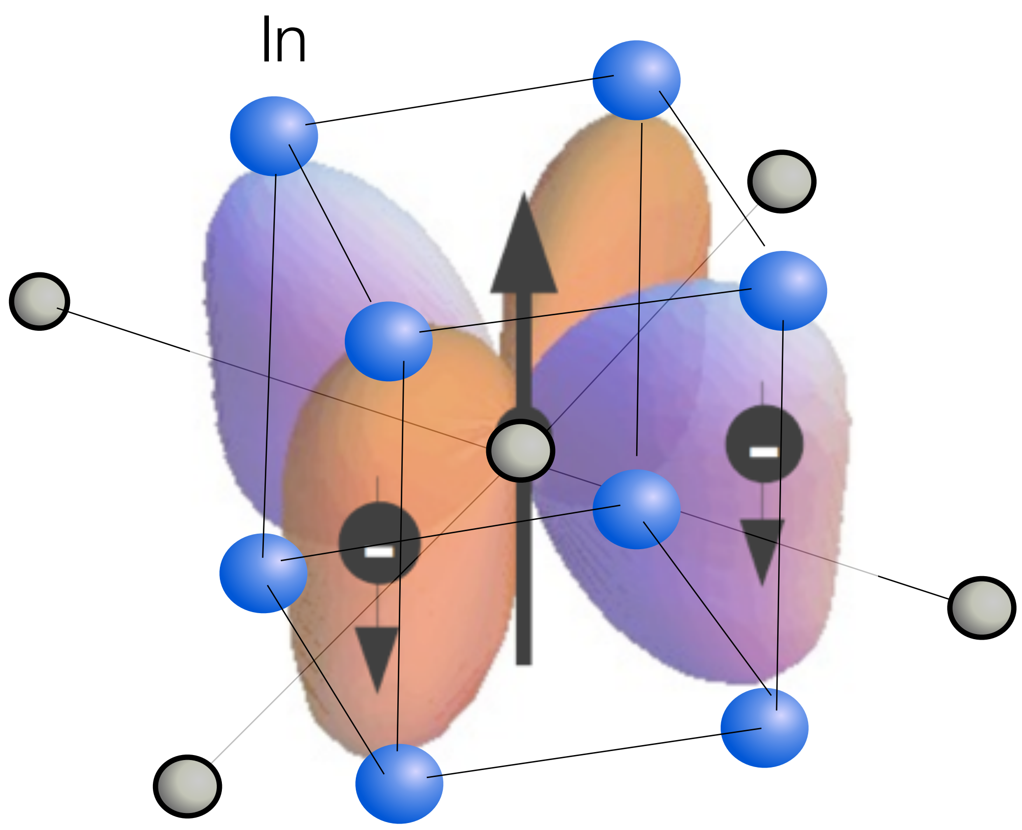

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Extreme Resilience to doping on Ce site.

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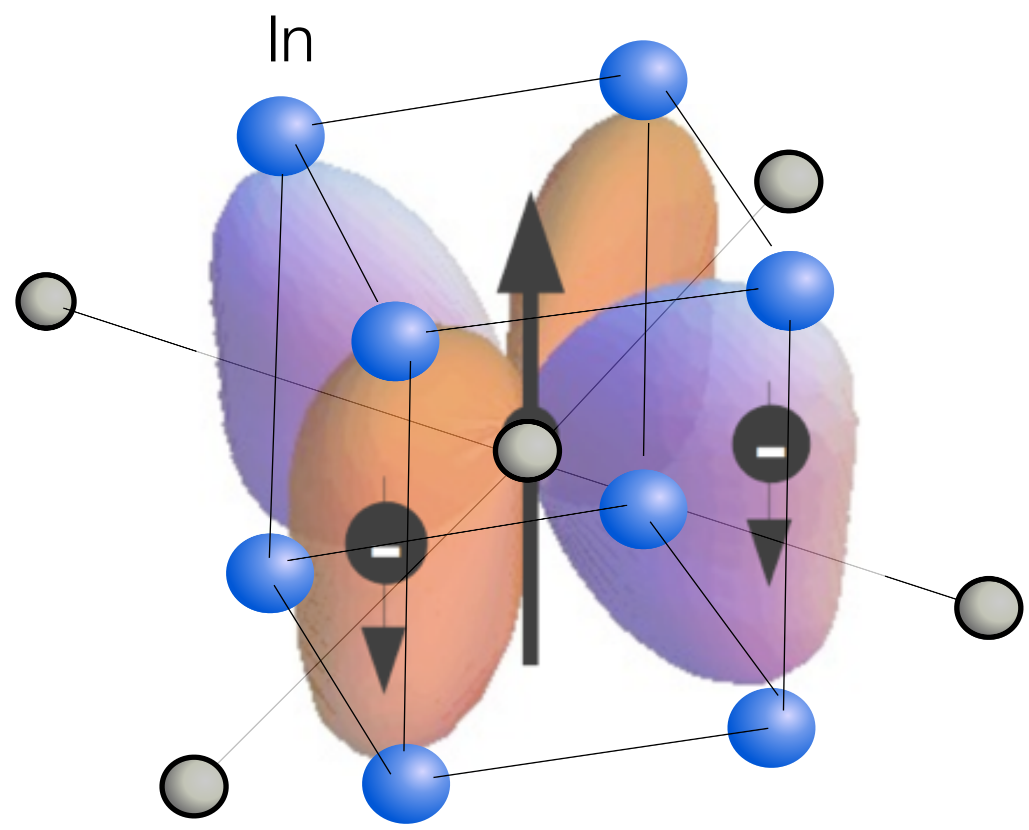

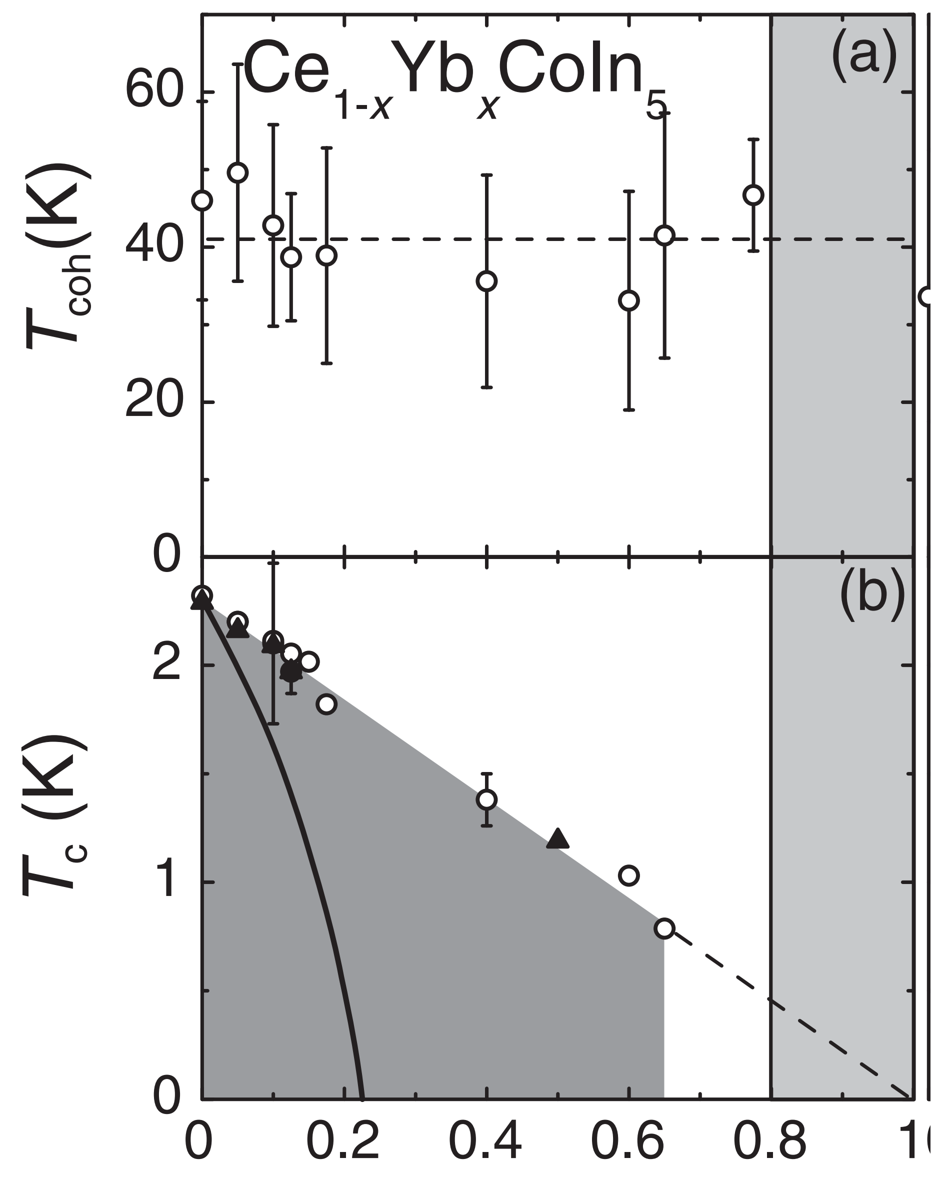

Lei Shu et al, PRL, (2011)

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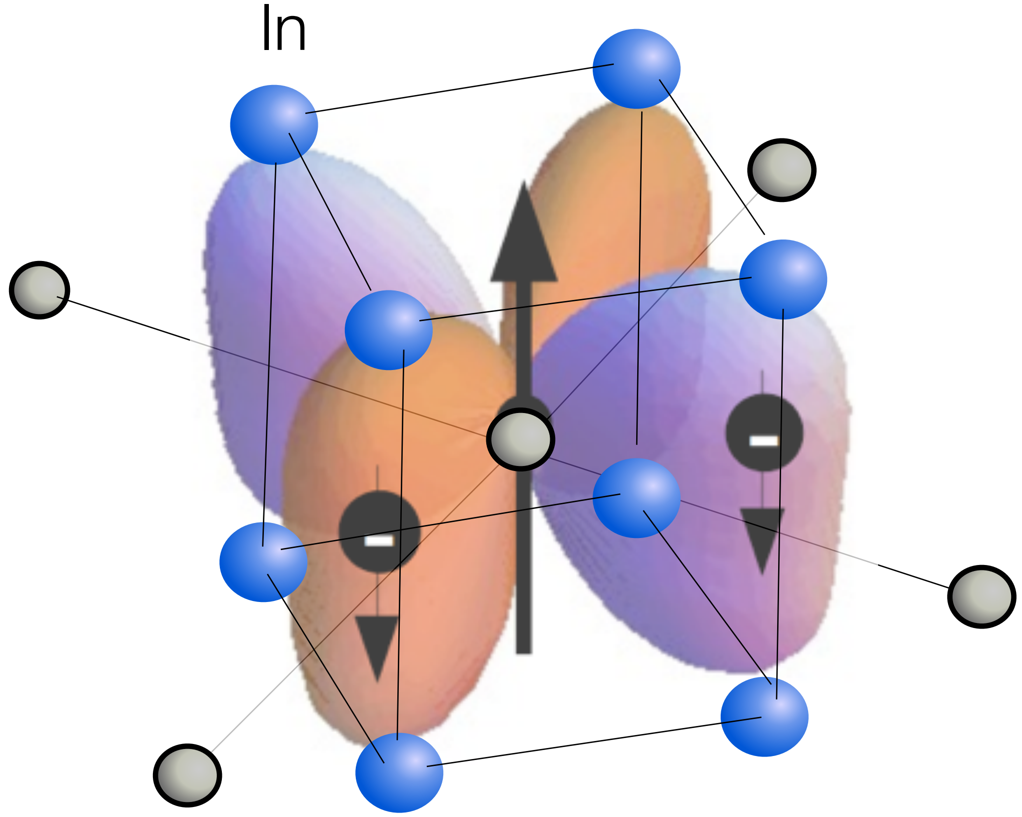

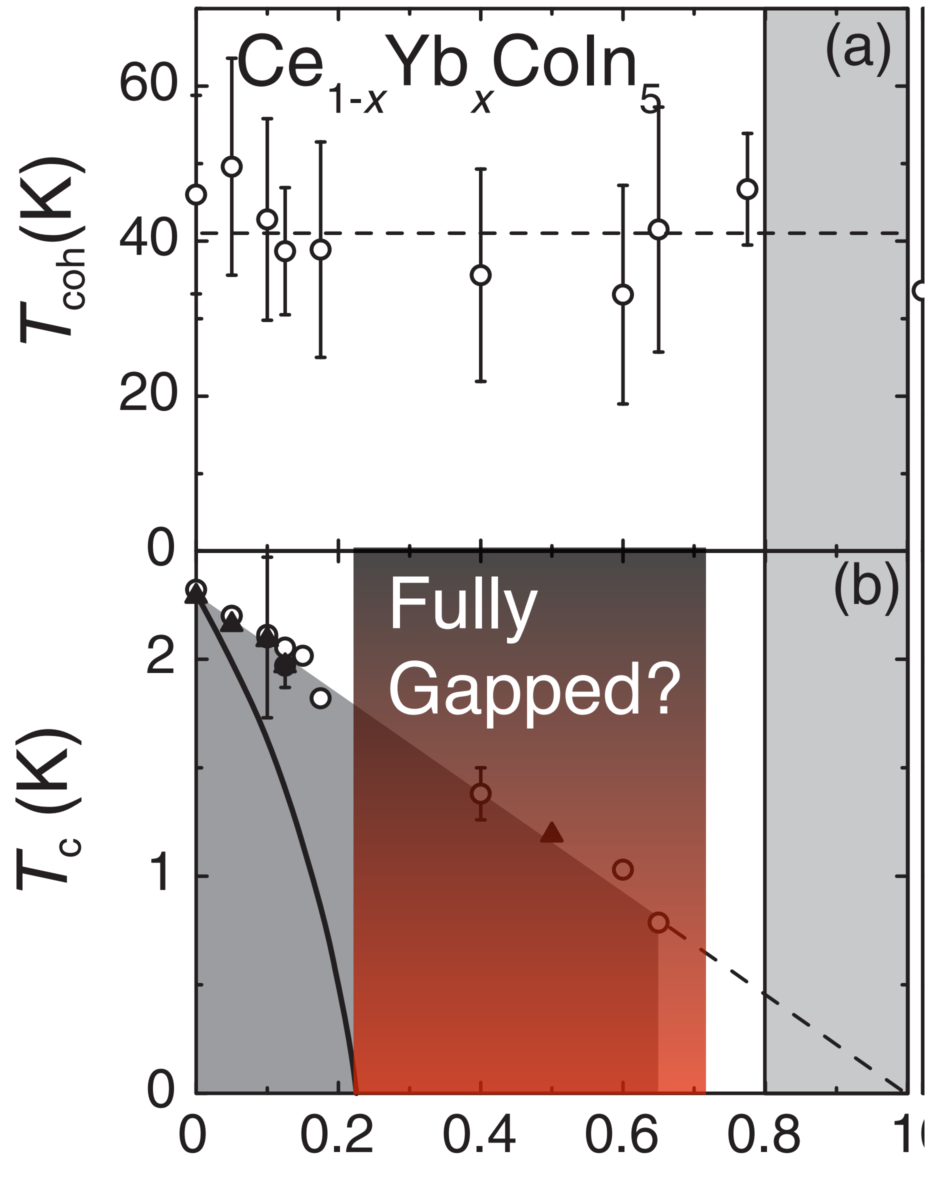

Lei Shu et al, PRL, (2011)

M. Tanatar et al (unpublished)

Erten and PC arXiv1402.7361

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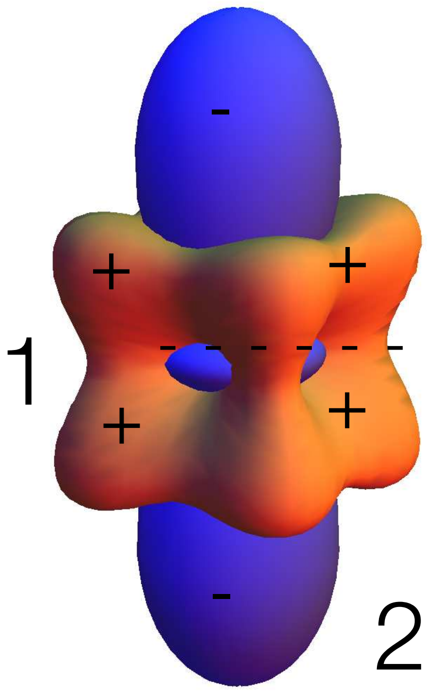

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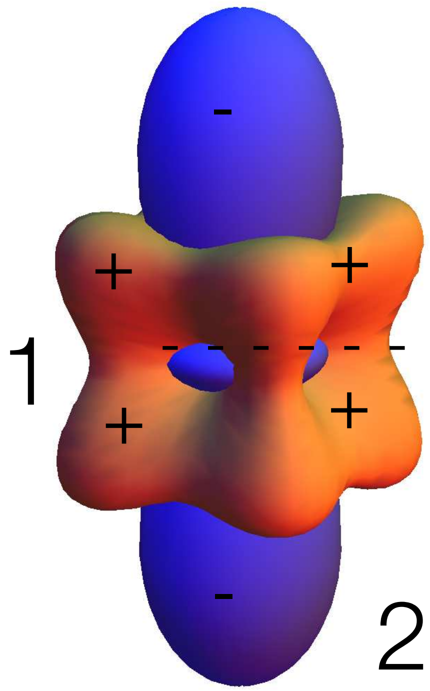

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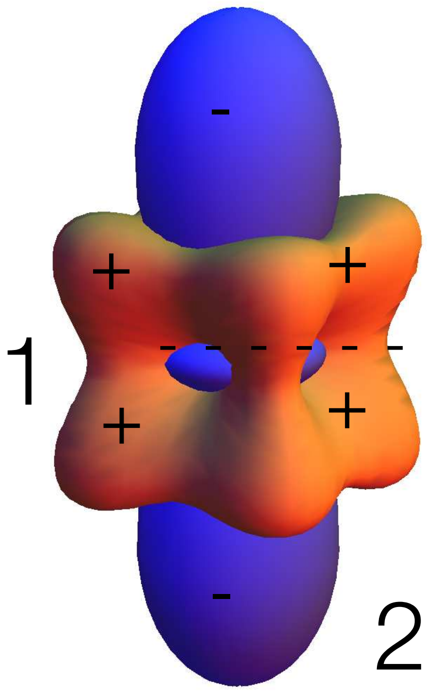

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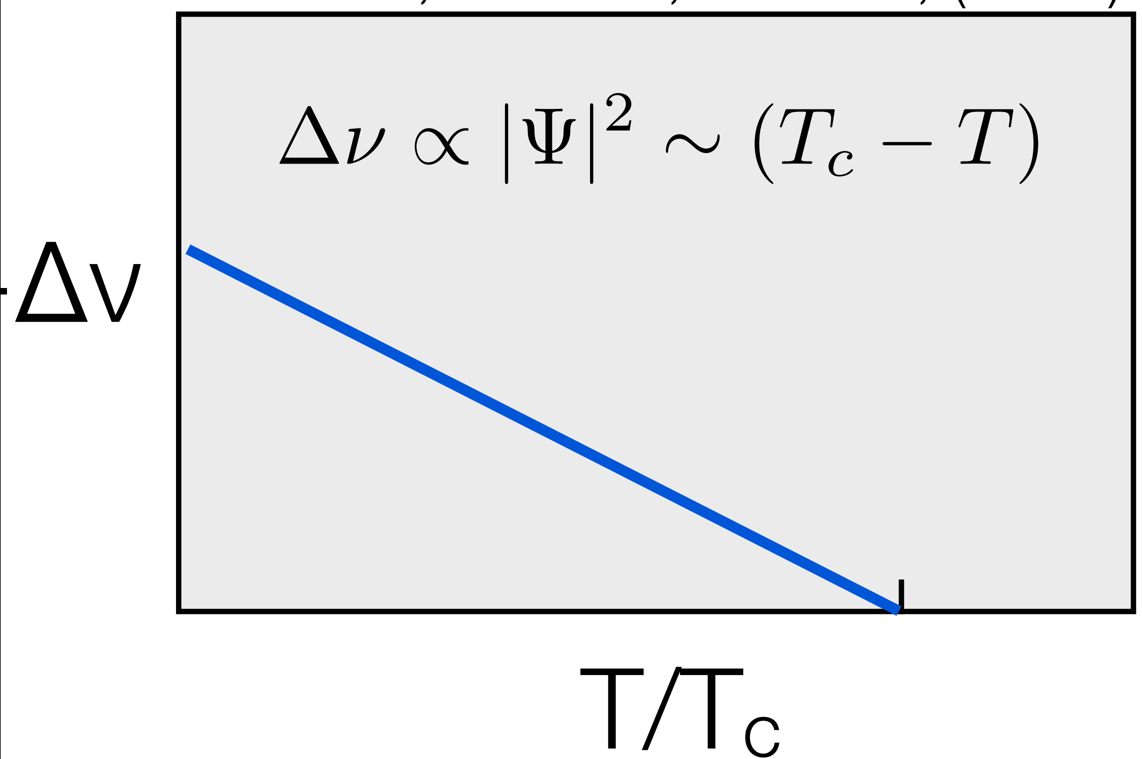
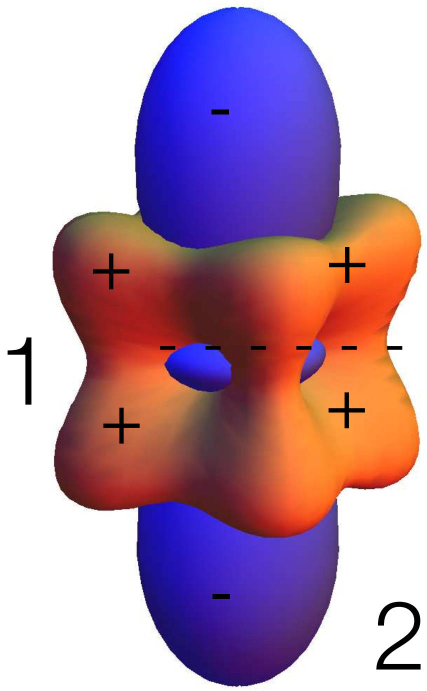
Flint et al, PRB 84, 064054, (2011)





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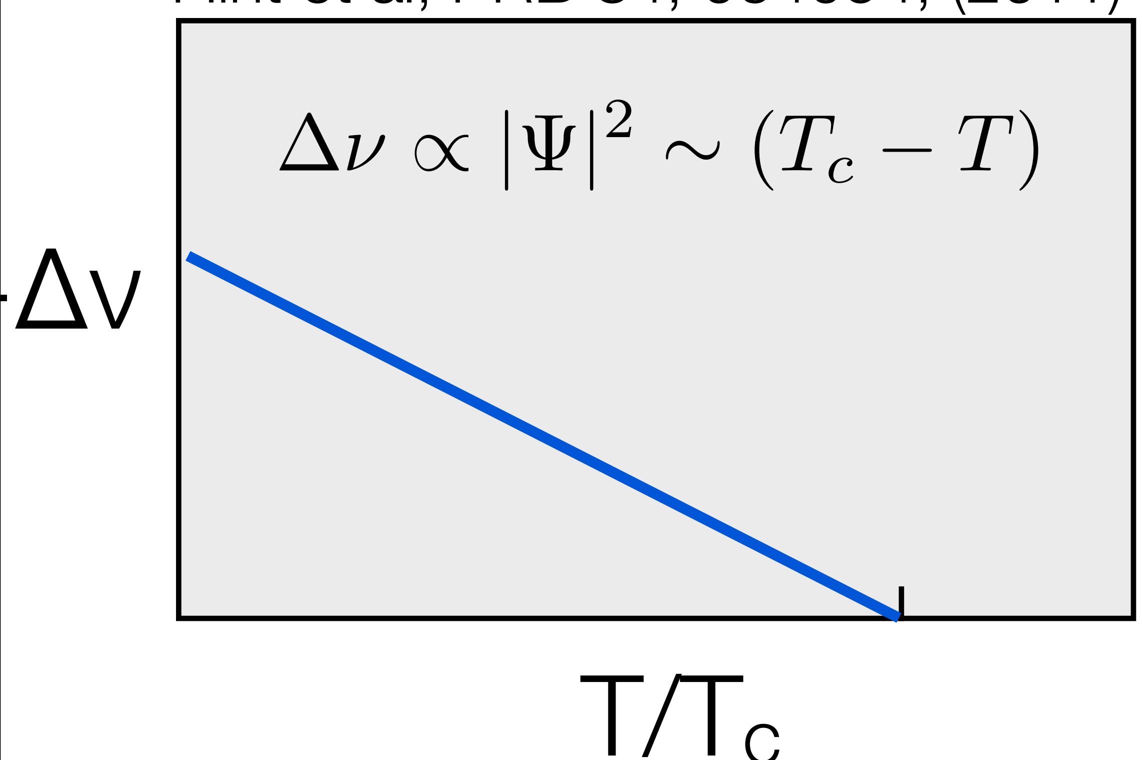
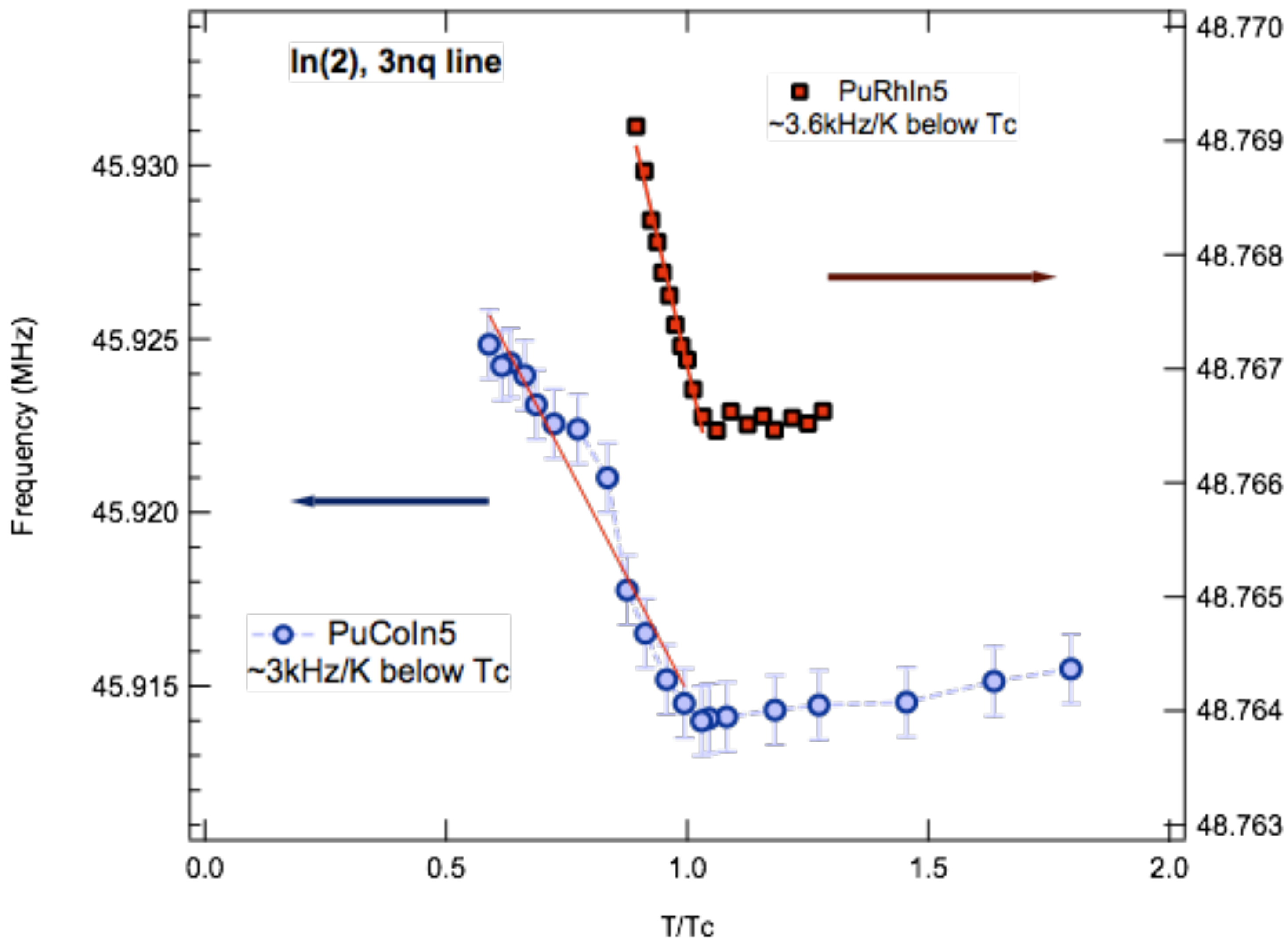


Bauer, G. Koutroulakis Yasuoko,(2014)

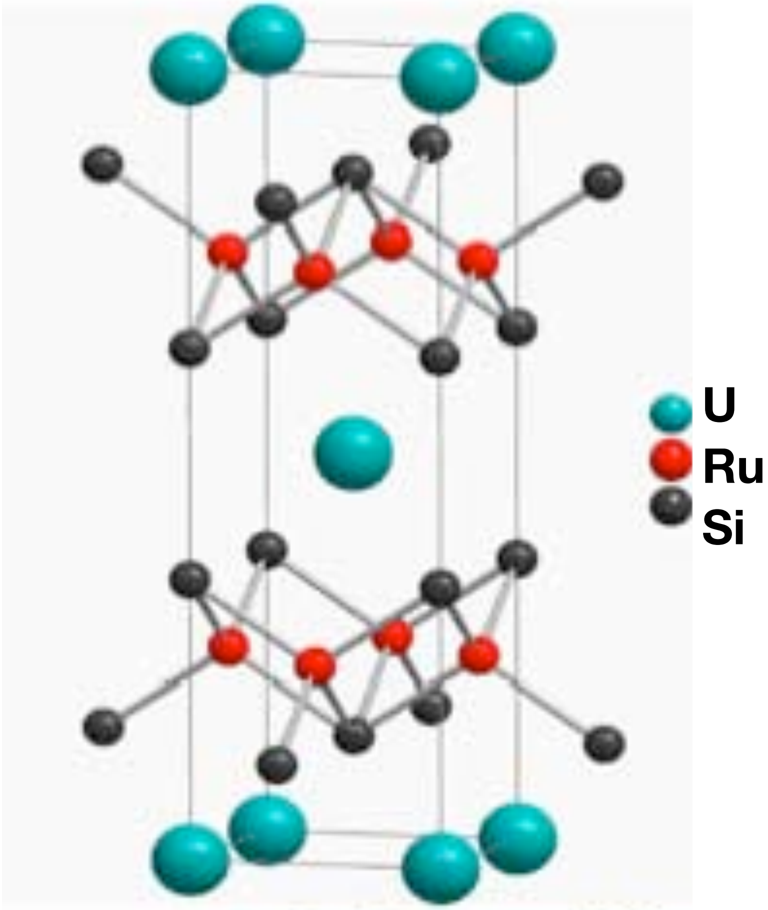


Open Challenges.

- HFSC: how is the spin incorporated into the condensate?
- Composite pairs?
- Possibility of molecular pairing. (see Onur Erten and Coleman, arXiv1402.7361

URu_2Si_2:
The Hidden Order Mystery

Hidden Order in URu_2Si_2



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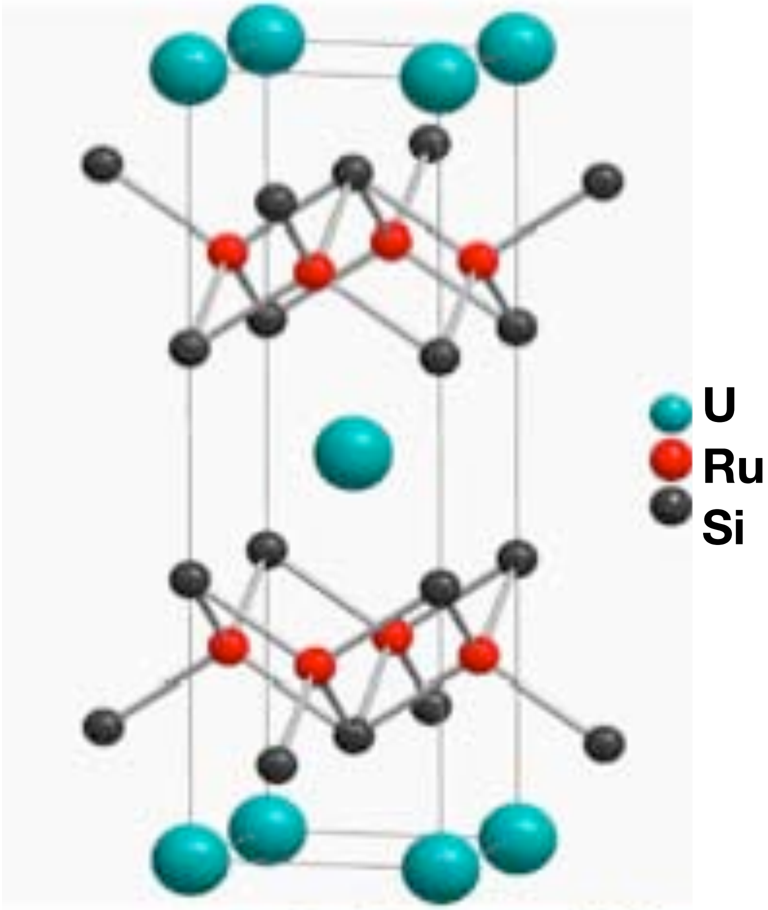

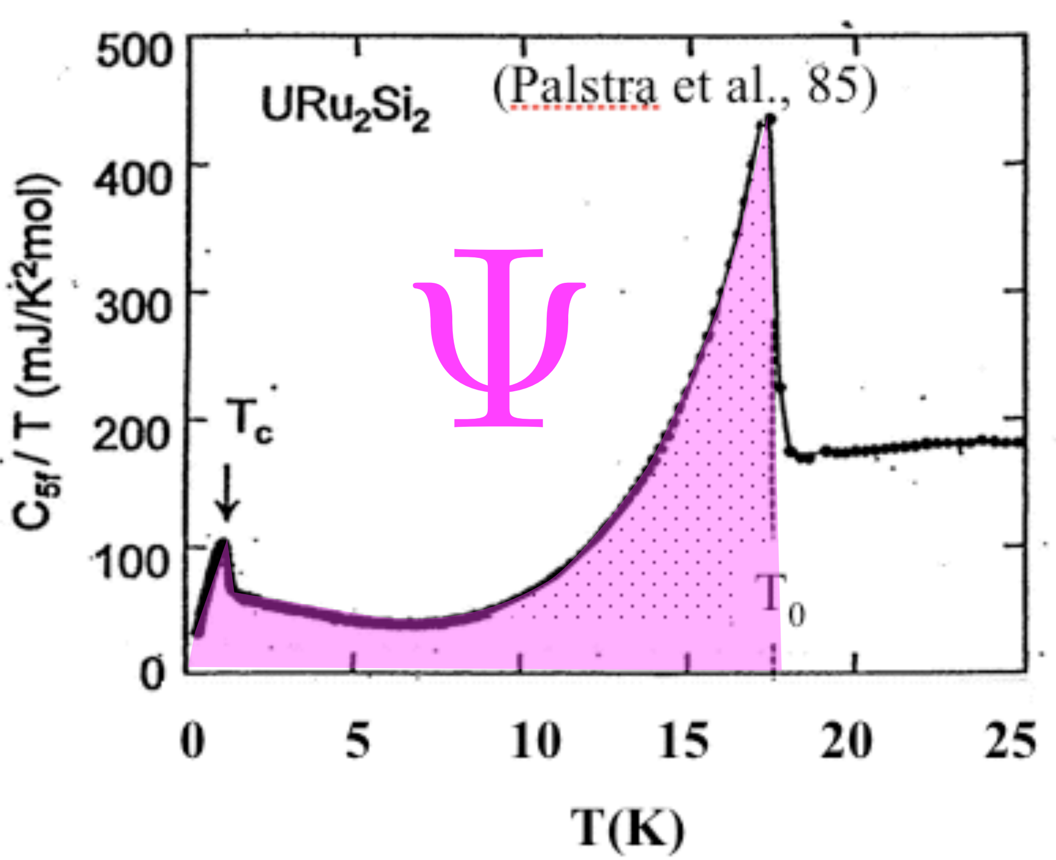

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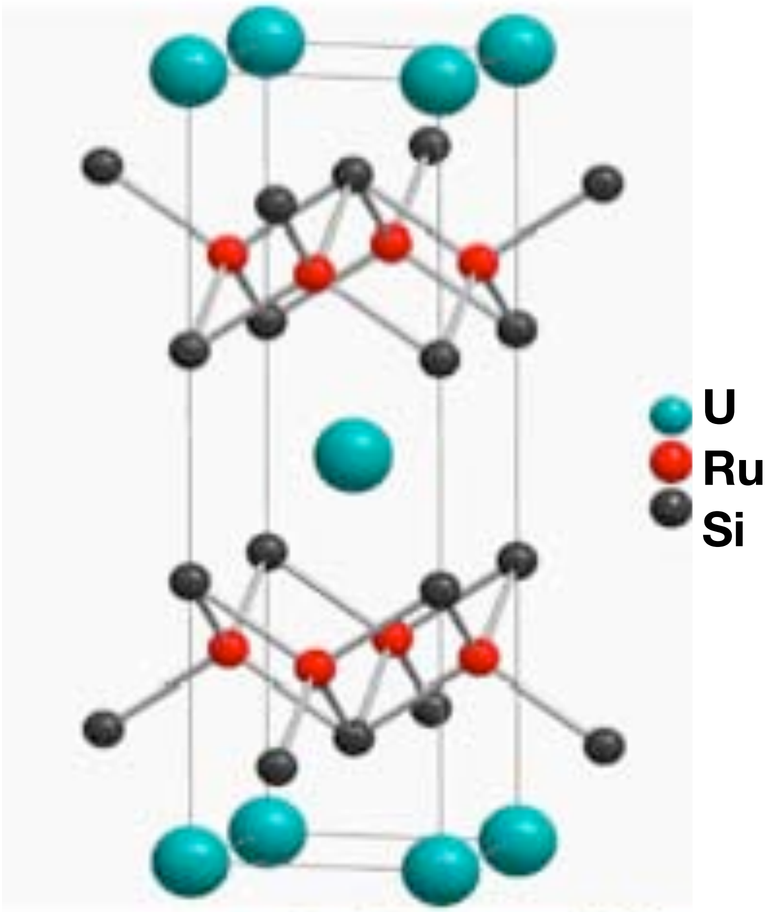
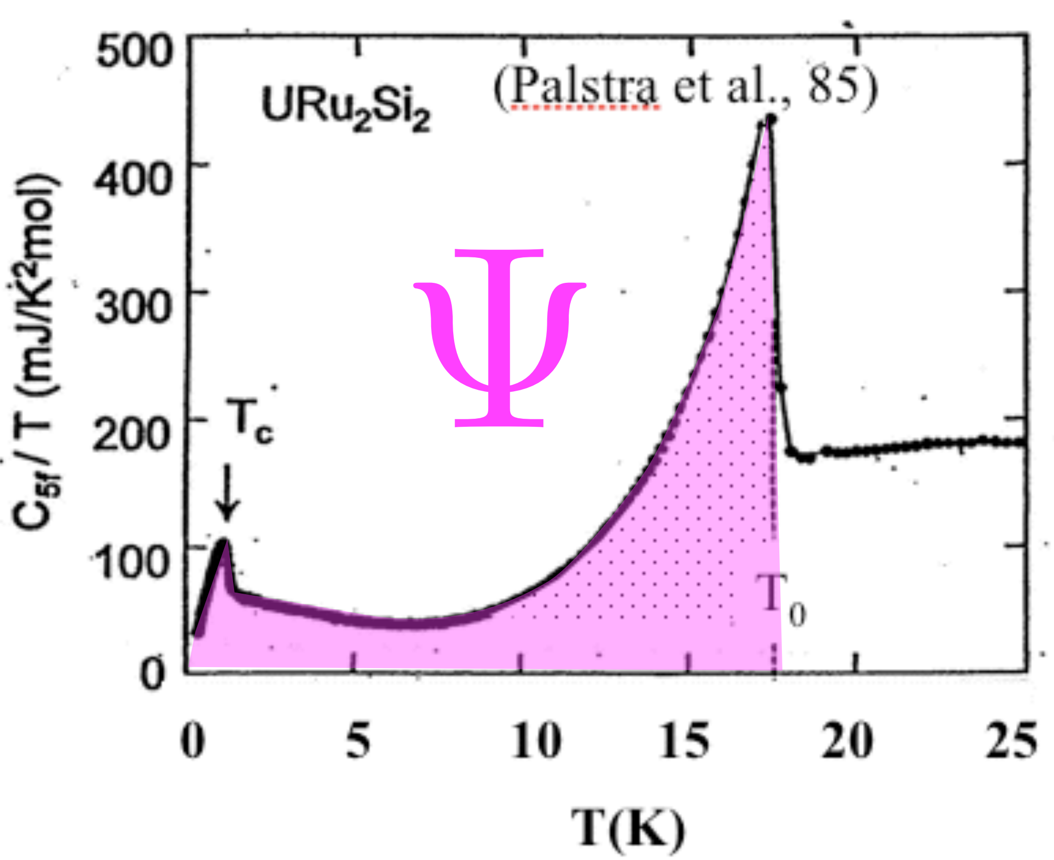
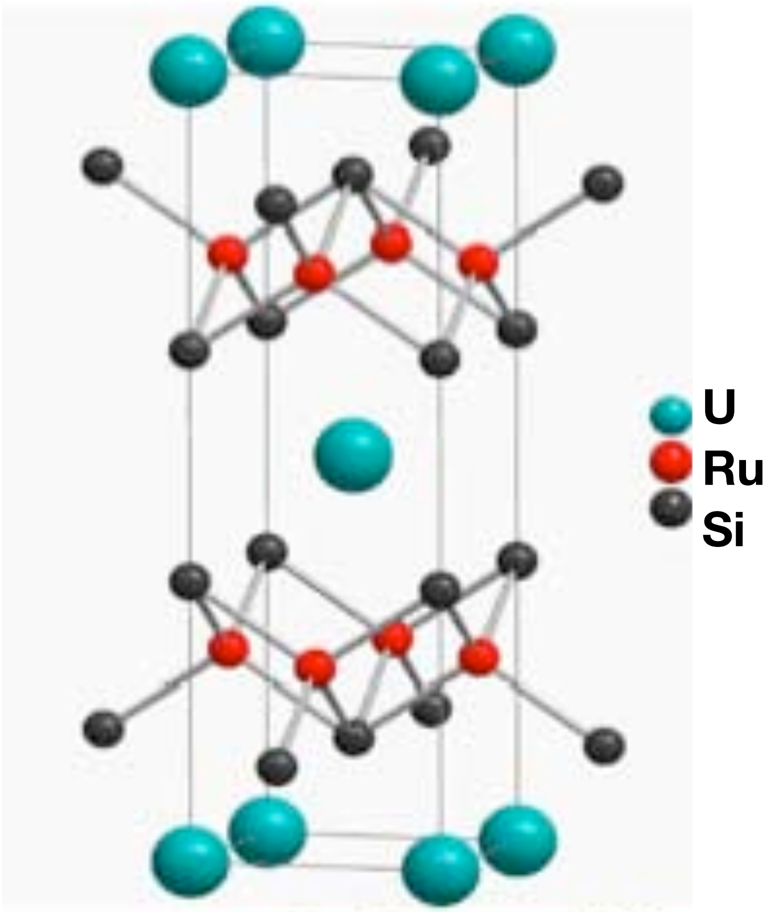


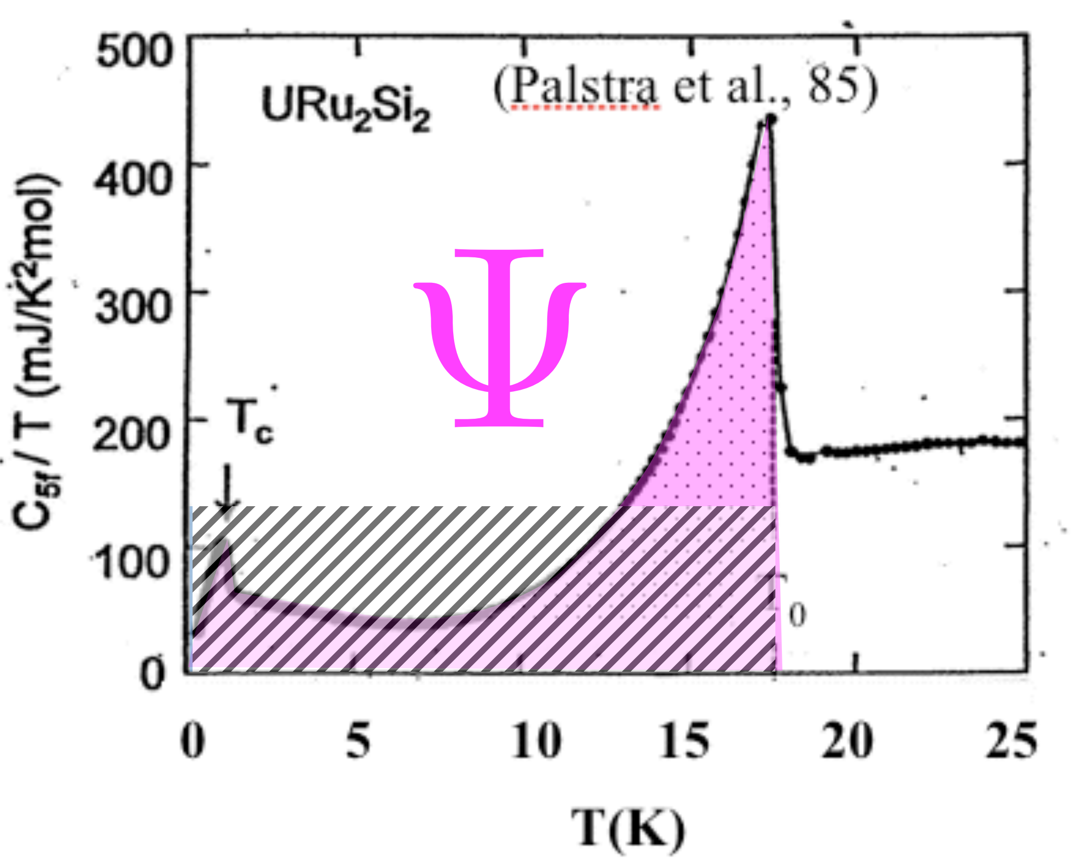
$$\Delta S = \int_0^{T_0} \frac{C_V}{T} dT$$



Hidden Order in URu_2Si_2



$$\Delta S = \int_0^{T_0} \frac{C_V}{T} dT$$



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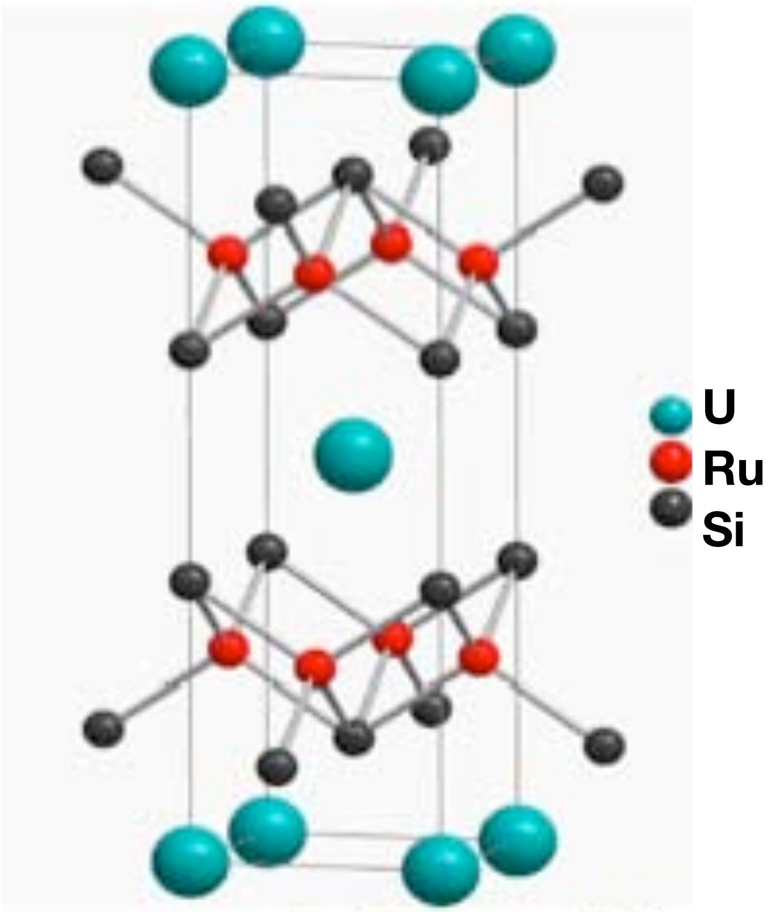
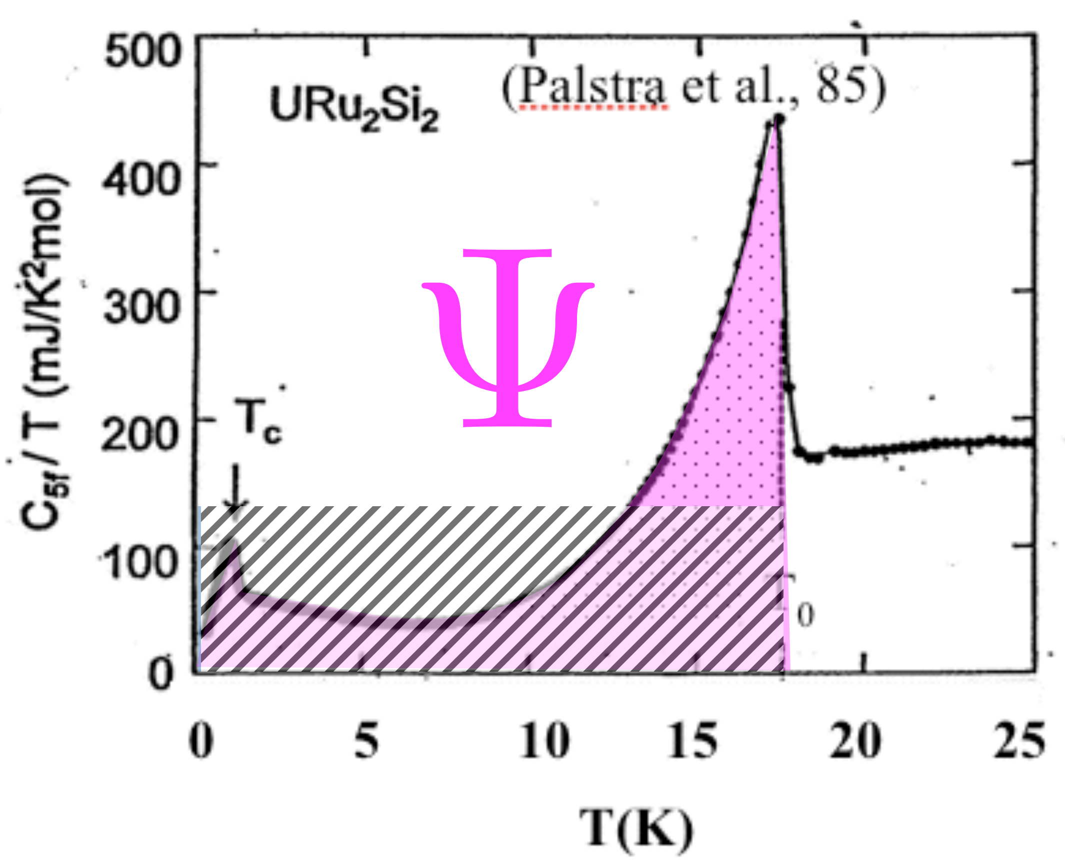


$$\Delta S = \int_0^{T_0} \frac{C_V}{T} dT$$

=0.14 x 17.5 K
=2.45 J/mol/K
=0.42 R ln 2



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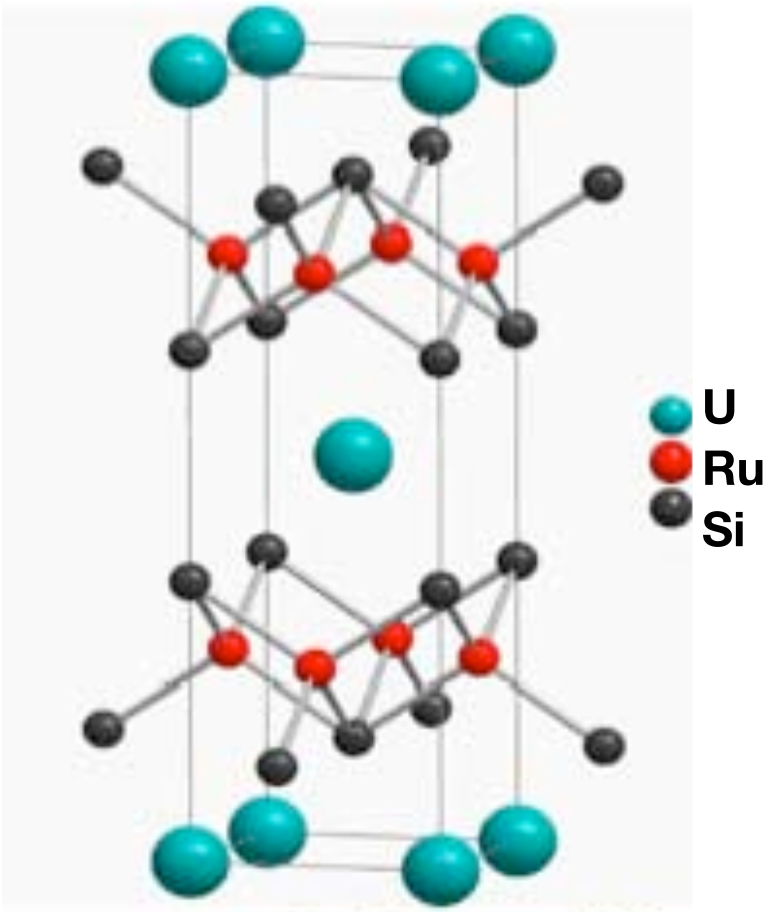

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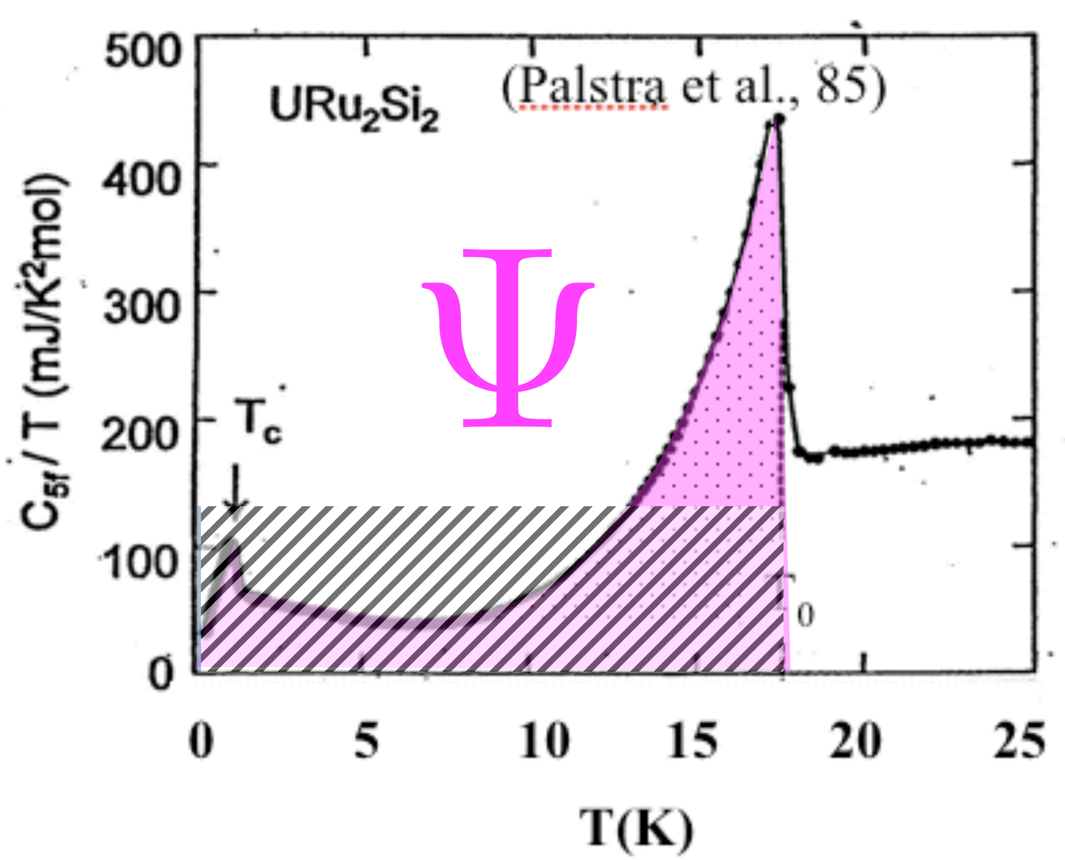
Large entropy of condensation.



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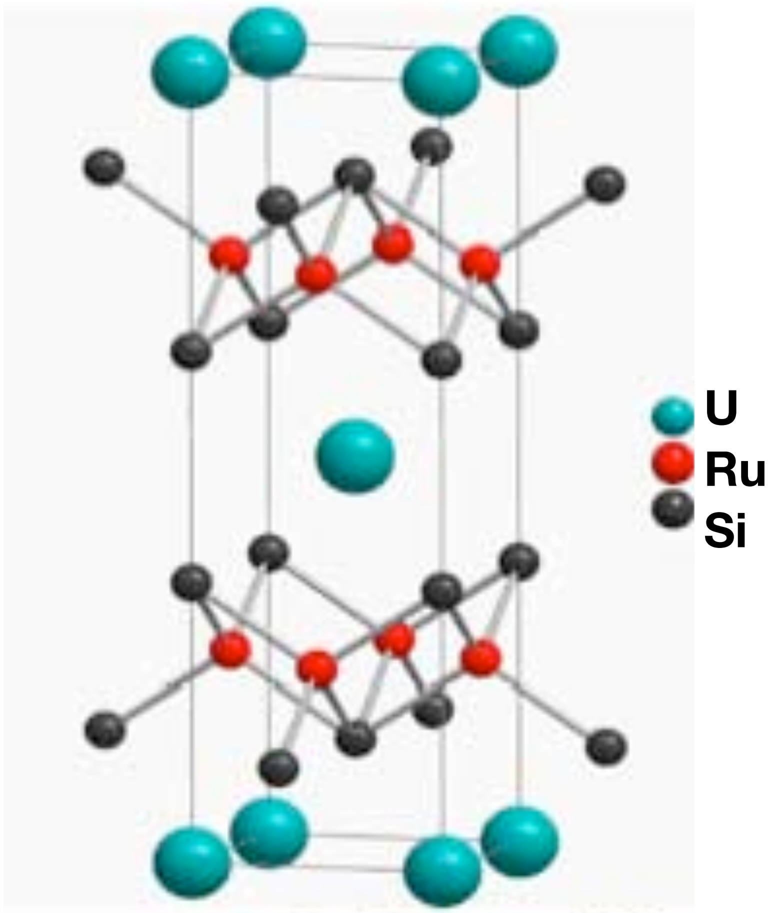

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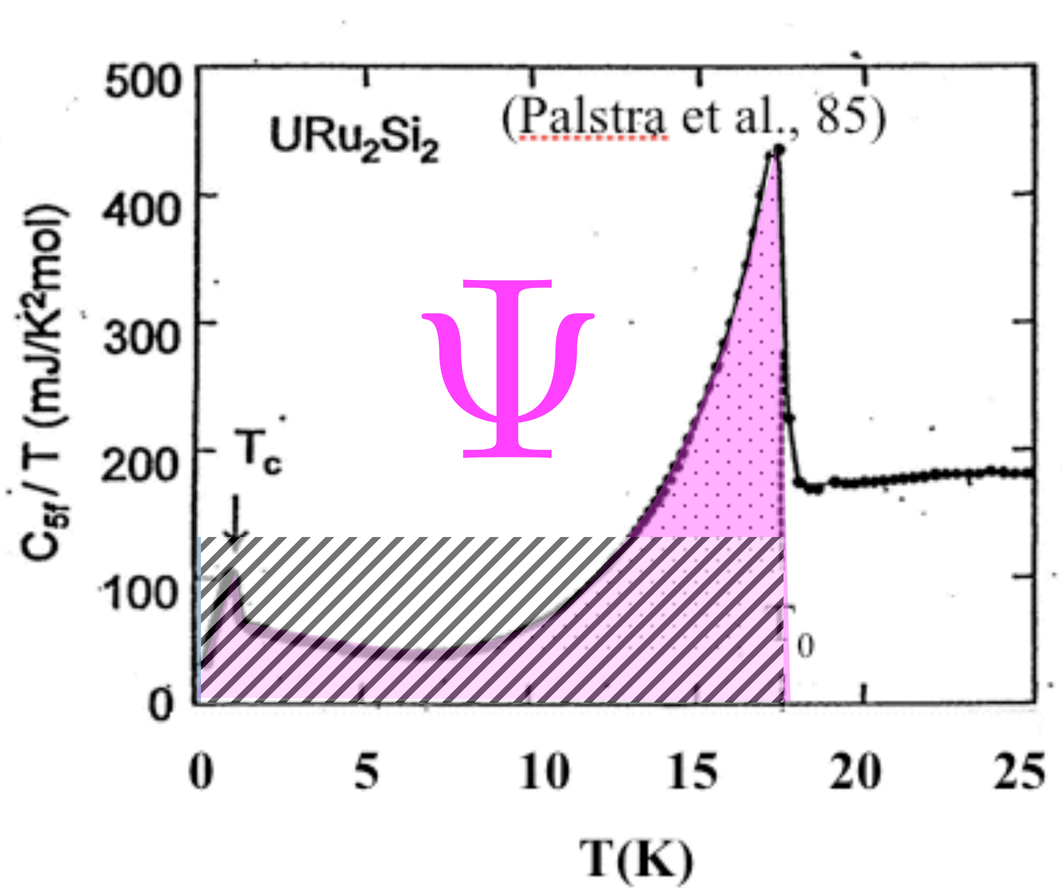
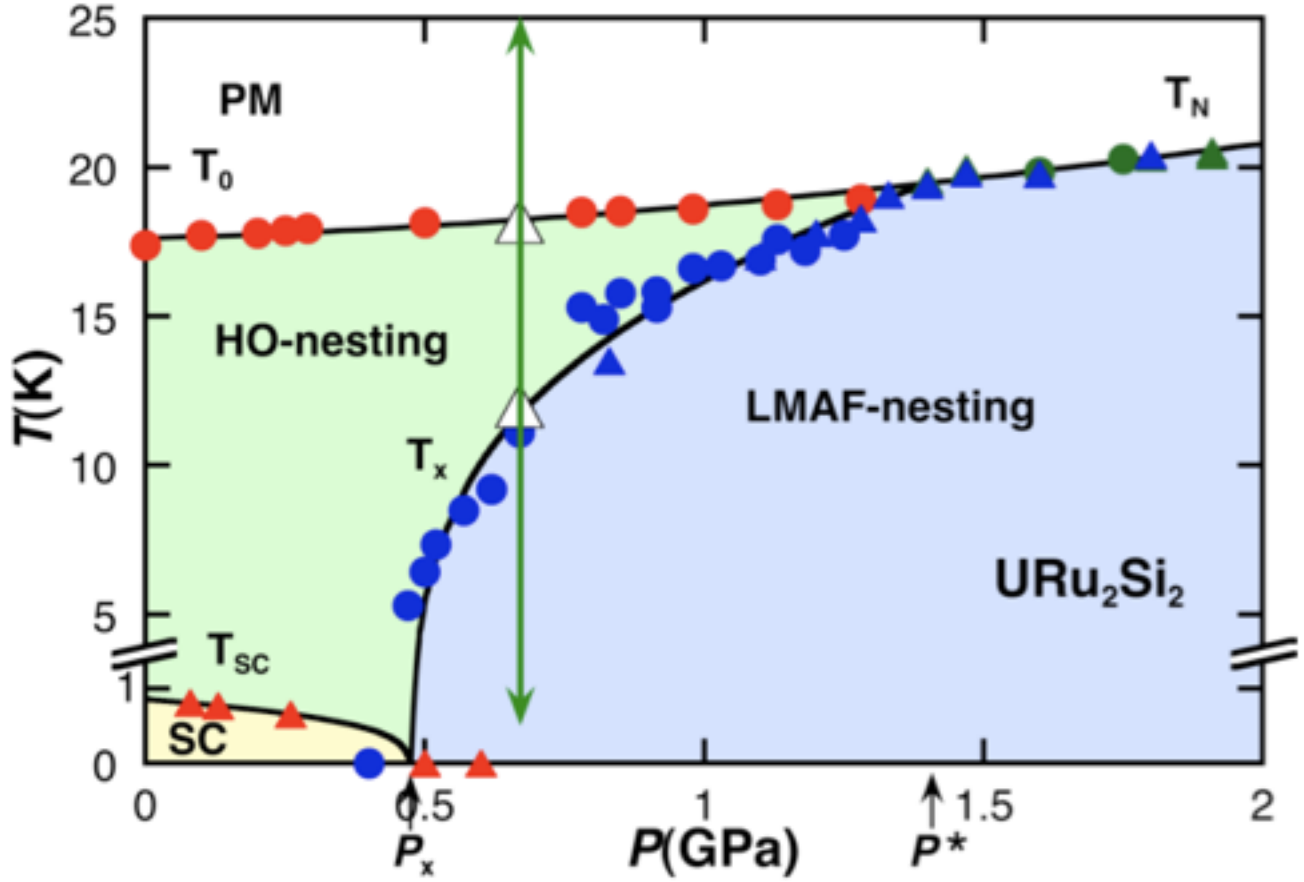


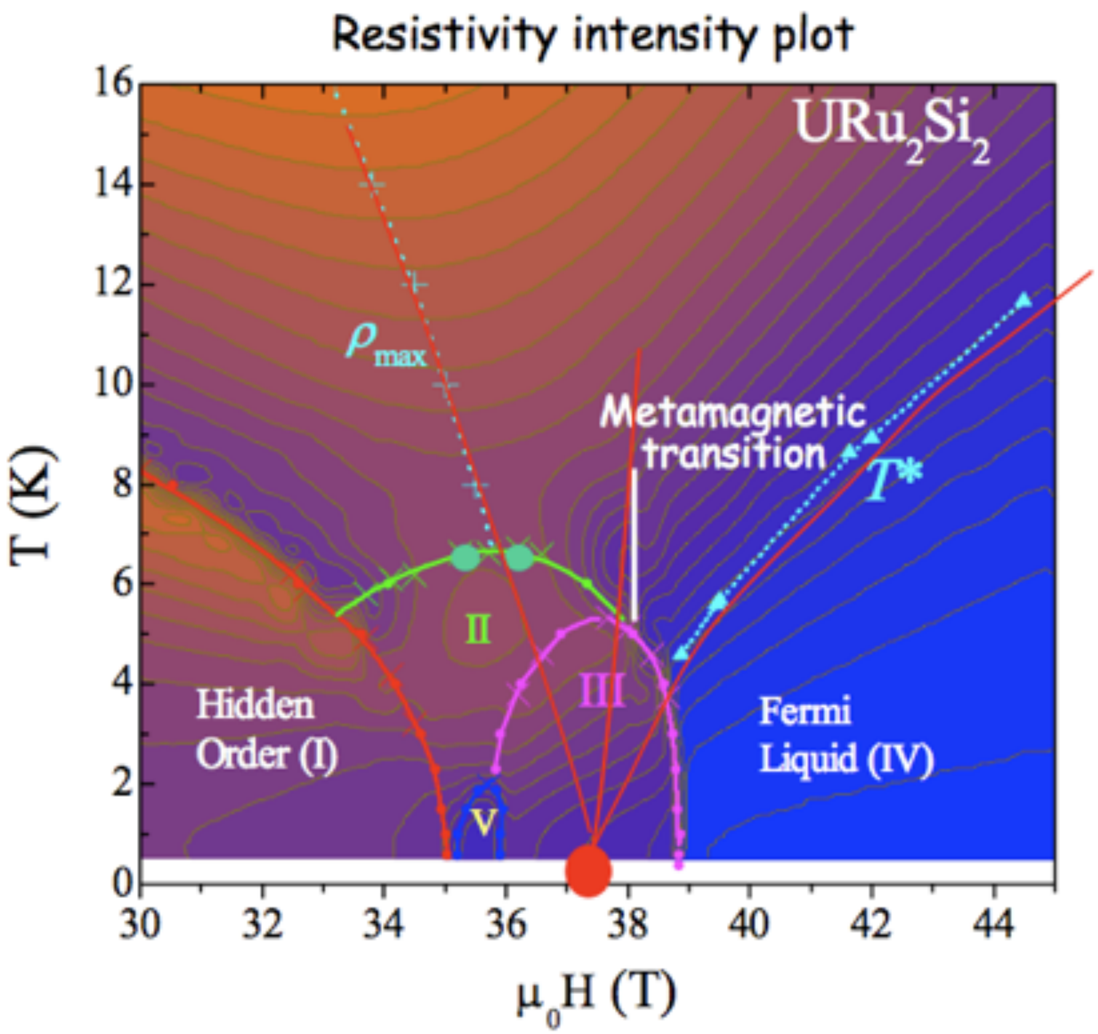
What is the nature of the hidden order?

Rutgers
Center for Materials Theory

High pressures, high fields

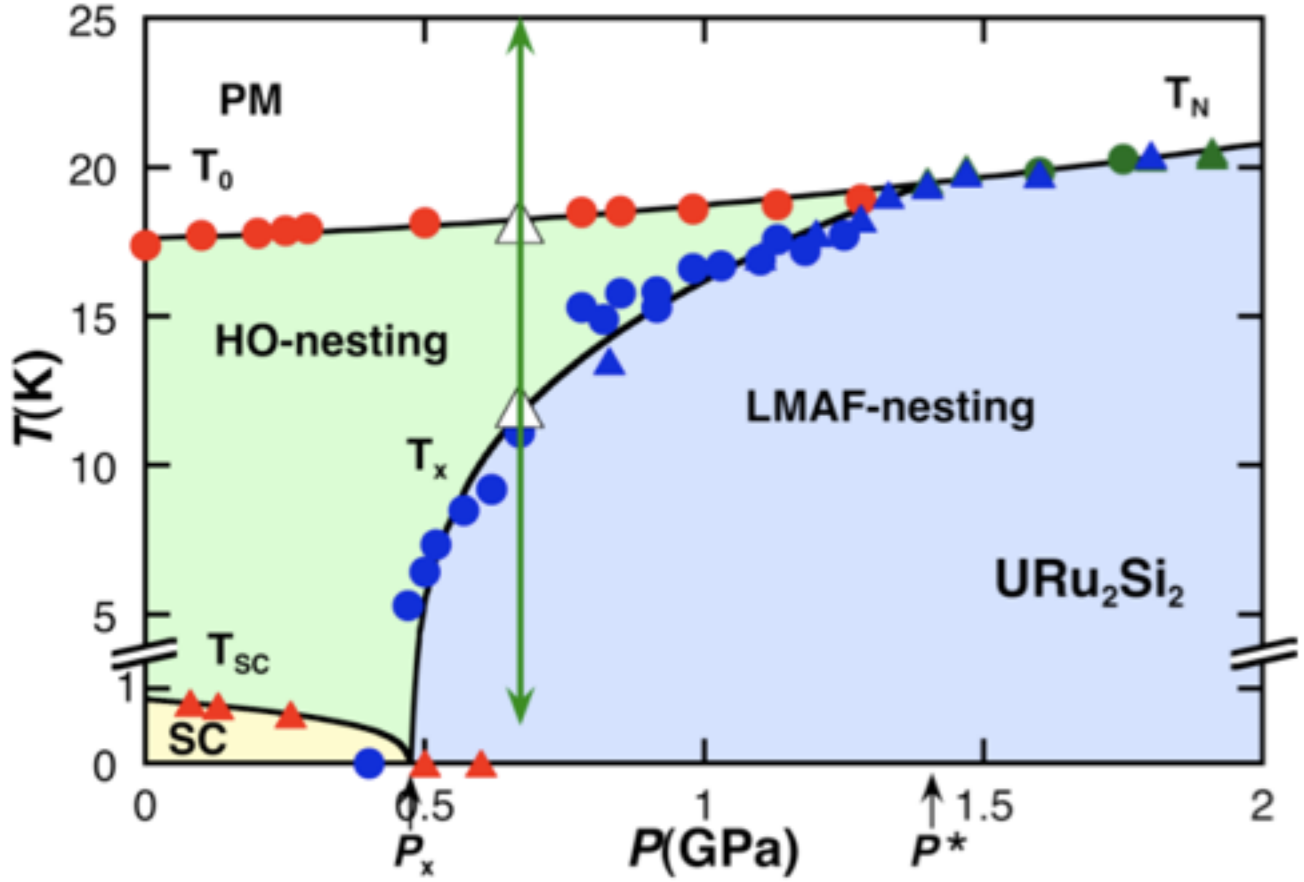


Villaume et al. (08)



Kim et al (03)

High pressures, high fields



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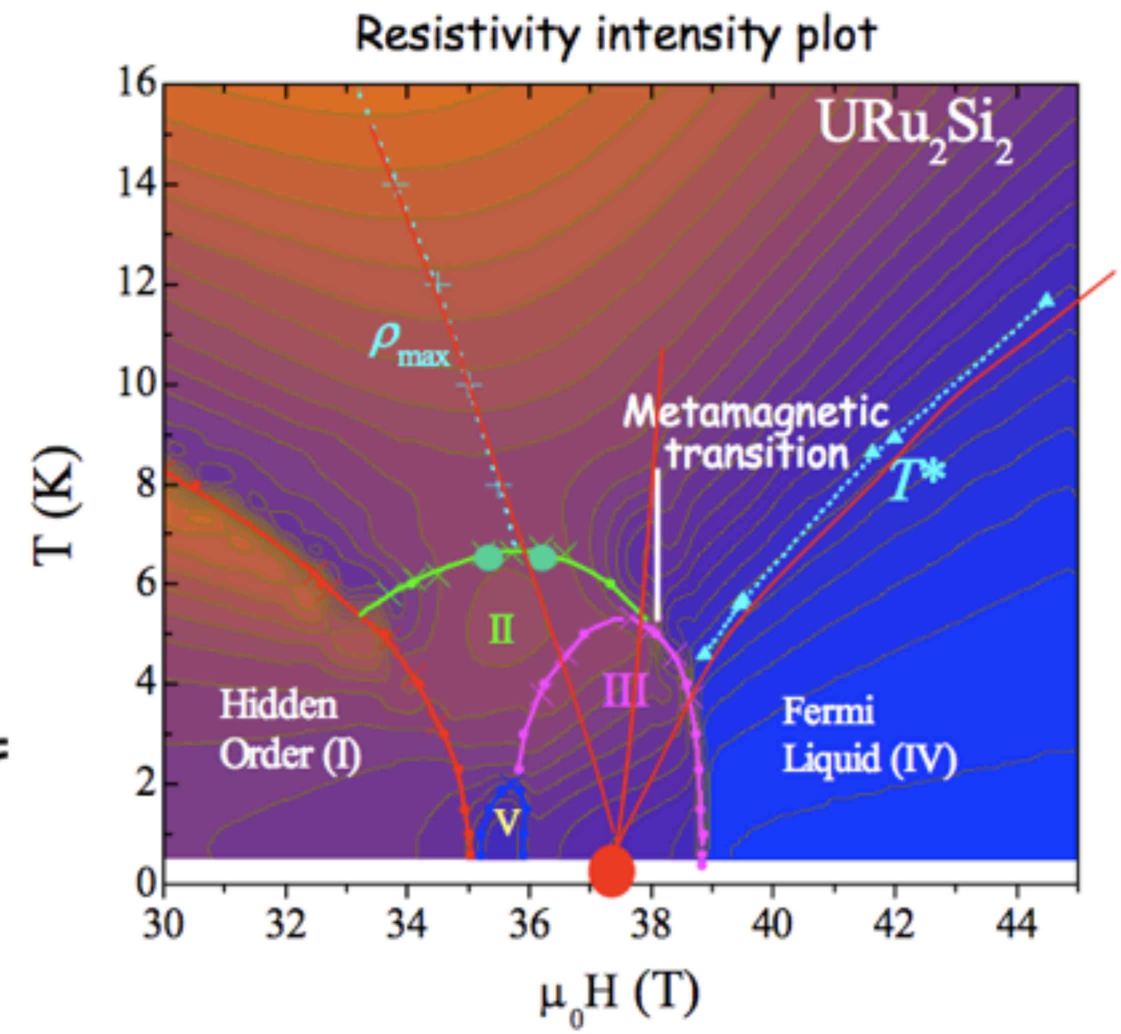


Kim et al (03)

Ising order, present in LMAF, vanishes in the hidden order state. (NMR,MuSR).

25 Years of Theoretical Proposals

Local

Barzykin & Gorkov, ’93 (three-spin correlation)
Santini & Amoretti, ’94, Santini (’98) (Quadrupole order)

Amitsuka & Sakihabara (Γ_5, Quadrupolar doublet, ‘94)
Kasuya, ’97 (U dimerization)
Kiss and Fazekas ’04, (octupolar order)
Haule and Kotliar ’09 (hexa-decapolar)

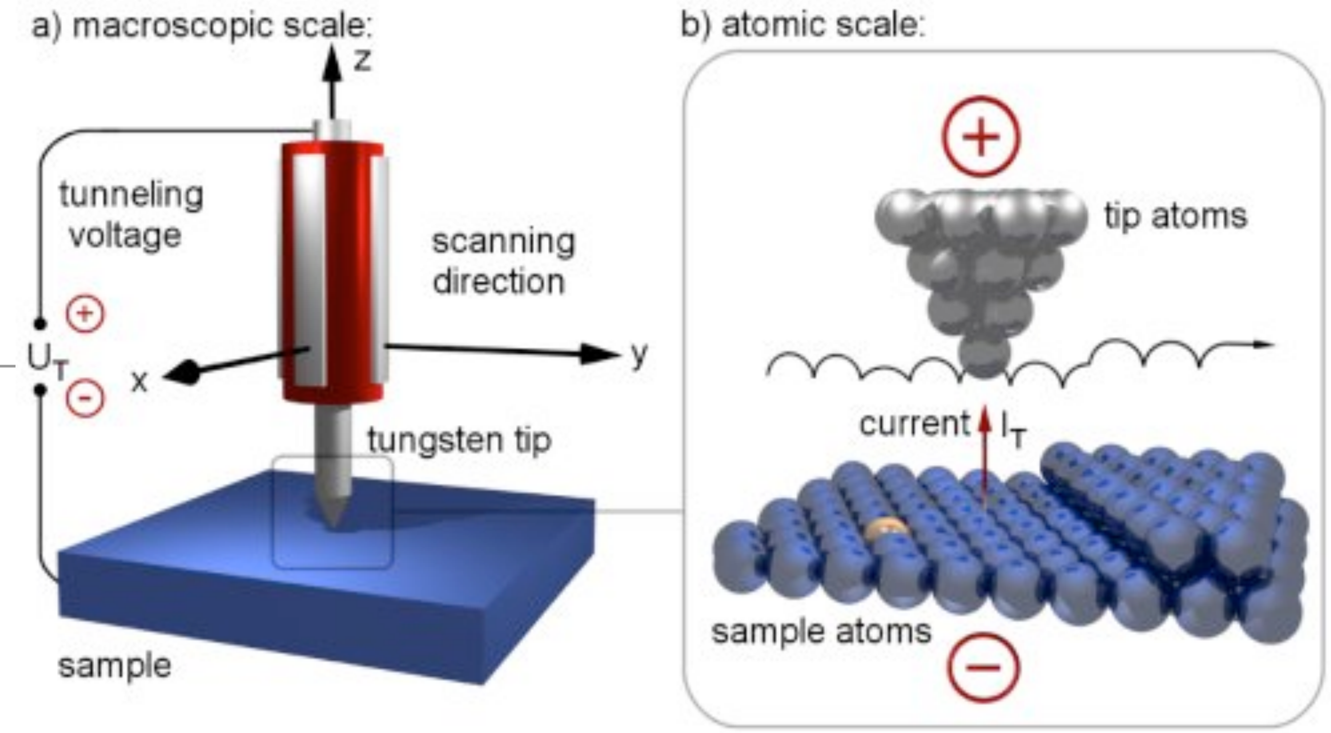
Landau Theory

Shah et al. (’00) “Hidden Order”,

Itinerant

Ramirez et al, ’92 (quadrupolar SDW)
Ikeda and Ohashi ’98 (d-density wave)
Okuno and Miyake ’98 (composite)
Tripathi, Chandra, PC and Mydosh, ’02 (orbital afm)
Dori and Maki, ’03 (unconventional SDW)
Mineev and Zhitomirsky, ’04 (SDW)
Varma and Zhu, ’05 (spin-nematic)
Ezgar et al ’06 (Dynamic symmetry breaking)
Pepin et al ’10 (Spin liquid/Kondo Lattice)
Dubi and Balatsky, ’10 (Hybridization density wave)
Fujimoto, 2011 (spin-nematic)
Rau and Kee 2012 (Rank 5 pseudo-spin vector)

Cause Célèbre: state of the art spectroscopies



Scanning Tunneling Microscopy

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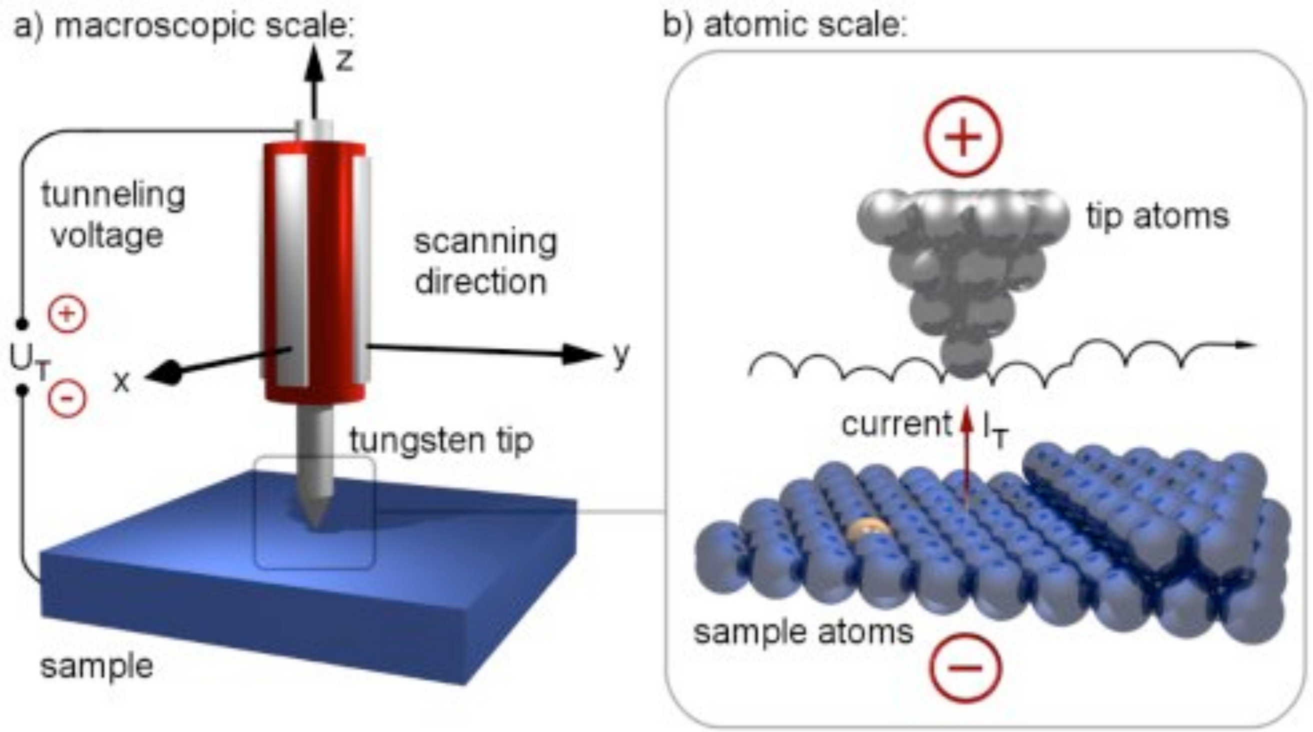
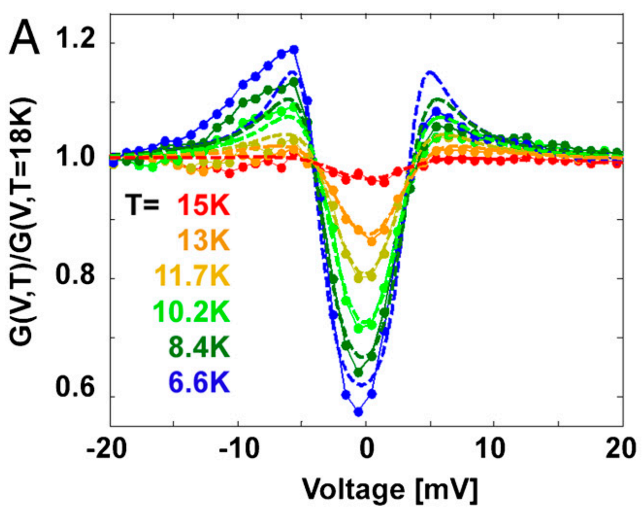
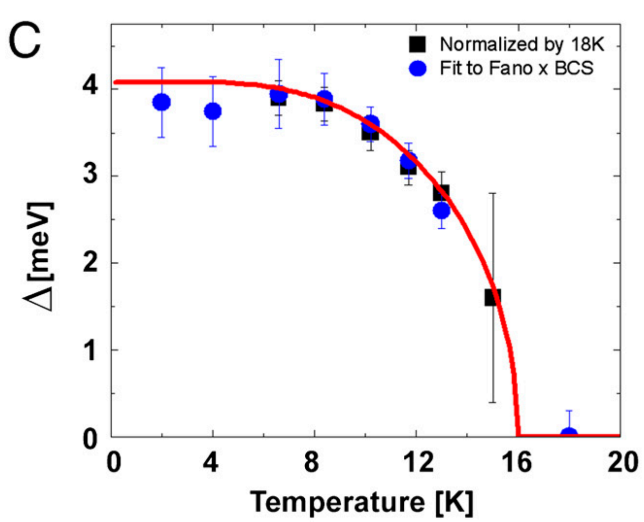


Scanning Tunneling Microscopy

Pegor Aynajian et al, PNAS (2010)





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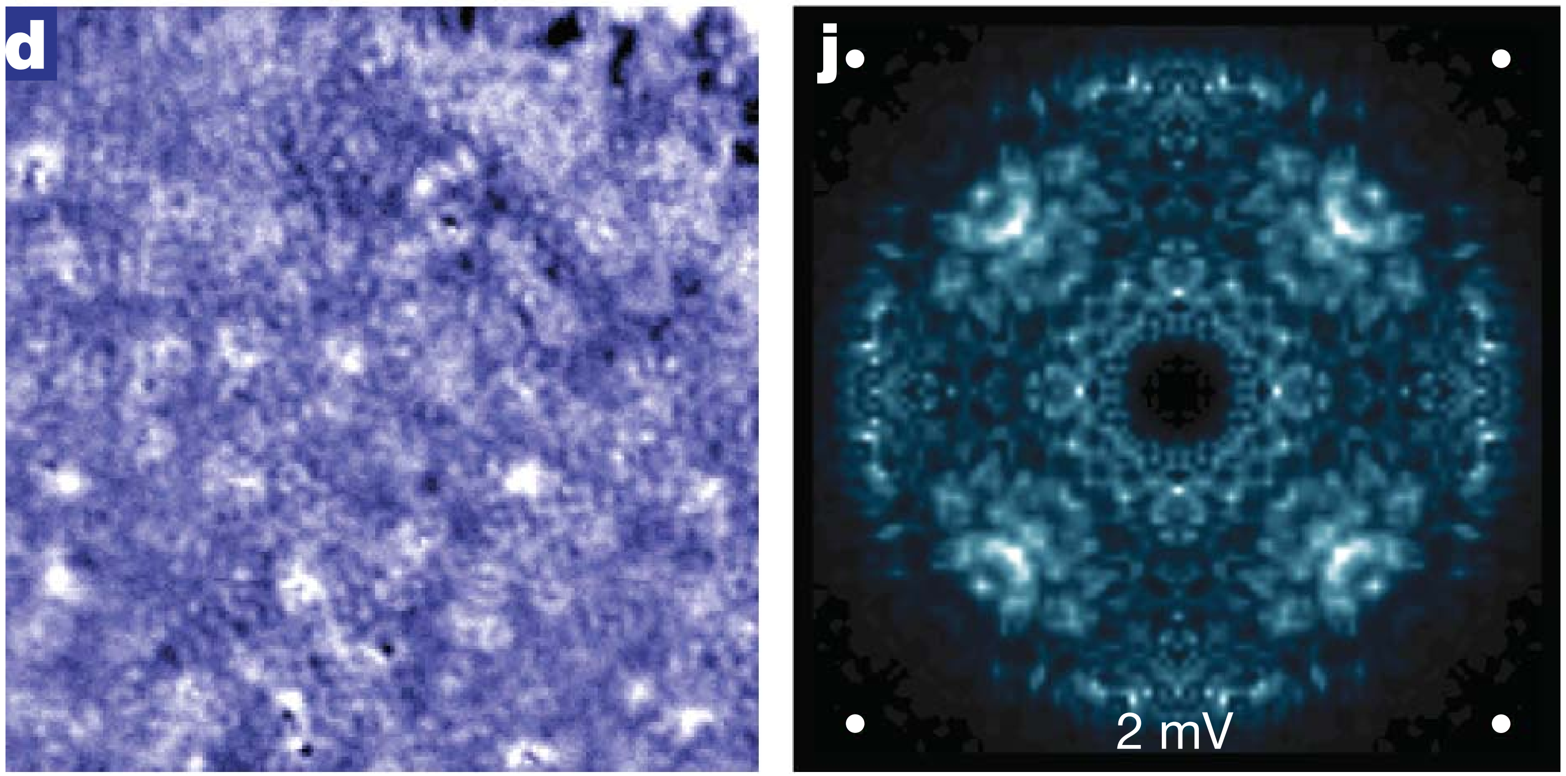

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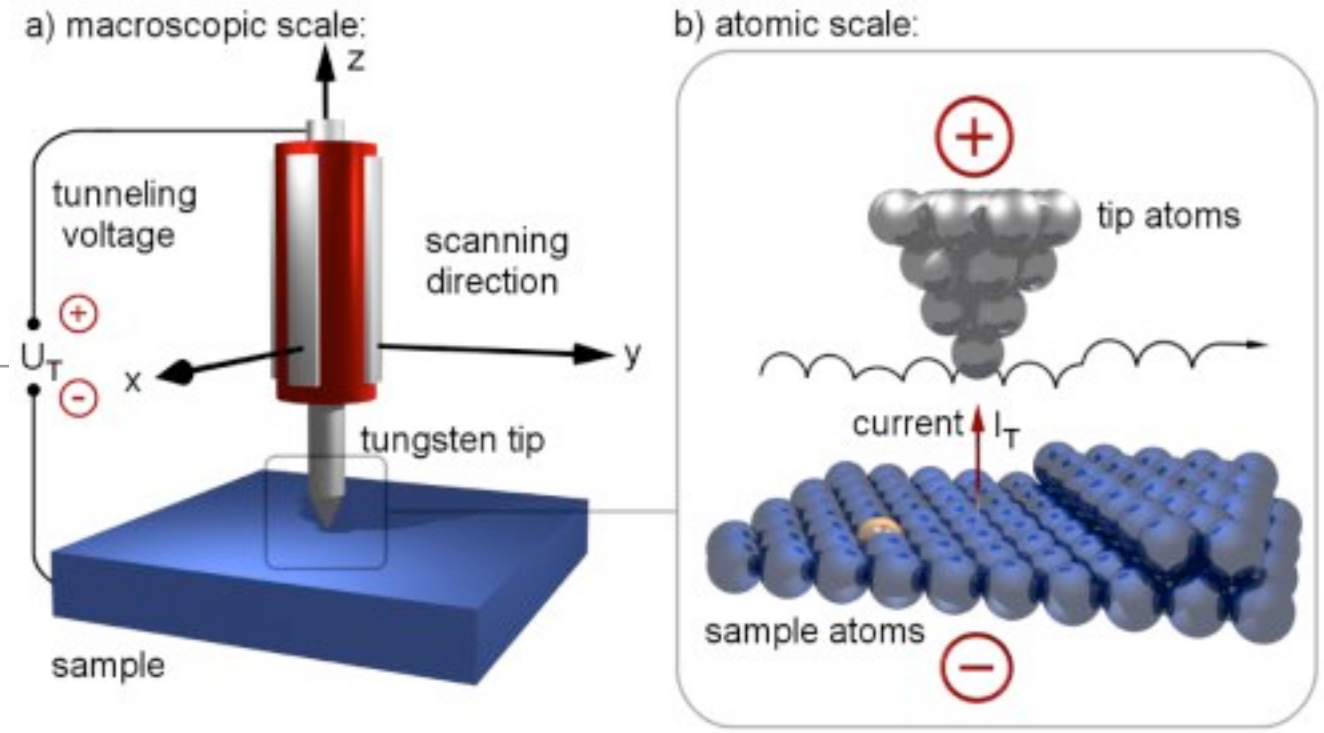

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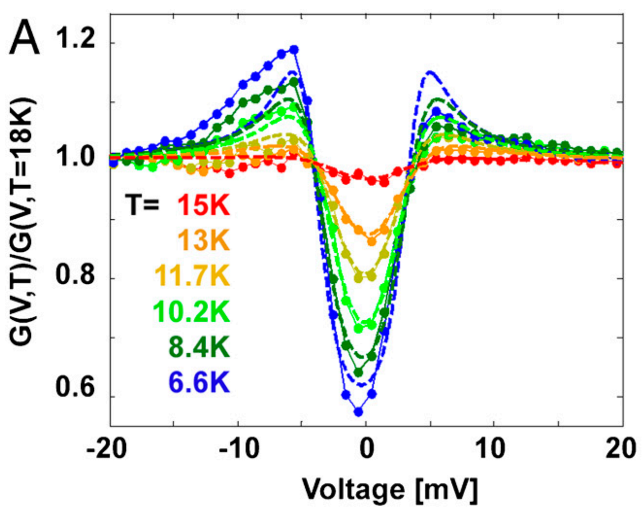

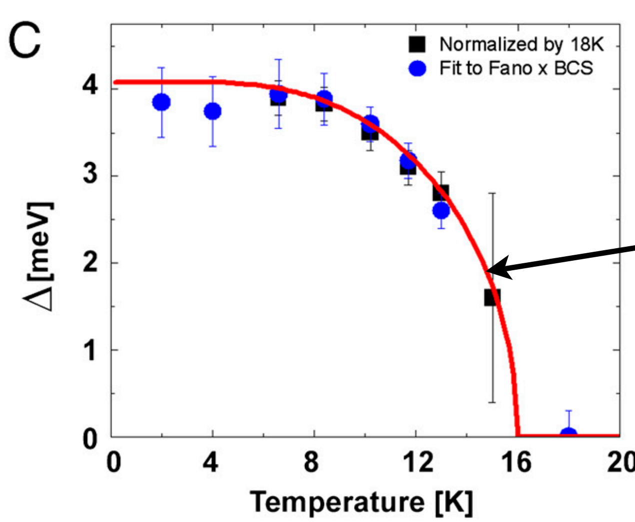

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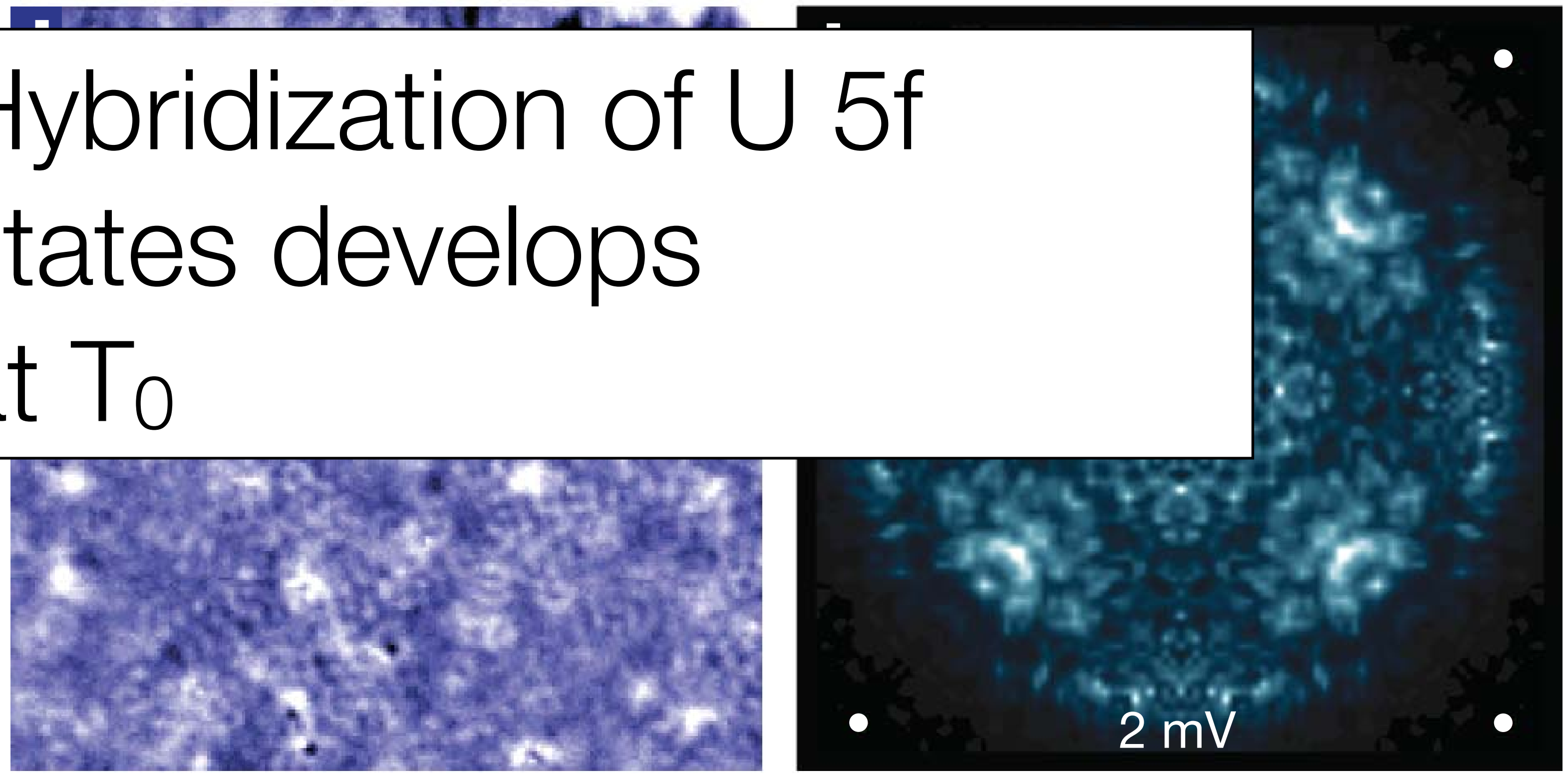
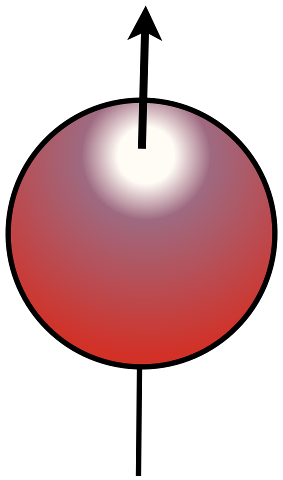


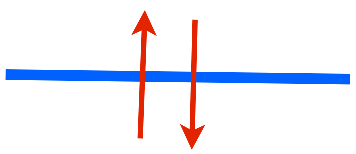
Hybridization of U 5f states develops at T_0

The Giant Ising Anisotropy.

Strange electron spin of URu_2Si_2



electron spin



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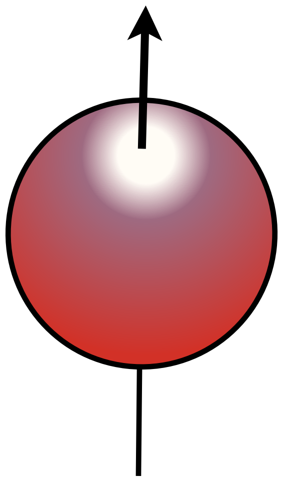
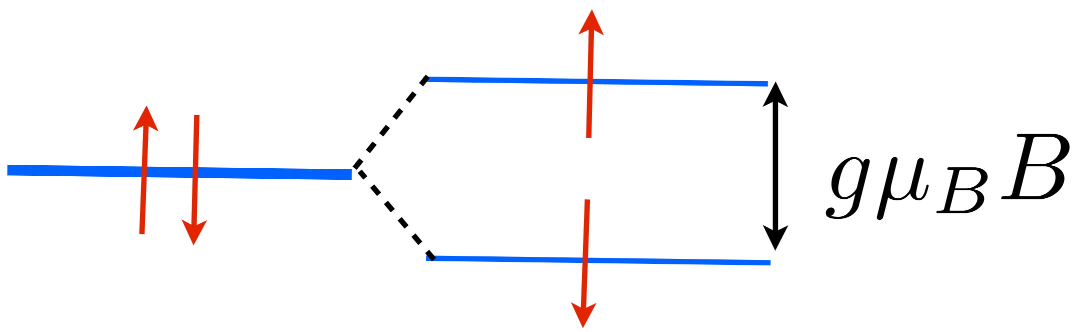


electron spin



magnetic field *B*

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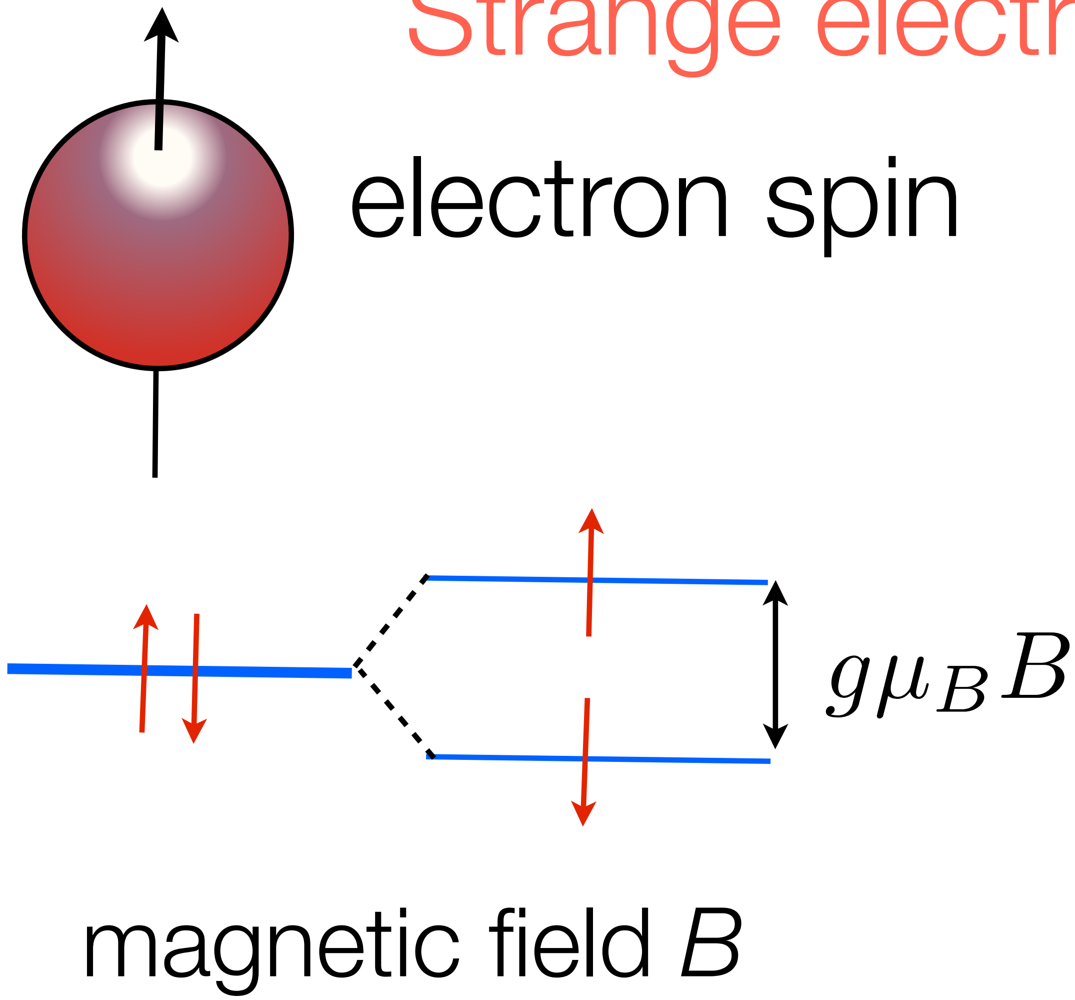
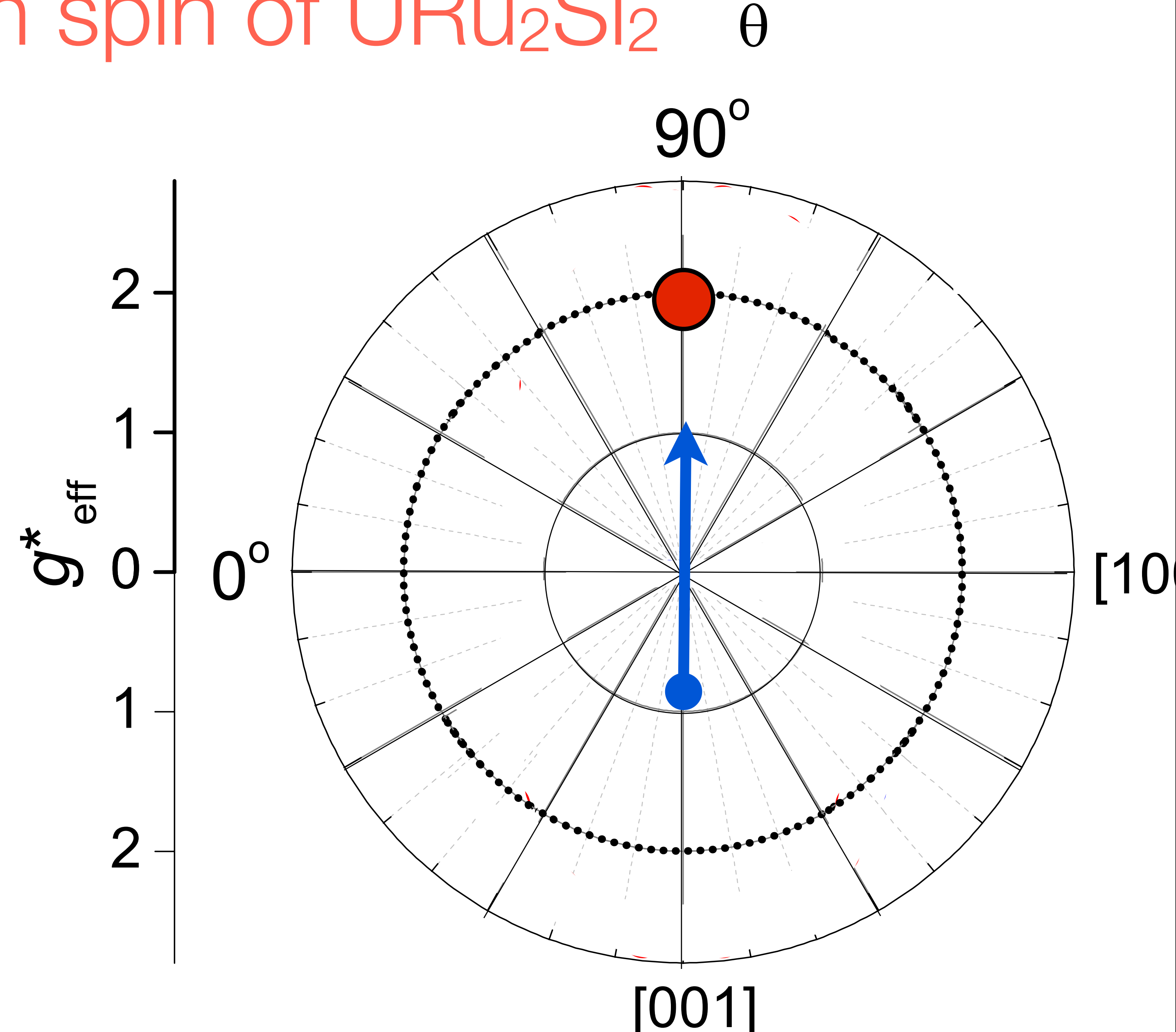
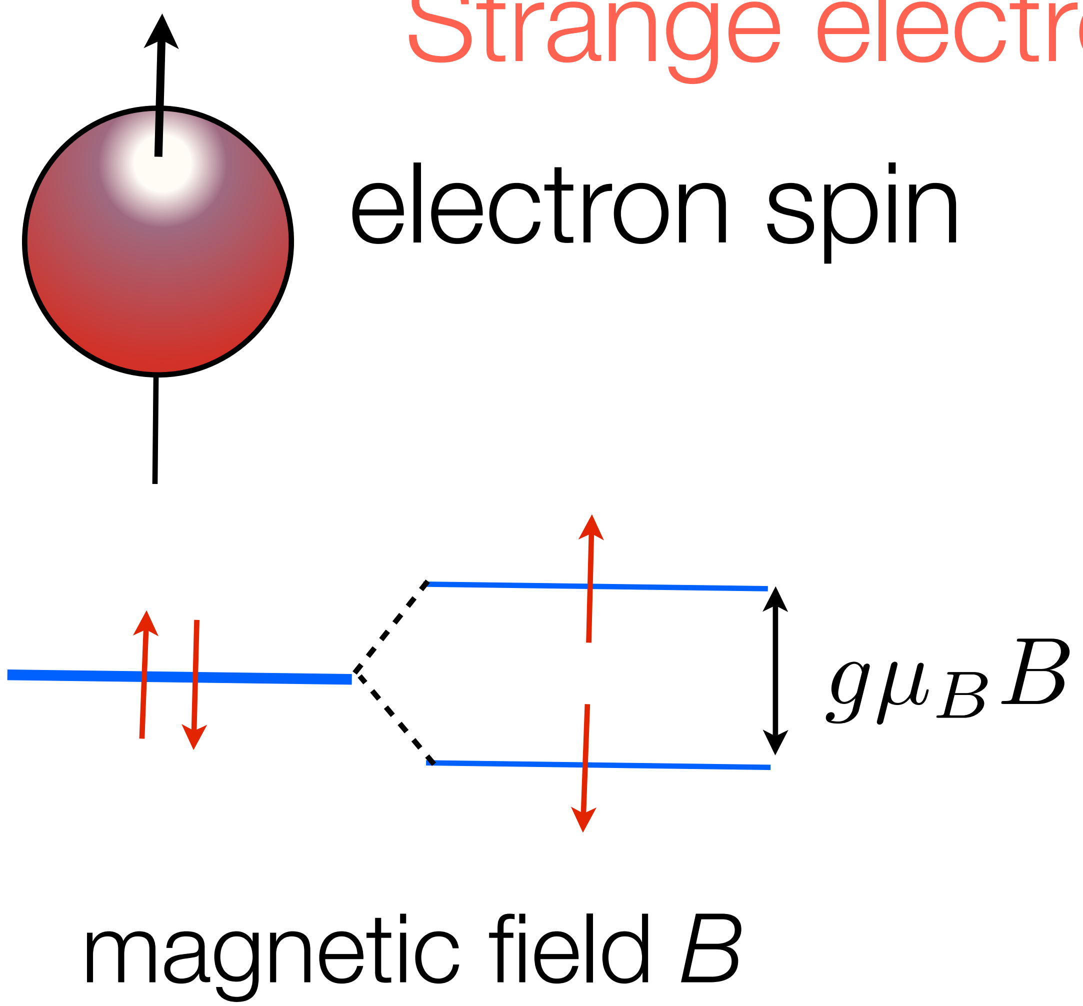
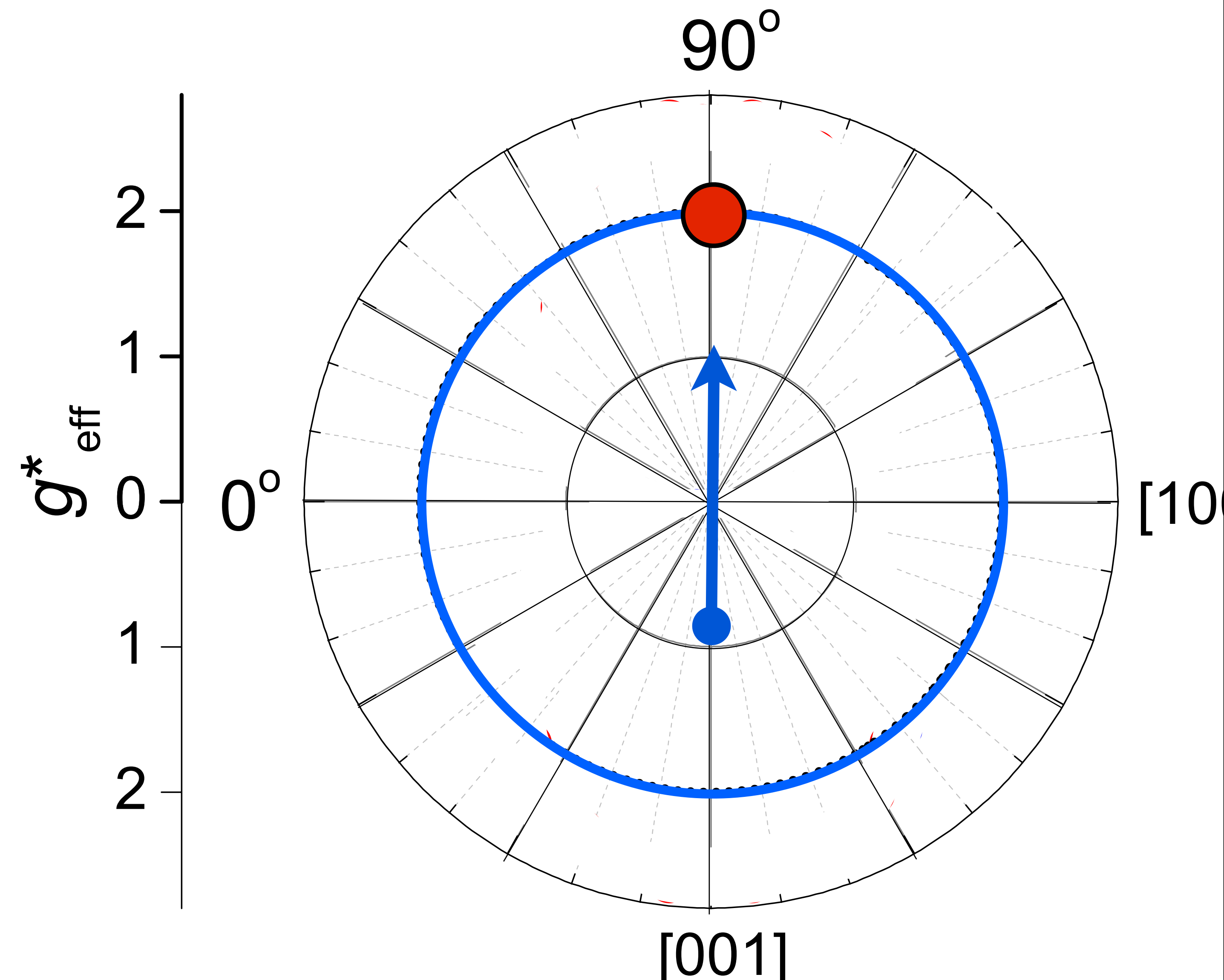




Strange electron spin of URu_2Si_2

θ

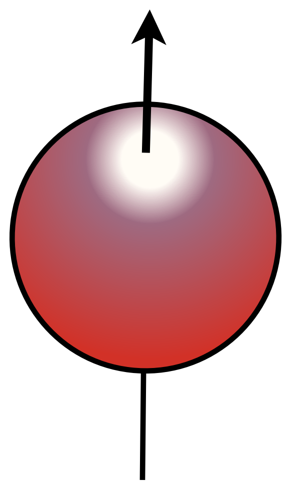




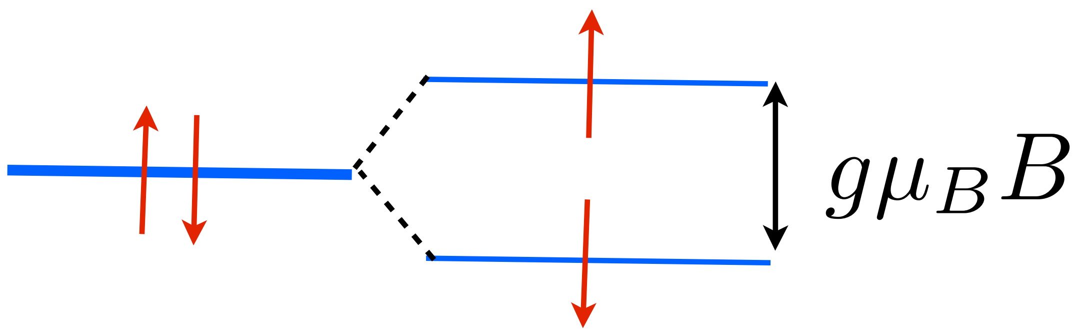
$$M = g(\theta)\mu_B = 2\mu_B$$

Isotropic moment

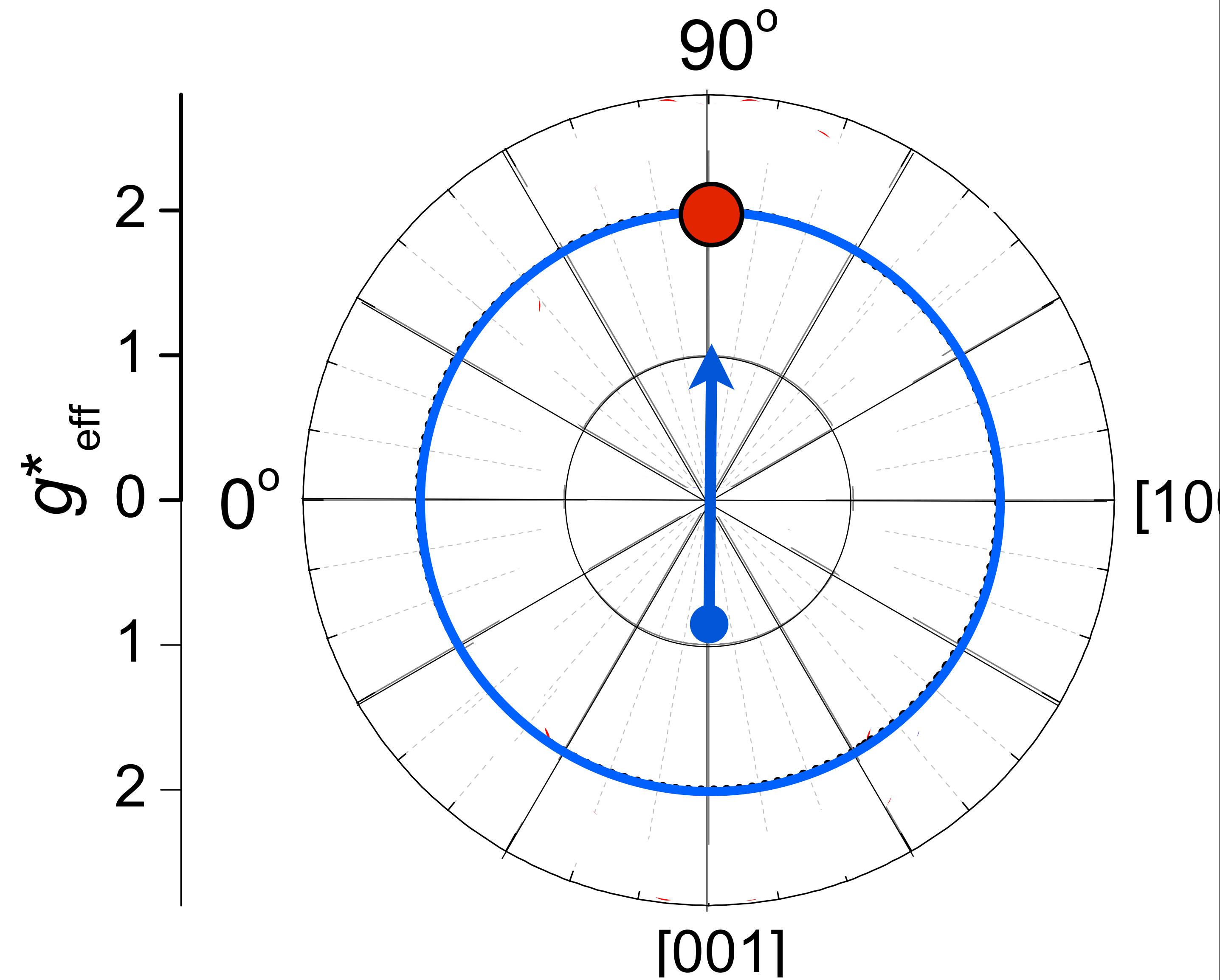
Strange electron spin of URu_2Si_2 θ



electron spin



magnetic field *B*



$$M = g(\theta)\mu_B = 2\mu_B$$

Isotropic moment

S=1/2

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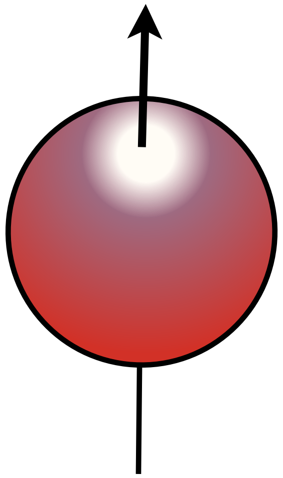
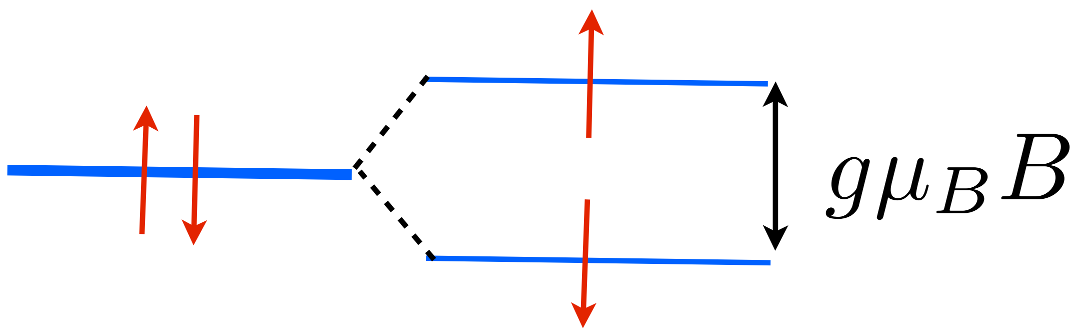
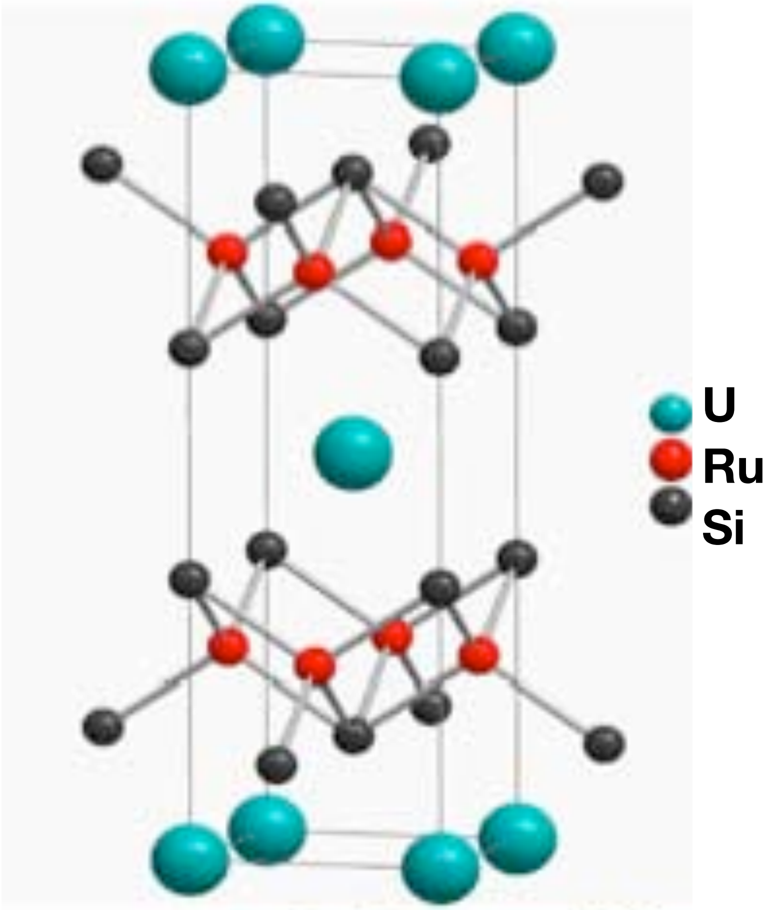
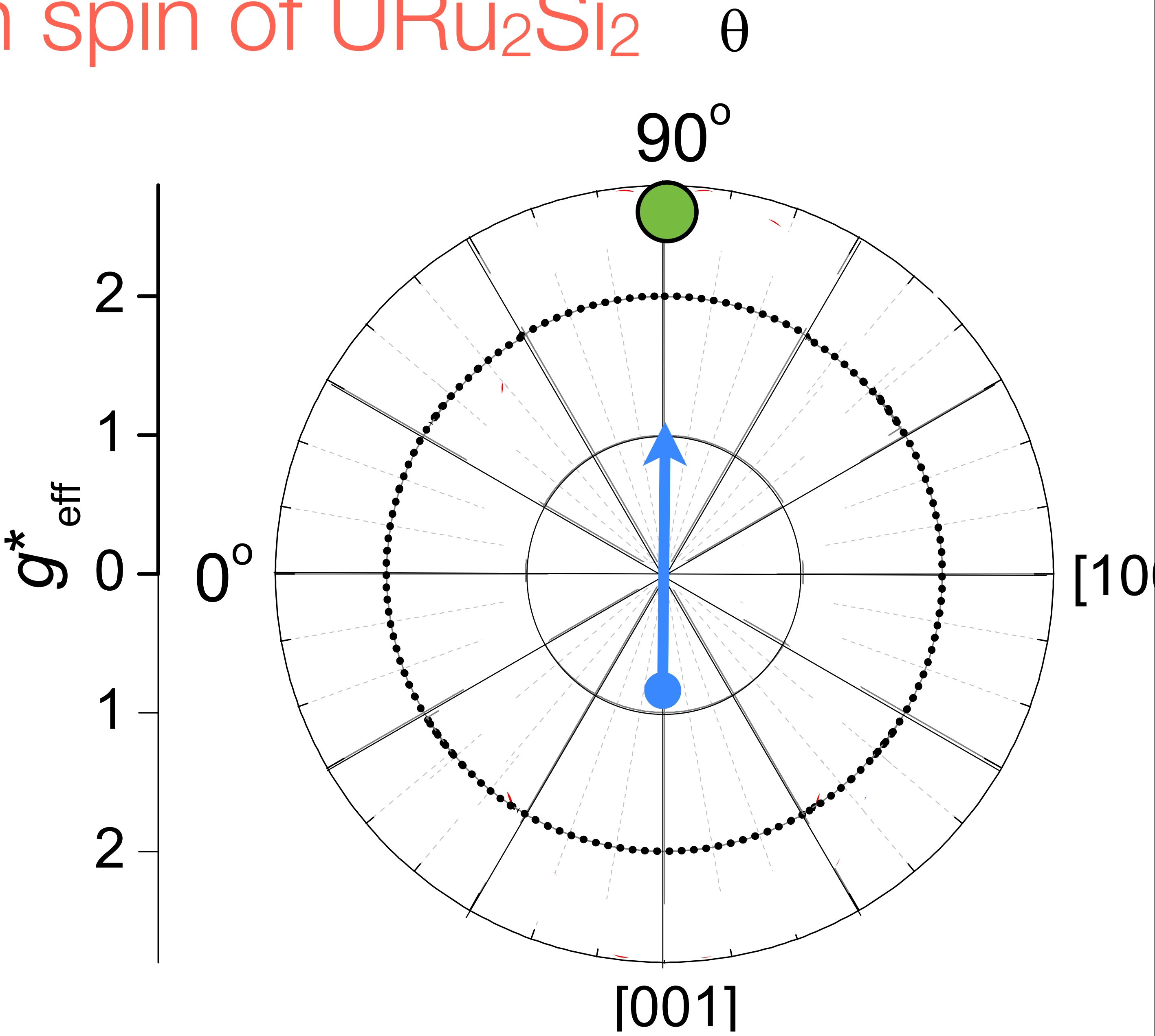


electron spin



URu_2Si_2





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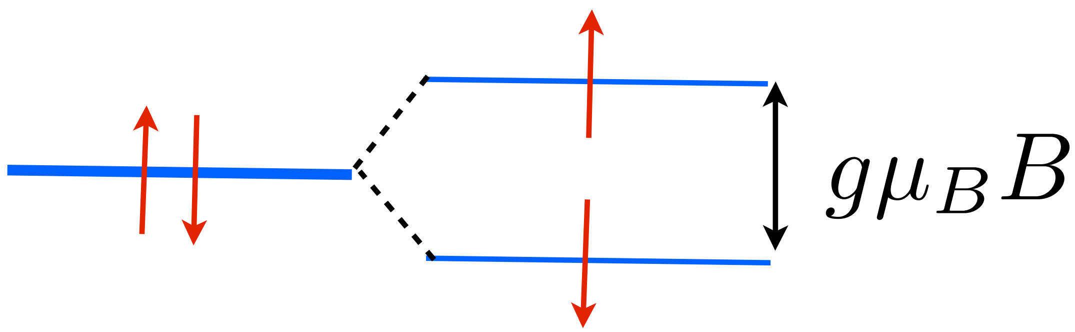

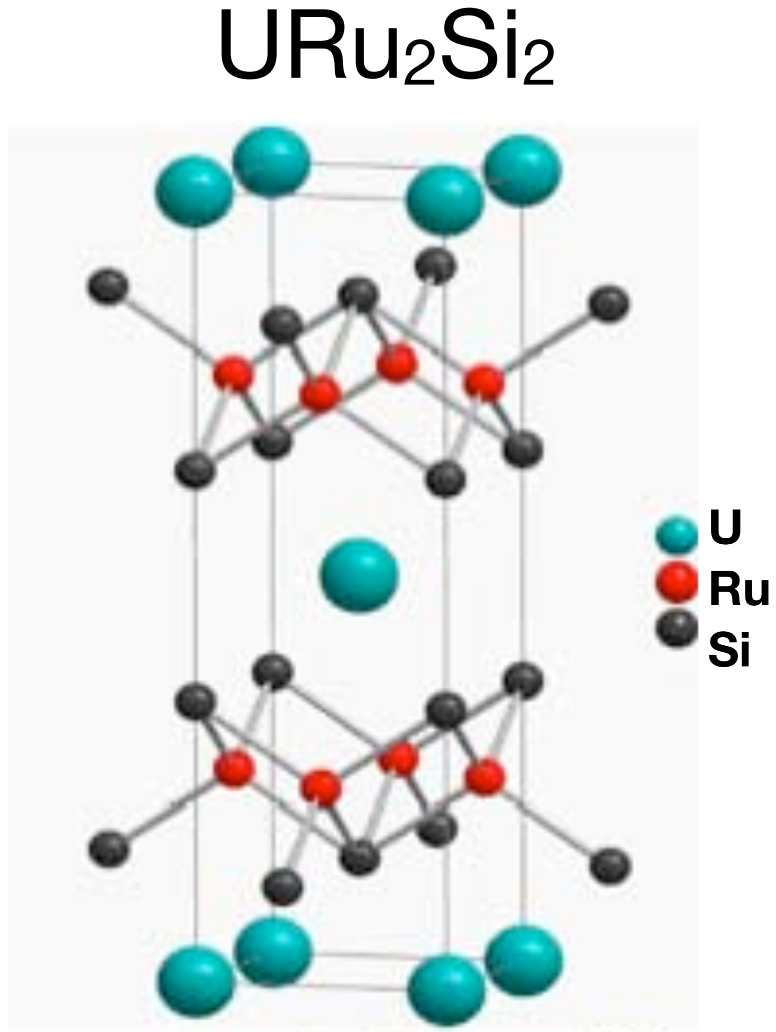

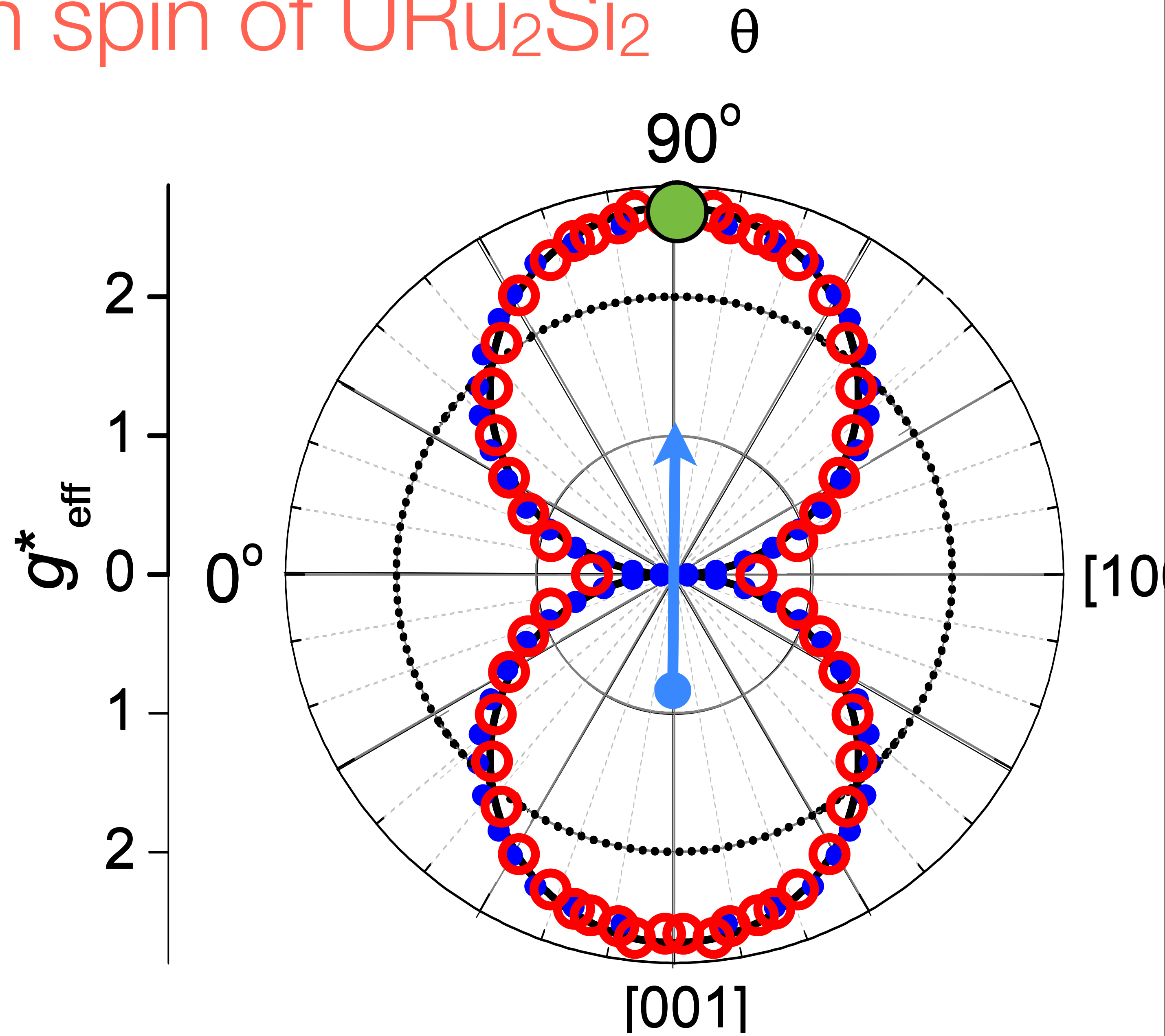

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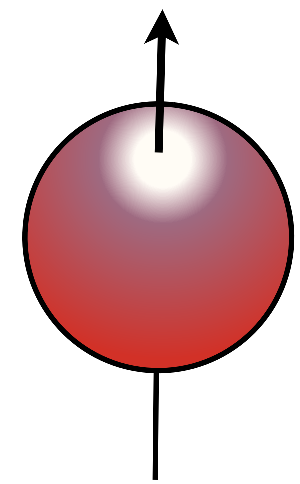
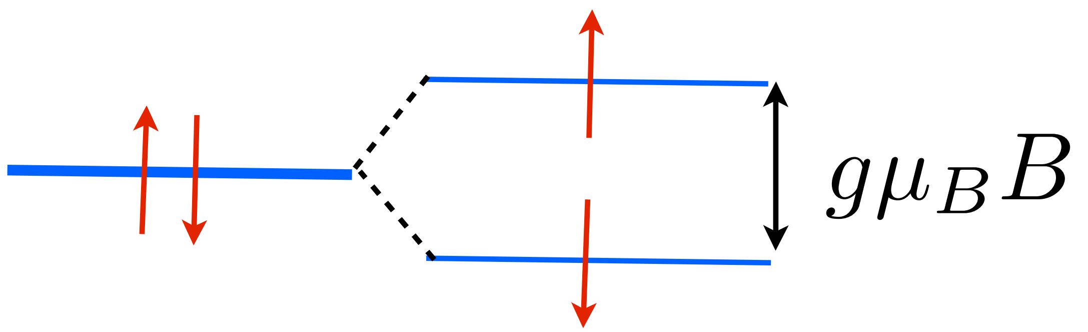
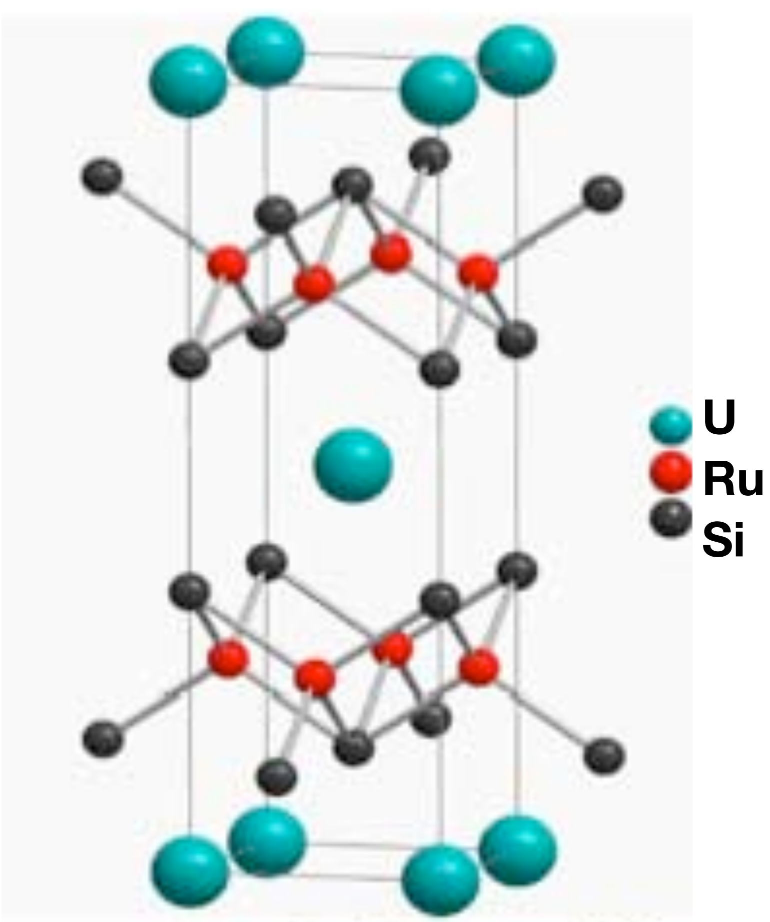
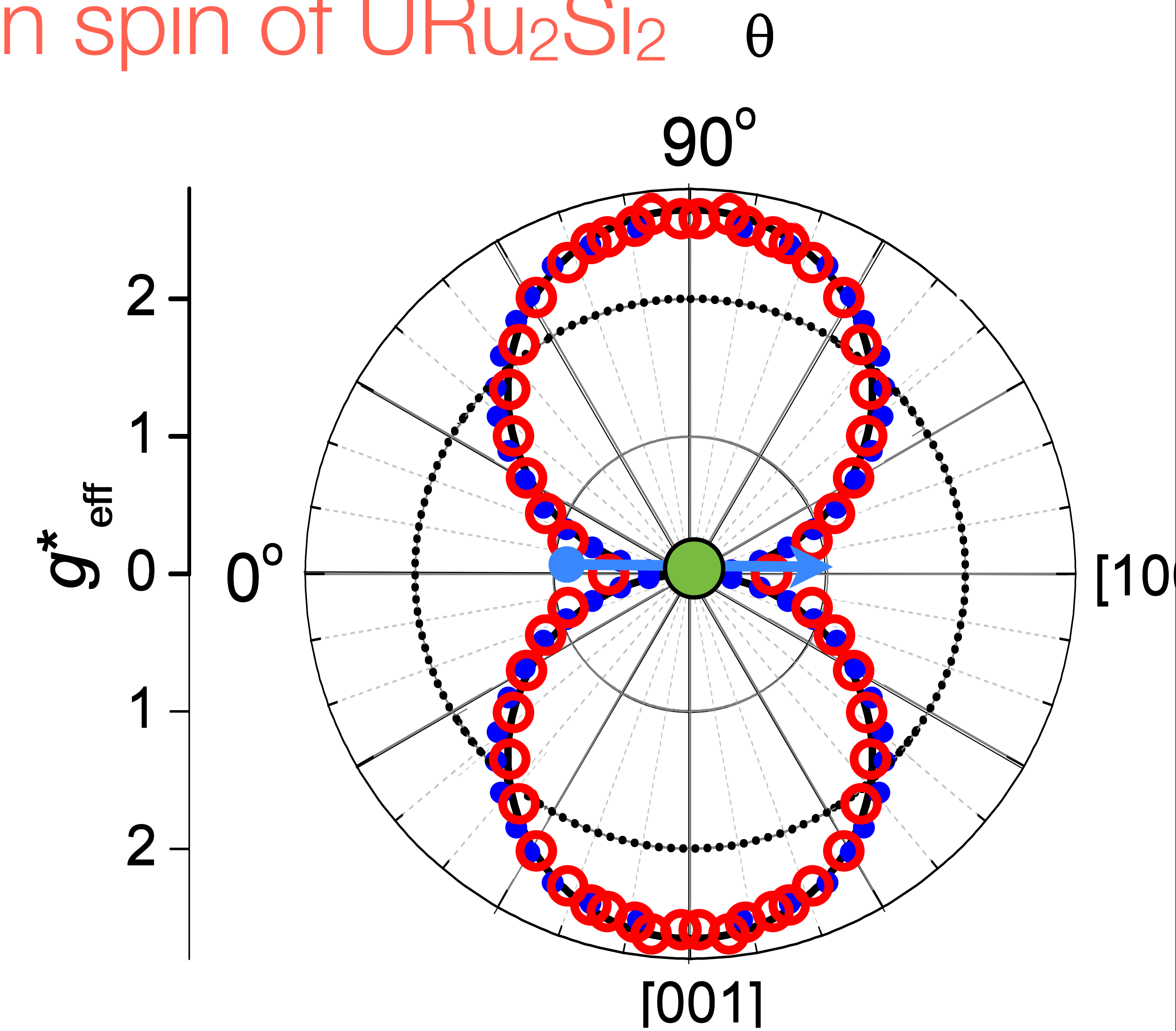


electron spin



URu_2Si_2

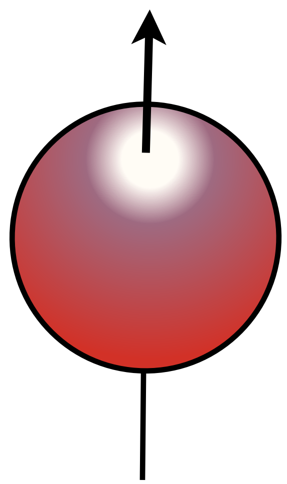




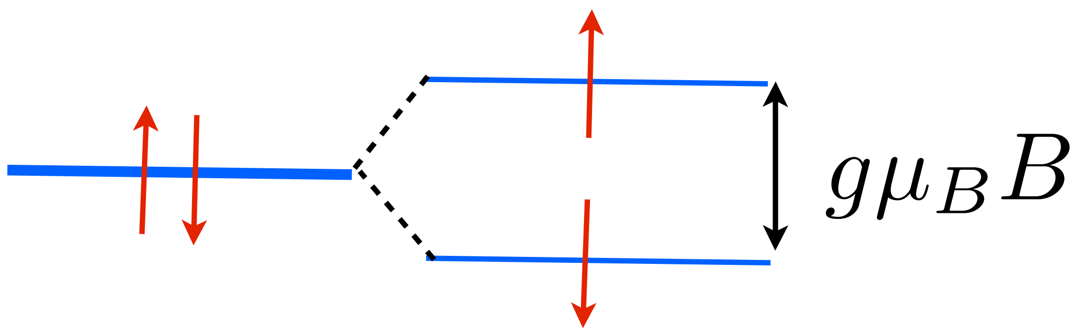
No splitting in transverse direction

Strange electron spin of URu_2Si_2

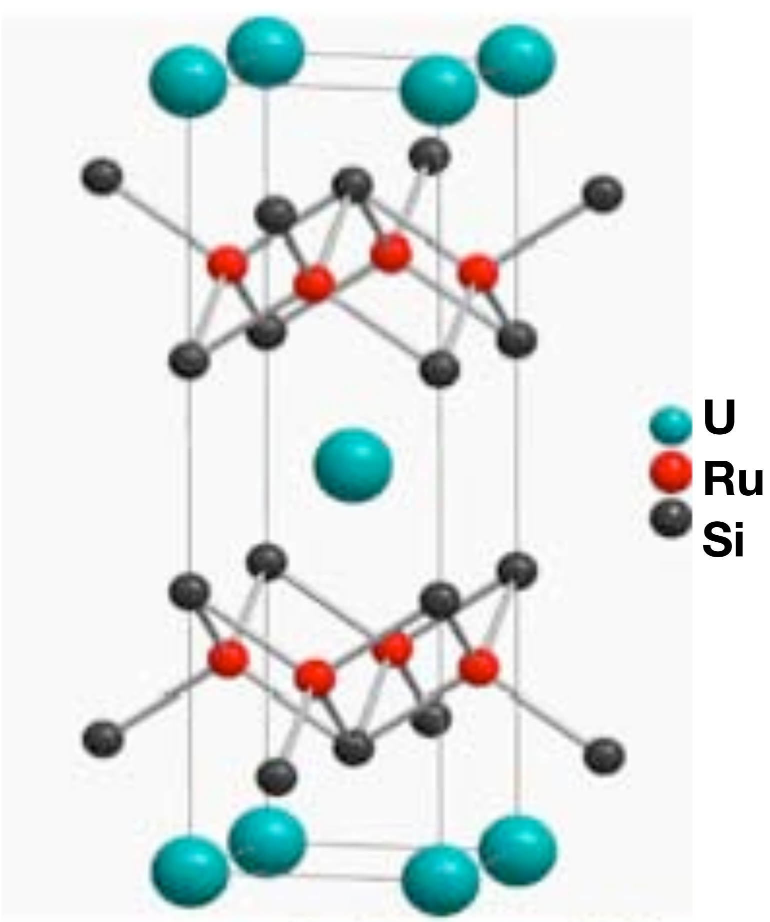
θ

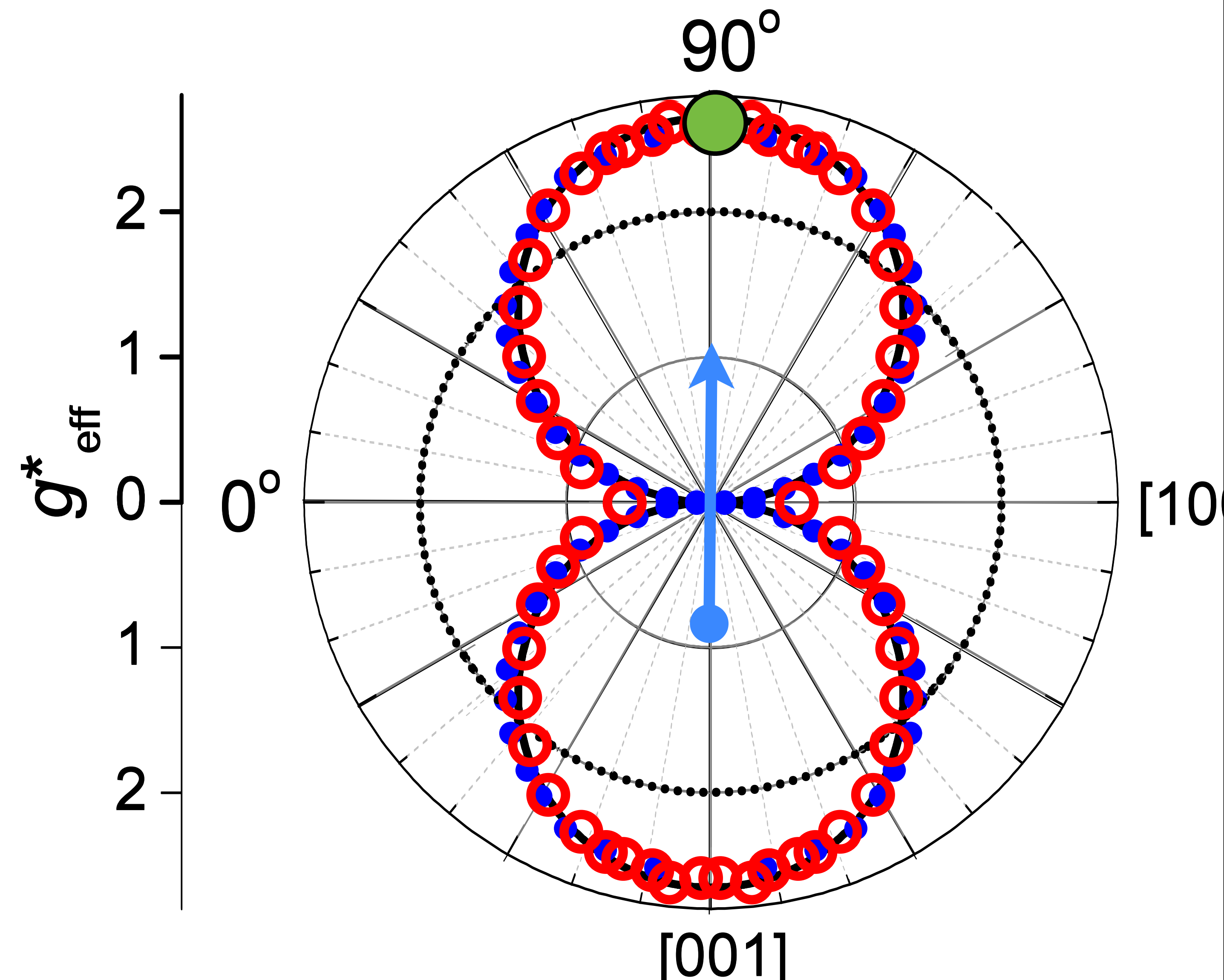


electron spin



URu_2Si_2



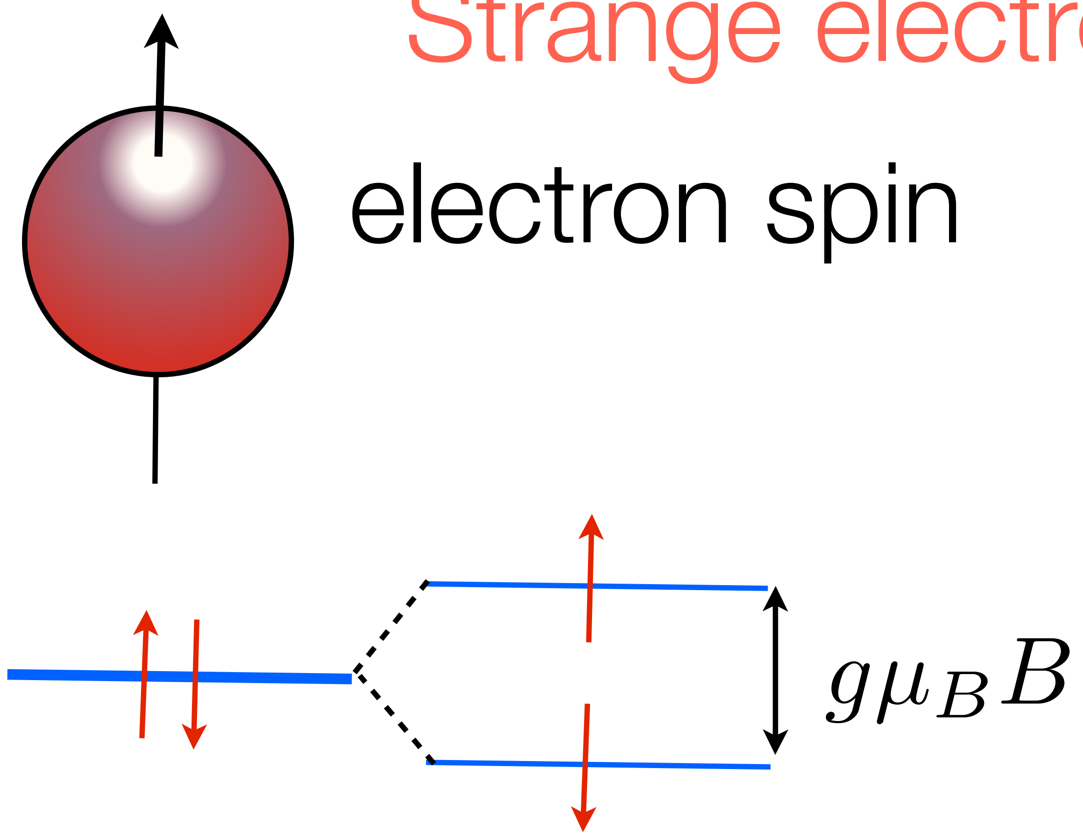


No splitting in transverse direction

$$M = g\mu_B \cos\theta = M_z$$

Magnetic moment only along z-axis

Strange electron spin of URu_2Si_2



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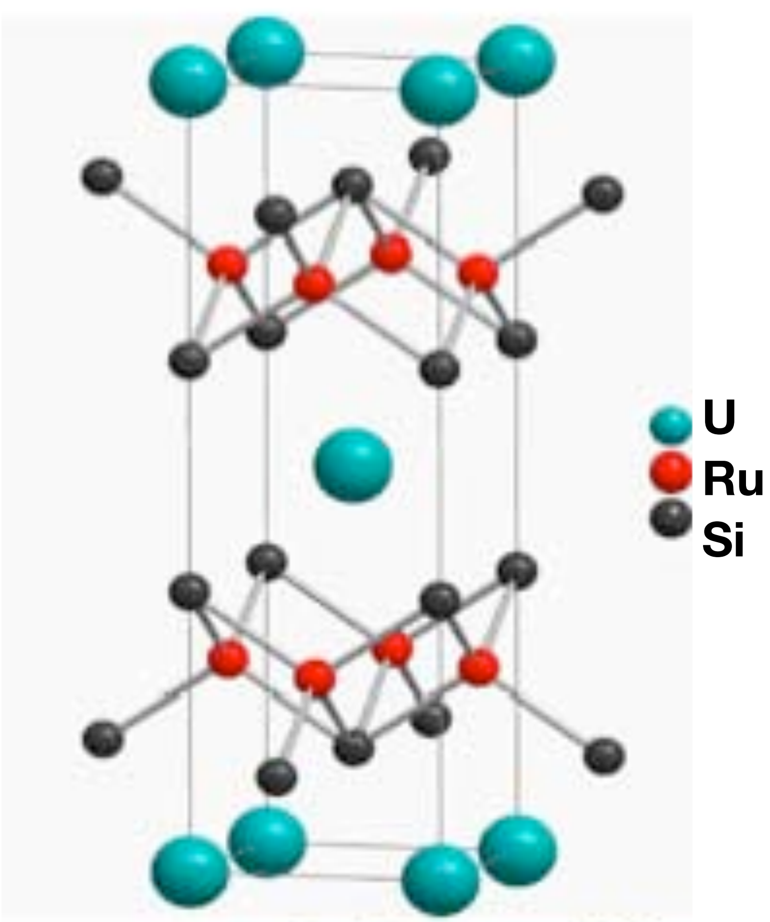

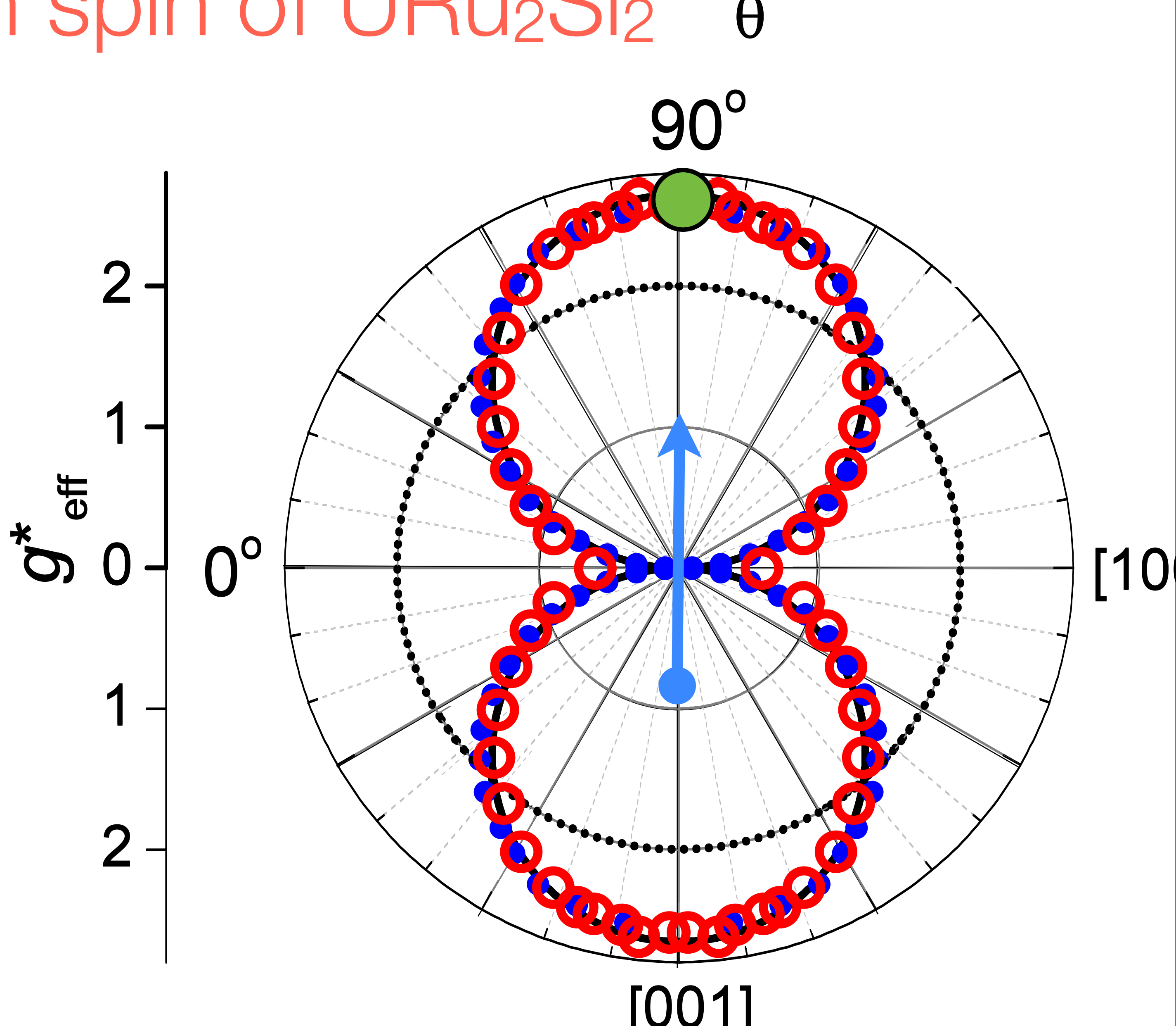

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Magnetic moment only along z-axis

"Ising moment"

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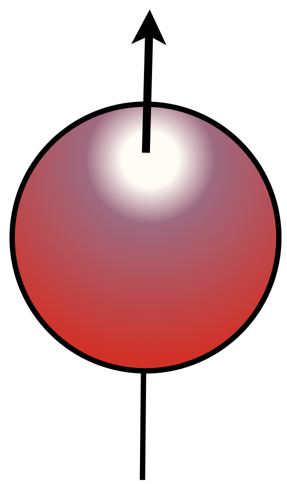
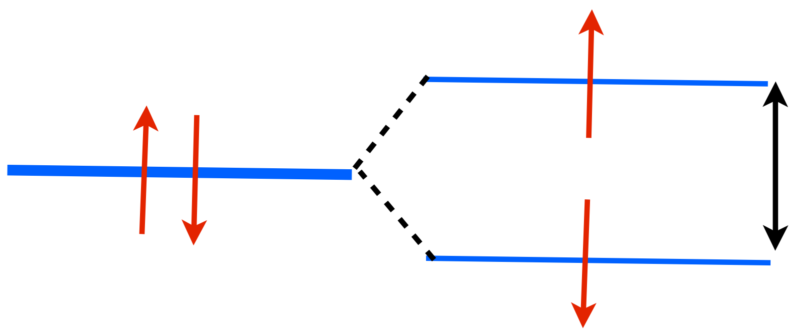
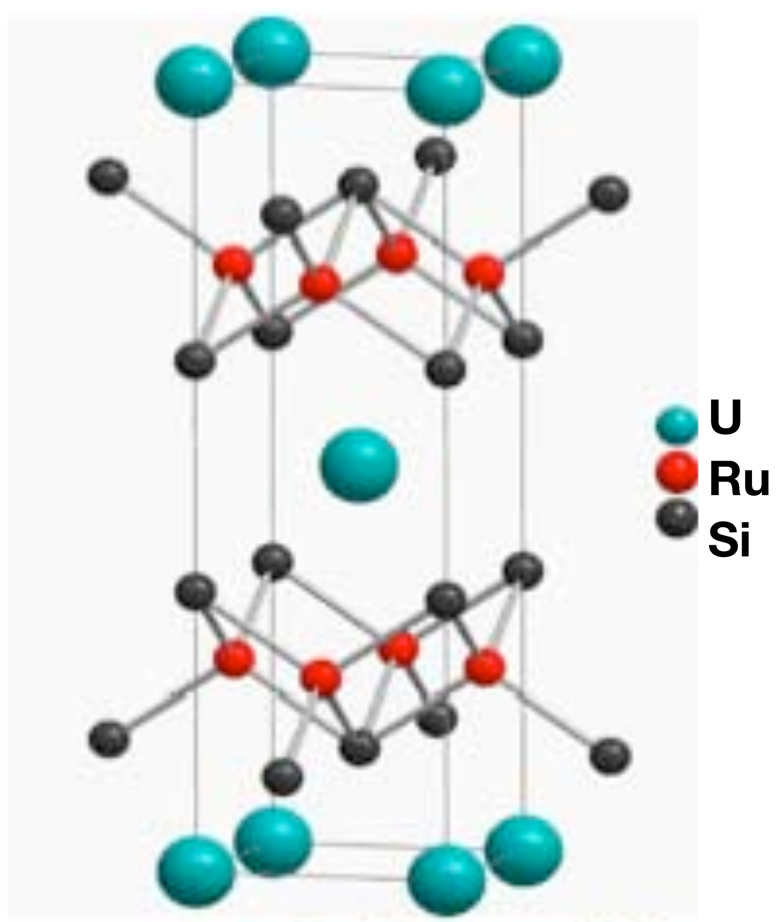
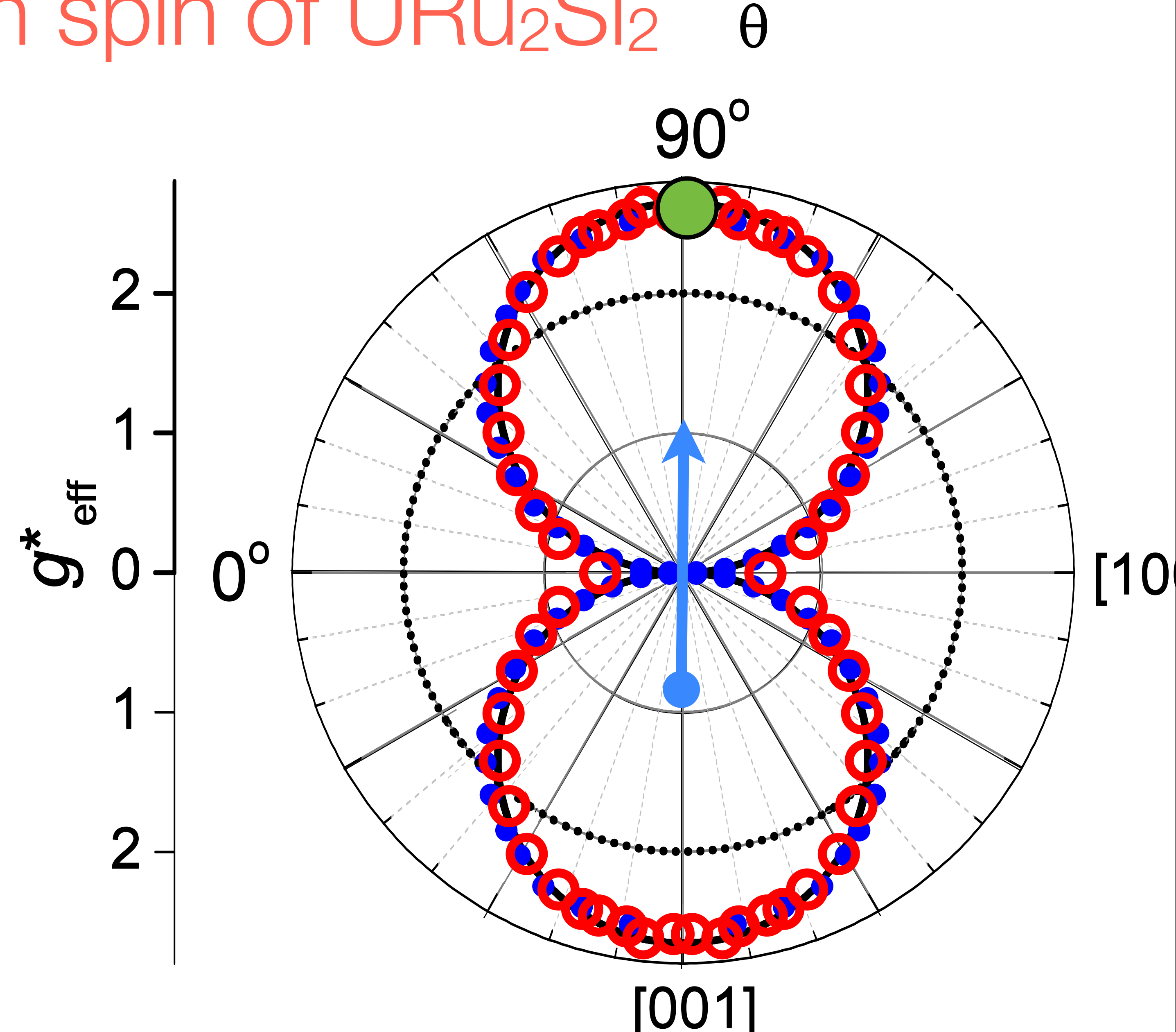


electron spin



URu_2Si_2





No splitting in transverse direction

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Magnetic moment only along z-axis

“Ising moment”

S~integer?

Quantum Oscillations: Giant Ising Anisotropy

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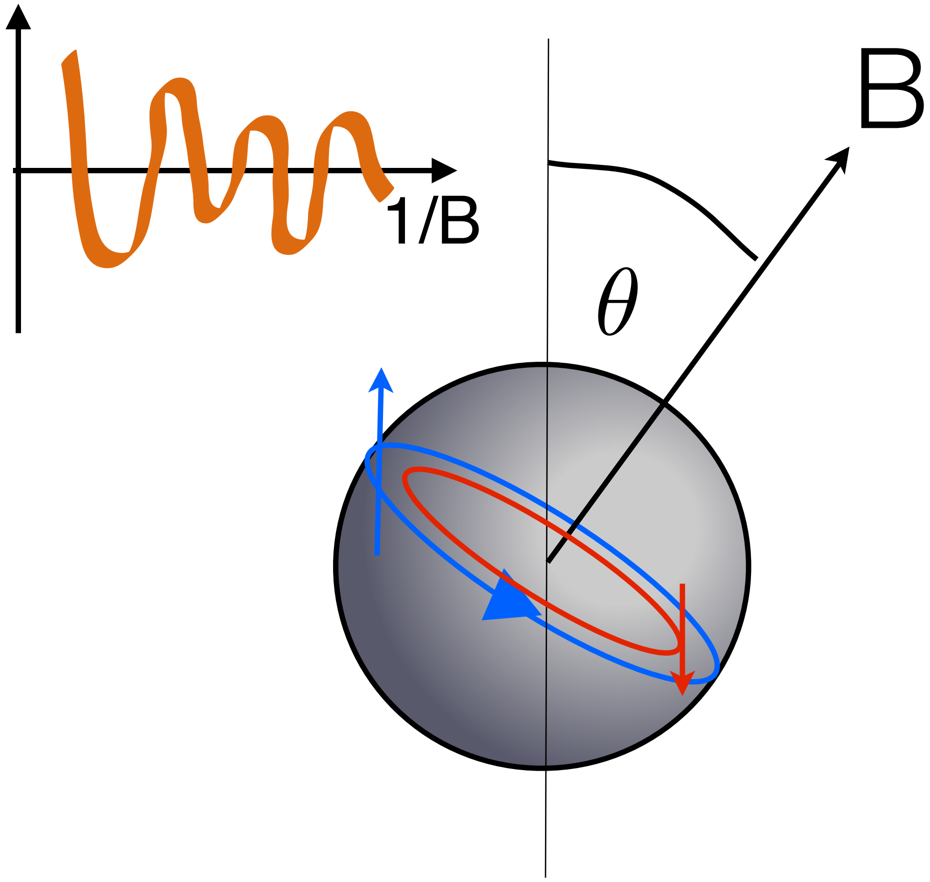
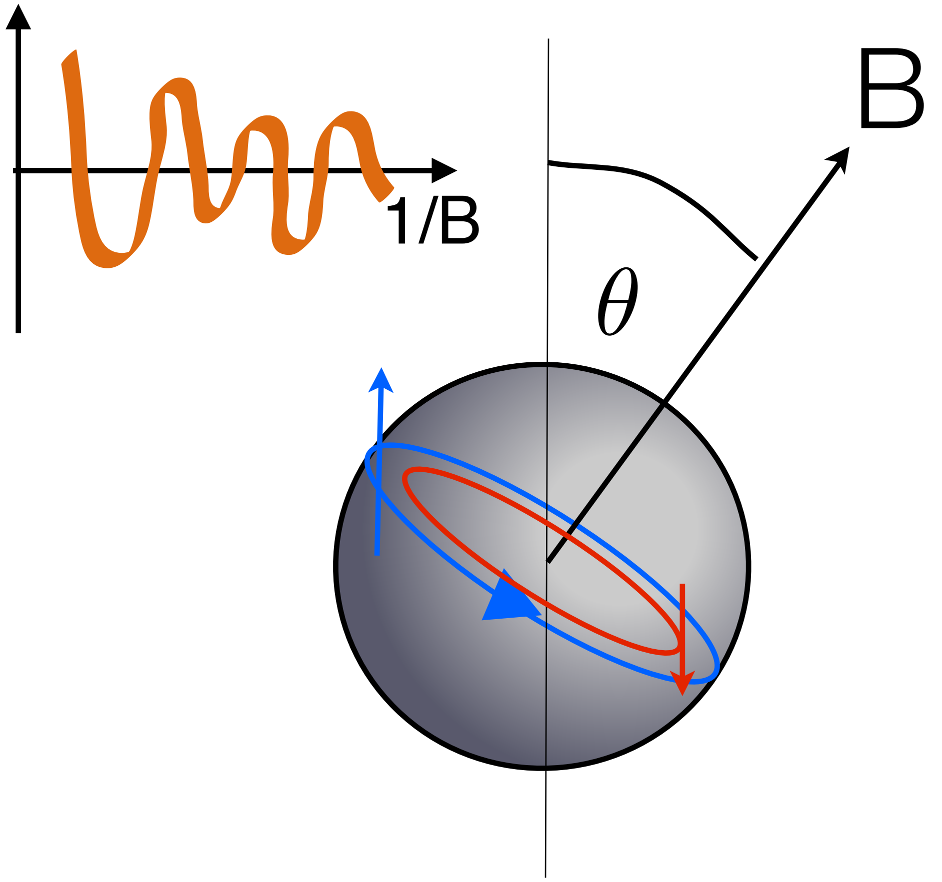


Quantum Oscillations: Giant Ising Anisotropy

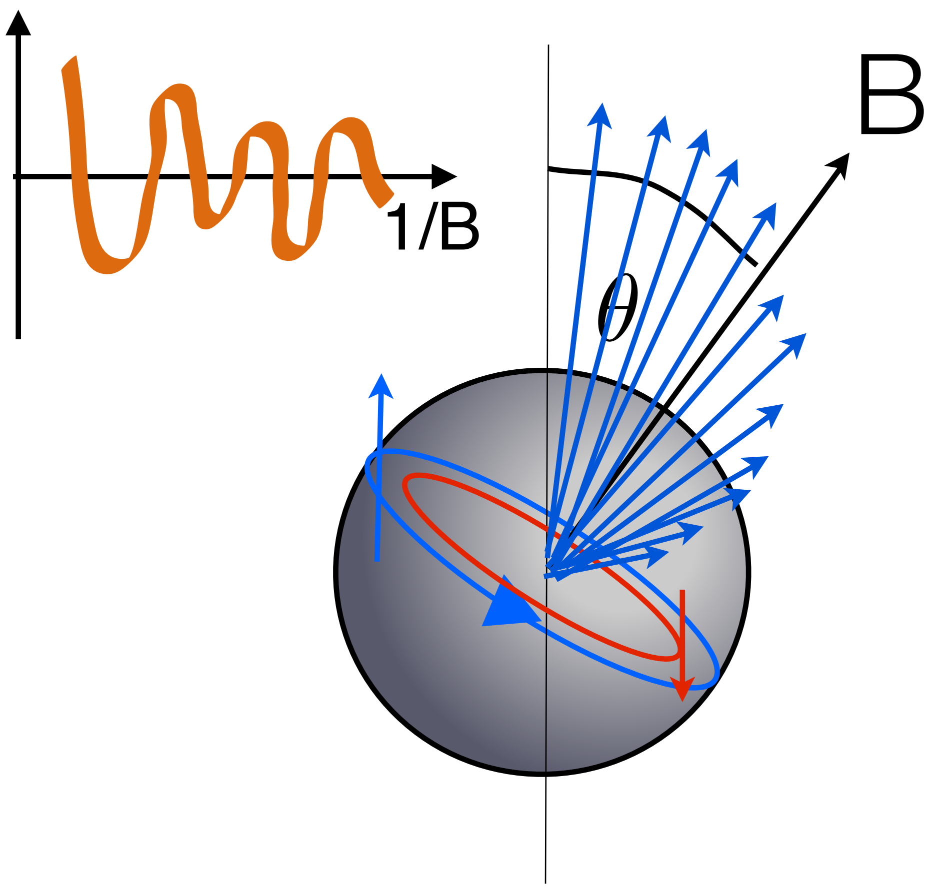


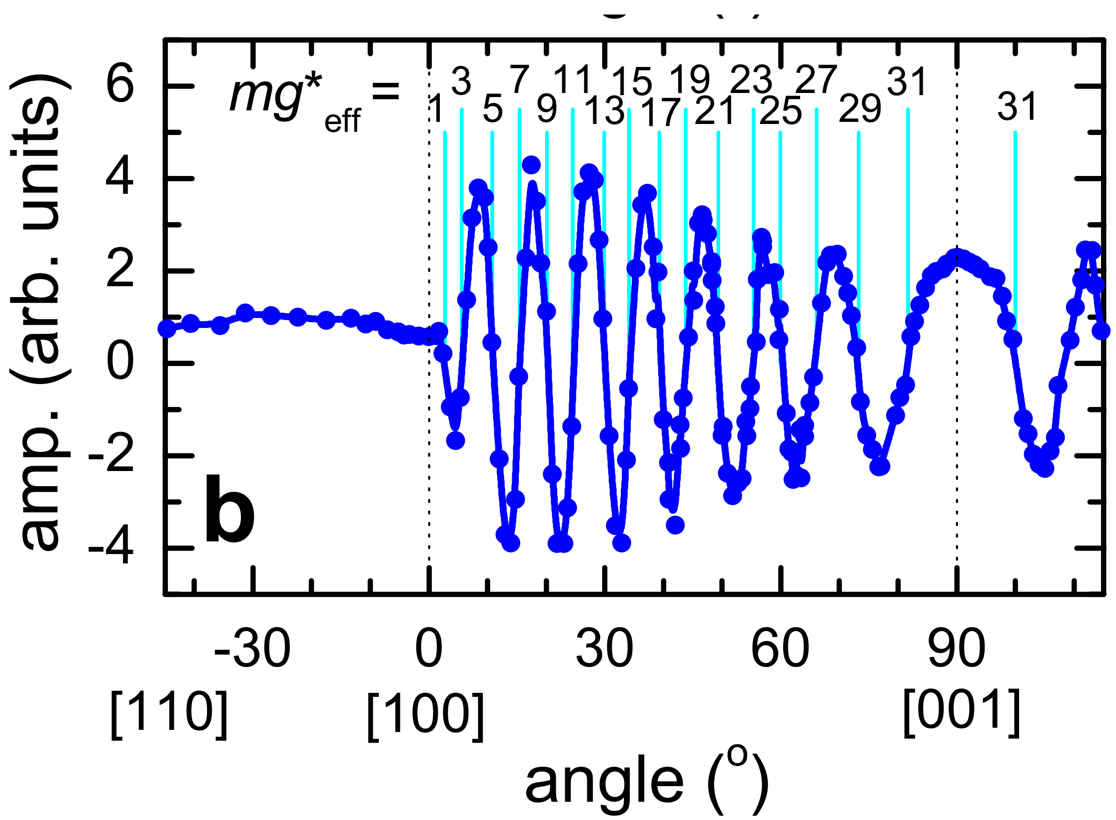
$$M \propto \cos\left[2\pi \frac{\text{Zeeman}}{\text{cyclotron}}\right]$$

$$\frac{m^*}{m_e} g(\theta) = 2n + 1$$

Spin Zero condition

Quantum Oscillations: Giant Ising Anisotropy





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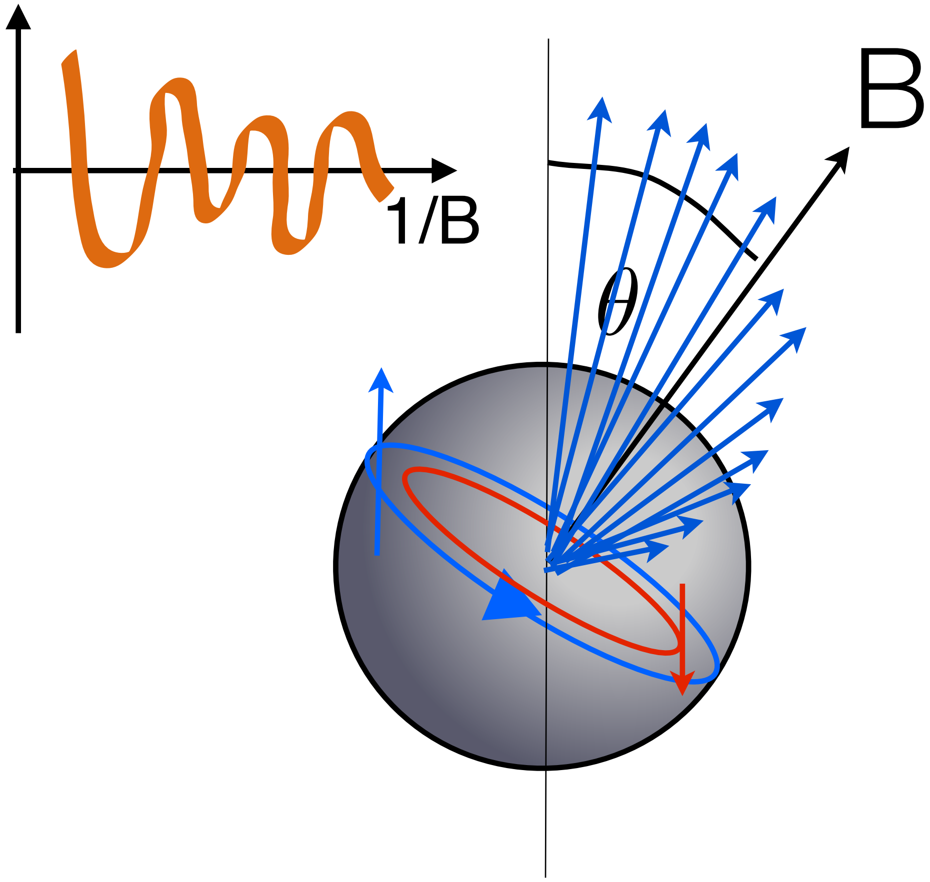
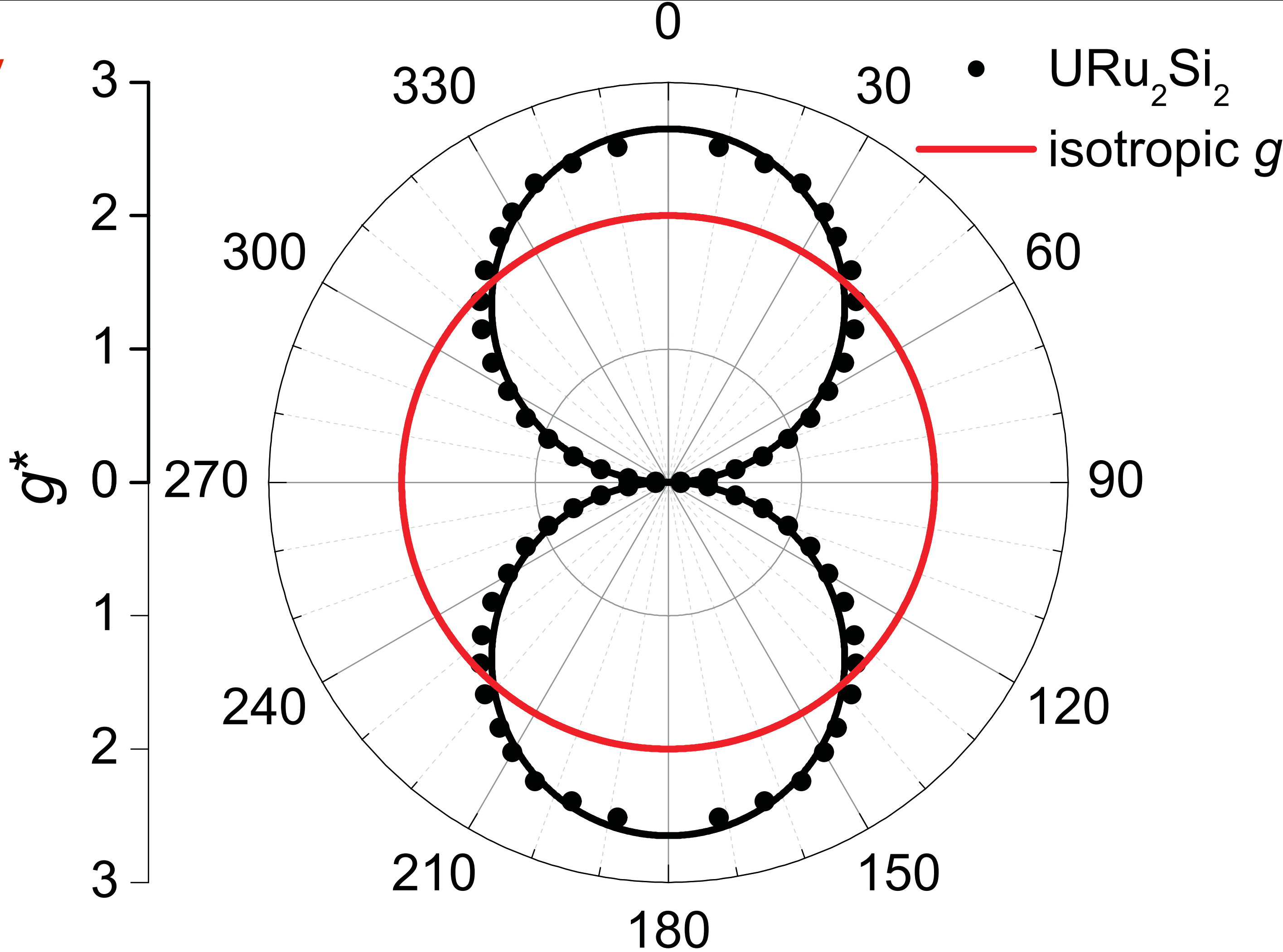
M. M. Altarawneh, N. Harrison, S. E. Sebastian, et al., PRL (2011).
H. Ohkuni *et al.*, Phil. Mag. B 79, 1045 (1999).

17 spin zeros!

Quantum Oscillations: Giant Ising Anisotropy





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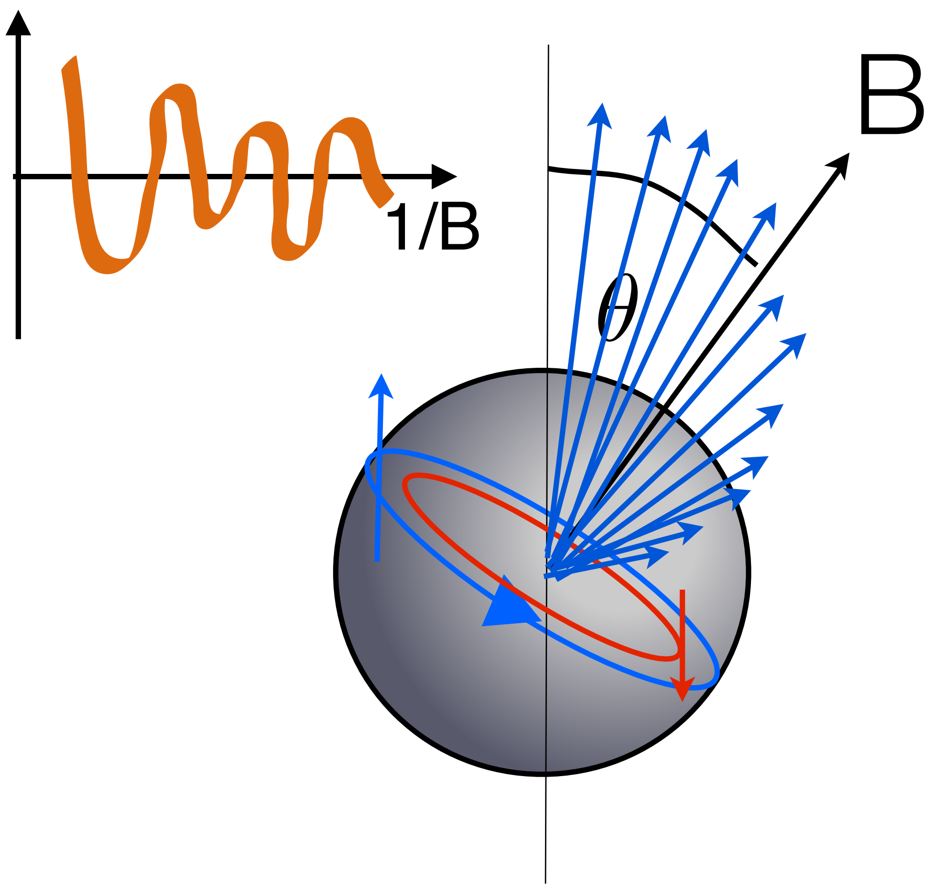

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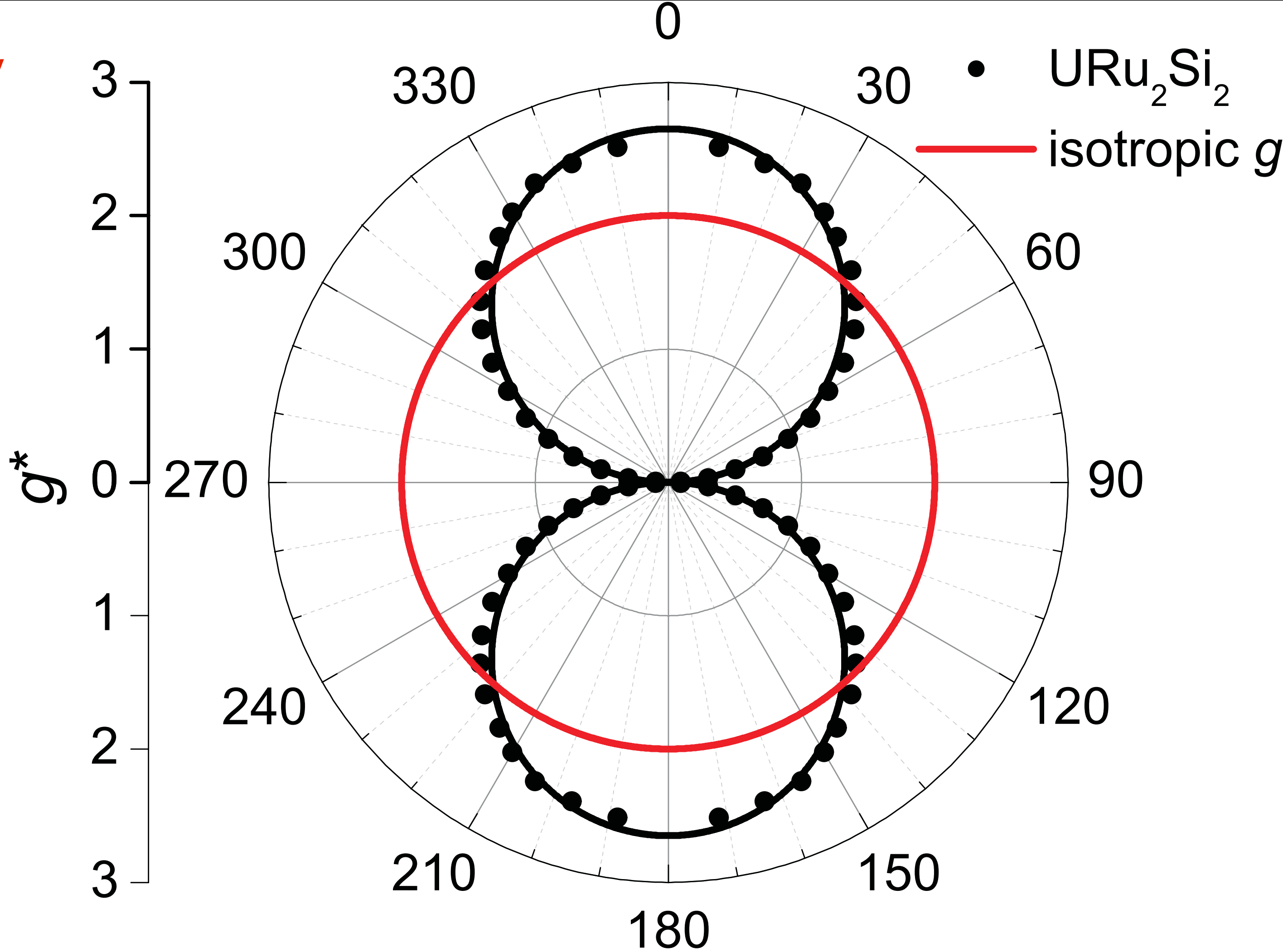
Spin Zero condition

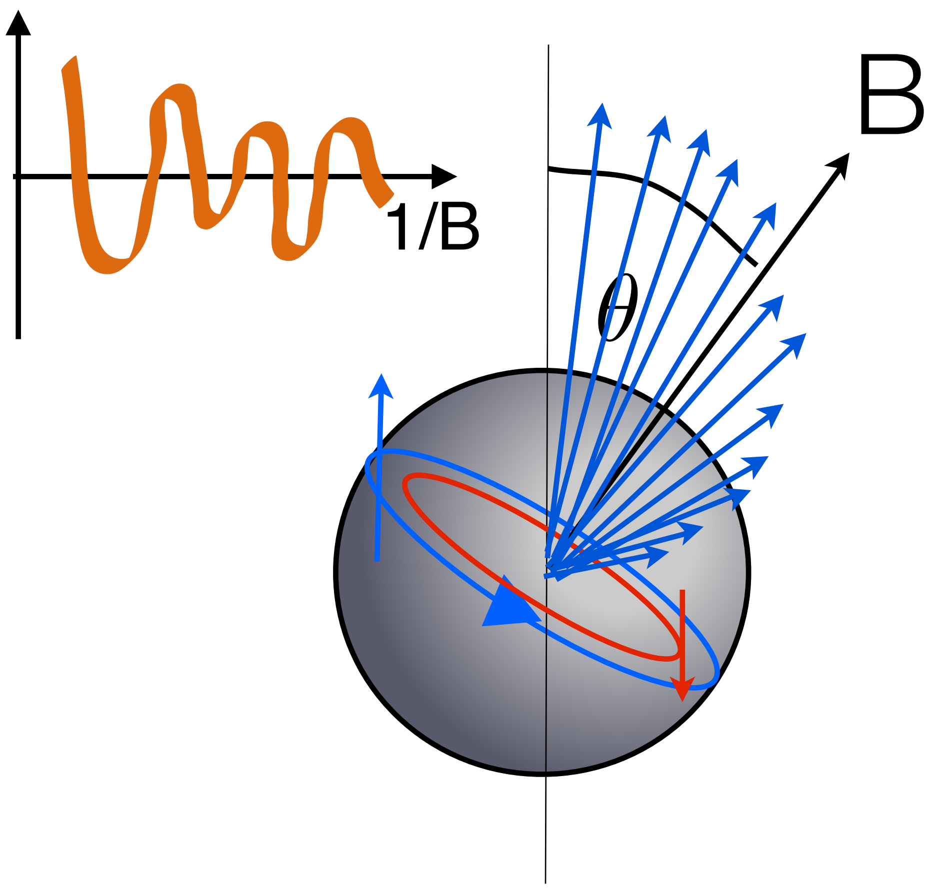


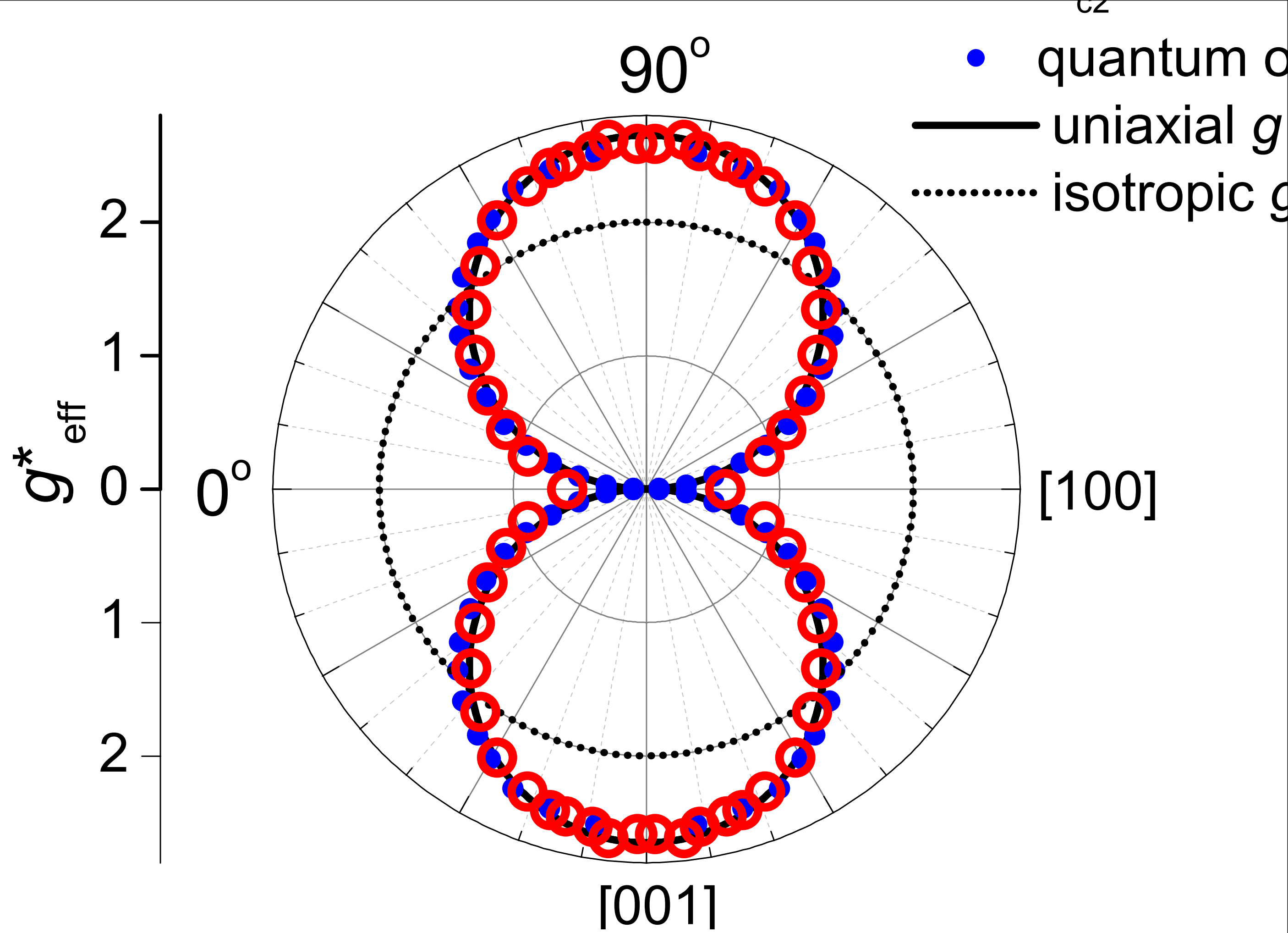
M. M. Altarawneh, N. Harrison, S. E. Sebastian, et al., PRL (2011).
H. Ohkuni *et al.*, Phil. Mag. B 79, 1045 (1999).

17 spin zeros!

Ising *quasiparticle* with giant Ising anisotropy > 30.
Pauli susceptibility anisotropy > 900

Quantum Oscillations: Giant Ising Anisotropy





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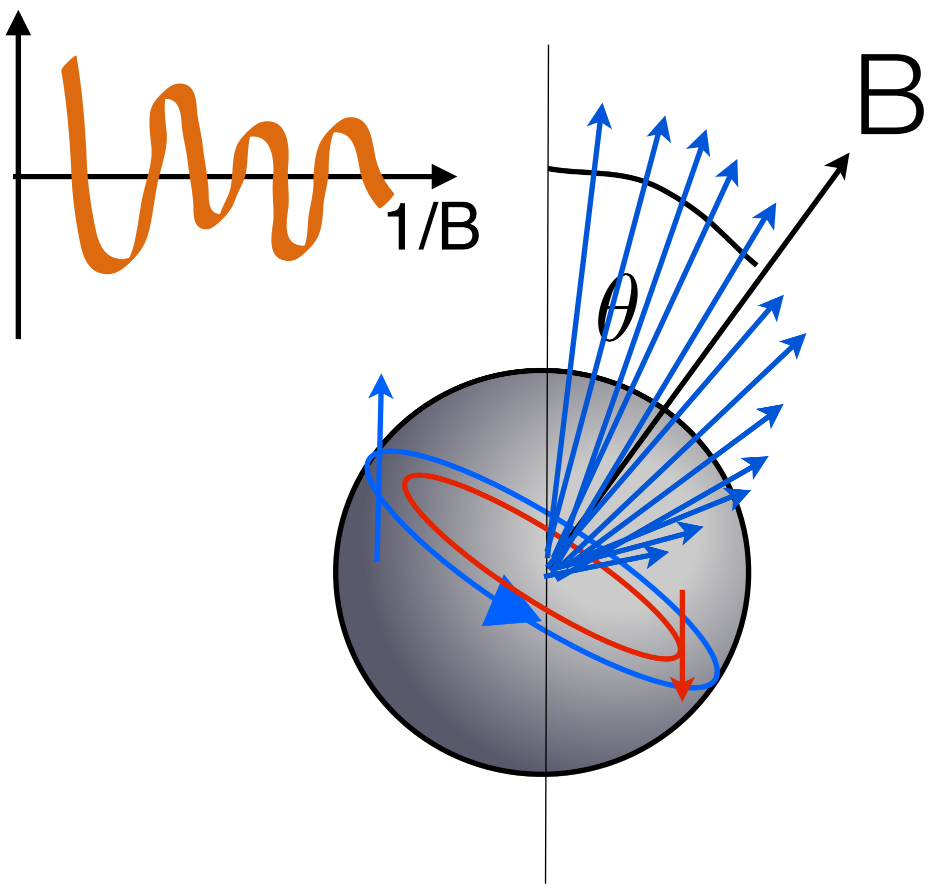

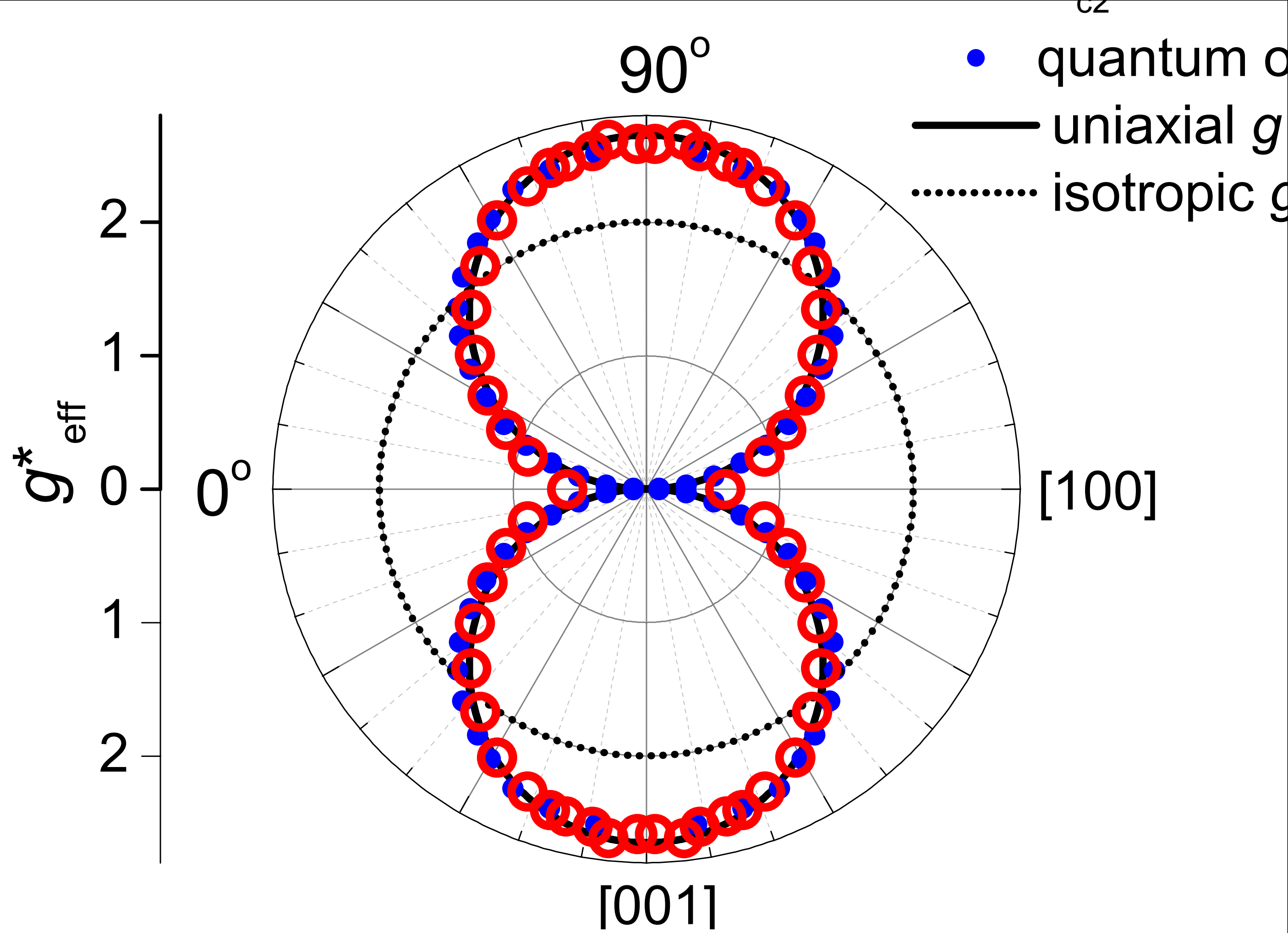

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Electrons hybridize with Ising 5f state to form Heavy Ising quasiparticles.

URu_2Si_2: Electronic Polaroid

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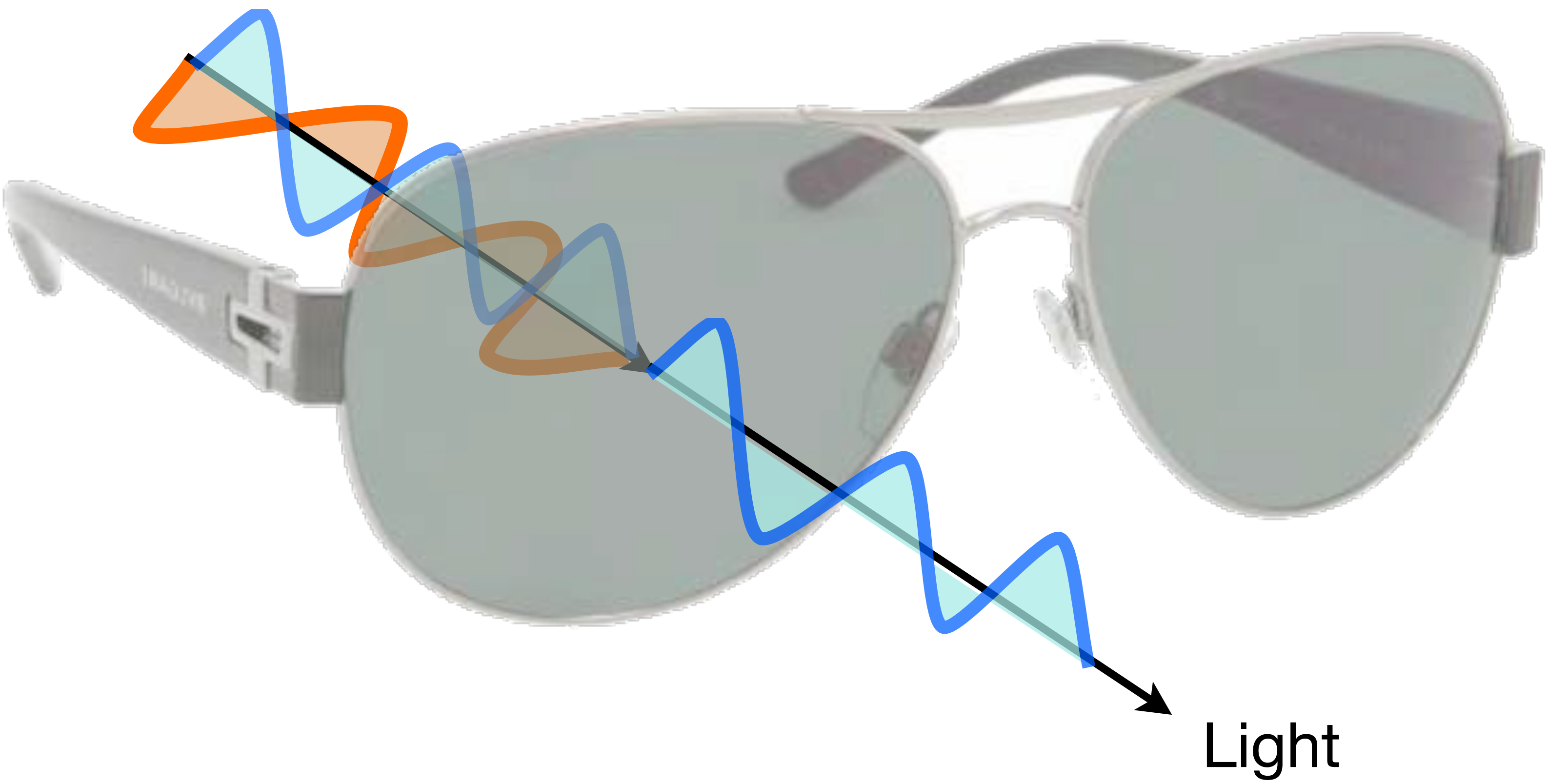

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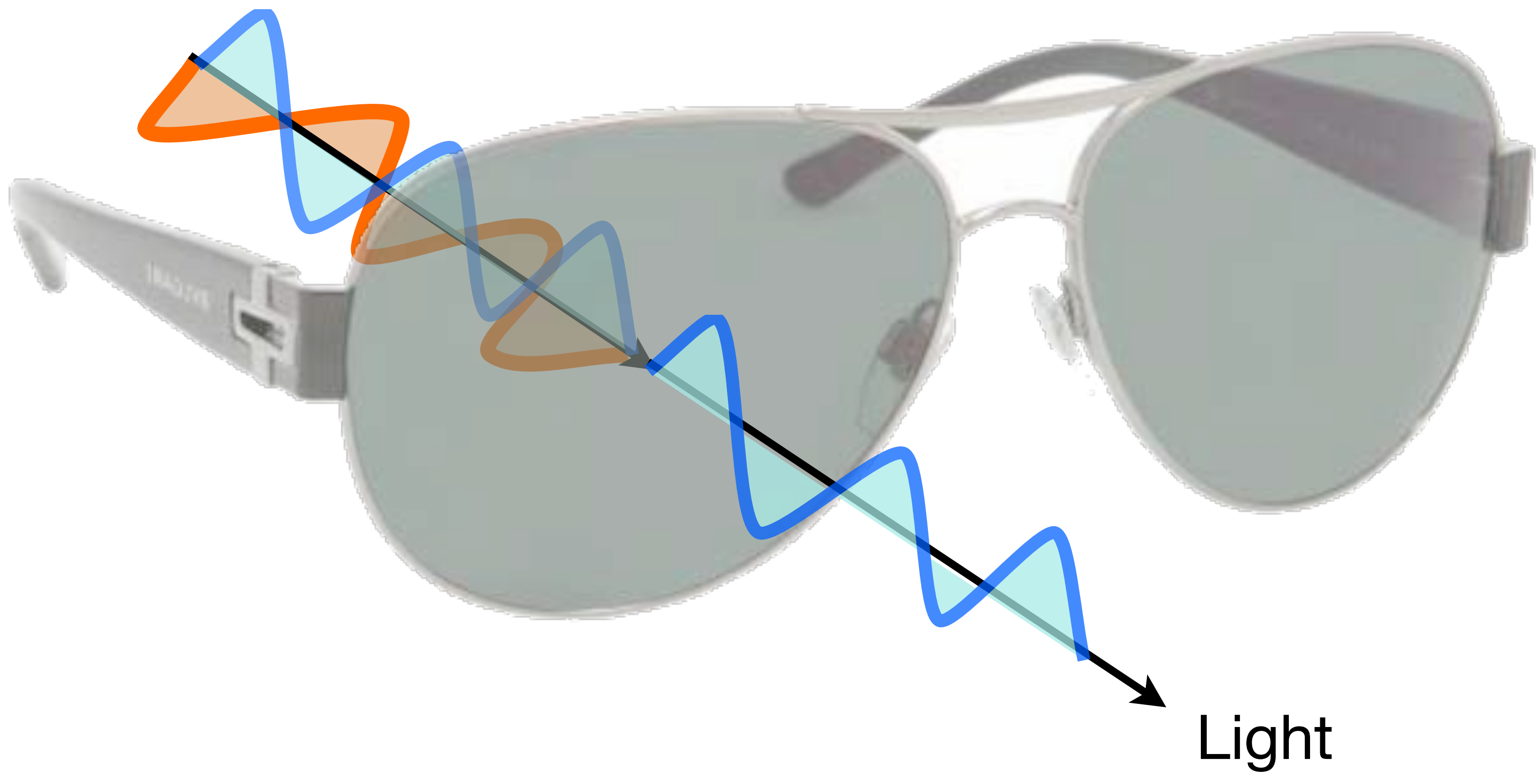

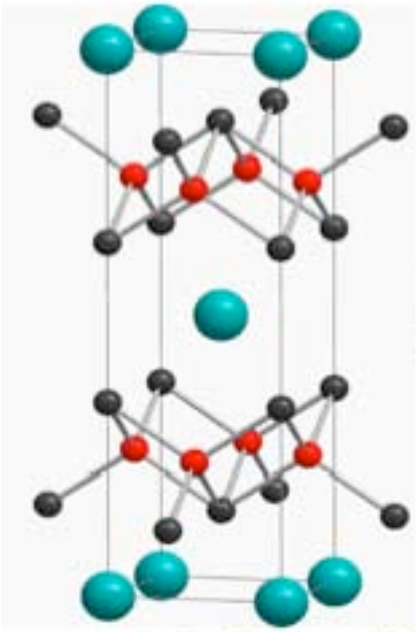

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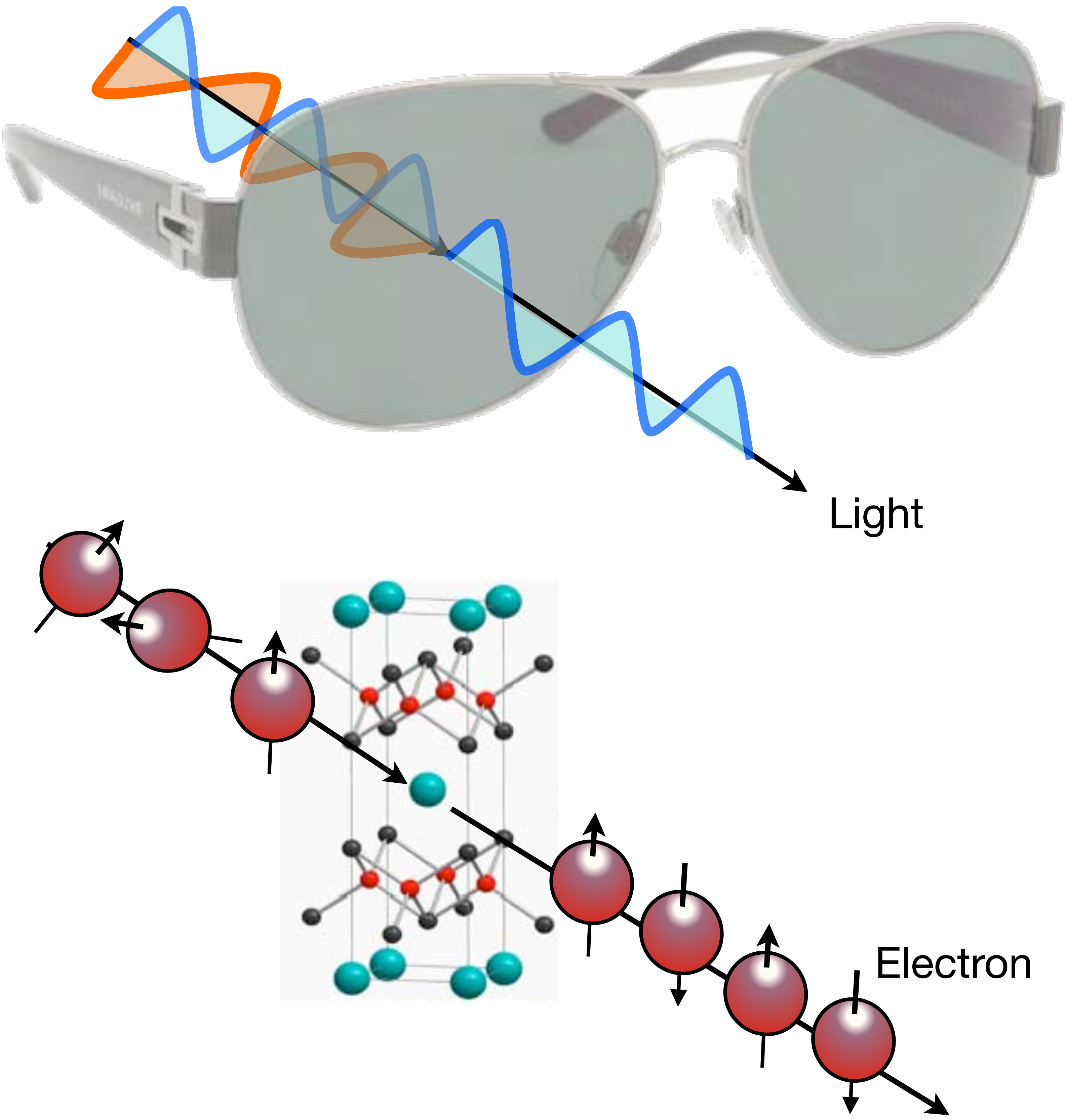

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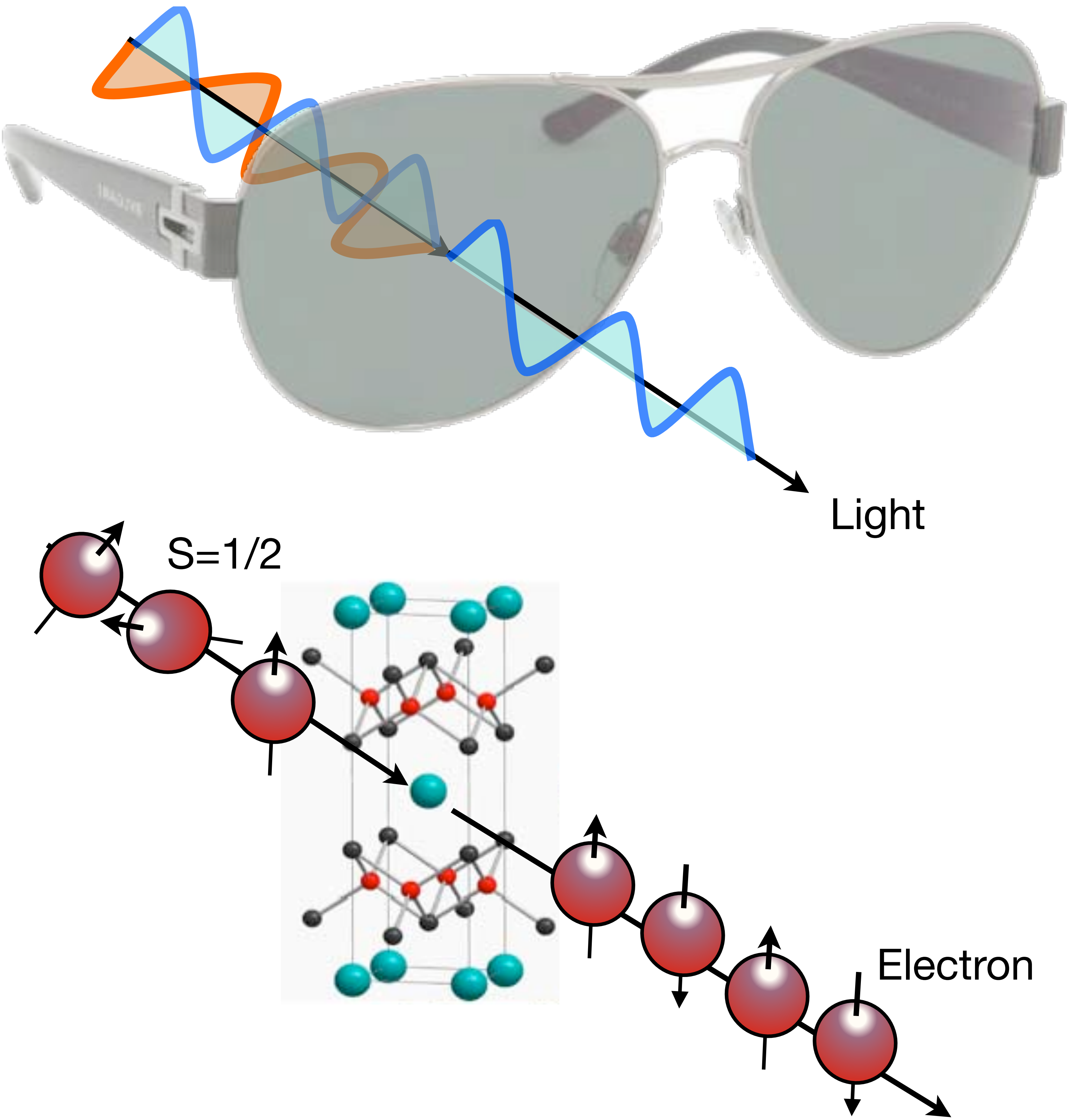

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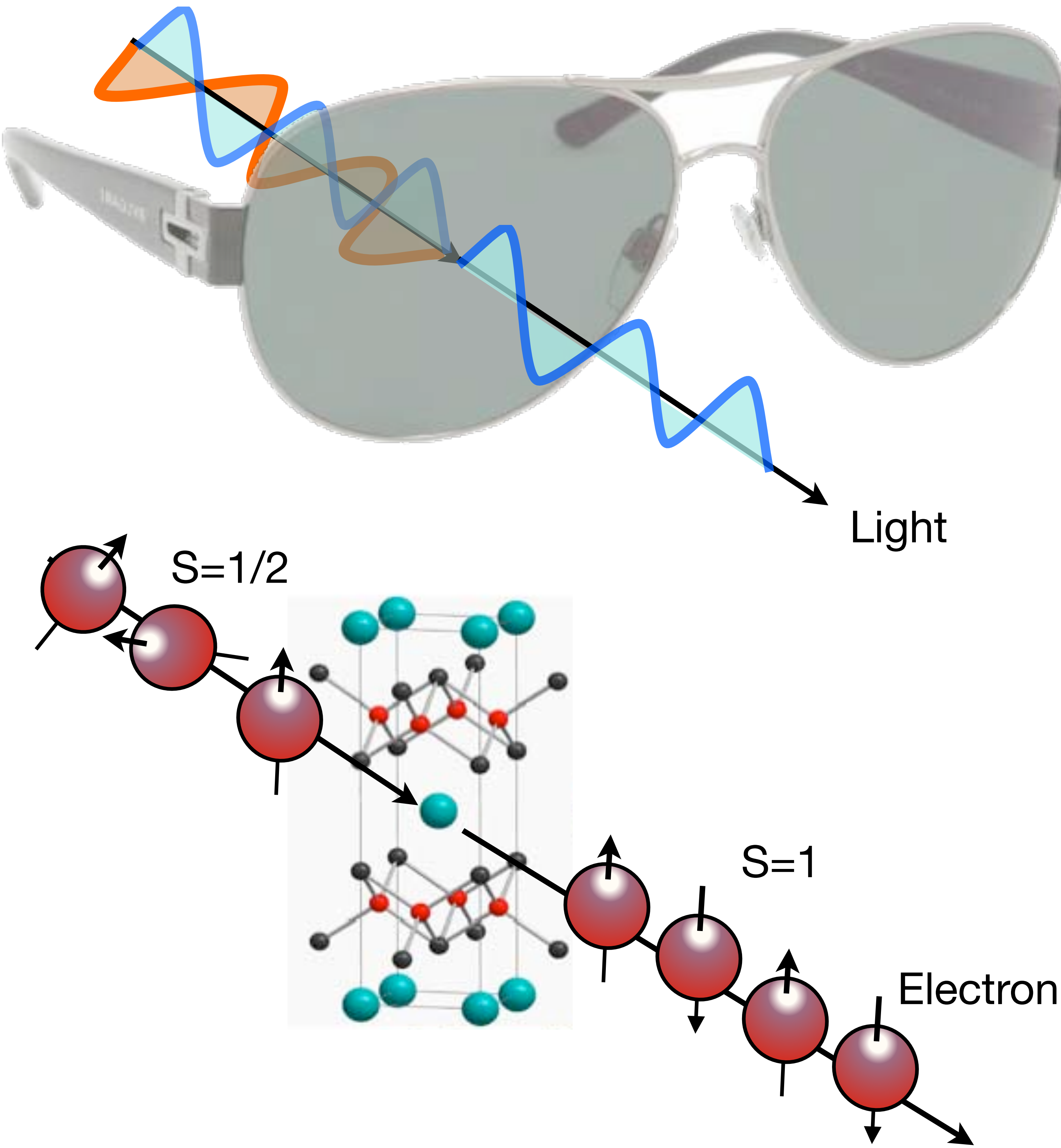

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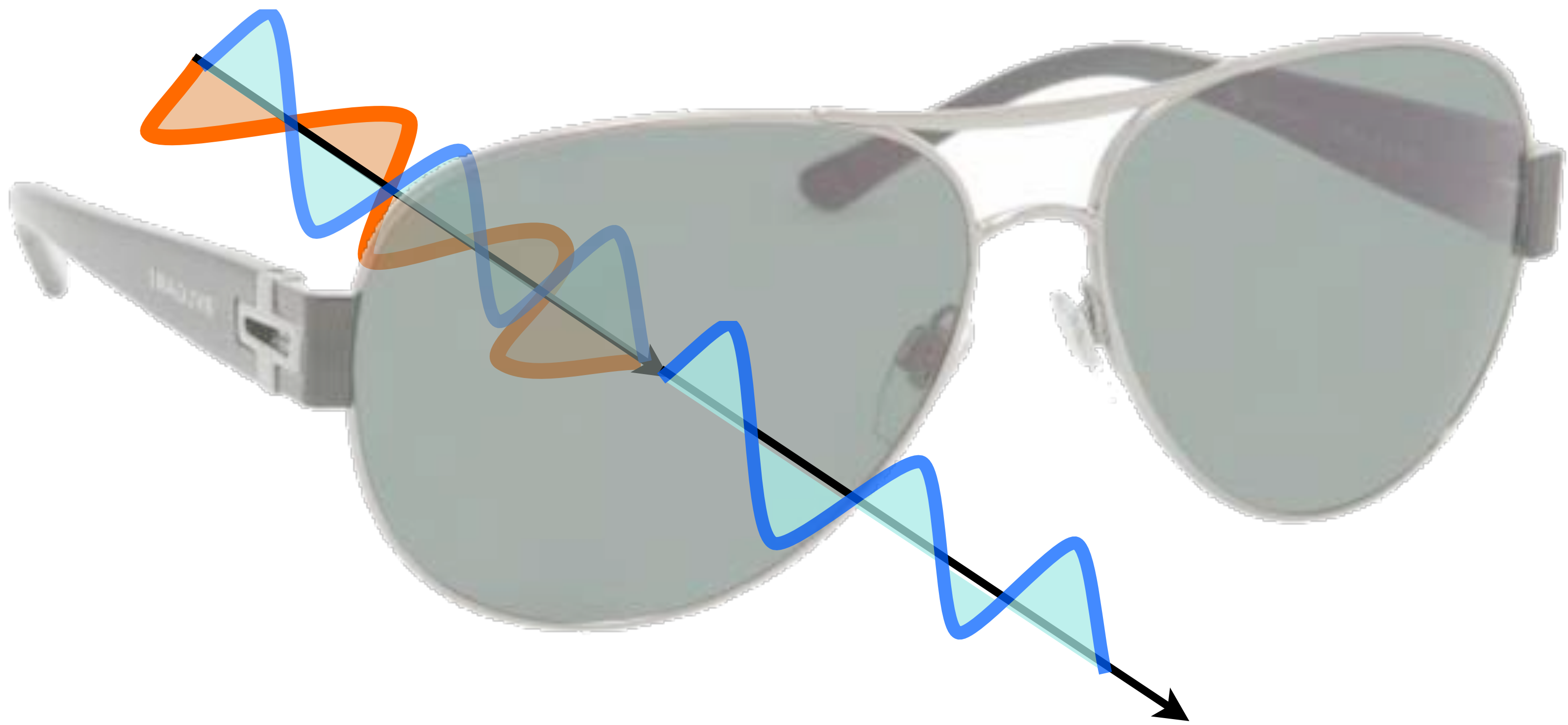

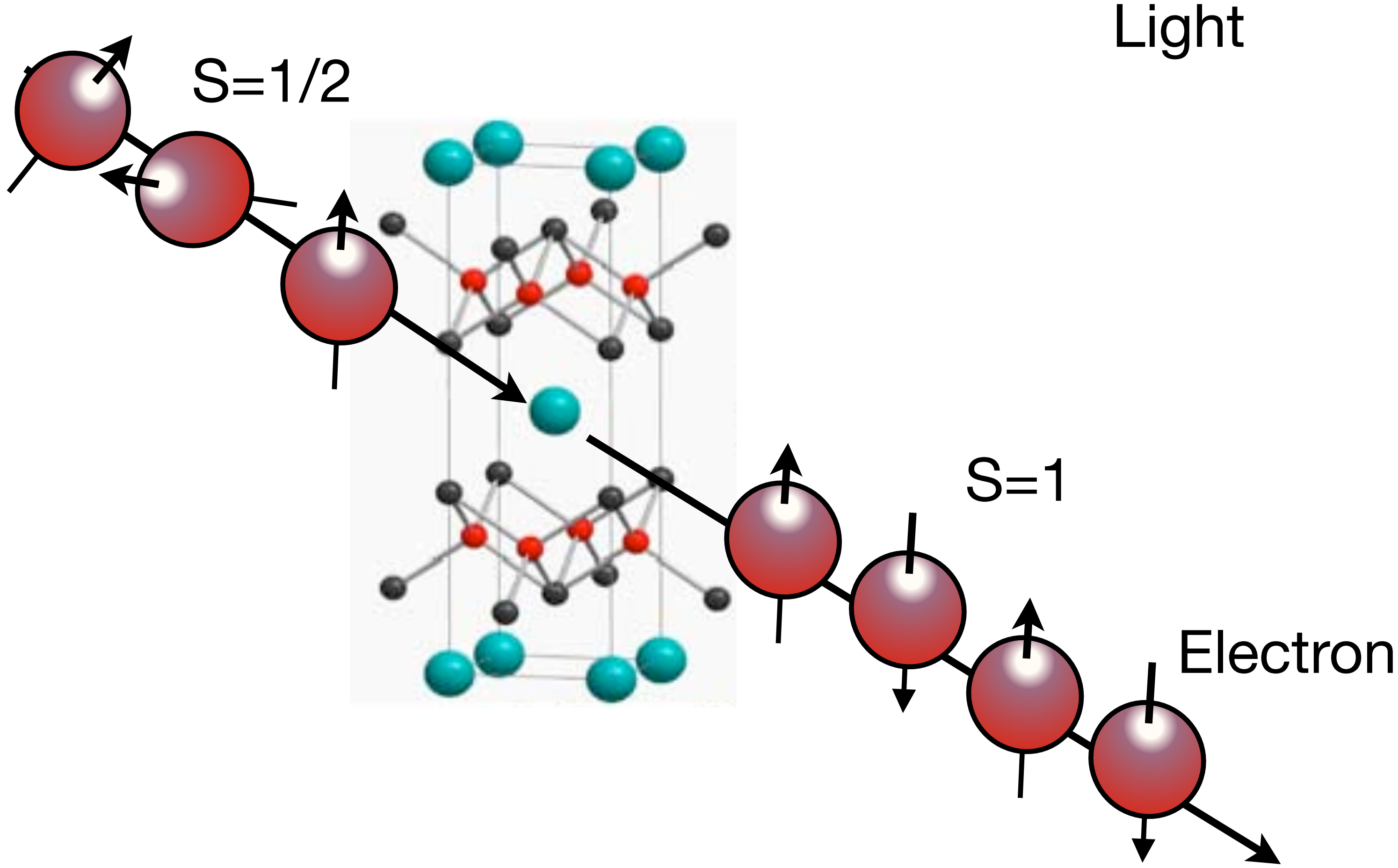

Ψ

Order parameter carries half-integer spin

"Spinor"

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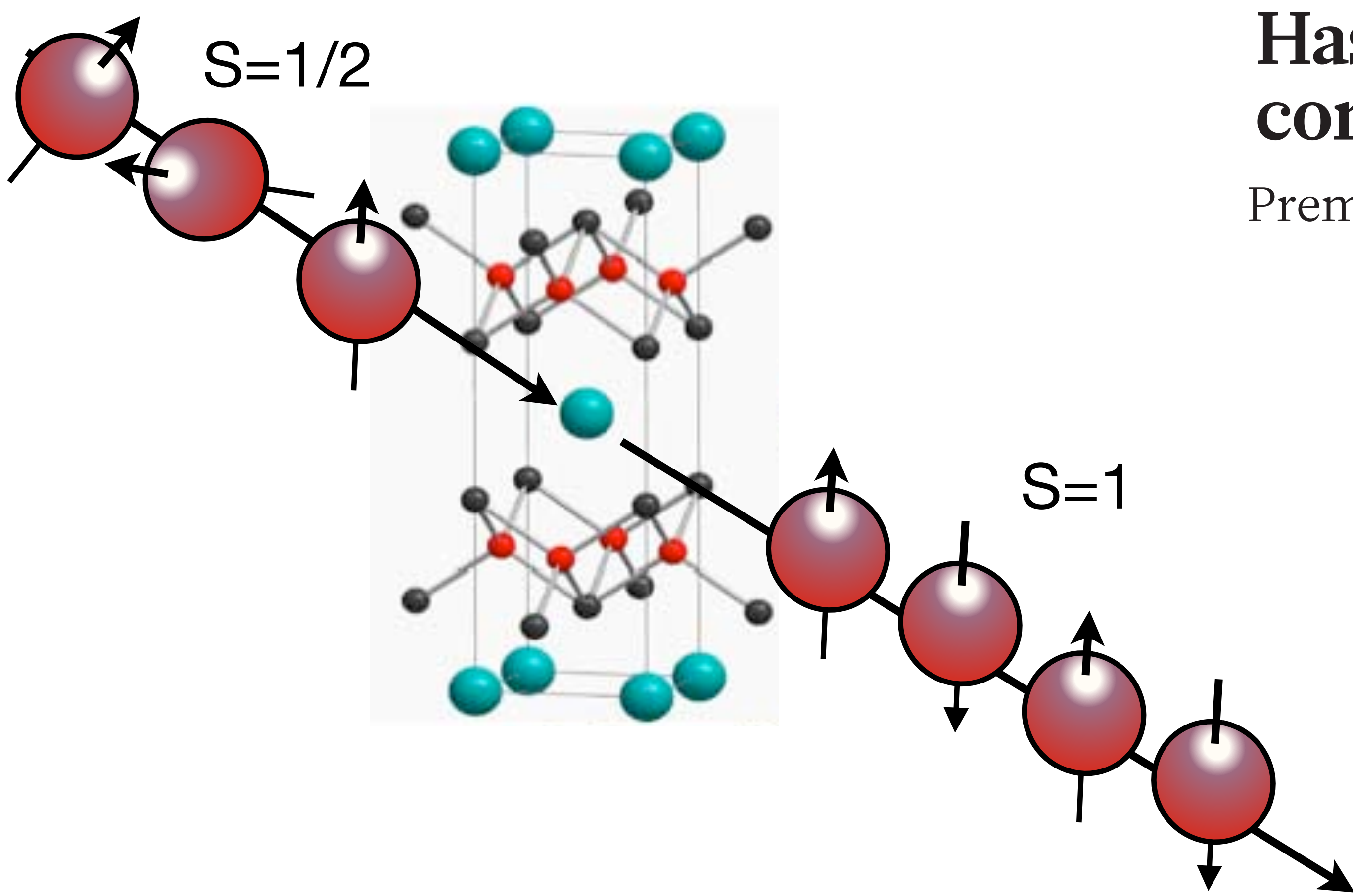

doi:10.1038/nature11820

Hastatic order in the heavy-fermion compound URu_2Si_2

Premala Chandra[1], Piers Coleman[1,2] & Rebecca Flint[3]

$$\Psi$$

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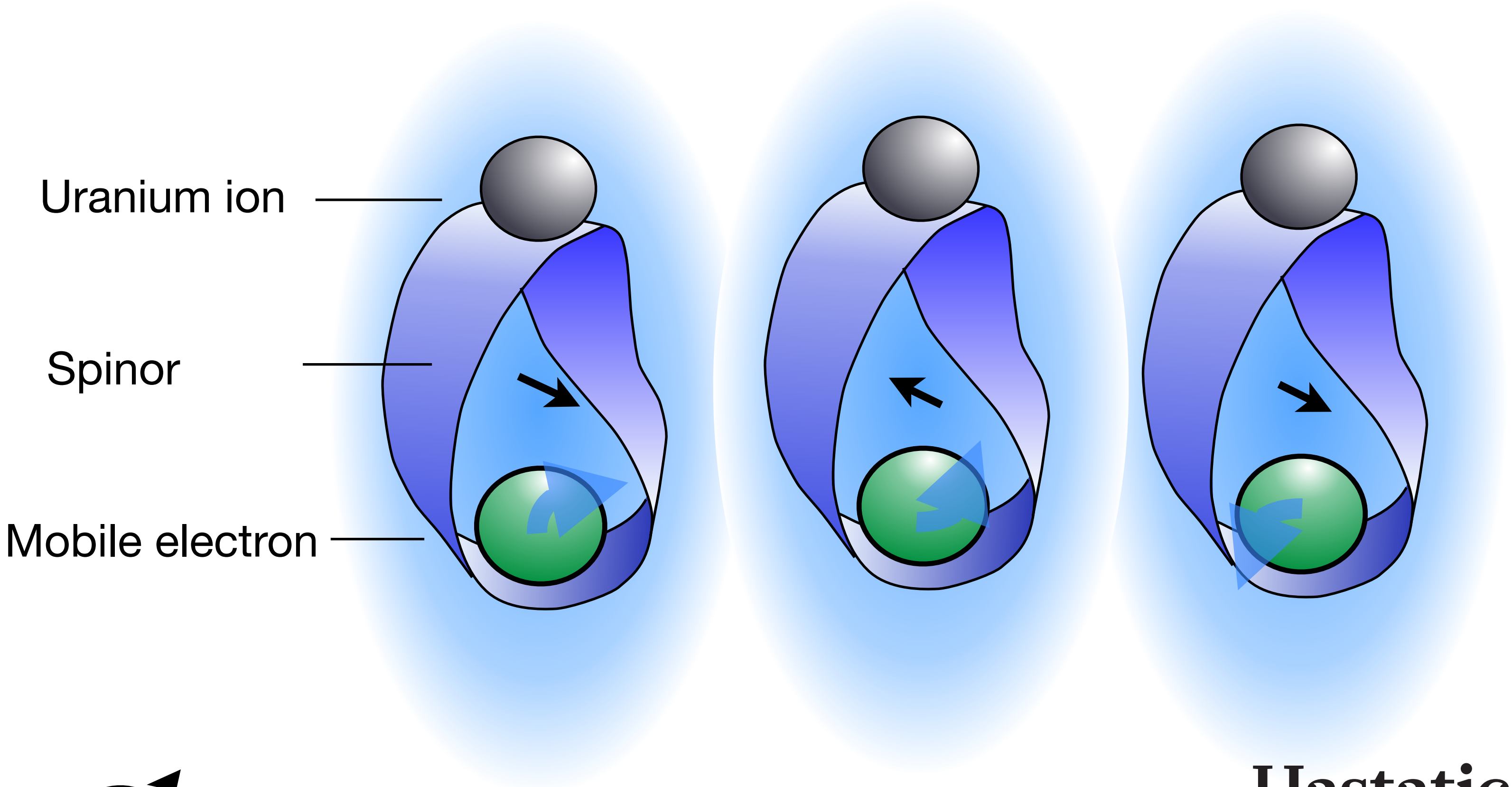

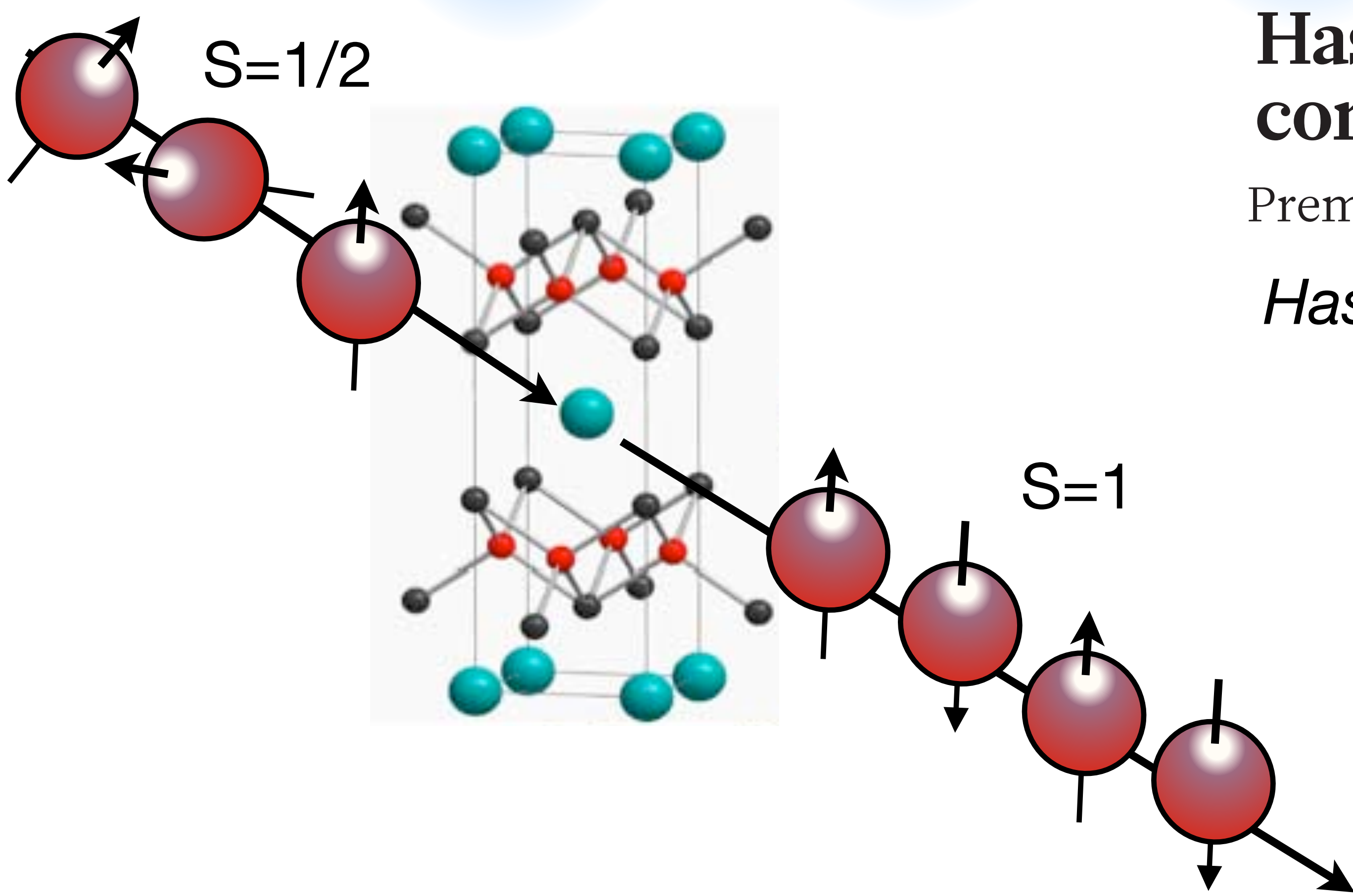

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Hasta: Spear (Latin)

Ψ

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Open Challenges.

- QCPs: Origin of $C/T \sim \mathrm{Log}(T_0/T)$? $\rho \sim T$?
- Co-existence heavy fermions & LM AFM = Two fluid behavior? [Supersymmetry? B/F]
- HFSC: how is the spin incorporated into the condensate? [Composite pairs?]
- Hidden order (HO). Origin of ISING qps? [1/2 integer spinor OP]
- HO: complex MDW (multipolar density wave) vs Fractional spinor order.

Thank you!