Spinors, Strings and Superconductors







P. Coleman CMT, Rutgers, USA & HTC, Royal Holloway, UK

U. Victoria, BC Mar 30, 2016

> Rutgers Center for Materials Theory



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4f	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Но	Er	Tm	Yb
5f	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No
3d	Sc		Ti	v	(Cr	Mn	F	e	Co	Ni	(Cu
4d	Y		Zr	Nb	N	10	Тс	R	u	Rh	Pd	/	Ag
5d	Lu		Hf	Та		w	Re	0	s	Ir	Pt	4	L u



2

No.

0.3

0.2

0.1

0.0

7 (K)

YbRh₂Si₂ B∥c

1

B (T)

4f	Ce I	Pr No	d Pm	Sm	Eu	Gd	Tb	Dy	Но	Er	Tm	Yb
5f	Th J	Pa U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No
3d	Sc	Ti	v	(Cr	Mn	F	e	Co	Ni	(Cu
4d	Y	Zr	Nb	N	10	Тс	R	u	Rh	Pd	1	lg
5d	Lu	Hſ	Та		w	Re	0	s	Ir	Pt	A	\u



Cooper Pair in Sn



the second second

2

0.3

0.2

0.1

0.0

7 (K)

YbRh₂Si₂ B∥c

1

B (T)

Mycoplasma mycoides





Cooper Pair in Sn





Cooper Pair in Sn





What are the principles that govern the emergence of collective behavior in matter between the Angstrom and the Micron?

Cooper Pair in Sn

B (T)



10⁵⁰⁰













Part I A curious link between String theory and Magnetism

Super

Part II

How a chance conversation with a particle physicist colleague led to a new idea about superconductivity.





Bhilahari Jeevanesan (KIT) Peter Orth (U. Minnesota) Premi Chandra (Rutgers CMT) Piers Coleman (Rutgers CMT) Joerg Schmalian (KIT)







discussions + Daniel Friedan Part I A curious link between String theory and Magnetism

Phys.Rev. Lett. 109, 237205 (2012), Phys. Rev. B 89, 0934417, (2014). Phys. Rev. Lett. 115, 177201 (2015).



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Hohenberg-Mermin-Wagner Theorem (1966) No Long-Range Order at Finite Temperatures

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Escape Clause: Frustration



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Sublattices classically decoupled



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"Order from Disorder" (Villain 1985)

















$$\vec{M}_1 \cdot \vec{M}_2 = \sigma = \pm 1$$

 $\xi_{Z_2}
ightarrow \infty$ T_{Z_2} Chandra, Coleman and Larkin (90)

Emergent Z₂ Phase Transition in a disordered Heisenberg System.



Emergent Z₂ Phase Transition in a disordered Heisenberg System.





Iron based superconductors (Hosono 2008).



Nominal F content x

Weber et al (2003) 0 \bigcirc 25 \ominus 20 (a) L=160 \circ $^{\circ}$ L=80 15 $\chi/{ m L}$ L=40 10 e L=20 \circ \bigcirc 5 0.18 0.2 0.21 0.19 0.22 ○ Fe/Co T/J, As (Top) As (Bottom) 160 $LaO_{1-x}F_{x}FeAs$ 0.15 orthorhombic 140 tetragonal 120 ξ^{-1} 0.1 T_s, T_n, T_c (K) 0 8 00 00 T_s from χ(T) $T_{c}/J_{1}=0.1965(5)$ 0.05 SDW ▲ T_N from µSR magnetic T_c from µSR order 0 0.195 0.2 0.205 0.215 0.21

 T/J_1

40

20

0.00

0.02 0.04

0.06

Iron based superconductors (Hosono 2008).

superconductivity

0.10

Nominal F content x

0.12

0.14 0.16 0.18 0.20

0.08

$p \ge 5$ \longrightarrow p-fold anisotropy becomes irrelevant Jose et al (77) Kosterlitz-Thouless Transition

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$$p \ge 5$$
 \longrightarrow p-fold anisotropy becomes
irrelevant
Jose et al (77) Kosterlitz-Thouless Transition



Can we find a 2D Frustrated Heisenberg model that has an emergent critical phase? (even though its underlying spin degrees of freedom have a finite correlation length?)

(Polyakov Conjecture: A. M. Polyakov, Phys. Lett. 59B, 79 (1975)).





Windmill in Strangnaes (Sweden)







2D Heisenberg Windmill Model



$$H = H_{hh} + H_{tt} + H_{th}$$
$$H_{\alpha\beta} = J_{\alpha\beta} \sum_{j=1}^{N_L} \sum_{\delta_{\alpha\beta}} S_{\alpha}(j) S_{\beta}(j + \delta_{\alpha\beta})$$

 $\alpha,\beta\in\{t,A,B\}$



Classically: two decoupled sublattices.



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 $\alpha,\beta\in\{t,A,B\}$



Classically: two decoupled sublattices.

Order from disorder drives coplanarity introducing a Z₆ anisotropy



$$H = H_{hh} + H_{tt} + H_{th}$$
$$H_{\alpha\beta} = J_{\alpha\beta} \sum_{j=1}^{N_L} \sum_{\delta_{\alpha\beta}} S_{\alpha}(j) S_{\beta}(j + \delta_{\alpha\beta}) \qquad \alpha, \beta \in \{t, A, B\}$$











 $X(x,y) = (\phi,\theta,\psi,\alpha)$



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Magnetization = 4D vector

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Magnetization = 4D vector

Regarding (x,y)=(x,t), then X(x,t) defines a string moving in a 4D "target" space.



$$X(x,y) = (\phi, \theta, \psi, \alpha)$$

$$S = \frac{1}{2} \int d^2 x g_{ij}(X(x)) \partial_\mu X^i \partial_\mu X^j$$



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Regarding (x,y)=(x,t), then X(x,t) defines a string moving in a 4D "target" space.



Long-wavelength action = 4D string theory.

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Covariance of the action under co-ordinate transformations in target space means that the scaling equations must also be covariant.

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Friedan '80, Hamilton '81, Perelmann '06

The decoupling of the U(1) degrees of freedom from the SO(3) degrees of freedom is a kind of compactification from a four to a one dimensional universe.

$$S = -\frac{1}{2} \int d^2 x \left(I_1 \Omega_{\mu,1}^2 + I_2 \Omega_{\mu,1}^2 + I_3 \Omega_{\mu,1}^2 \right) + \frac{I_\alpha}{2} \int d^2 x (\partial_\mu \alpha)^2 + \frac{\kappa}{2} \int d^2 x \partial_\mu \alpha \Omega_{\mu,3}$$

$$S = -\frac{1}{2} \int d^2 x \left(I_1 \Omega_{\mu,1}^2 + I_2 \Omega_{\mu,1}^2 + I_3 \Omega_{\mu,1}^2 \right) \longrightarrow S = \frac{1}{2} \int d^2 x \ g_{ij} \left[\nabla_\mu X^i \nabla_\mu X^j \right]$$
$$+ \frac{I_\alpha}{2} \int d^2 x (\partial_\mu \alpha)^2 + \frac{\kappa}{2} \int d^2 x \partial_\mu \alpha \Omega_{\mu,3}$$

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 $rac{dg_{ij}}{dl} = -rac{1}{2\pi}R_{ij}$ 15 line mathematica code for Ricci tensor

Input Metric Tensor

$$gl = \begin{pmatrix} \sin^{2}(\theta) \left(II \sin^{2}(\psi) + I2 \cos^{2}(\psi) \right) + I3 \cos^{2}(\theta) \sin(\theta) \left(II - I2 \right) \sin(\psi) \cos(\psi) & I3 \cos(\theta) & \frac{1}{2} \times \cos(\theta) \\ \sin(\theta) \left(II - I2 \right) \sin(\psi) \cos(\psi) & I1 \cos^{2}(\psi) + I2 \sin^{2}(\psi) & 0 & 0 \\ I3 \cos(\theta) & 0 & I3 & \frac{\kappa}{2} \\ & \frac{1}{2} \times \cos(\theta) & 0 & \frac{\kappa}{2} & I\varphi \end{pmatrix};$$

Compute Cristoffel Symbol
(rⁱ)_{k1} =
$$\frac{1}{2}$$
g^{ij} ($\nabla_1 g_{jk} + \nabla_k g_{j1} - \nabla_j g_{k1}$)

$$rac{dg_{ij}}{dl} = -rac{1}{2\pi}R_{ij}$$
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```
For [i = 1, i \le 4, i++,

For [k = 1, k \le 4, k++,

For [1 = 1, 1 \le 4, 1++,

\Gamma[[i, k, 1]] =

\sum_{j=1}^{4} \frac{1}{2} \star gu[[i, j]] \star

(D[g1[[j, k]], x[[1]]] + D[g1[[j, 1]], x[[k]]] -

D[g1[[k, 1]], x[[j]]))]]]

da: 15 line mathematica code for
```

 $\frac{dg_{ij}}{dl} = -\frac{1}{2\pi}R_{ij} \quad \begin{array}{l} \mbox{15 line mathematica code for} \\ \mbox{Ricci tensor} \end{array}$

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Compute Ricci Tensor

 $\mathbf{R^{k}_{ij1}} = \mathbf{\Gamma^{k}_{i1,j}} - \mathbf{\Gamma^{k}_{ij,1}} + \mathbf{\Gamma^{k}_{jn}} \mathbf{\Gamma^{n}_{li}} - \mathbf{\Gamma^{k}_{ln}} \mathbf{\Gamma^{n}_{ij}}$

 $R_{ij} = R^{k}_{ikj}$

$$rac{dg_{ij}}{dl} = -rac{1}{2\pi}R_{ij}$$
 15 line mathematica code for Ricci tensor

Input Metric Tensor

$$gl = \begin{pmatrix} \sin^{2}(\theta) \left(II \sin^{2}(\psi) + I2 \cos^{2}(\psi) \right) + I3 \cos^{2}(\theta) \sin(\theta) (II - I2) \sin(\psi) \cos(\psi) & I3 \cos(\theta) & \frac{1}{2} \times \cos(\theta) \\ \sin(\theta) (II - I2) \sin(\psi) \cos(\psi) & I1 \cos^{2}(\psi) + I2 \sin^{2}(\psi) & 0 & 0 \\ I3 \cos(\theta) & 0 & I3 & \frac{\kappa}{2} \\ & \frac{1}{2} \times \cos(\theta) & 0 & \frac{\kappa}{2} & I\varphi \end{pmatrix};$$

For
$$[i = 1, i \le 4, i++,$$
 Compute Ricci Tensor
For $[k = 1, k \le 4, k++,$ $R^{k}_{ij1} = \Gamma^{k}_{i1,j} - \Gamma^{k}_{ij,1} + \Gamma^{k}_{jn} \Gamma^{n}_{1i} - \Gamma^{k}_{1n} \Gamma^{n}_{ij}$
Riccill $[[i, k]] =$

$$\sum_{l=1}^{4} \left(p[r[[l, i, k]], x[[l]]] - p[r[[l, i, 1]], x[[k]]] + R_{ij} = R^{k}_{ikj} \right)$$

$$\sum_{m=1}^{4} \left(r[[m, 1, m]] * r[[1, i, k]] - r[[m, i, 1]] * r[[1, k, m]]) \right)]];$$

$$\frac{dg_{ij}}{dl} = -\frac{1}{2\pi} R_{ij}$$
 I5 line mathematica code for
Ricci tensor

Input Metric Tensor

$$gl = \begin{pmatrix} \sin^{2}(\theta) \left(II \sin^{2}(\psi) + I2 \cos^{2}(\psi) \right) + I3 \cos^{2}(\theta) \sin(\theta) (II - I2) \sin(\psi) \cos(\psi) & I3 \cos(\theta) & \frac{1}{2} \times \cos(\theta) \\ \sin(\theta) (II - I2) \sin(\psi) \cos(\psi) & I1 \cos^{2}(\psi) + I2 \sin^{2}(\psi) & 0 & 0 \\ I3 \cos(\theta) & 0 & I3 & \frac{x}{2} \\ & \frac{1}{2} \times \cos(\theta) & 0 & \frac{x}{2} & I\varphi \end{pmatrix};$$

For [i = 1, i ≤ 4, i++, Compute Ricci Tensor
For [k = 1, k ≤ 4, k++, R^k_{ij1} = T^k_{i1,j} - T^k_{ij,1} + T^k_{jn} Tⁿ_{1i} - T^k_{in} Tⁿ_{ij}
Riccill[[i, k]] =

$$\sum_{i=1}^{4} \left(\mathbb{D}[r[[1, i, k]], x[[1]]] - \mathbb{D}[r[[1, i, 1]], x[[k]]] + R_{ij} = Rk_{ikj}\right)$$

$$\frac{dg_{ij}}{dl} = -\frac{1}{2\pi} R_{ij} \quad 15 \text{ line mathematica code for } This is the renormalization of I3 \\ Ricci tensor - FullSimplify[\frac{1}{2\pi} Riccill[[3, 3]] - \frac{13^2 - (11 - 12)^2}{4\pi 11 12}$$
Input Metric Tensor
$$gl = \left(\begin{cases} \sin^2(\theta) (11 \sin^2(\theta) + 12 \cos^2(\theta)) + 13 \cos^2(\theta) \sin(\theta) (11 - 12) \sin(\theta) \cos(\theta) - \frac{13}{2} \times \cos(\theta) \\ 13 \cos(\theta) & 0 & 13 & \frac{x}{2} \\ \frac{1}{2} \times \cos(\theta) & 0 & \frac{x}{2} & I\varphi \end{cases} \right),$$

For
$$[i = 1, i \le 4, i++, Compute Ricci Tensor$$

For $[k = 1, k \le 4, k++, R^{k}_{ij1} = \Gamma^{k}_{i1,j} - \Gamma^{k}_{ij,1} + \Gamma^{k}_{jn} \Gamma^{n}_{1i} - \Gamma^{k}_{1n} \Gamma^{n}_{ij}$
Riccill $[[i, k]] = a^{k}_{ij} \left[P[\Gamma[[1, i, k]], x[[1]]] - P[\Gamma[[1, i, 1]], x[[k]]] + R_{ij} = R^{k}_{ikj} e^{A^{*}} \right]$

$$\sum_{l=1}^{4} \left(P[\Gamma[[n, 1, m]] * \Gamma[[1, i, k]] - \Gamma[[m, i, 1]] * \Gamma[[1, k, m]]) \right)]];$$

$$\frac{dg_{ij}}{dl} = -\frac{1}{2\pi} R_{ij}$$
I5 line mathematica code for This is the renormalization of I3
Ricci tensor -FullSimplify $\left[\frac{1}{2\pi} \operatorname{Riccill}[[3, 3]]\right]$

$$-\frac{I3^2 - (I1 - I2)^2}{4\pi I1 I2}$$

$$\begin{aligned} & \text{For} \Big[\mathbf{i} = 1, \ \mathbf{i} \le 4, \ \mathbf{i} + \mathbf{i}, \\ & \text{For} \Big[\mathbf{k} = 1, \ \mathbf{k} \le 4, \ \mathbf{k} + \mathbf{i}, \\ & \text{Riccill} \big[[\mathbf{i}, \mathbf{k}] \big] = \\ & \sum_{i=1}^{4} \left(\mathsf{P} \big[\mathsf{r} [[\mathbf{i}, \mathbf{k}]], \mathbf{x} [[\mathbf{i}]] \big] - \mathsf{P} \big[\mathsf{r} [[\mathbf{i}, \mathbf{i}, \mathbf{1}]], \mathbf{x} [[\mathbf{k}]] \big] + \\ & \sum_{m=1}^{4} \left(\mathsf{P} \big[\mathsf{r} [[\mathbf{i}, \mathbf{i}, \mathbf{k}]], \mathbf{x} [[\mathbf{i}]] \big] - \mathsf{P} \big[\mathsf{r} [[\mathbf{i}, \mathbf{i}, \mathbf{1}] \big], \mathbf{x} [[\mathbf{k}]] \big] + \\ & \sum_{m=1}^{4} \left(\mathsf{r} [[\mathbf{m}, \mathbf{1}, \mathbf{m}]] * \mathsf{r} [[\mathbf{i}, \mathbf{i}, \mathbf{k}] \big] - \mathsf{P} \big[\mathsf{r} [[\mathbf{m}, \mathbf{i}, \mathbf{1}] \big] * \mathsf{r} [[\mathbf{1}, \mathbf{k}, \mathbf{m}]] \big) \Big] \Big]; \\ & \frac{dg_{ij}}{dl} = -\frac{1}{2\pi} R_{ij} \quad \frac{15 \text{ line mathematica code for}}{\mathsf{Ricci tensor}} \\ & \text{This is the renormalization of I3} \\ & -\mathsf{FullSimplify} \Big[\frac{1}{2\pi} \operatorname{Riccill} [[\mathbf{3}, \mathbf{3}]] \Big]; \\ & \frac{dI_1}{dl} = \frac{(I_2 - I_3)^2 - I_1^2}{4\pi I_2 I_3} - \frac{(I_1^2 - I_2^2) \kappa^2}{16\pi I_2 I_3^2 (I_\varphi - \frac{\kappa^2}{4I_3})} \\ & \frac{dI_2}{dl} = \frac{(I_1 - I_3)^2 - I_2^2}{4\pi I_1 I_3} + \frac{(I_1^2 - I_2^2) \kappa^2}{16\pi I_1 I_3^2 (I_\varphi - \frac{\kappa^2}{4I_5})} \\ & \frac{dI_3}{dl} = \frac{(I_1 - I_2)^2 - I_3^2}{4\pi I_1 I_2} \\ & \frac{dI_4}{dl} = -\frac{\kappa^2}{16\pi I_1 I_2} \\ & \frac{d$$

$$\begin{aligned} & \text{For}[\mathbf{i} = 1, \ \mathbf{i} \le 4, \ \mathbf{i} + \mathbf{i}, \\ & \text{For}[\mathbf{k} = 1, \ \mathbf{k} \le 4, \ \mathbf{k} + \mathbf{i}, \\ & \text{Riccill}[[\mathbf{i}, \mathbf{k}]] = \\ & \sum_{i=1}^{4} \left(\mathbb{P}[\mathbf{r}[[\mathbf{i}, \mathbf{i}, \mathbf{k}]], \mathbf{x}[[\mathbf{i}]]] - \mathbb{P}[\mathbf{r}[[\mathbf{1}, \mathbf{i}, \mathbf{1}]], \mathbf{x}[[\mathbf{k}]]] + \\ & \sum_{i=1}^{4} \left(\mathbb{P}[\mathbf{r}[[\mathbf{1}, \mathbf{i}, \mathbf{k}]], \mathbf{x}[[\mathbf{1}]]] - \mathbb{P}[\mathbf{r}[[\mathbf{1}, \mathbf{i}, \mathbf{1}]], \mathbf{x}[[\mathbf{k}]]] + \\ & \sum_{i=1}^{4} \left(\mathbb{P}[\mathbf{r}[[\mathbf{n}, \mathbf{1}, \mathbf{m}]] + \mathbb{P}[[\mathbf{1}, \mathbf{i}, \mathbf{k}]] - \mathbb{P}[\mathbf{r}[[\mathbf{n}, \mathbf{i}, \mathbf{1}]], \mathbf{x}[[\mathbf{k}]]] + \\ & \sum_{i=1}^{4} \left(\mathbb{P}[\mathbf{r}[[\mathbf{n}, \mathbf{1}, \mathbf{m}]] + \mathbb{P}[[\mathbf{1}, \mathbf{i}, \mathbf{k}]] - \mathbb{P}[\mathbf{r}[[\mathbf{n}, \mathbf{i}, \mathbf{1}]] + \mathbb{P}[[\mathbf{n}, \mathbf{k}, \mathbf{m}]]) \right) \right]]; \\ & \frac{dg_{ij}}{dl} = -\frac{1}{2\pi} R_{ij} \quad \text{15 line mathematica code for} \\ & \text{Ricci tensor} \\ & -\mathbb{Pullsimplify}[\frac{1}{2\pi} \operatorname{Riccill}[[\mathbf{3}, \mathbf{3}]] \\ & \frac{dI_1}{dl} = \frac{(I_2 - I_3)^2 - I_1^2}{4\pi I_2 I_3} - \frac{(I_1^2 - I_2^2) \kappa^2}{16\pi I_1 I_3^2} (I_{\varphi} - \frac{\kappa^2}{4I_3}) \\ & \frac{dI_2}{dl} = \frac{(I_1 - I_3)^2 - I_2^2}{4\pi I_1 I_2} + \frac{(I_1^2 - I_2^2) \kappa^2}{16\pi I_1 I_3^2} (I_{\varphi} - \frac{\kappa^2}{4I_3}) \\ & \frac{dI_3}{dl} = \frac{(I_1 - I_2)^2 - I_3^2}{16\pi I_1 I_2} \end{aligned}$$

Detailed Analysis: Decoupling of U(1) Degrees of Freedom for all flows.

Poincare Conjecture

Every simply connected, closed 3-manifold is homeomorphic to the 3-sphere.



$$\frac{\delta S}{\delta g^{ab}} = 0 = \dot{g}_{ab} - \frac{1}{2\pi} R_{ab}$$

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Every simply connected, closed 3-manifold is homeomorphic to the 3-sphere.



$$\frac{dg^{ab}}{dl} = -\frac{1}{2\pi}R^{ab}$$

Friedan '80, Hamilton '81, Perelmann '06



Trivially connected: Polyakov collapse of stiffness to Zero



collapse of stiffness to Zero

U(1) stiffness survives.

Monte-Carlo simulations

B. Jeevanesan, P. Chandra, P. Coleman, P. Orth Phys. Rev. Lett. 115, 177201 (2015).



Phase diagram

Monte-Carlo phase diagram



RG phase diagram disordered $T_1 \sim \bar{J}$ uncoupled $T_{\rm cp}$ coplanar $T_{\rm BKT}^{>} \sim \frac{\pi \bar{J}}{2 + \frac{4\pi \bar{J}^2}{J_{\mu_b}^2} e}$ critical $T_{\rm BKT}^{<}$

 \mathbb{Z}_6 broken

Phase diagram

Monte-Carlo phase diagram



Monte-Carlo simulations provide unbiased verification of long-wavelength picture.





Friedan '80, Hamilton '81, Perelmar







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$$\frac{\delta S}{\delta g^{ab}} = 0 = \dot{g}_{ab} - \frac{1}{2\pi} R_{ab}$$
$$S = \int d\tau \left[\frac{1}{2} g^{ab} \dot{g}_{ab} - \frac{1}{2\pi} R \right]$$





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• Realization of mapping of RG into time. (cf AdSCFT)





Friedan '80, Hamilton '81, Perelmar

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- Realization of mapping of RG into time. (cf AdSCFT)
- Can this mapping of renormalization onto time be generalized?



Part II

Part II: COMPOSITE PAIRS

How a chance conversation with a particle physicist colleague led to a new idea about superconductivity.



Temperature (K)









Mott Mechanism. Anderson *U* (Anderson 1959)





No double occupancy: strongly correlated

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- No double occupancy: strongly correlated
- Residual valence fluctuations induce AFM Superexchange.





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Many things are possible at the brink of magnetism.

Heavy Fermions + Kondo



Heavy Fermions + Kondo



Heavy Fermions + Kondo







$$\begin{split} H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + J \quad \vec{S} \cdot \vec{\sigma}(0) \\ \text{J. Kondo, 1962} \end{split}$$





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$$\uparrow \downarrow - \downarrow \uparrow$$

$$S(T) = \int_0^T \frac{C_V}{T'} dT'$$

Spin entanglement entropy



$$\uparrow \downarrow - \downarrow \uparrow$$

$$S(T) = \int_0^T \frac{C_V}{T'} dT'$$









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Spin entanglement entropy







"Kondo Lattice (Doniach 1978)"









Entangled spins and electrons













The remarkable case of NpPd₅Al₂ [001] Pd(1) $NpPd_5Al_2 T_C = 4.5K$ Np 40Pd(2) NpPd5Al2 30 Al χ (x10⁻³ emu/mol) H // [100] 20108 [001] 0 100300 0 200[010] Temperature (K) [100]

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4.5K Heavy Fermion S.C NpAl₂Pd₅ Aoki et al 2007



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Np

Pd(2)

[010]

Al



How does the spin form the condensate?

4.5K Heavy Fermion S.C NpAl₂Pd₅ Aoki et al 2007



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D. Aoki et al., J. Phys. Soc. Jpn. **76** (2007) 063701.





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T. Tayama et al., RPB 65, 180504R (2002)



Paradox:

How can a neutral magnetic moments form a charged superconducting condensate?





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How can a neutral magnetic moments form a charged superconducting condensate?

 $\prod_{\otimes j} \{ = \begin{array}{c} & & & \\ & & \\ & & \\ & & \\ & \\ & & \\$

Composite pairing Hypothesis.



















Temperature (K)



$$\Psi^{\dagger} = c_{1\downarrow}^{\dagger} c_{2\downarrow}^{\dagger} S_{+}$$



 $\Psi^{\dagger} = c_{1 \perp}^{\dagger} c_{2 \perp}^{\dagger} S_{+}$



Abrahams, Balatsky, Scalapino, Schrieffer 1995

$$H = \sum_{k} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \frac{1}{N} \sum_{\mathbf{k},\mathbf{k}'} \left(J_{1} \psi_{1a}^{\dagger}(j) \psi_{1b}(j) + J_{2} \psi_{2a}^{\dagger}(j) \psi_{2b}(j) \right) S^{ba}(j)$$

$$\psi_{\Gamma}(j) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \gamma_{\Gamma \mathbf{k}} \ c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}_{j}}$$



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Wild quantum fluctuations!

$$H = \sum_{k} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \frac{1}{N} \sum_{\mathbf{k},\mathbf{k}'} \left(J_1 \psi_{1a}^{\dagger}(j) \psi_{1b}(j) + J_2 \psi_{2a}^{\dagger}(j) \psi_{2b}(j) \right) S^{ba}(j)$$

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$$\int S[\psi]$$

$$\int \tau, x$$

$$\int \frac{1}{N} \sim \hbar_{eff}$$

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PC: why don't you ever use the group SP(N)?



$$H = \sum_{k} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + \frac{1}{N} \sum_{\mathbf{k},\mathbf{k}'} \left(J_{1} \psi_{1a}^{\dagger}(j) \psi_{1b}(j) + J_{2} \psi_{2a}^{\dagger}(j) \psi_{2b}(j) \right) S^{ba}(j)$$

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SU(N):MesonsBaryons $\bar{q}q$ $q_1q_2 \dots q_N$



 $\rightarrow \infty$

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Since ES two channels

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"Symplectic Large N" R. Fint and PC '08

$$S^{ba} = f_b^{\dagger} f_a - \operatorname{sgn}(a) \operatorname{sgn}(b) f_{-b}^{\dagger} f_{-a}$$

$$SU(N): \quad Mesons \quad Baryons$$

$$\overline{q}q \quad q_1 q_2 \dots q_N$$

$$Cooper pairs$$

$$g_q \quad q_a q_{-a}$$

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Single FS, two channels.

 $\psi_{\Gamma}(j) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \gamma_{\Gamma \mathbf{k}} c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}_{j}}$

λτ
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Impurity: quantum critical point for $J_1 = J_2$



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Nozieres and Blandin 1980



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Magnetic pair: intercell

 $\Psi_M^{\dagger} = \Delta_d (1-2) f_{\uparrow}^{\dagger}(1) f_{\downarrow}^{\dagger}(2)$



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Composite pair $\Psi_C^\dagger = c_{1\downarrow}^\dagger c_{2\downarrow}^\dagger S_+$

Magnetic pair: intercell

 $\Psi_M^{\dagger} = \Delta_d (1-2) f_{\uparrow}^{\dagger}(1) f_{\downarrow}^{\dagger}(2)$



Composite pair: intra-cell boson

$$\Psi_C^{\dagger} = c_{1\downarrow}^{\dagger} c_{2\downarrow}^{\dagger} S_+$$

Magnetic pair: intercell

 $\Psi_M^{\dagger} = \Delta_d (1-2) f_{\uparrow}^{\dagger}(1) f_{\downarrow}^{\dagger}(2)$



Extreme Resilience to doping on Ce site.



Composite pair: intra-cell boson $\Psi_C^\dagger = c_{1\downarrow}^\dagger c_{2\downarrow}^\dagger S_+$

Lei Shu et al, PRL, (2011)

Magnetic pair: intercell

 $\Psi_M^{\dagger} = \Delta_d (1-2) f_{\uparrow}^{\dagger}(1) f_{\downarrow}^{\dagger}(2)$



Composite pair: intra-cell boson

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Extreme Resilience to doping on Ce site.



Lei Shu et al, PRL, (2011)

H. Kim et al PRL (2014) Erten, Flint and PC PRL (2014) Part III

How an Ising anisotropy in the electrons Suggests a new kind of spinor order.



Hasta: Spear (Latin)

Part III

How an Ising anisotropy in the electrons Suggests a new kind of spinor order.



Rebecca FlintPremi Chandra(lowa St.)(Rutgers)

Hastatic order in the heavy-fermion compound URu_2Si_2

Premala Chandra¹, Piers Coleman^{1,2} & Rebecca Flint³

Hasta: Spear (Latin)

doi:10.1038/nature11820



Part III

How an Ising anisotropy in the electrons Suggests a new kind of spinor order.

Can order parameters Fractionalize?



Rebecca FlintPremi Chandra(lowa St.)(Rutgers)

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L. Boltzmann









L. Boltzmann $S = k. \log W$

Heat lost on cooling = k log W = amount of entanglement



 $\Delta S = \int_0^{T_0} \frac{C_V}{T} dT$



L. Boltzmann S = k. log W

Heat lost on cooling = k log W = amount of entanglement











 $\Delta S = \int_0^{T_0} \frac{C_V}{T} dT$

=0.14 x 17.5 K =2.45 J/mol/K =0.42 R ln 2





Huge entropy of condensation!

L. Boltzmann





 $\Delta S = \int_{0}^{T_0} \frac{C_V}{T} dT$

=0.14 x 17.5 K =2.45 J/mol/K =0.42 R ln 2





Huge entropy of condensation!

L. Boltzmann



What is the nature of the hidden order?



High pressures, high fields



Villaume et al. (08)



High pressures, high fields



Villaume et al. (08)

Ising order, present in LMAF, vanishes in the hidden order state. (NMR,MuSR).

25 Years of Theoretical Proposals

Local	Barzykin & Gorkov, '93 (three-spin correlation) Santini & Amoretti, '94, Santini ('98) (Quadrupole order)
	Amitsuka & Sakihabara (F 5, Quadrupolar doublet, '94) Kasuya, '97 (U dimerization)
	Kiss and Fazekas '04, (octupolar order)
	Haule and Kotliar '09 (hexa-decapolar)
Landau Theory	Shah et al. ('00) "Hidden Order",
Itinerant	Ramirez et al, '92 (quadrupolar SDW)
	Ikeda and Ohashi '98 (d-density wave)
	Okuno and Miyake '98 (composite)
	Tripathi, Chandra, PC and Mydosh, '02 (orbital afm)
	Dori and Maki, '03 (unconventional SDW)
	Mineev and Zhitomirsky, '04 (SDW)
	Varma and Zhu, '05 (spin-nematic)
	Ezgar et al '06 (Dynamic symmetry breaking)
	Pepin et al '10 (Spin liquid/Kondo Lattice)
	Dubi and Balatsky, '10 (Hybridization density wave)
	Fujimoto, 2011 (spin-nematic)
	Rau and Kee 2012 (Rank 5 pseudo-spin vector)

Cause Célèbre: state of the art spectroscopies



Scanning Tunneling Microscopy

Cause Célèbre: state of the art spectroscopies





A. R. Schmidt et al., Nature (2010).







6.6K

0.6



tip atoms




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 $rac{m^*}{m_e} g(\theta) = 2n + 1$
Spin Zero condition



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Ising *quasiparticle* with giant anisotropy > 30. Pauli susceptibility anisotropy > 900

Confirmed from upper critical field measurements

Electrons hybridize with Ising 5f state to form Heavy Ising quasiparticles.

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 $rac{m^*}{m_e} g(\theta) = 2n + 1$
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doi:10.1038/nature11820 Hastatic order in the heavy-fermion compound URu_2Si_2

Premala Chandra¹, Piers Coleman^{1,2} & Rebecca Flint³

Ψ

Order parameter carries half-integer spin







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Hasta: Spear (Latin)

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Part II

How a chance conversation with a particle physicist colleague led to a new idea about superconductivity.



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Thank You.