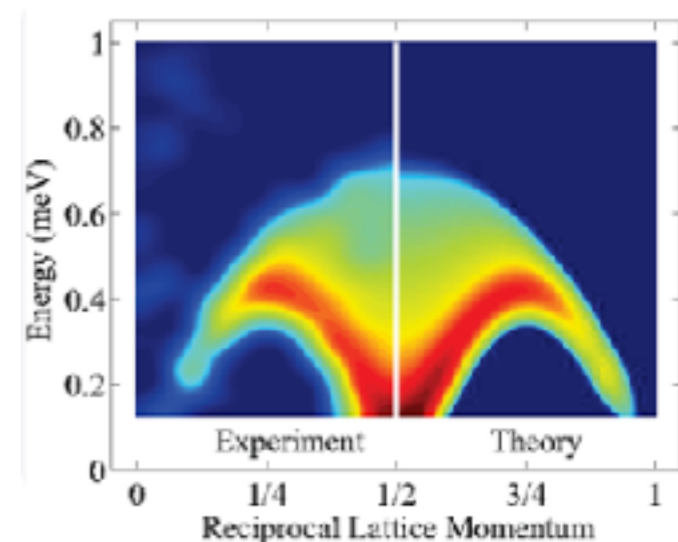
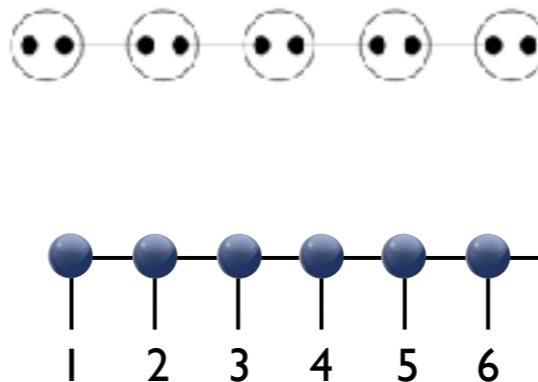
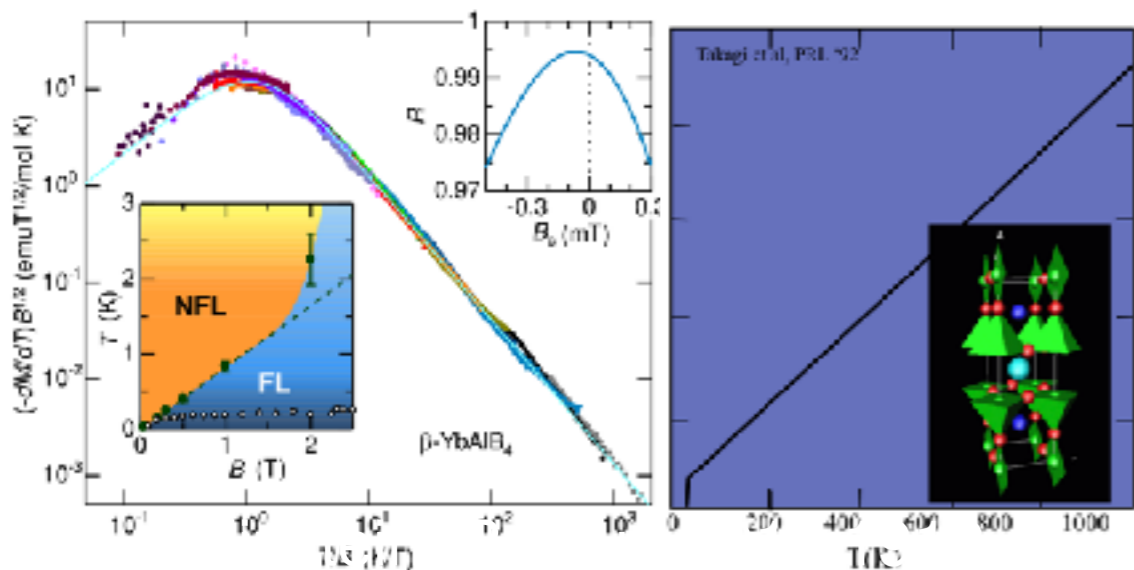


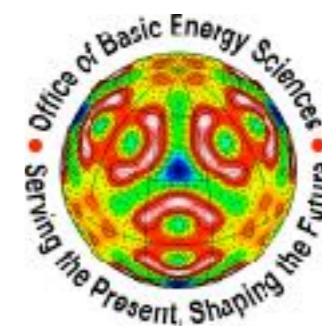
SCES in the Quantum Information era: New challenges and paradigms.

Piers Coleman

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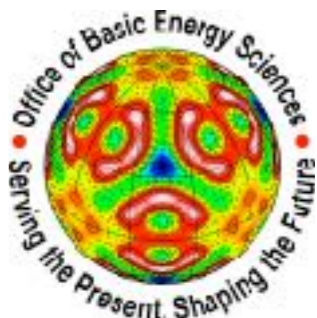
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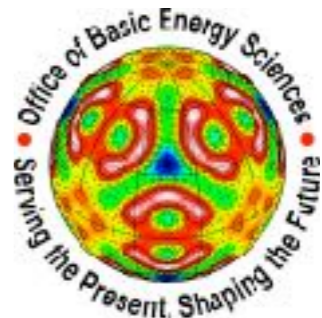
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Einstein: Prague 1911-1912

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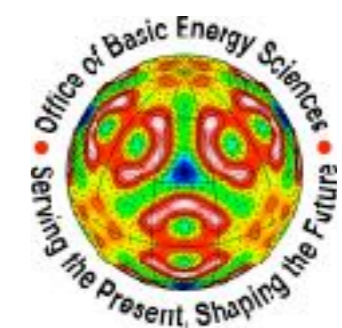
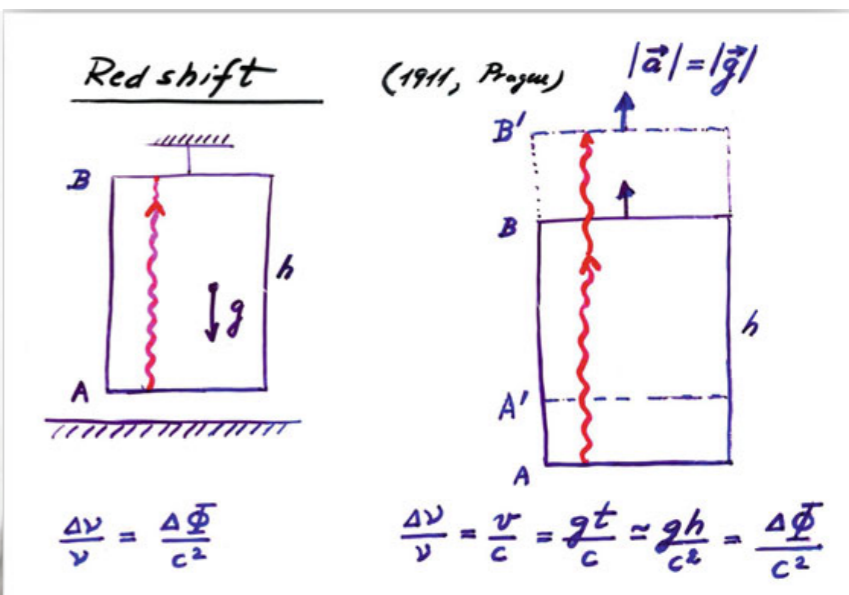
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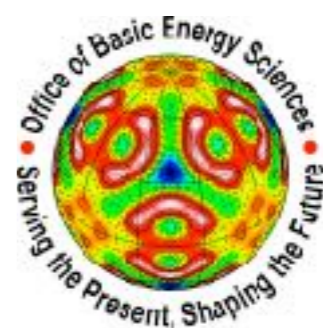
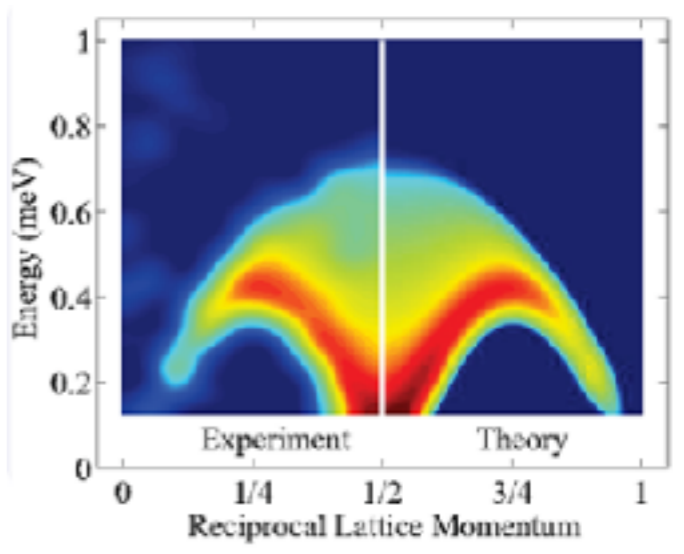
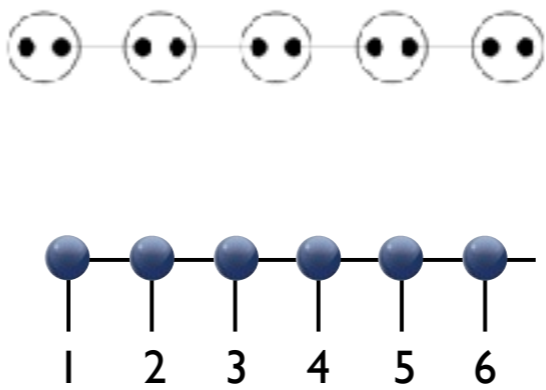
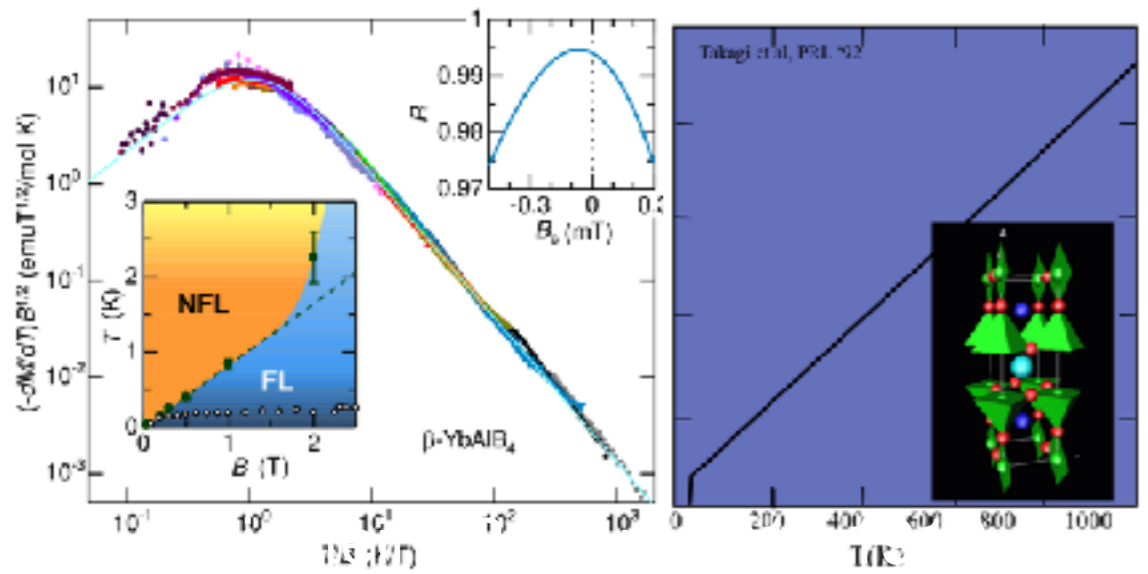
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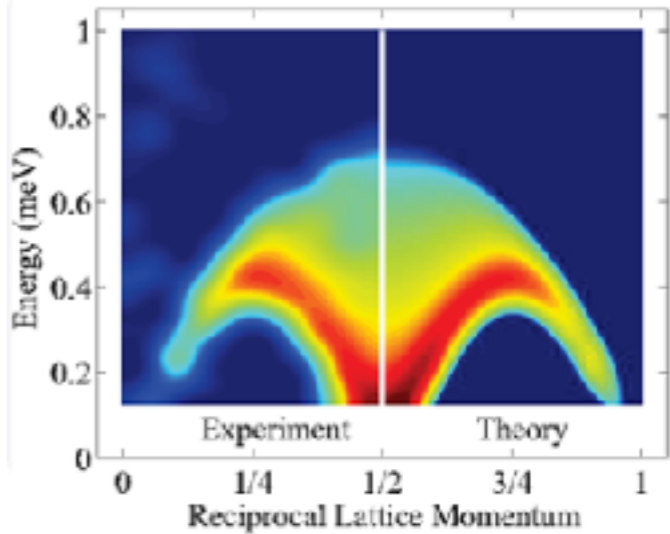
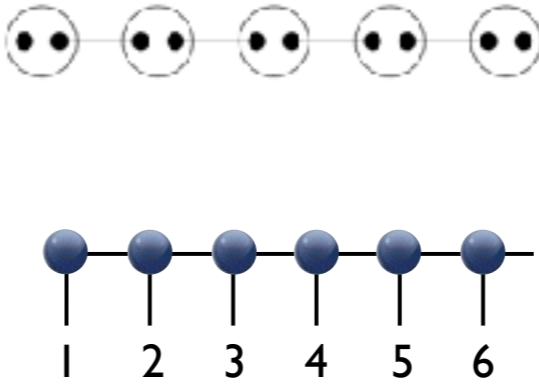
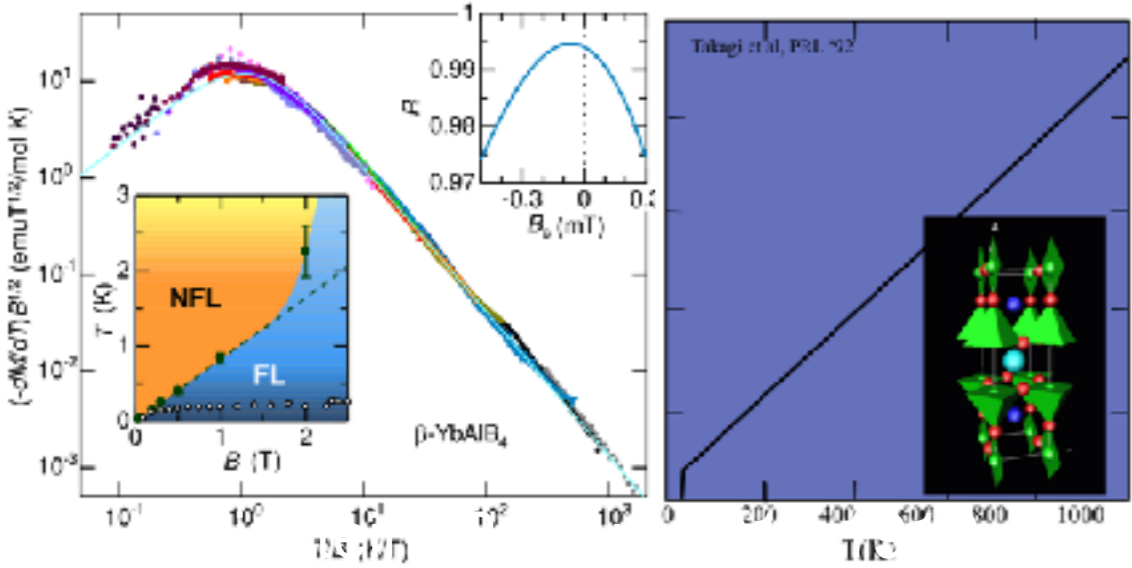
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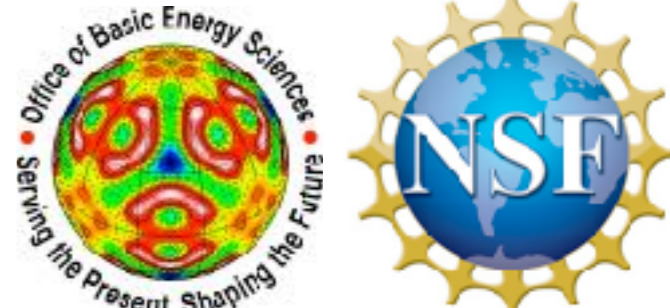
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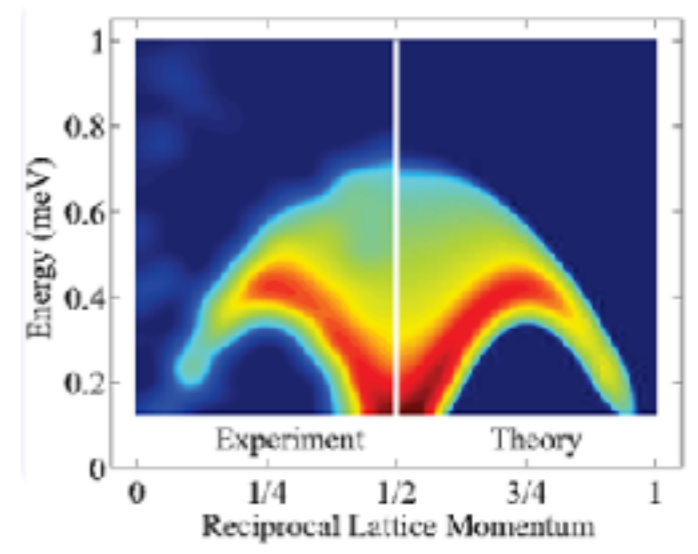
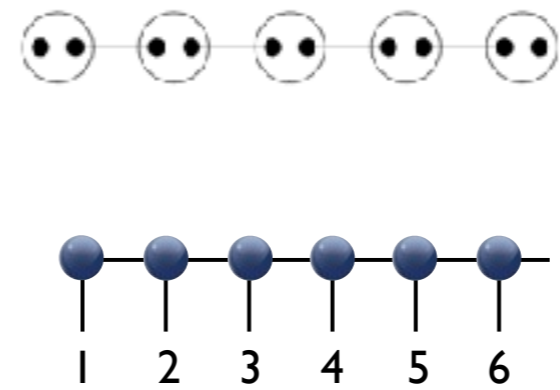
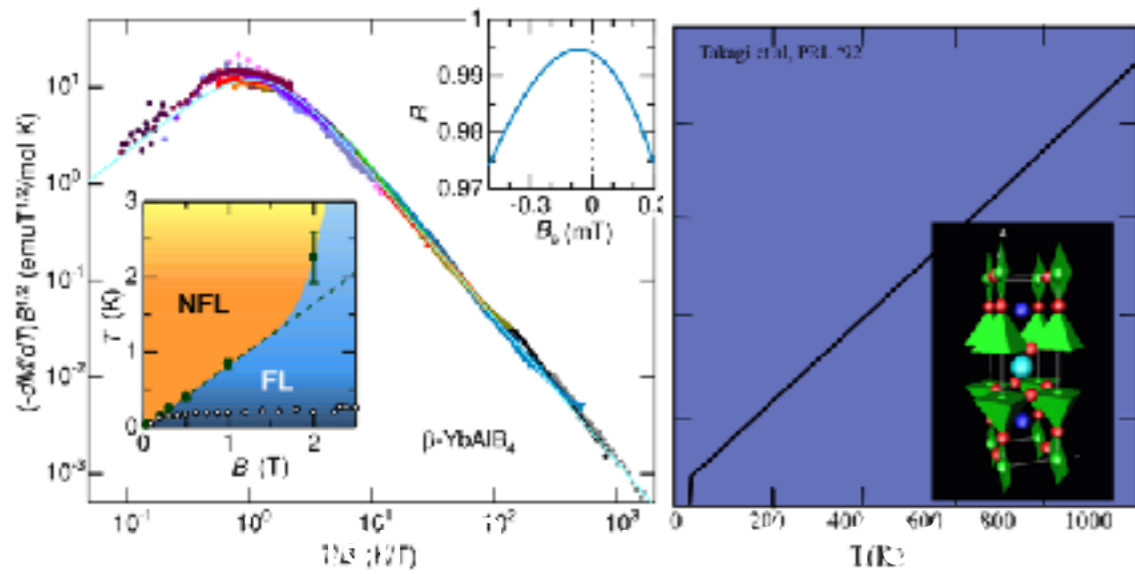
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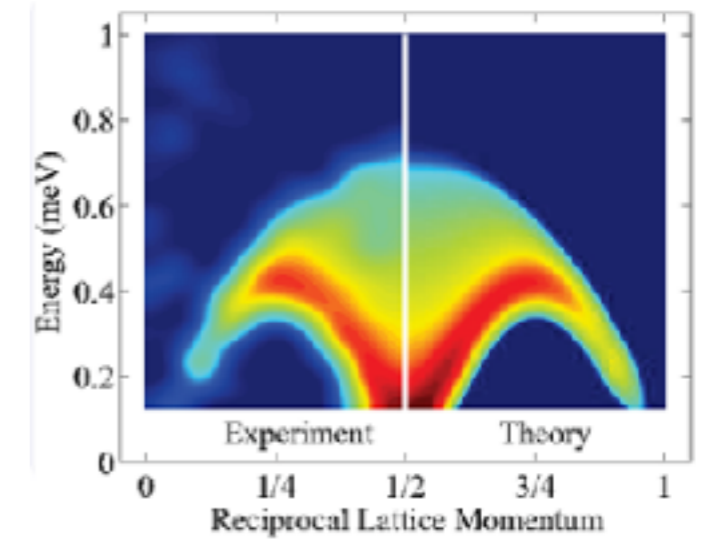
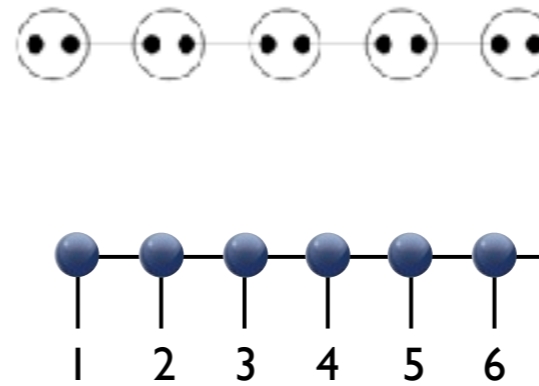
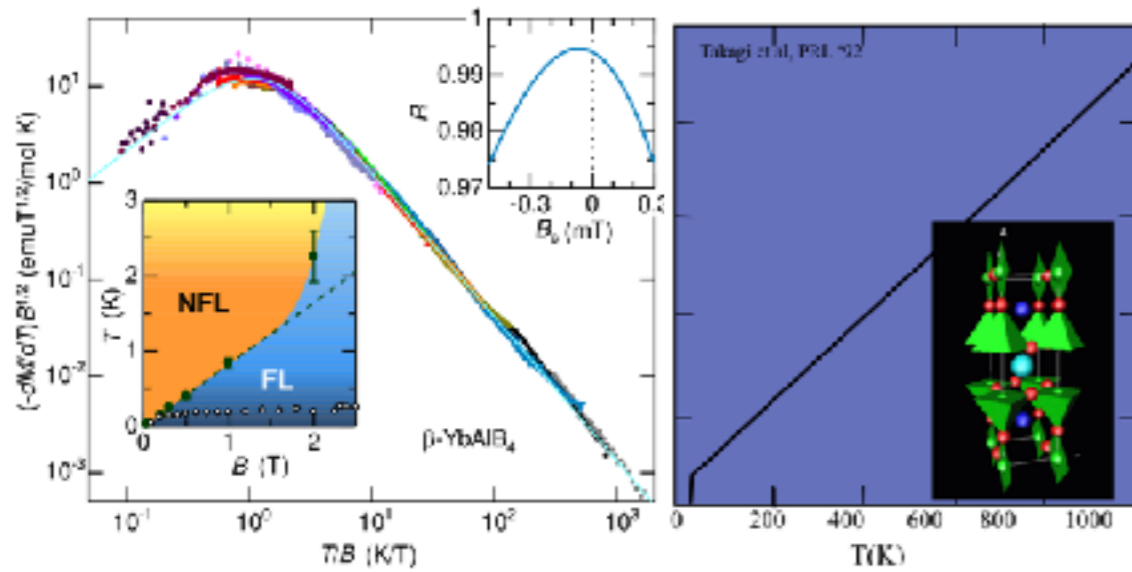


An era of extraordinary opportunity for our field.





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Collaborators

Po Yao Chang
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Maxim Dzero
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Kai Sun

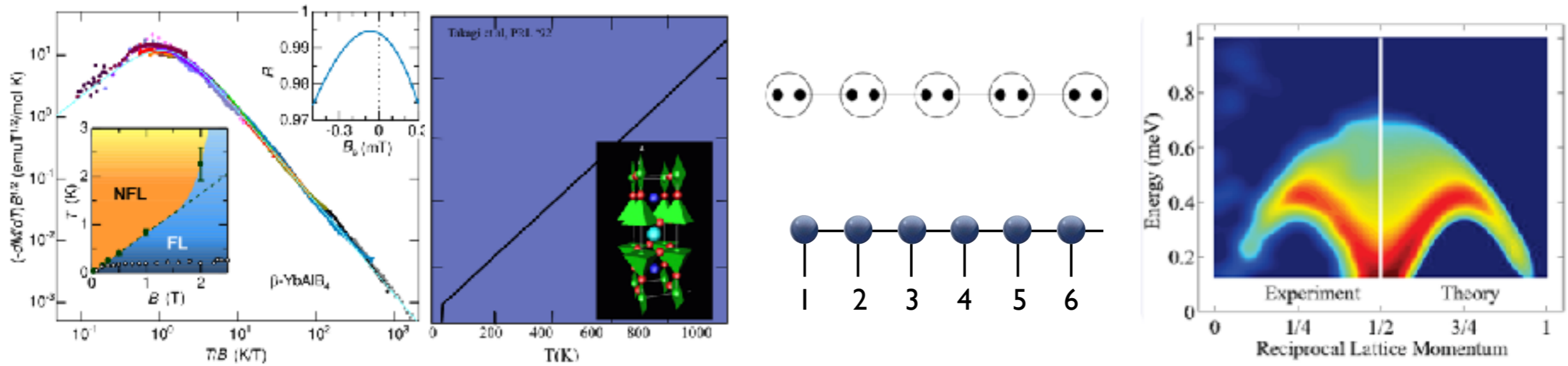
ISSP, Japan
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Kent U.
U. Maryland
U. Michigan

Thanks to:

Ganpathy Baskaran
Girsh Blumberg
Qiuyun Chen
Neil Harrison
Gabriel Kotliar
Gilbert Lonzarich
Frank Pollmann
Filip Ronning
Patricia Rosa
Suchitra Sebastian
Todadri Senthil
Toshio Takabatake

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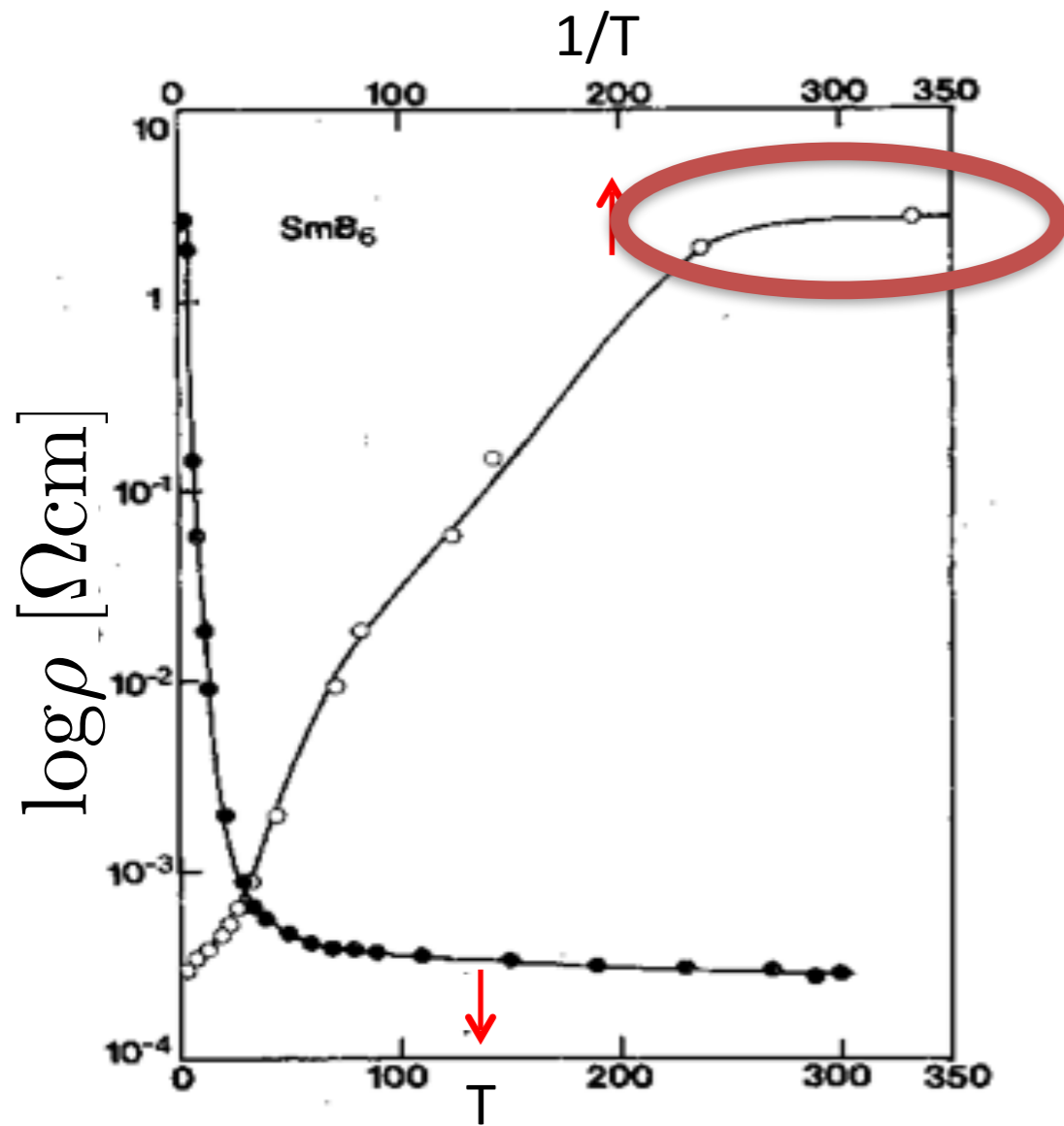
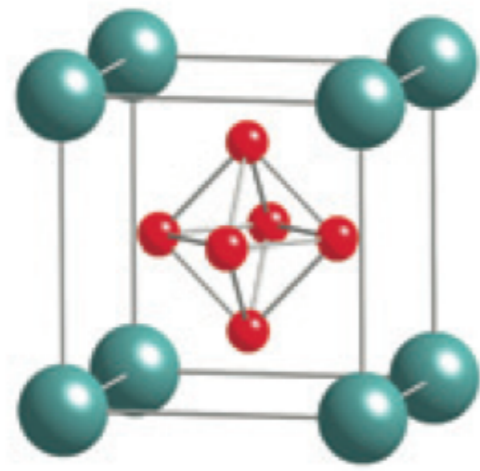


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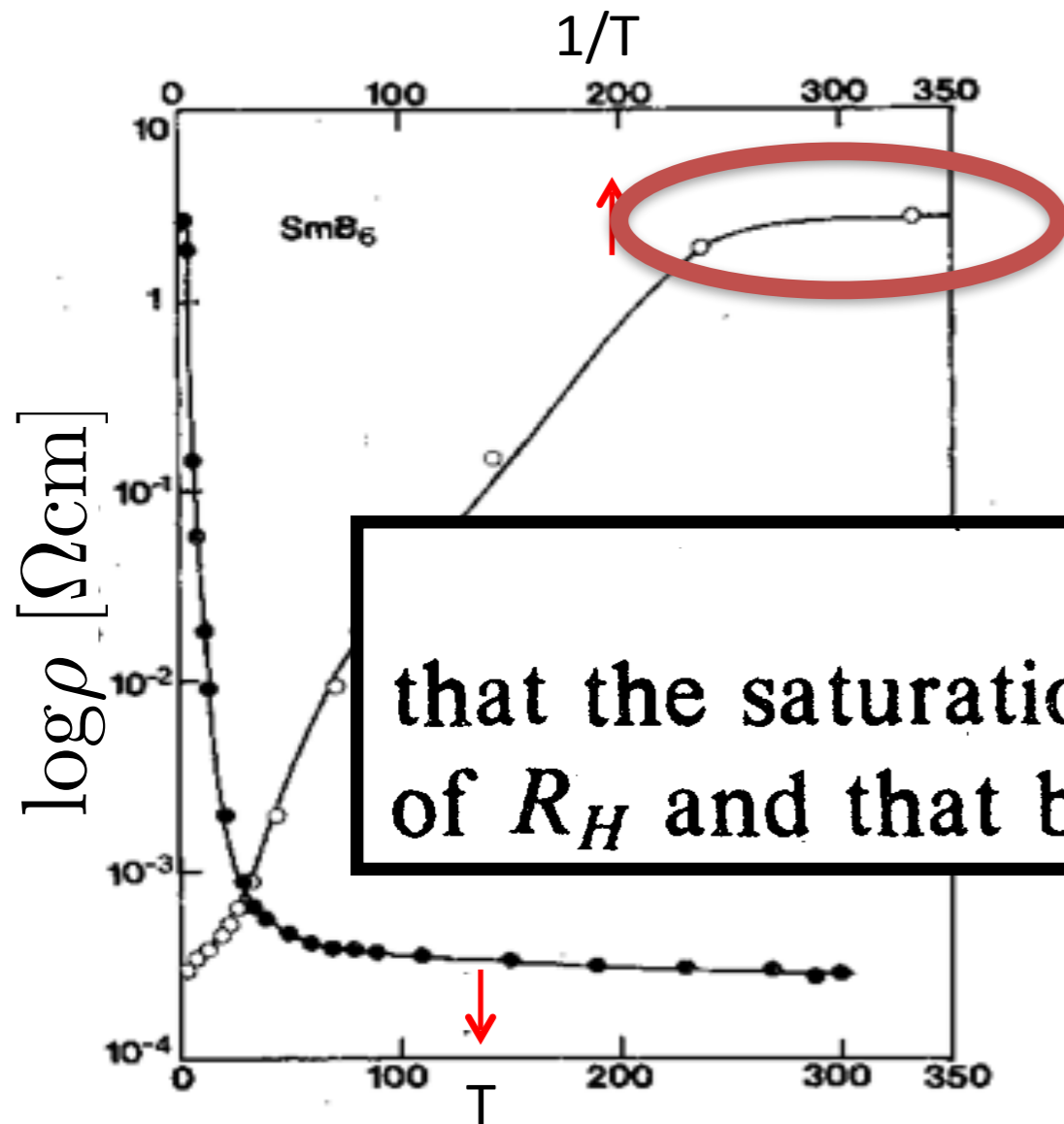
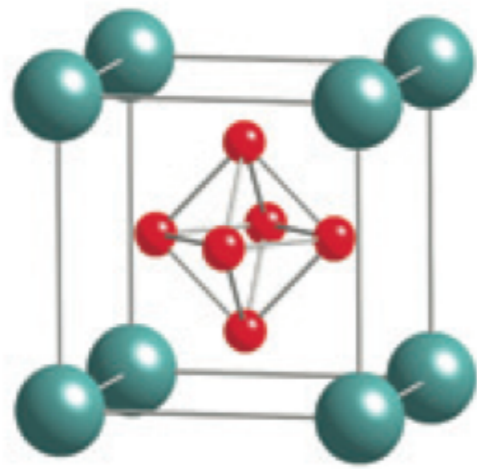
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SmB₆ 1969



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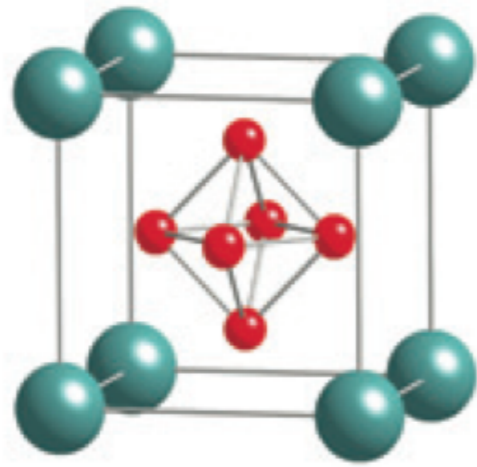
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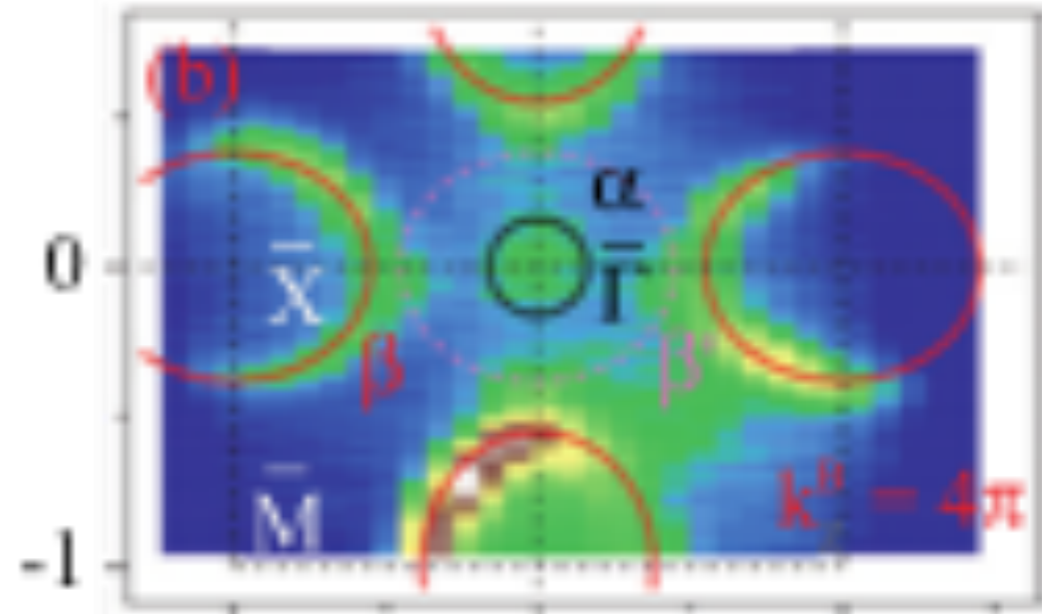
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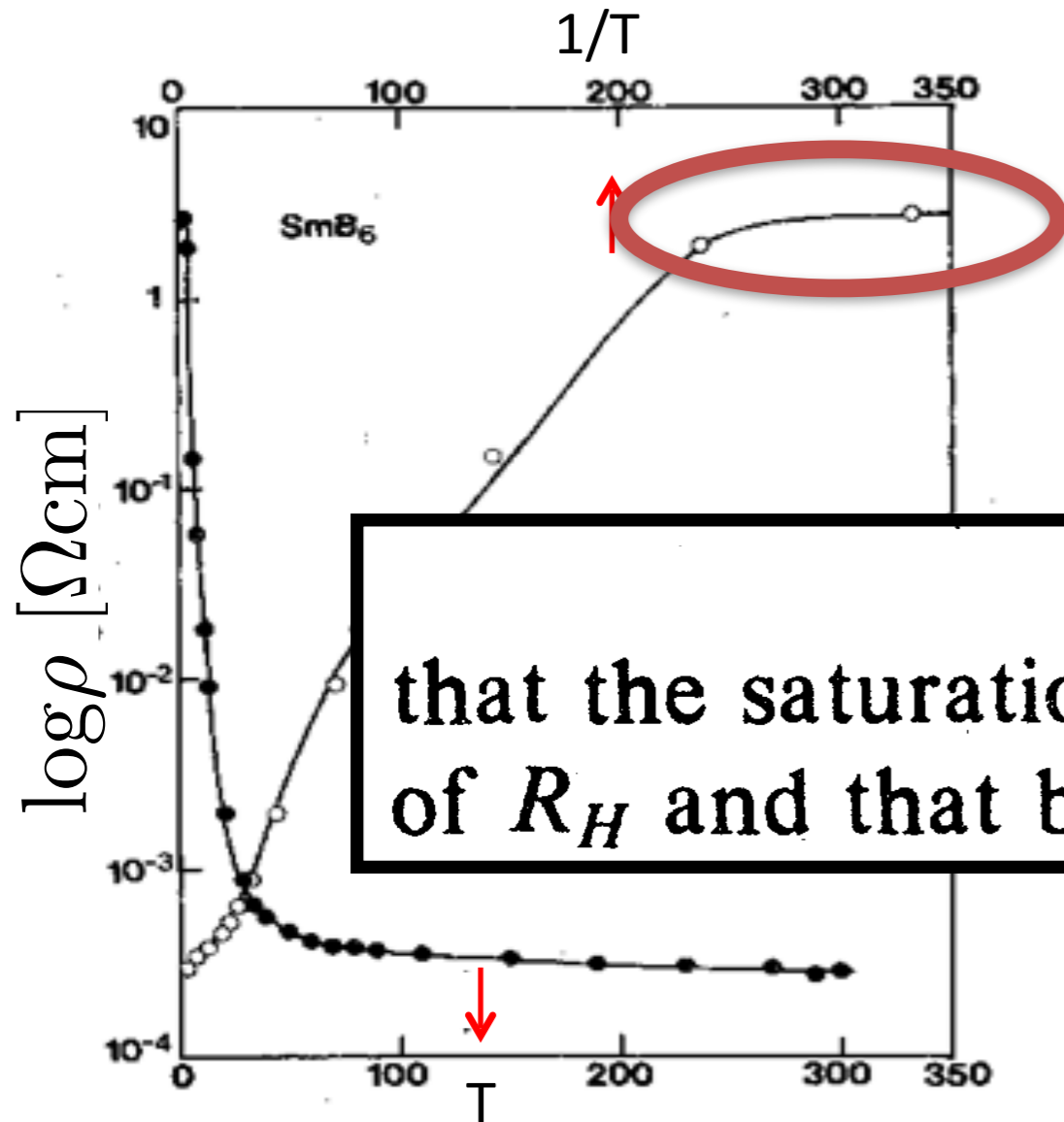
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ARPES → Surface states



Xu et. al PRB **88**, 121102(R) (2013)

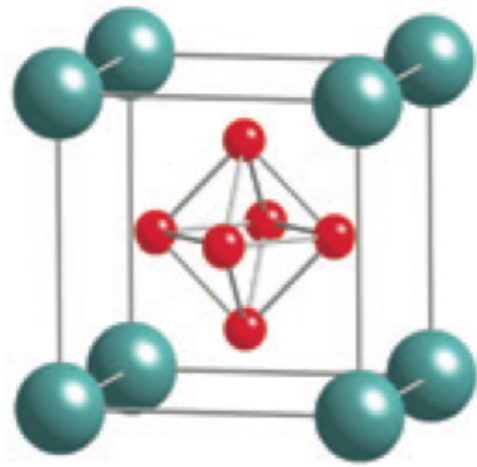


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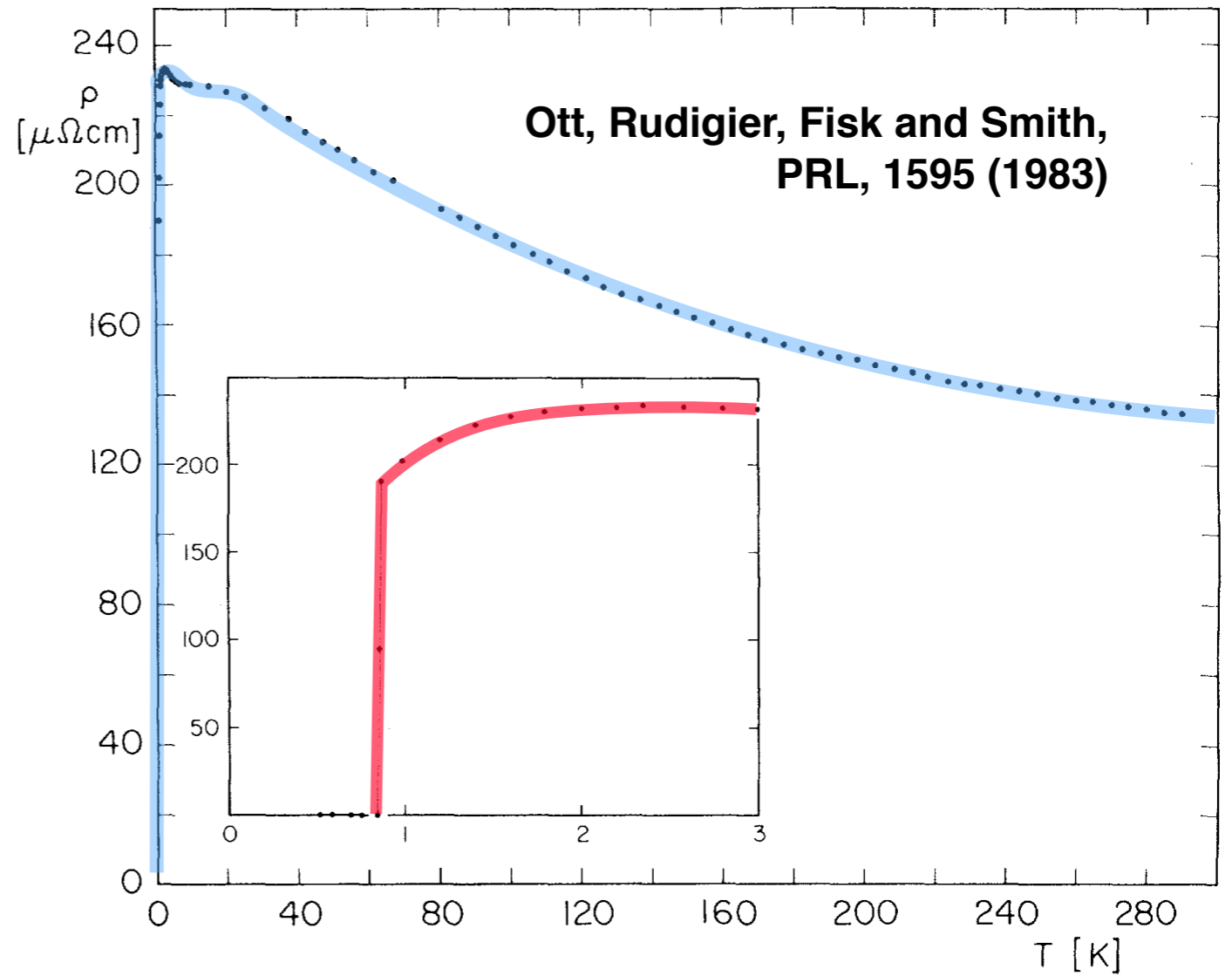
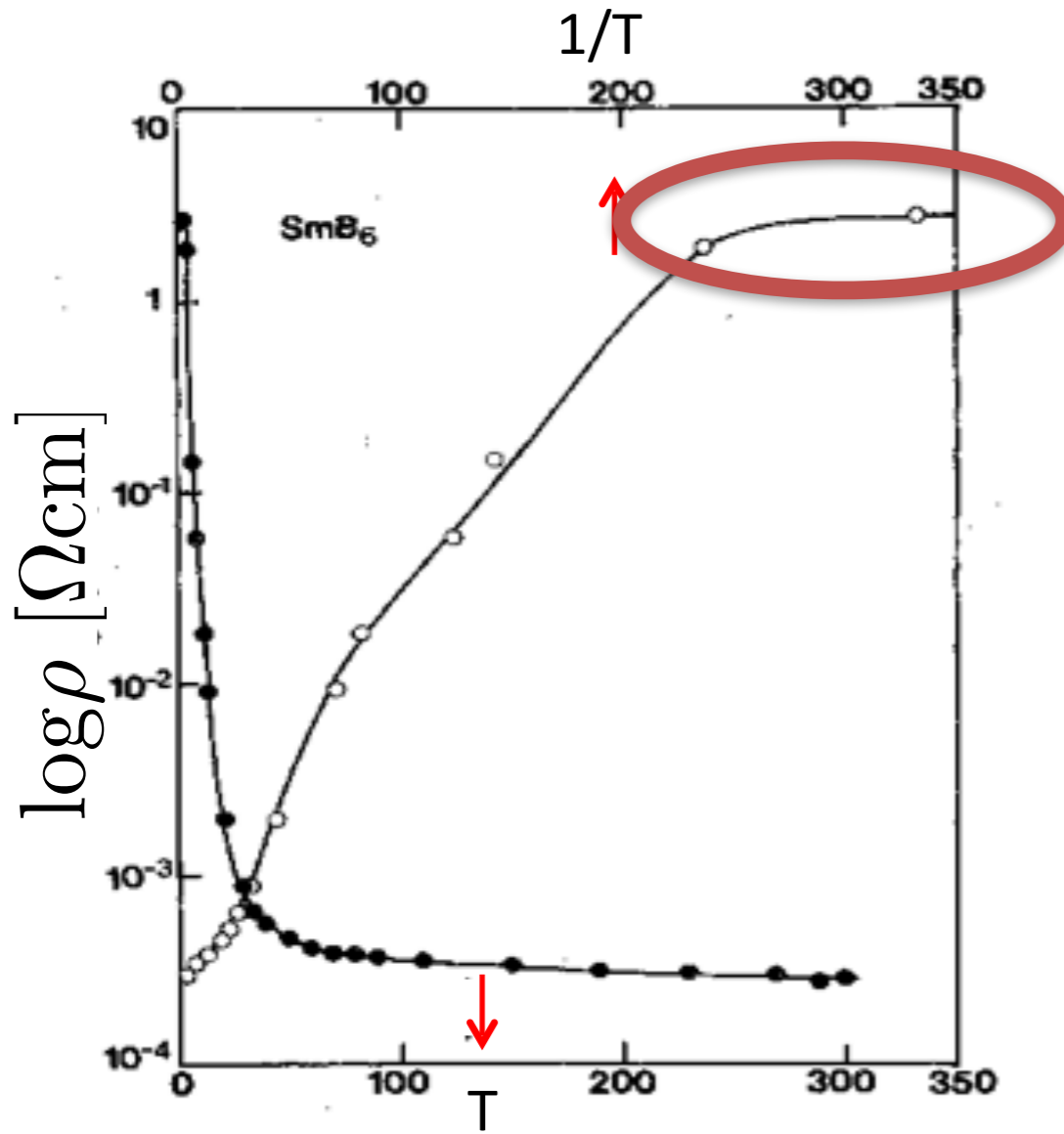
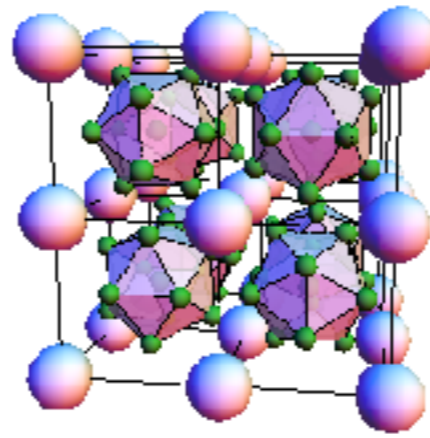
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UBe₁₃
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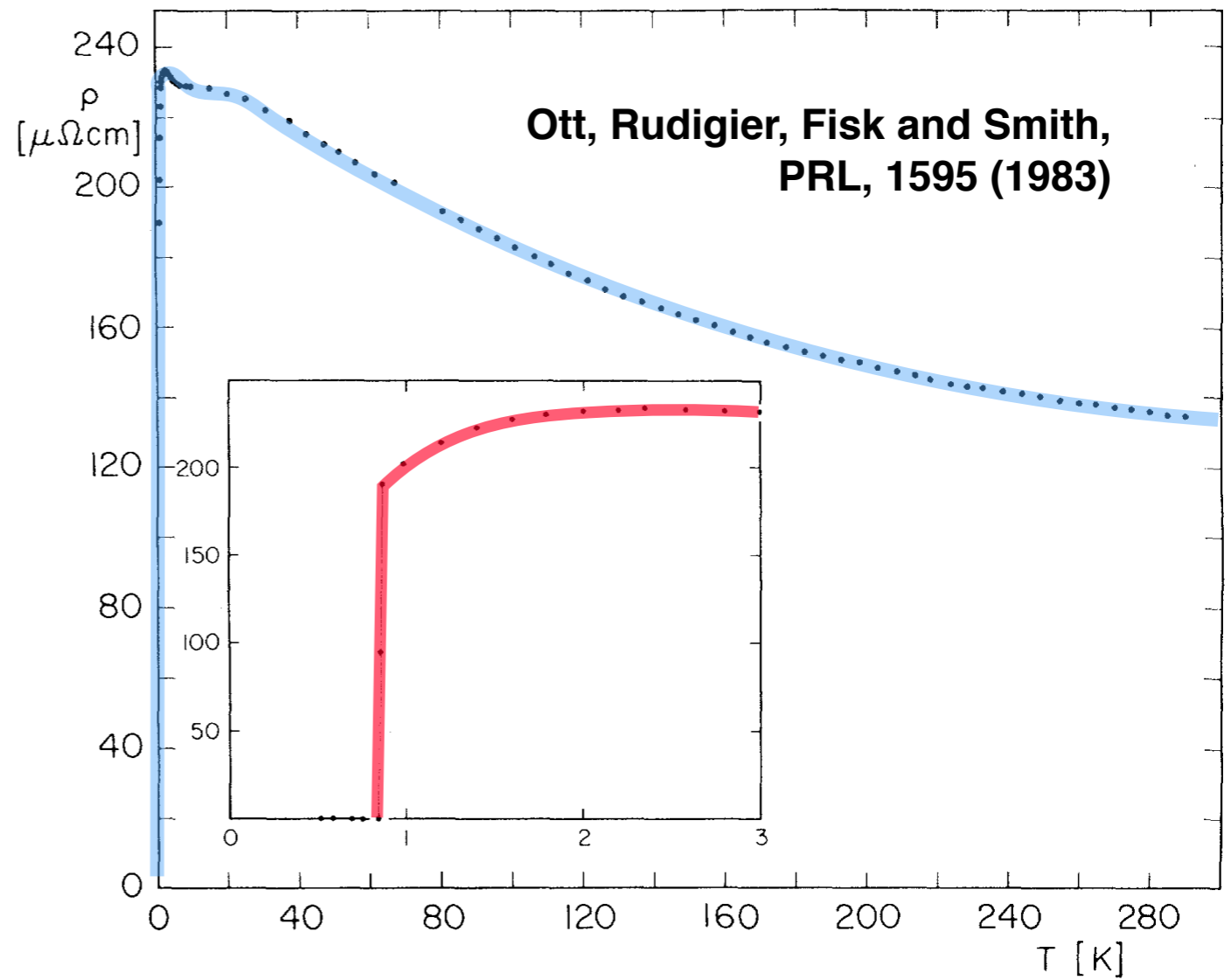
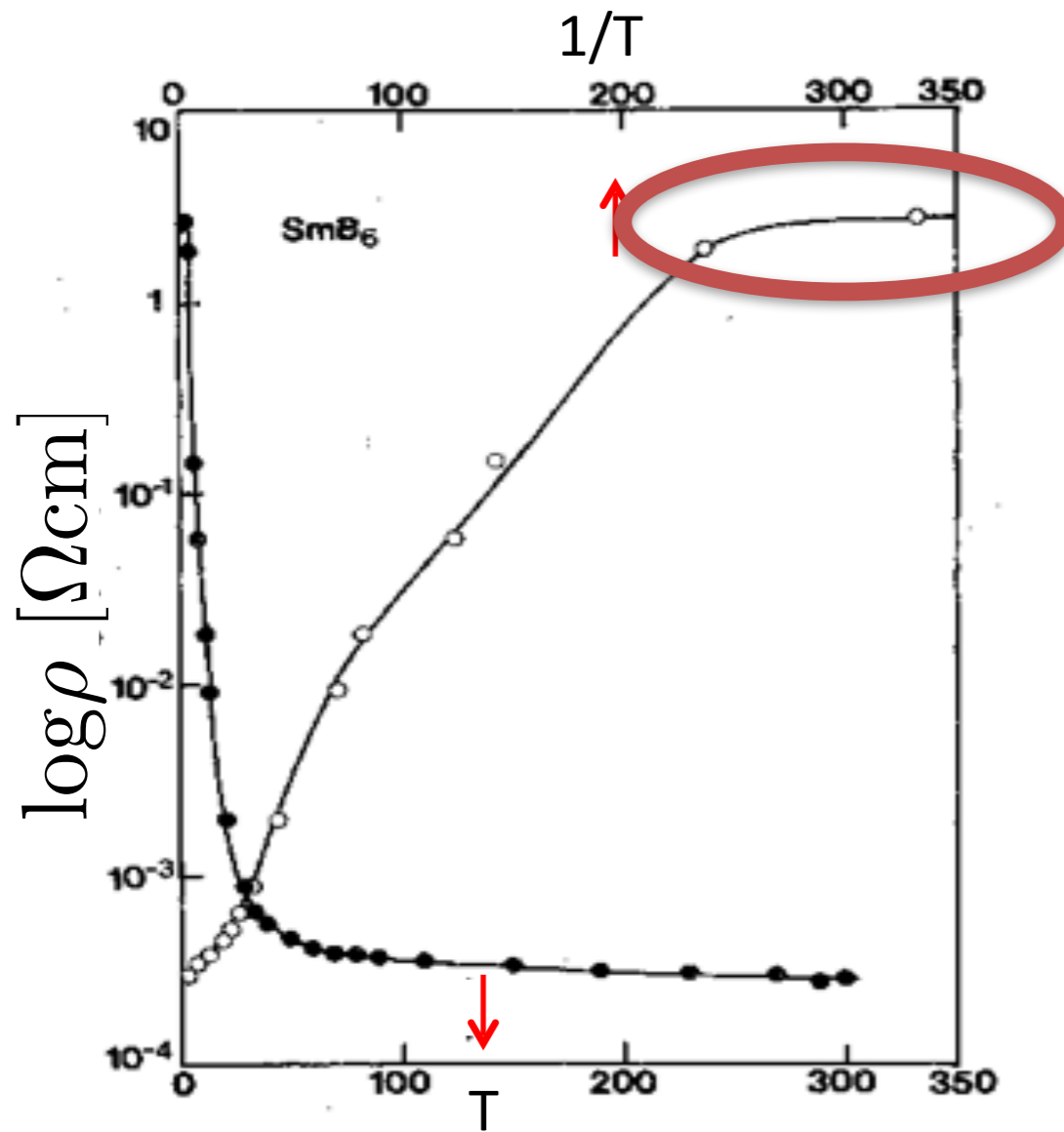
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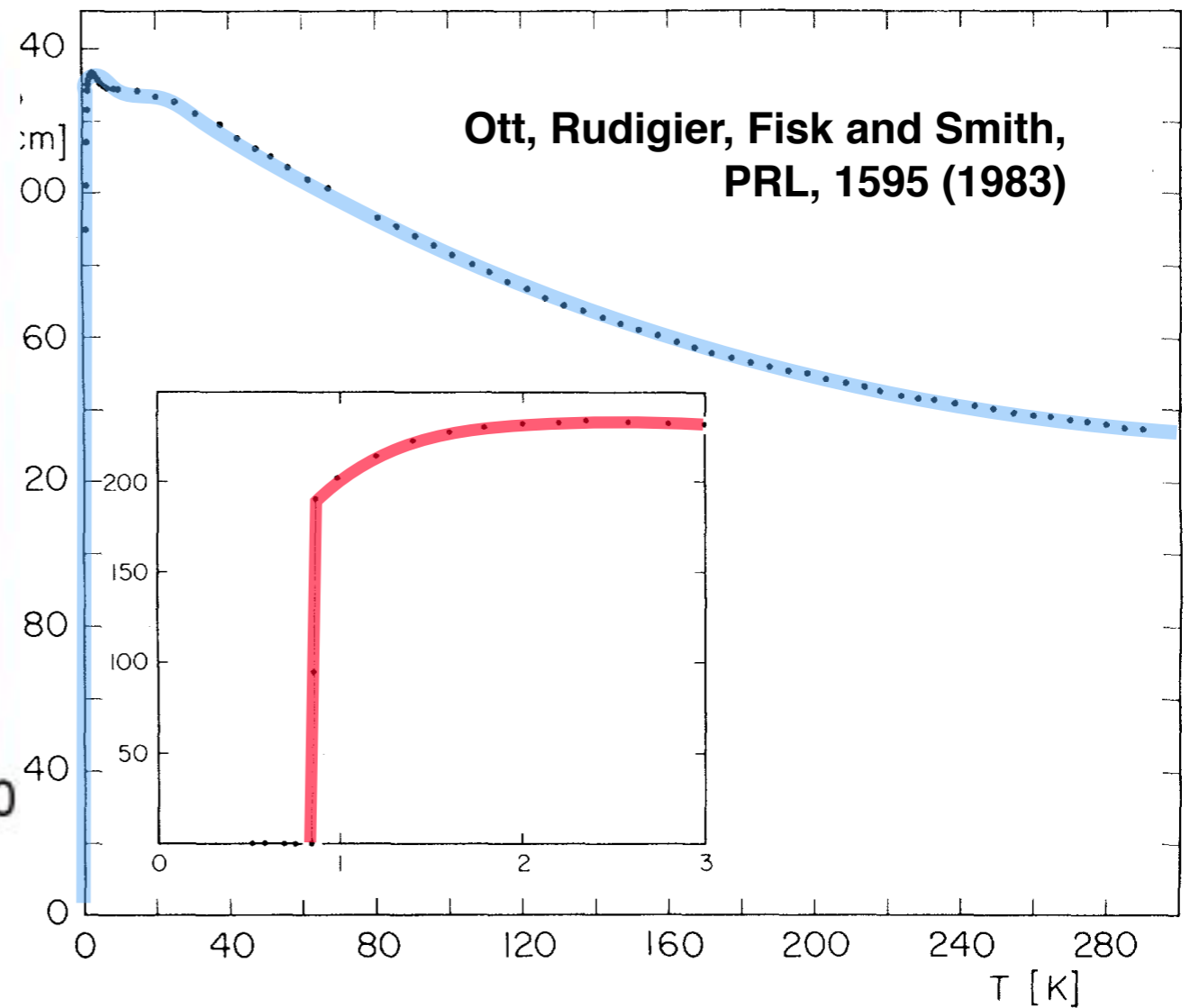
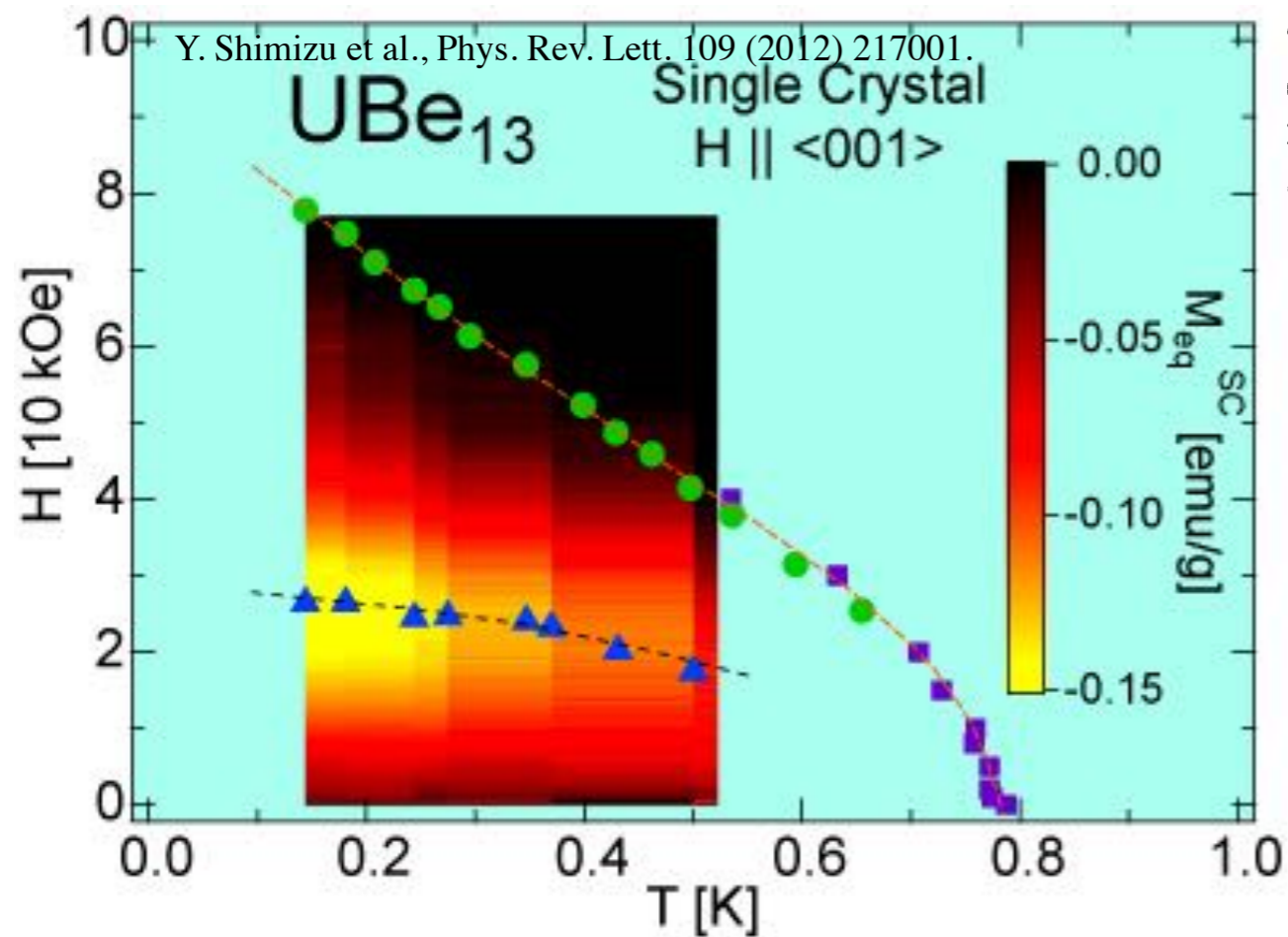
Ott, Rudigier, Fisk and Smith,
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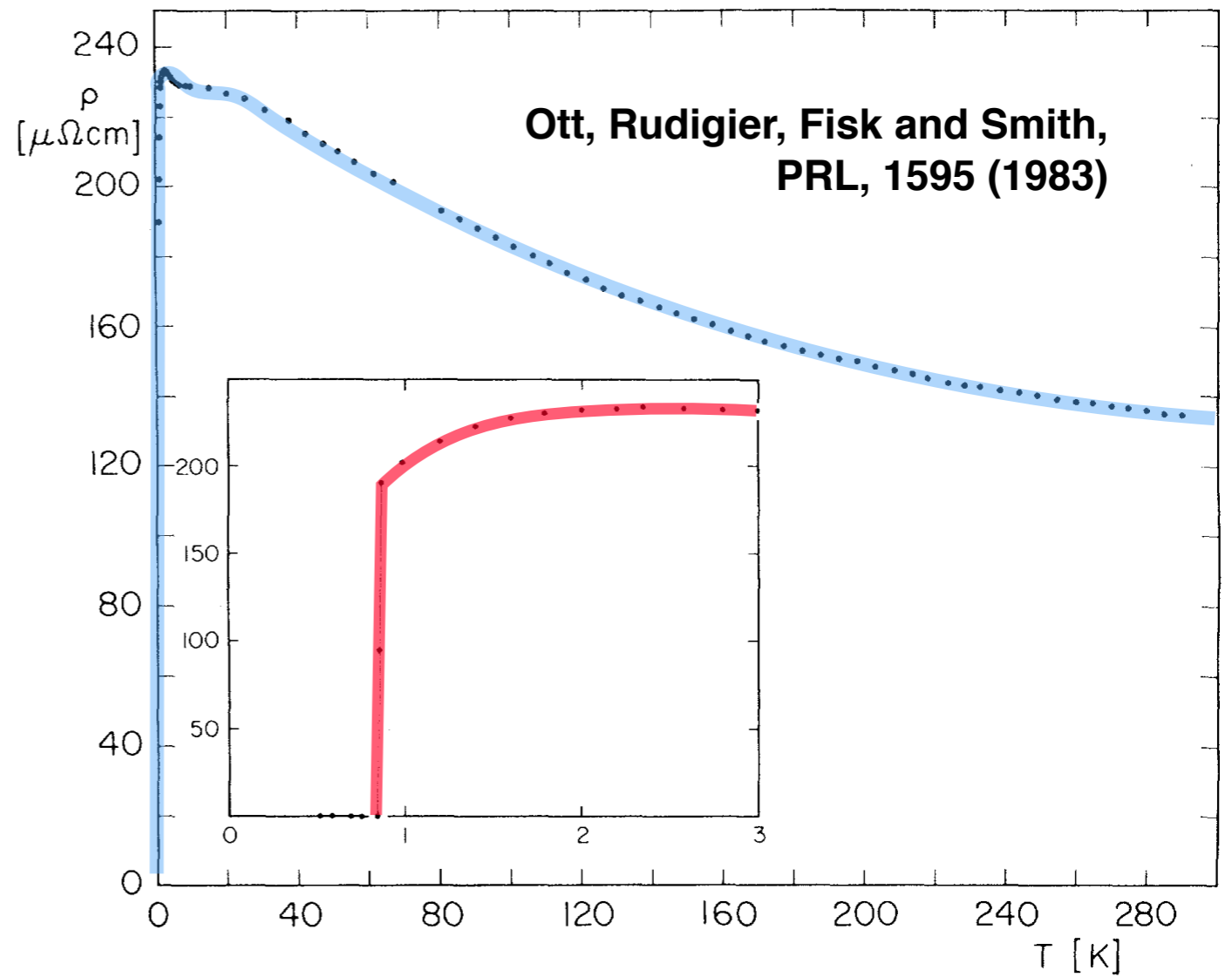
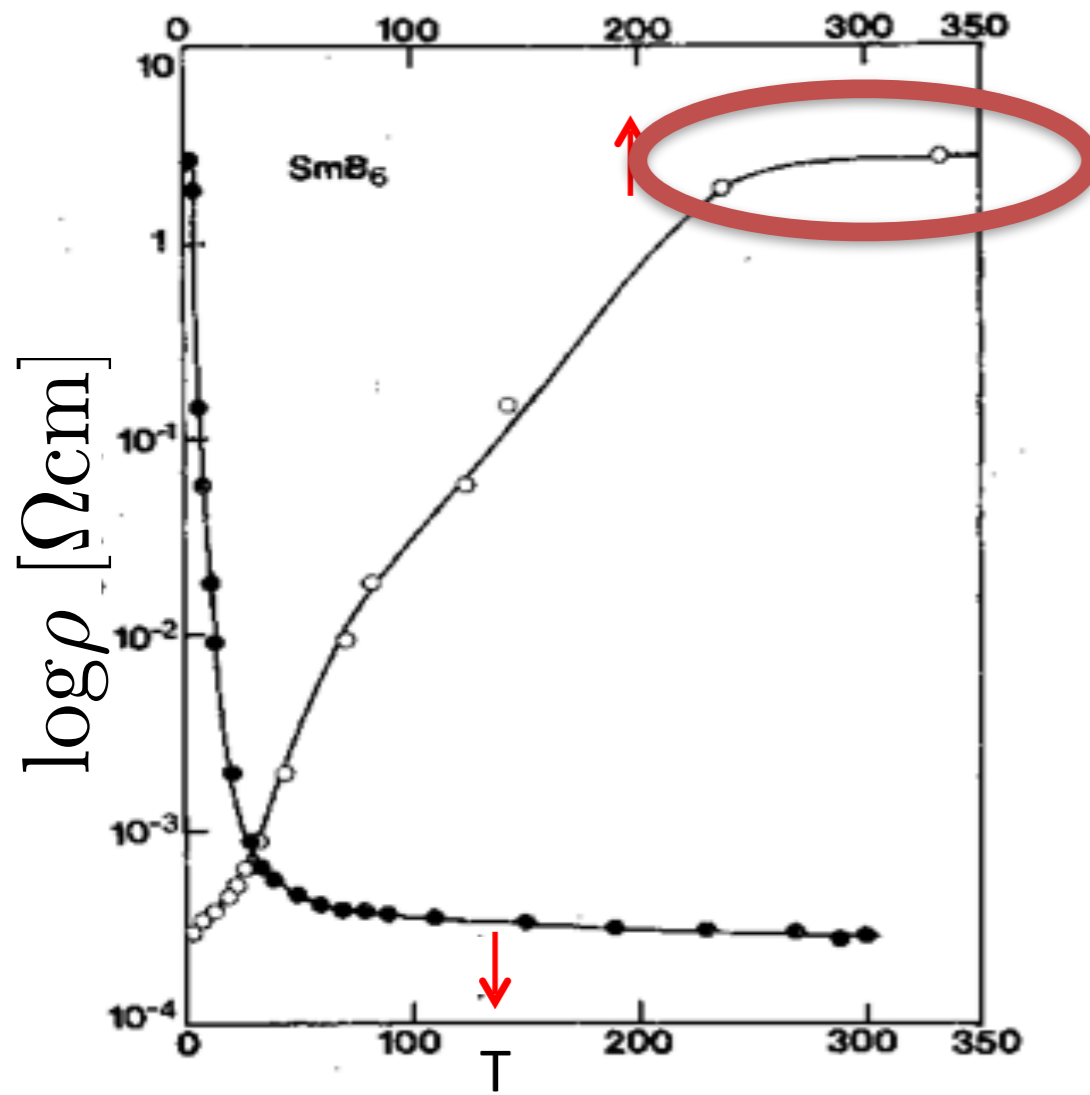
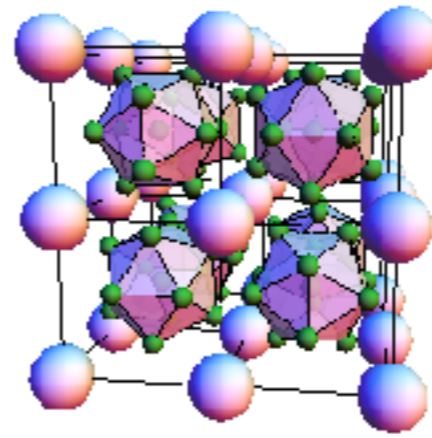
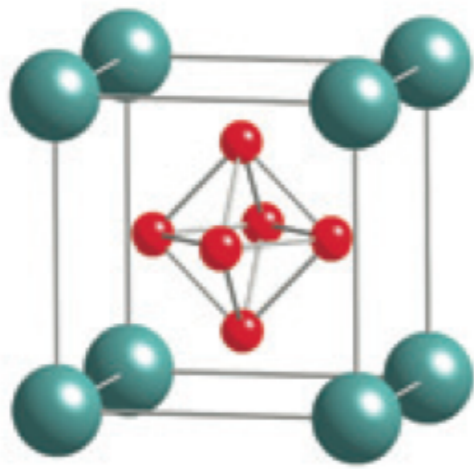
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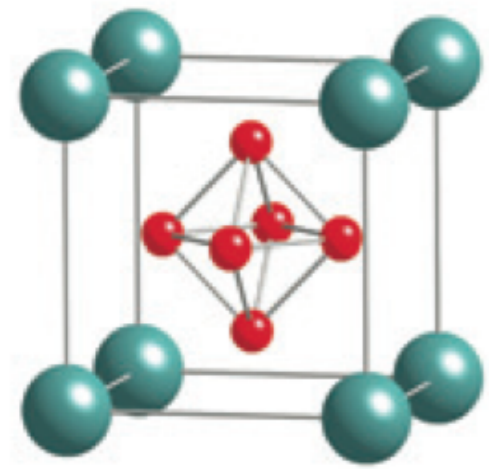
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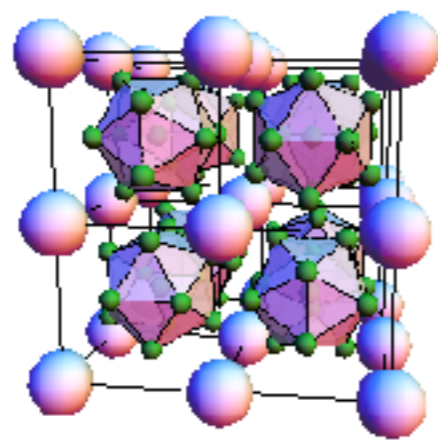




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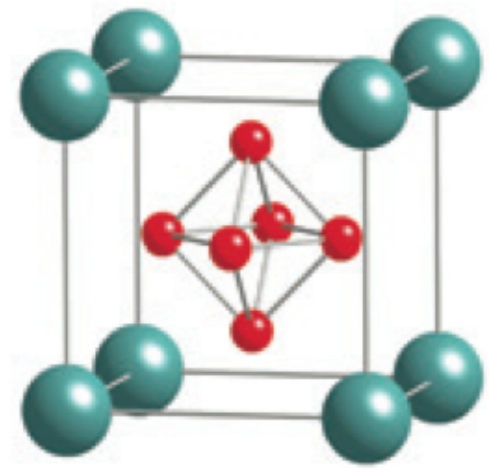


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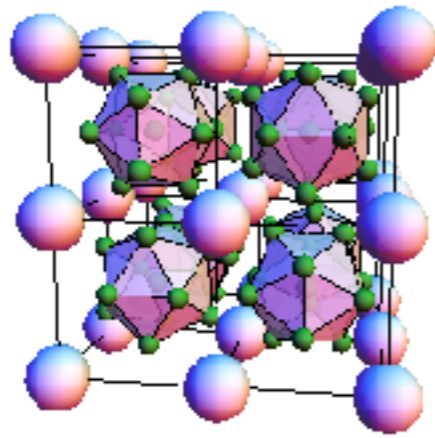


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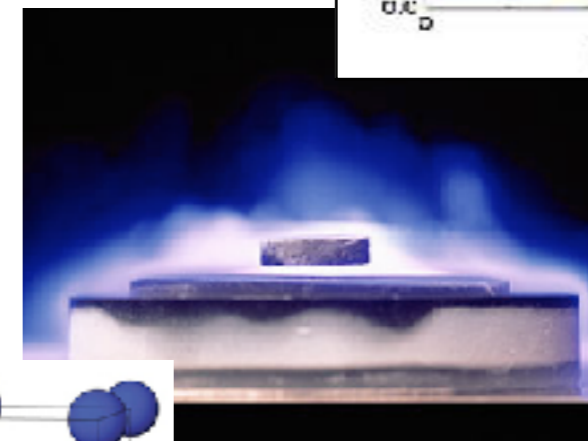
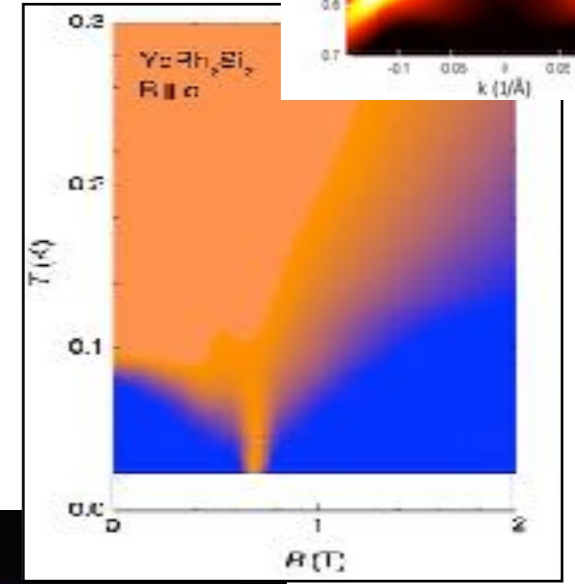
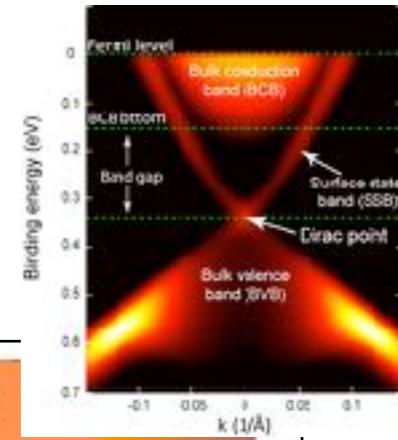
Discovery often awaits new concepts, new consensus



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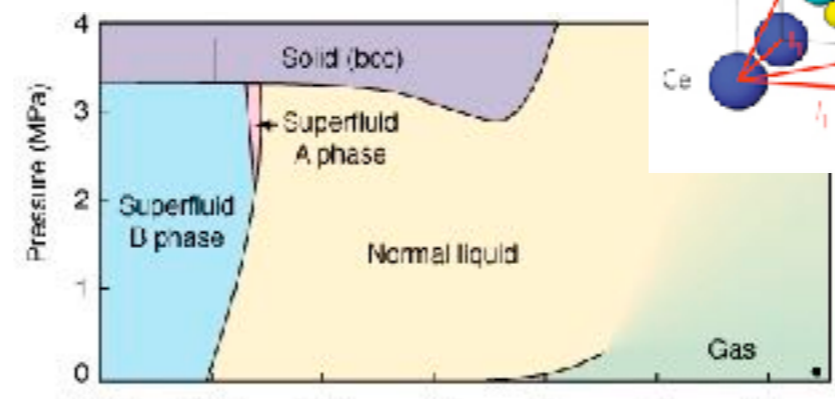
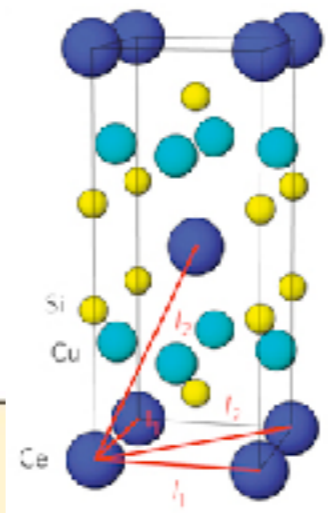
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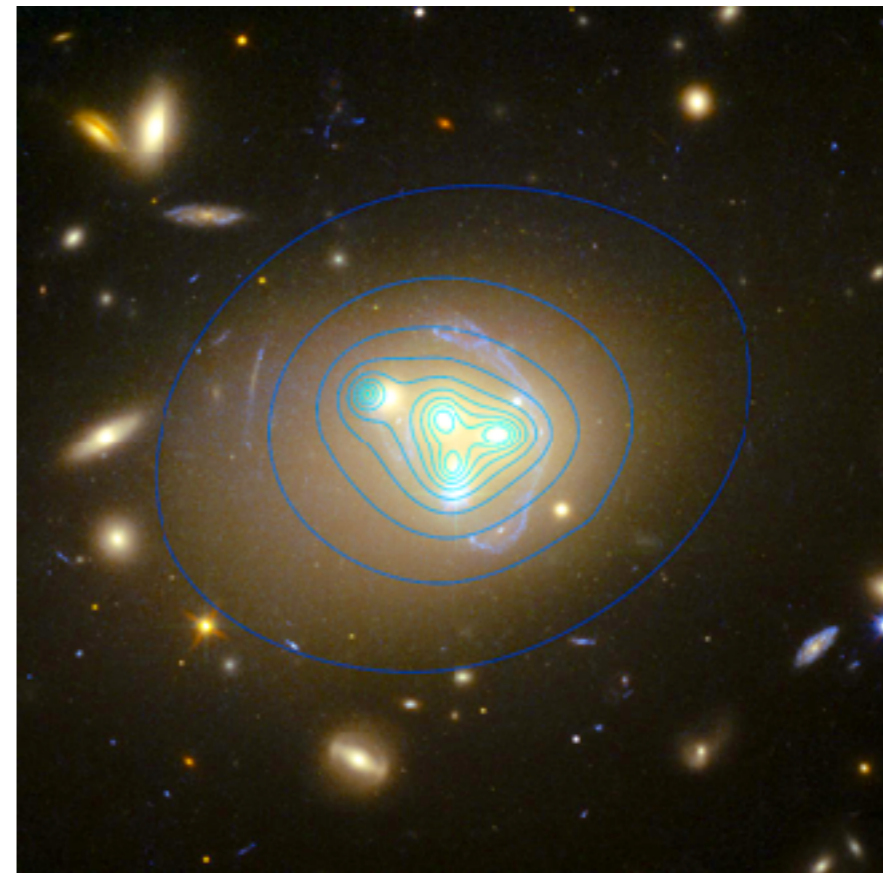
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- ~5 yrs before **Superfluid He-3**
- 10 yrs before **Heavy Fermion SC**
- 20 yrs before **High Temperature SC**
- 30 yrs before **Quantum criticality**
- 40 yrs before **Topological insulators, Fe based SC**



Dark Matter Challenges of the Solid State.

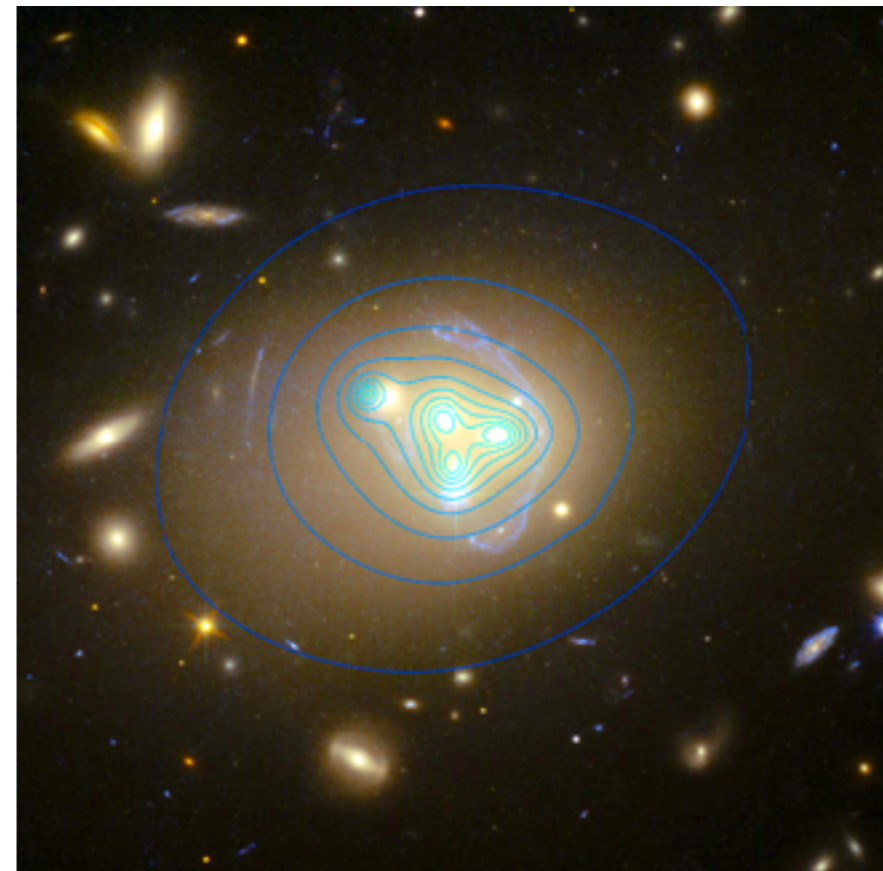


Abell Cluster 3827 (ESO/R Massey)

Dark Matter Challenges of the Solid State.

- Linear resistivity in strange metals
- Strange Insulator SmB_6

- Pairing Mechanism of Cu/Fe/HFSC
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- Nature of the pseudogap
- Sign Problem in QMC
- Topology in SCES
- Uemura Scaling $\rho_s \sim T_c$
(overdoped) cuprates
- Ground-state of Spin Liquids

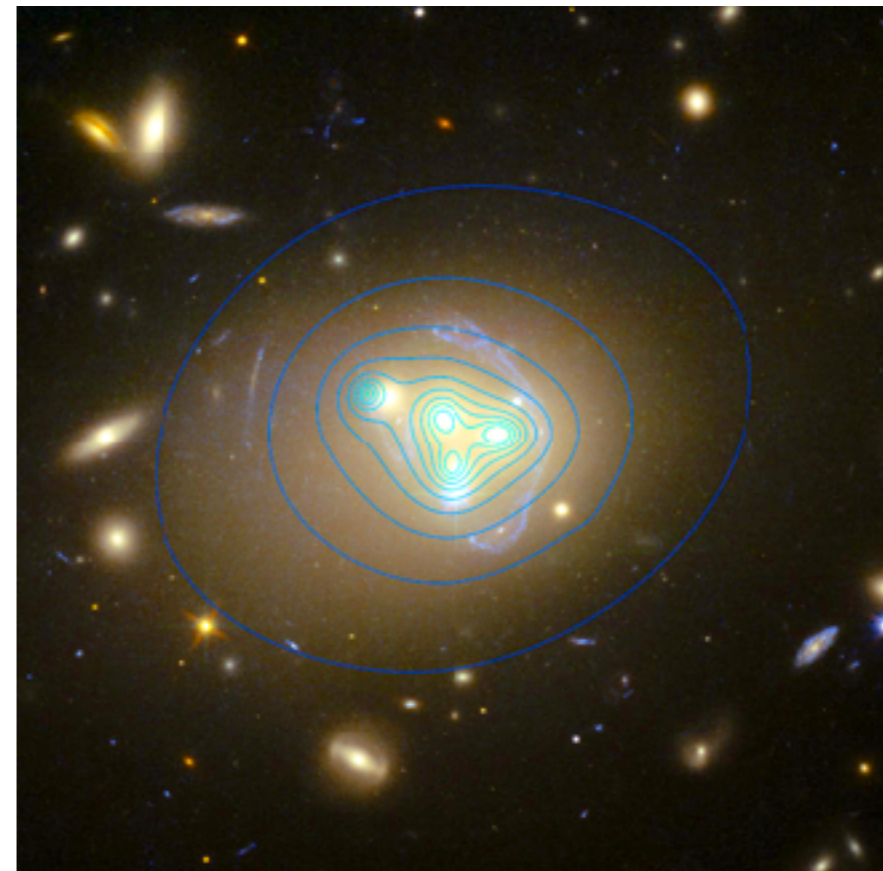


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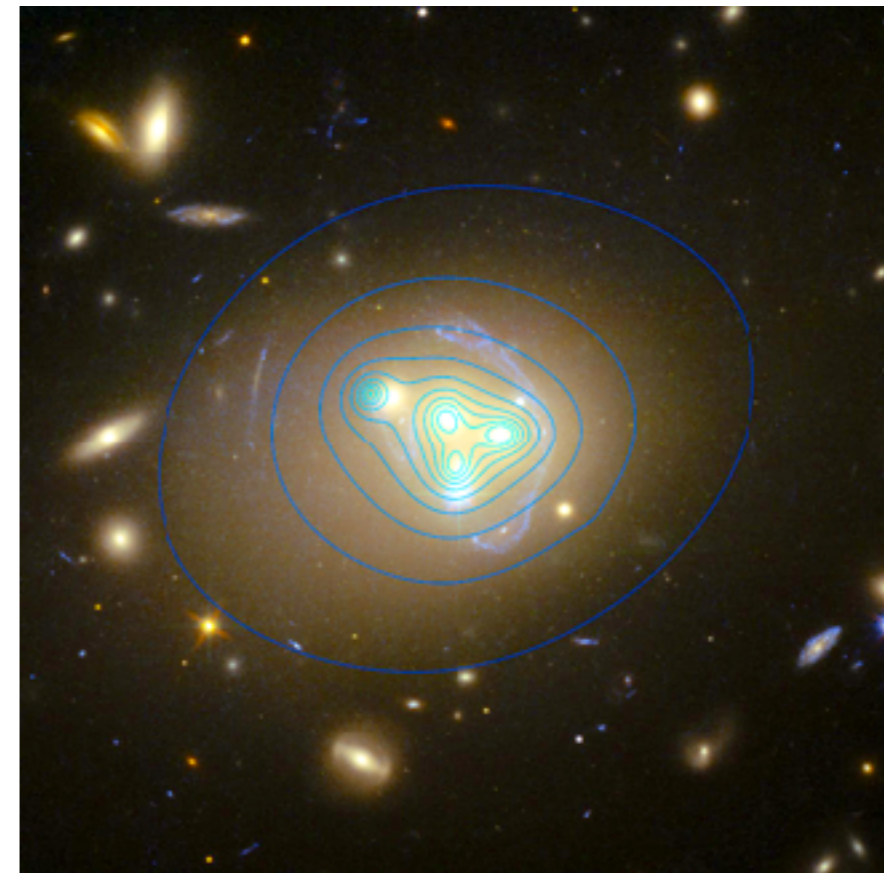
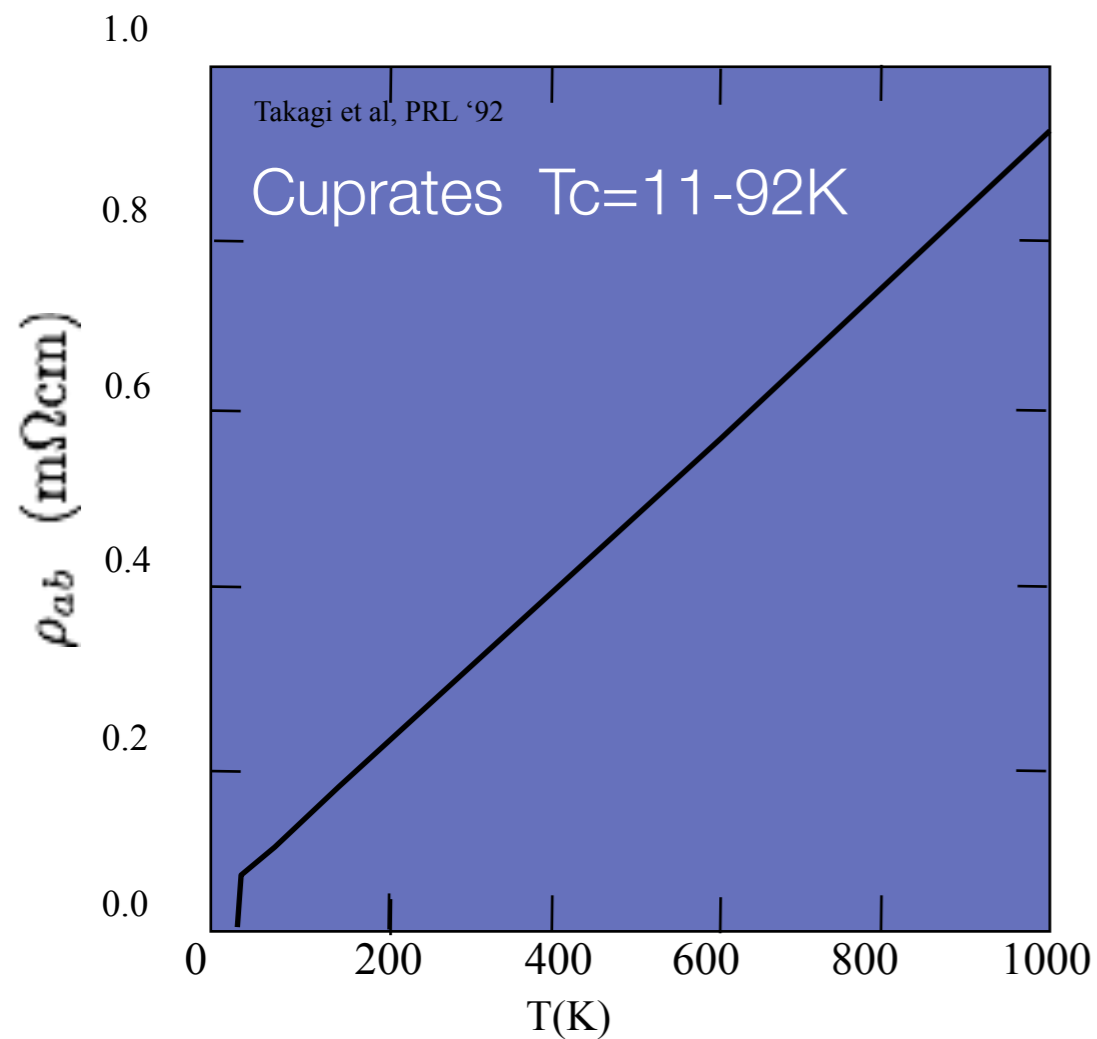


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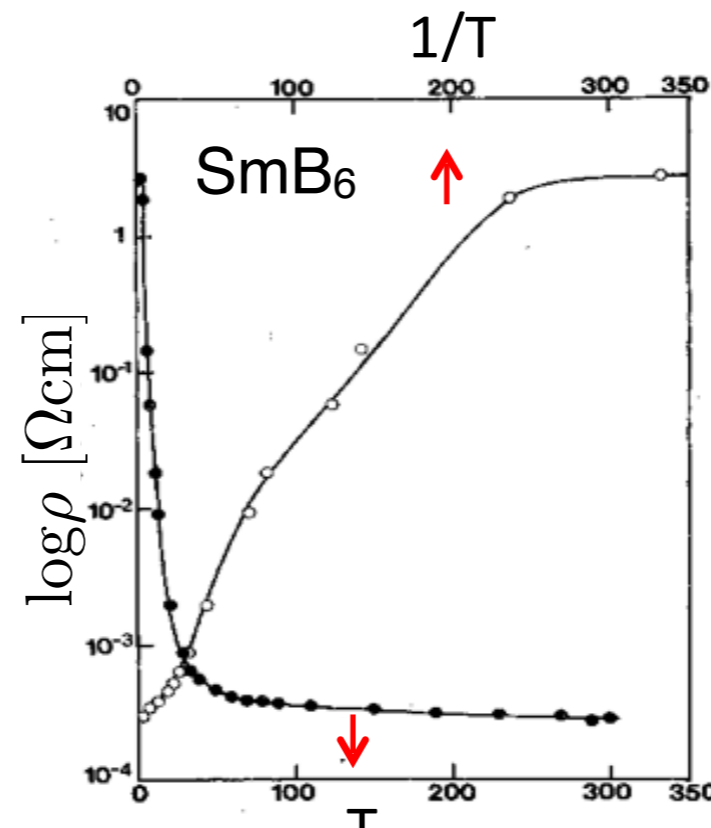
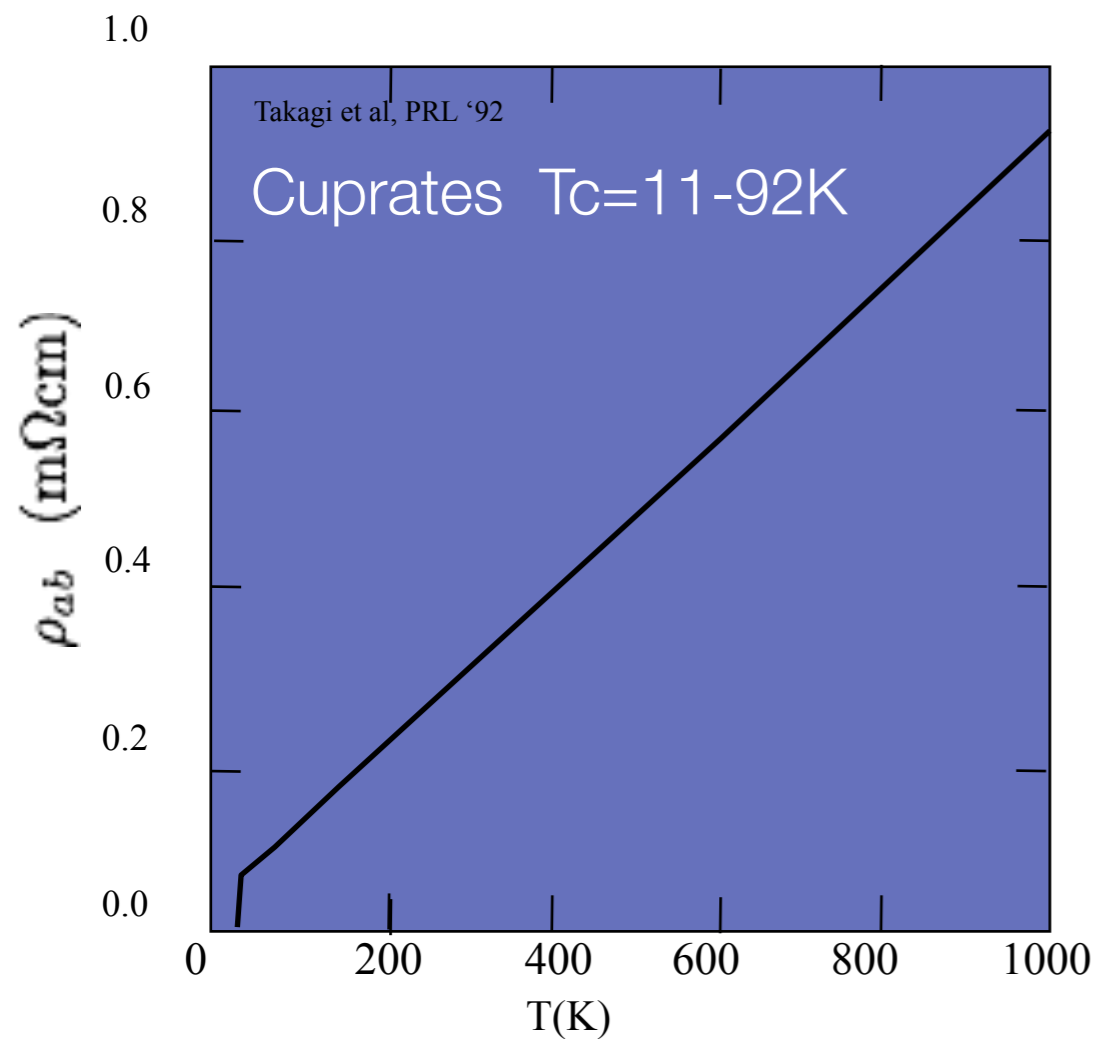


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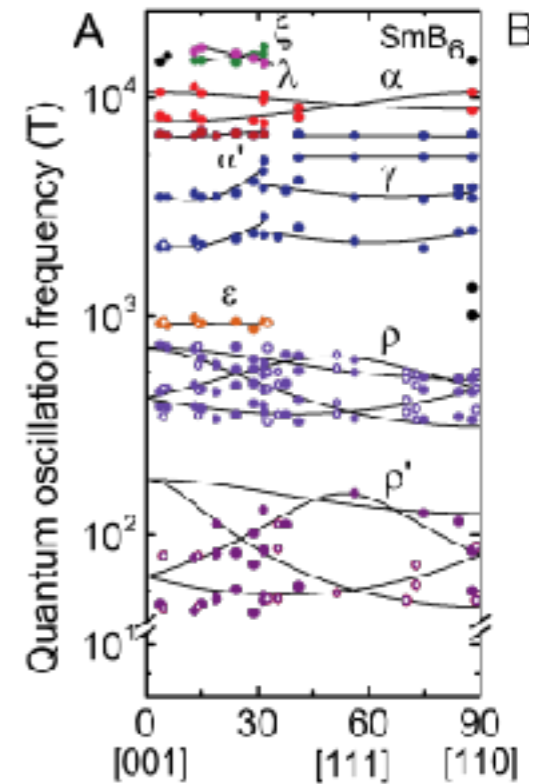
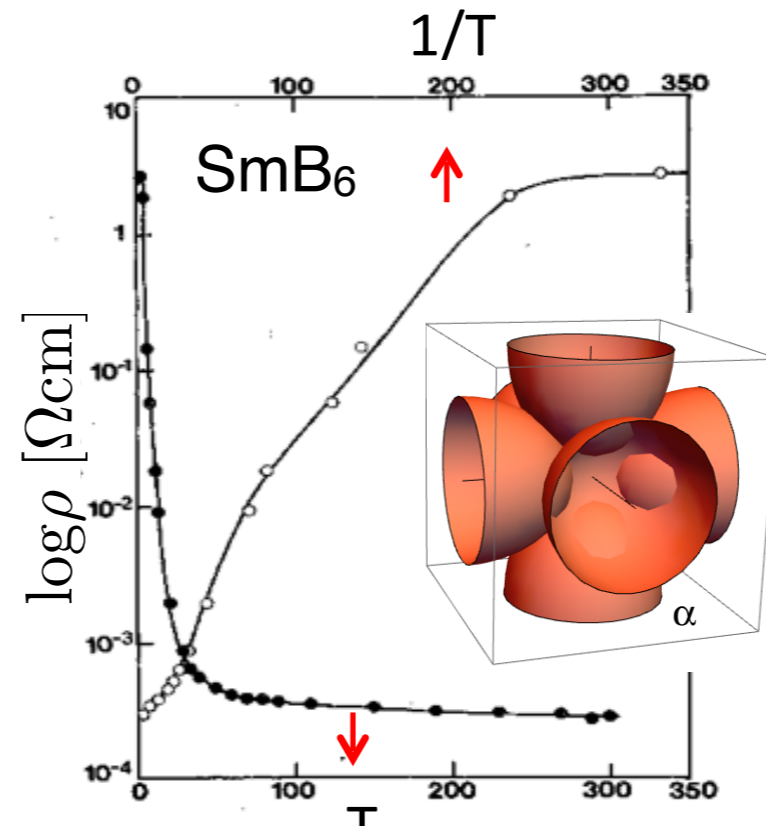
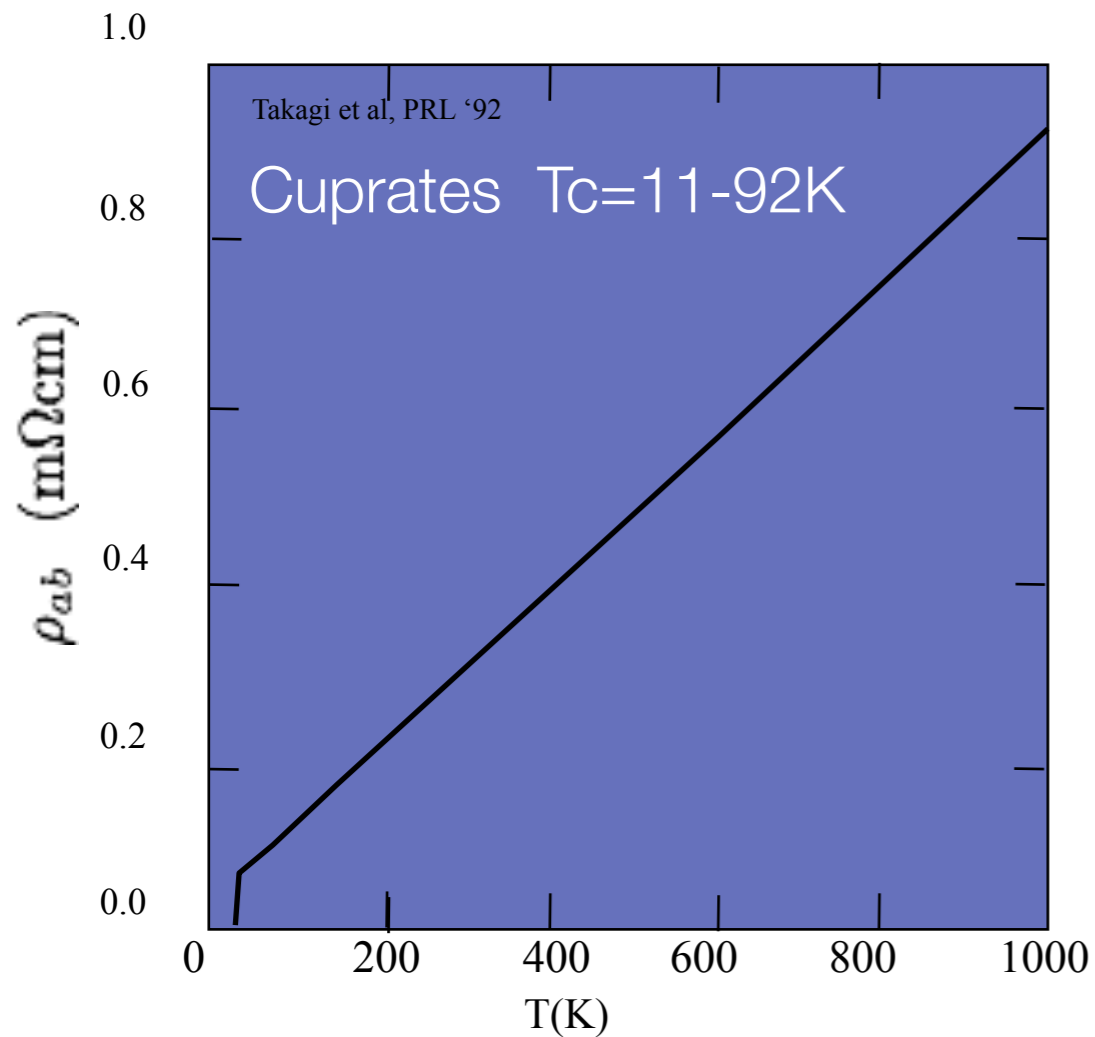


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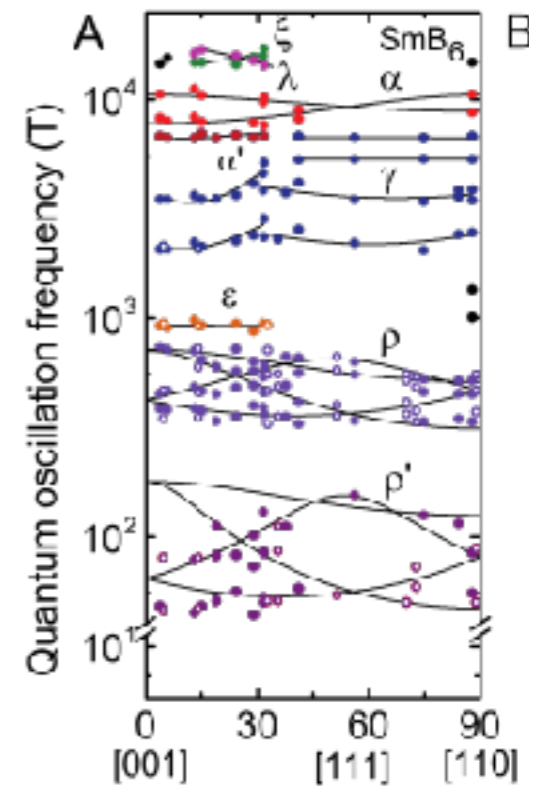
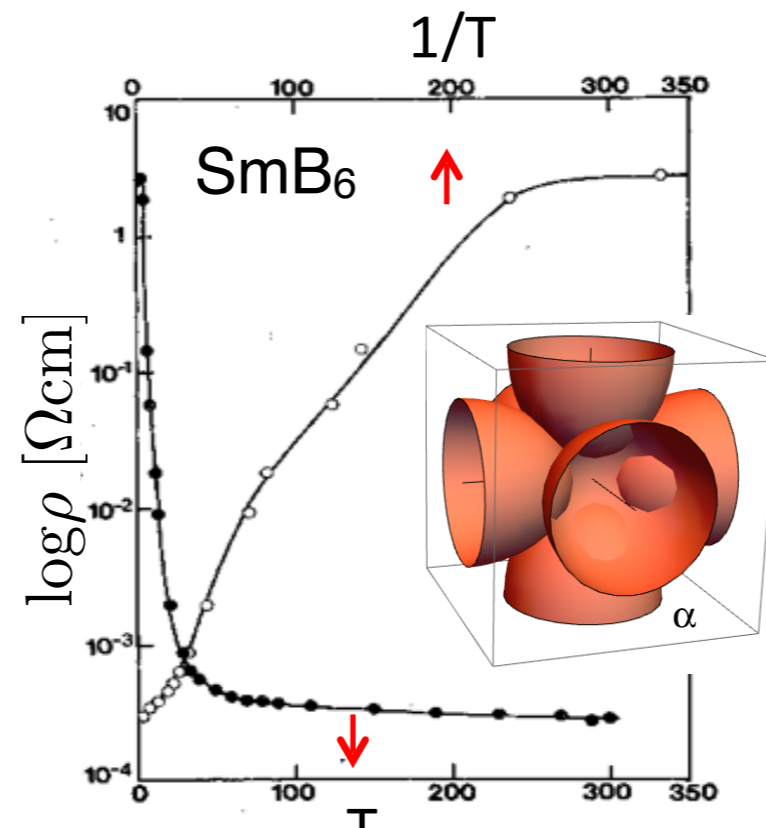
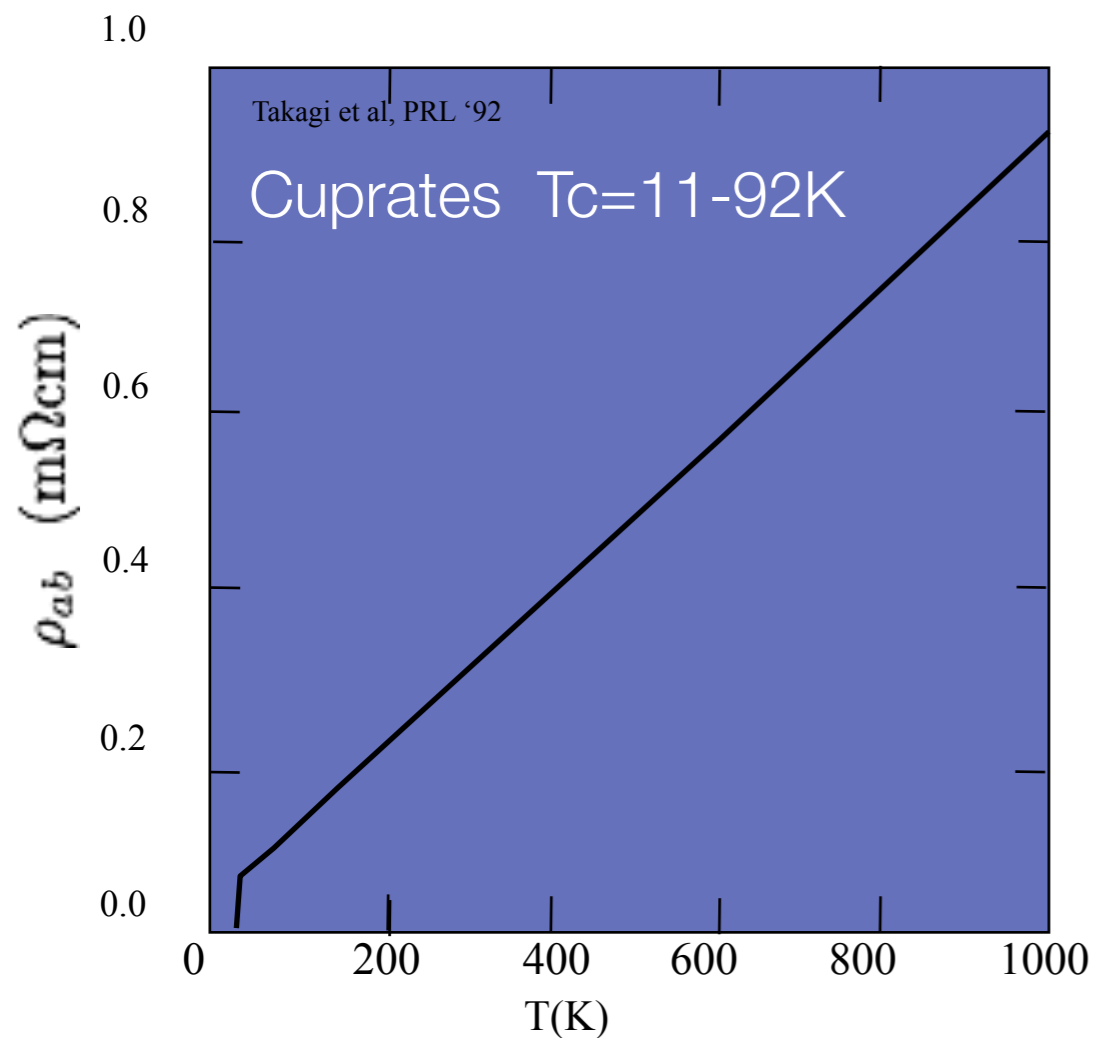
Tan *et al.* Science 349, 287 (2015)

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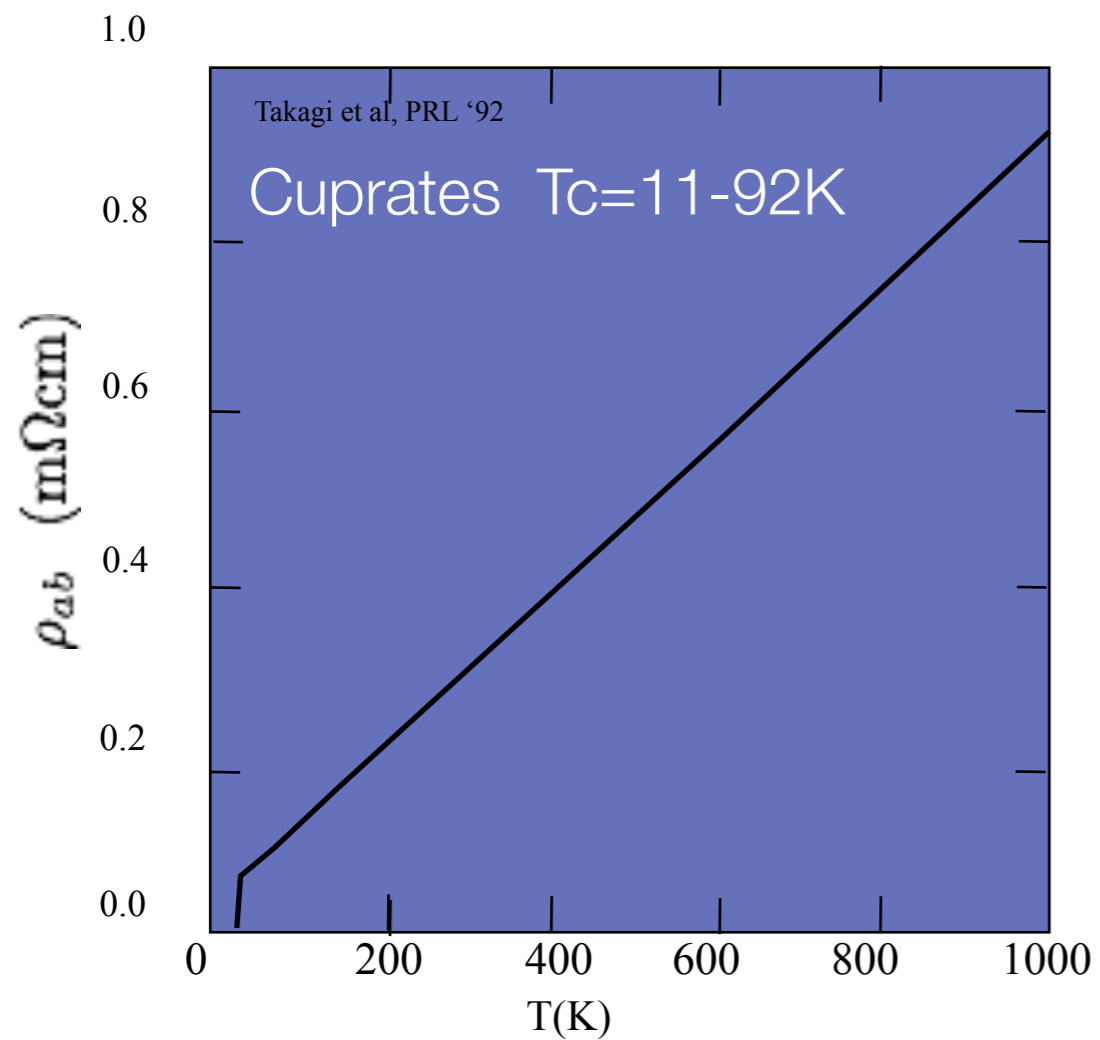
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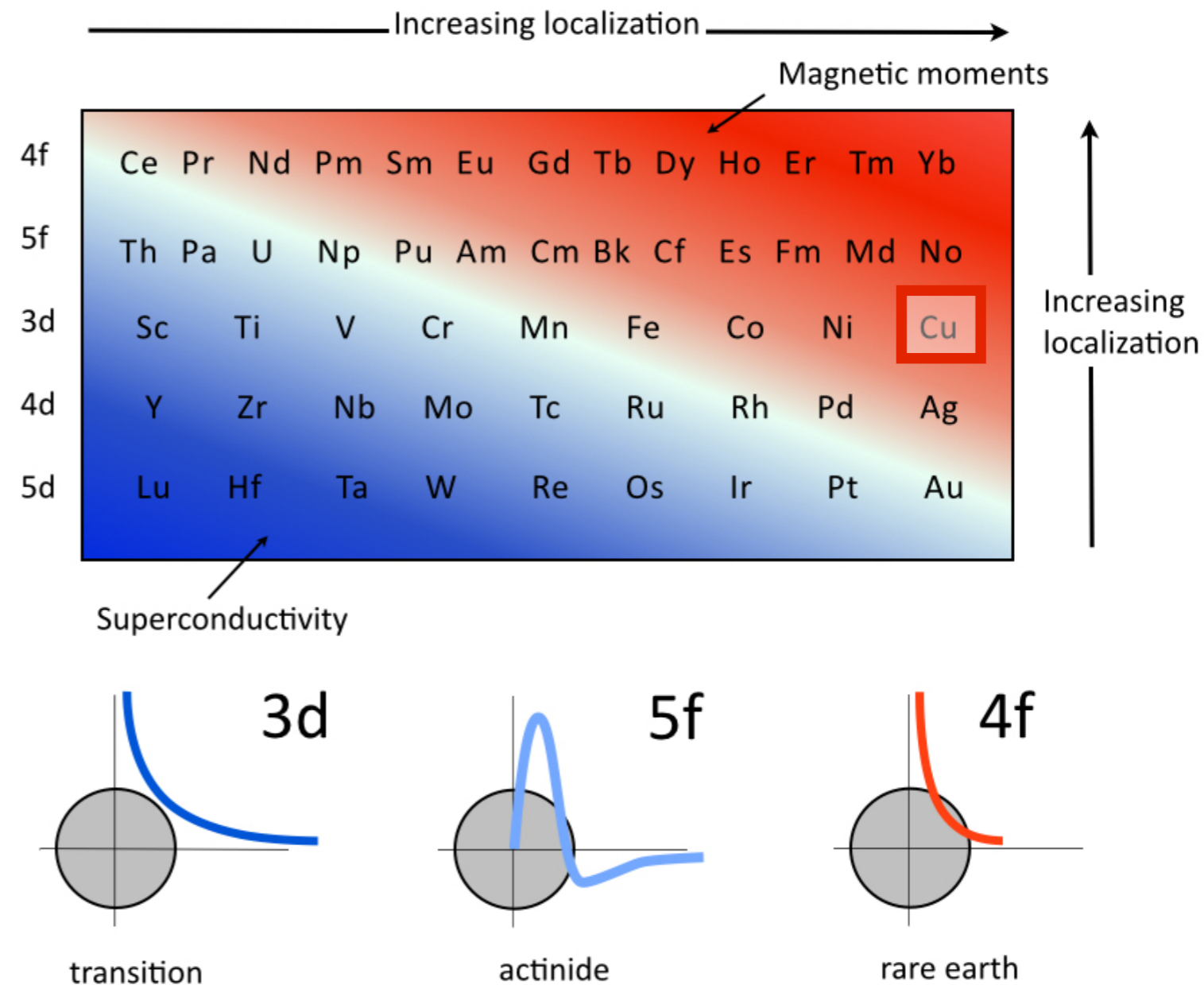
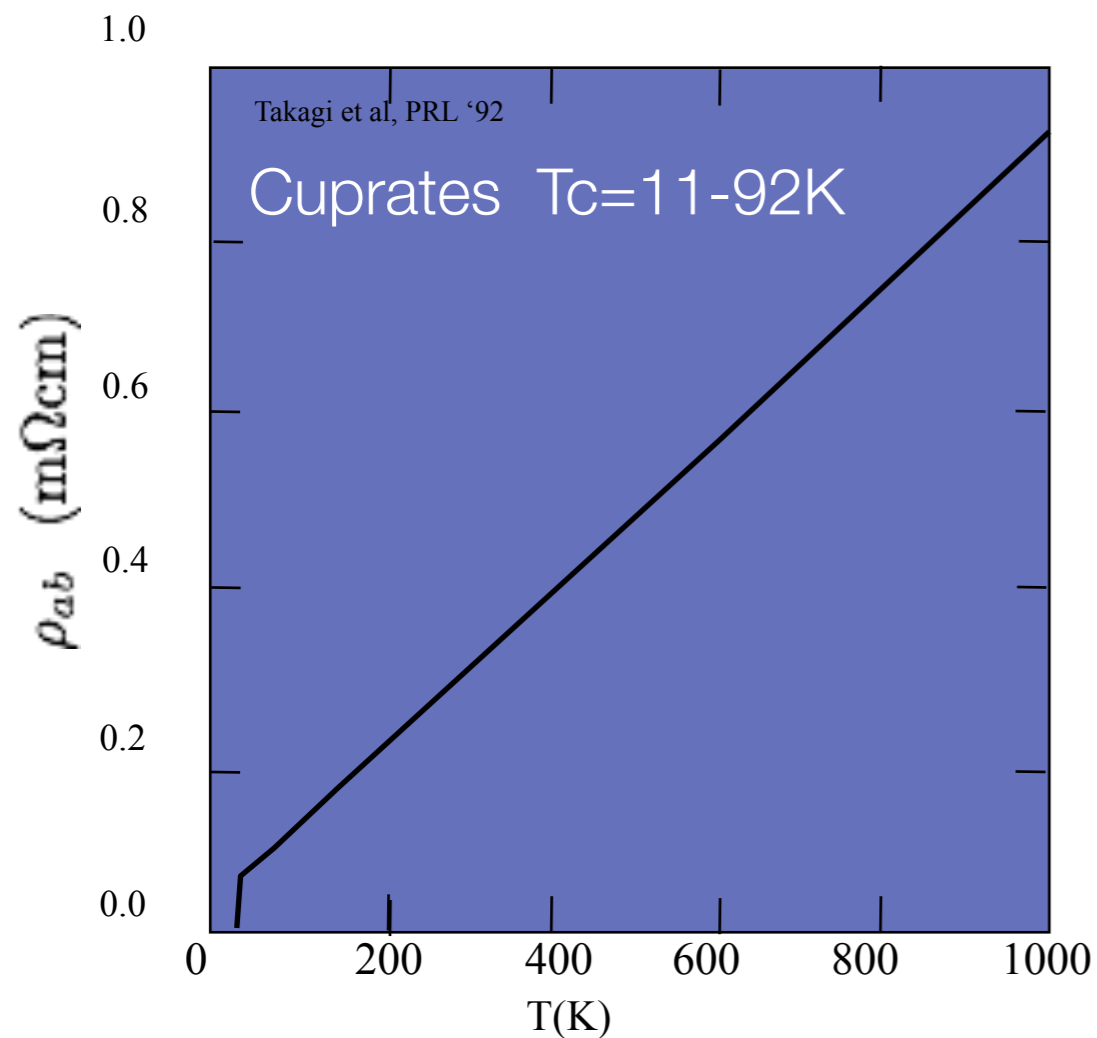
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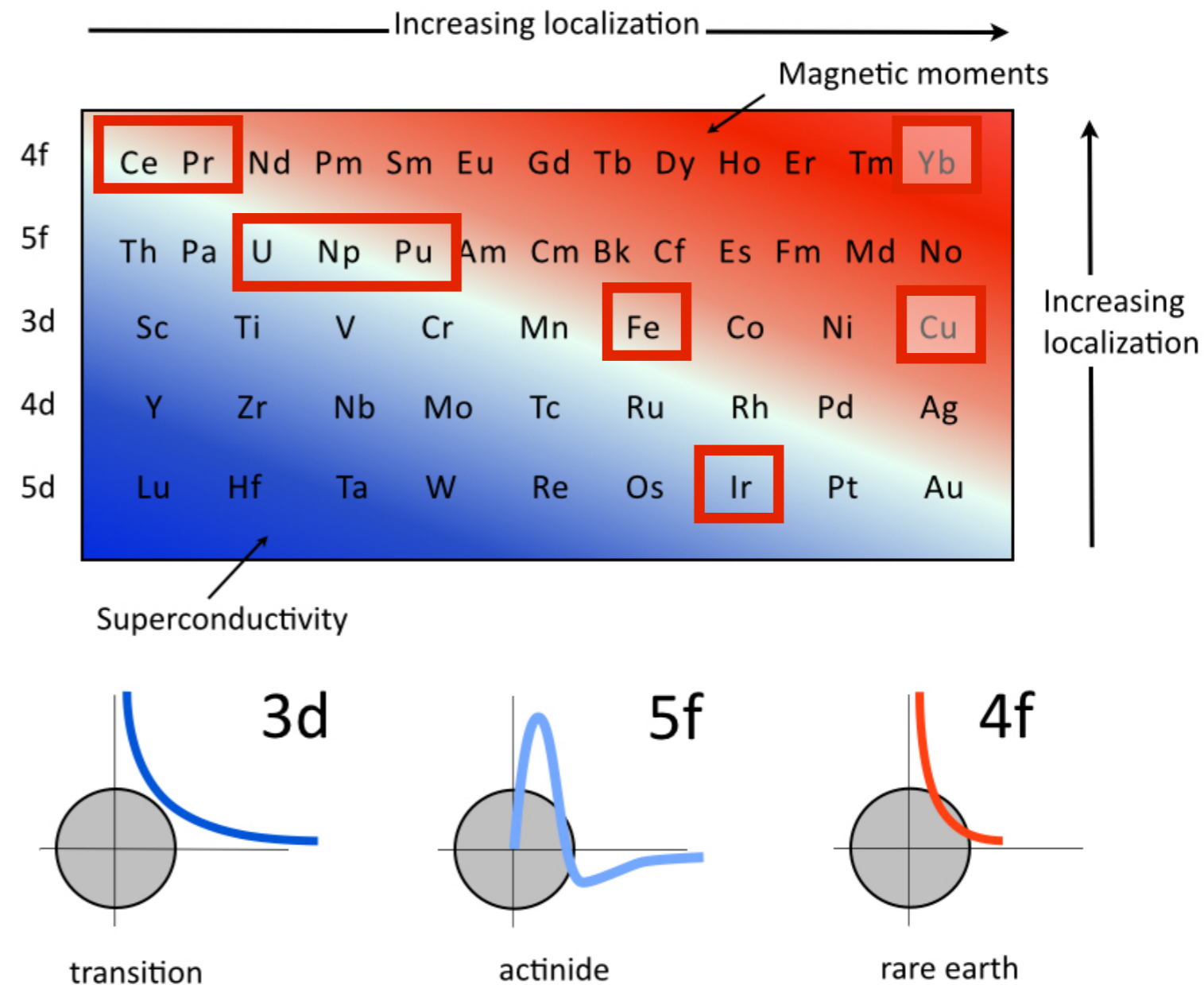
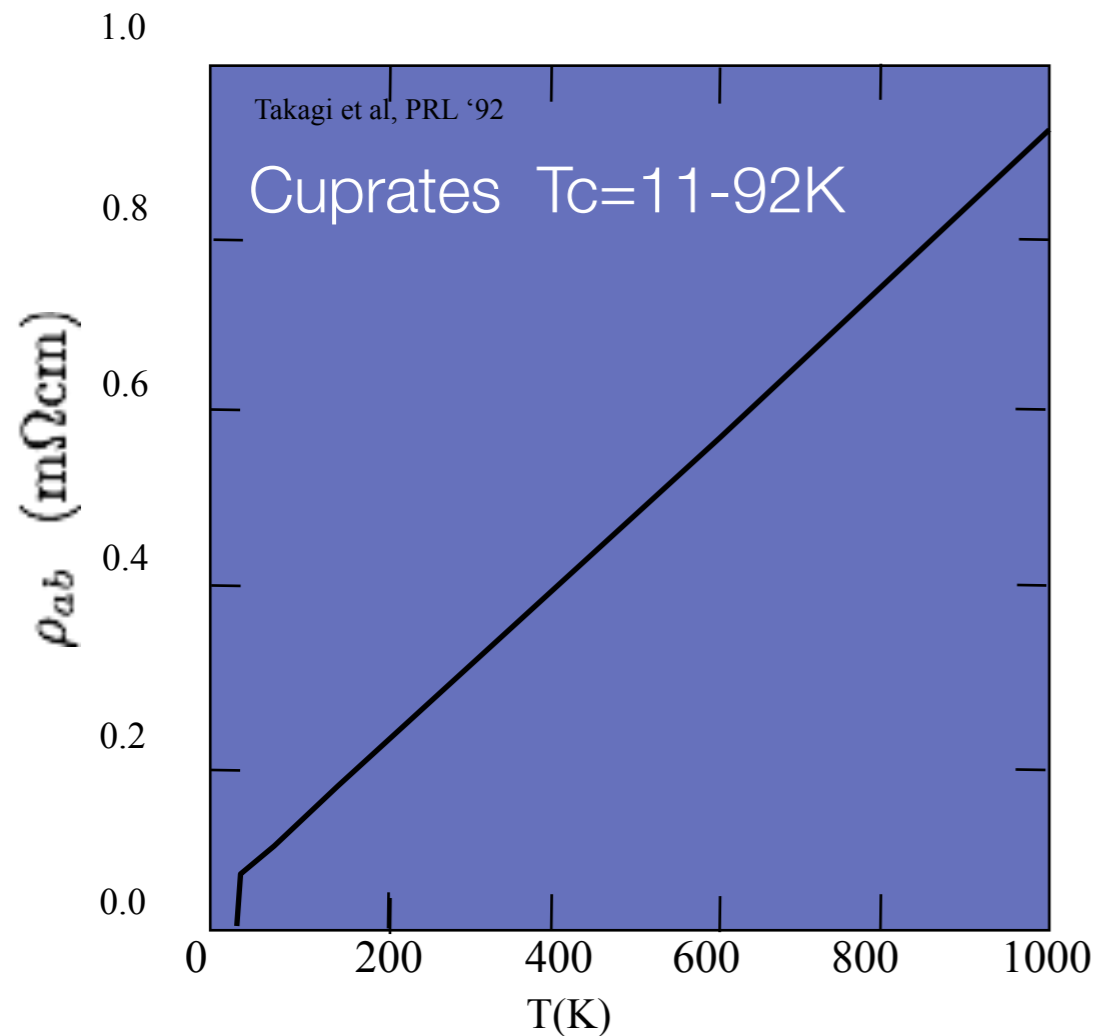
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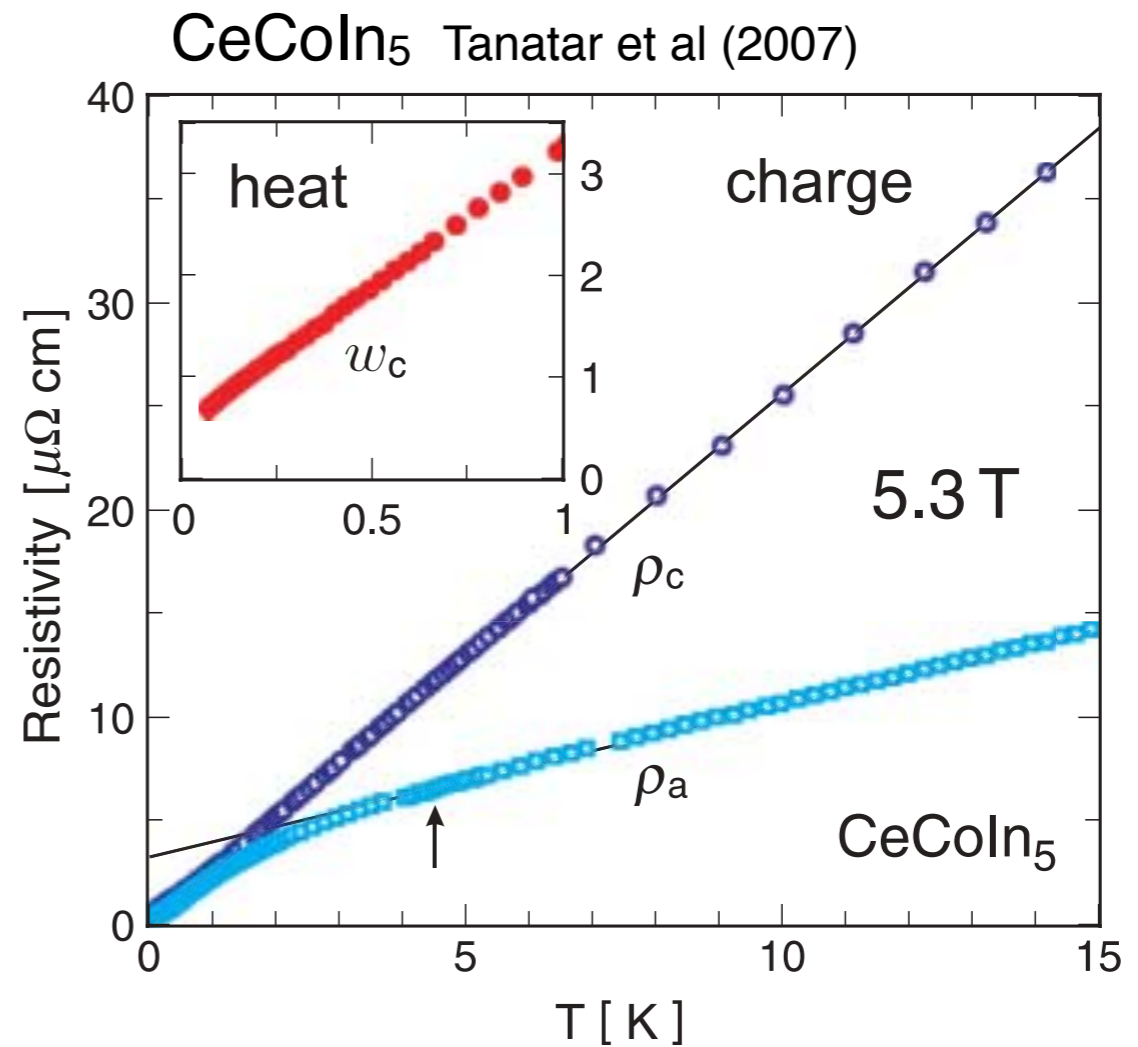
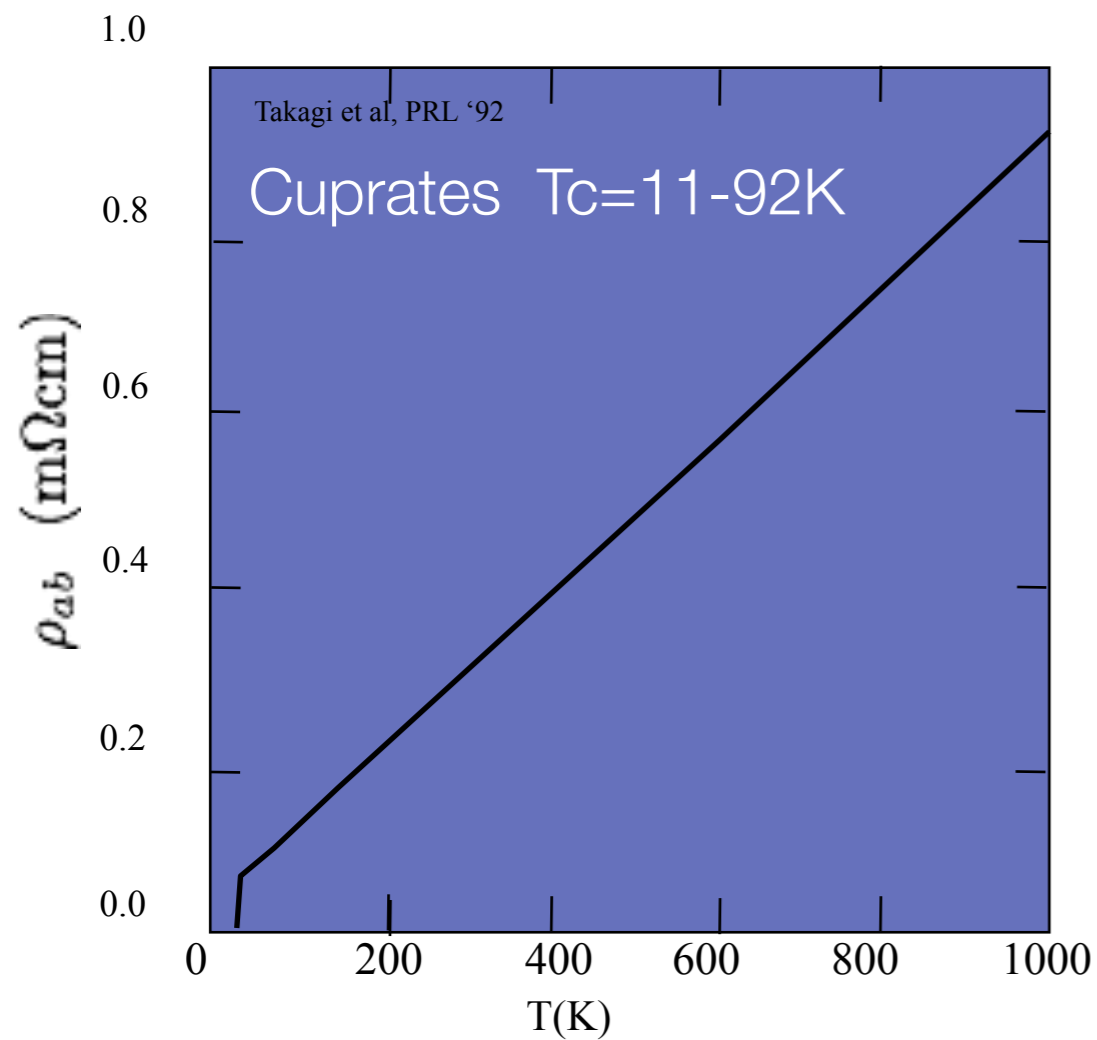
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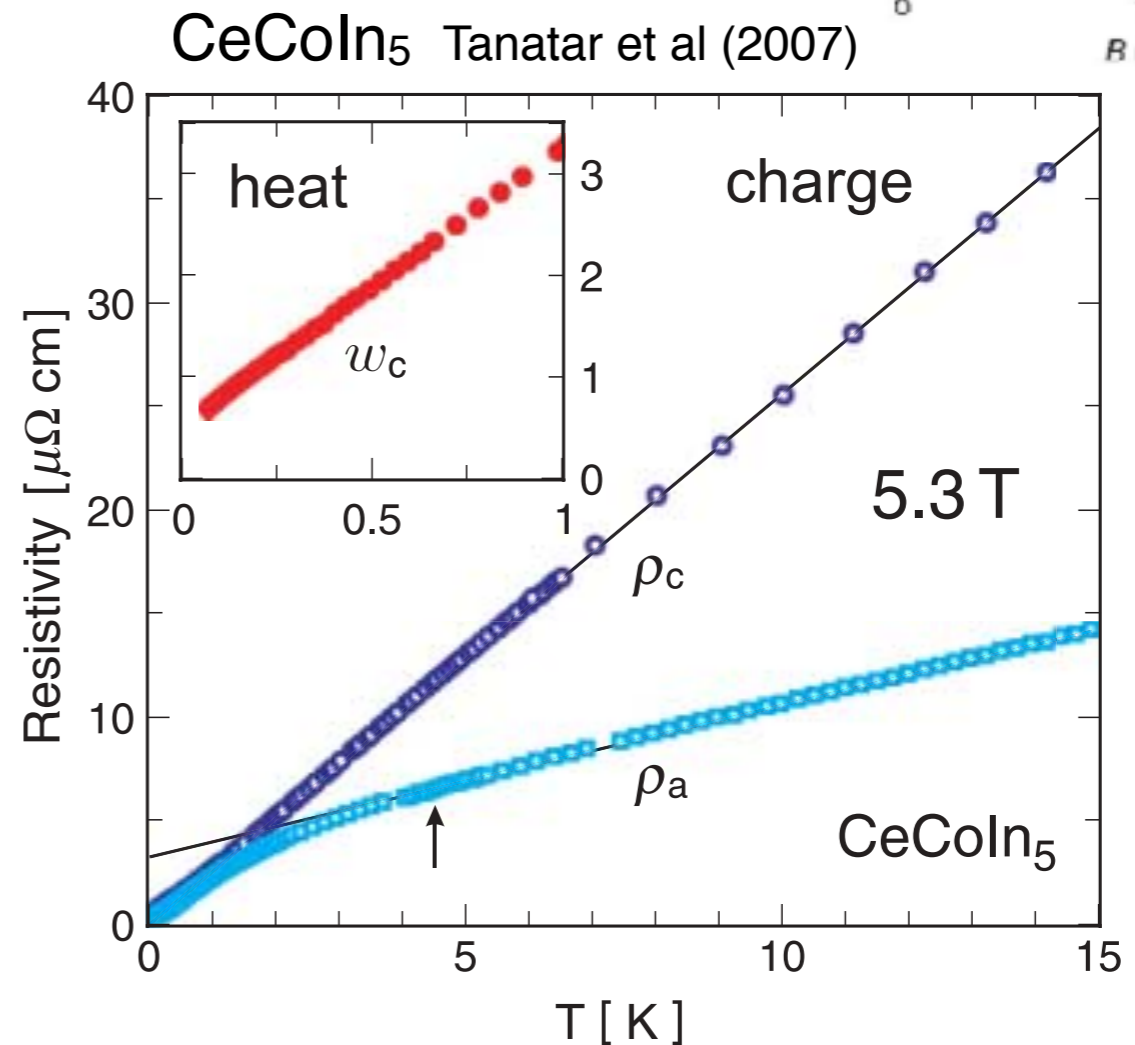
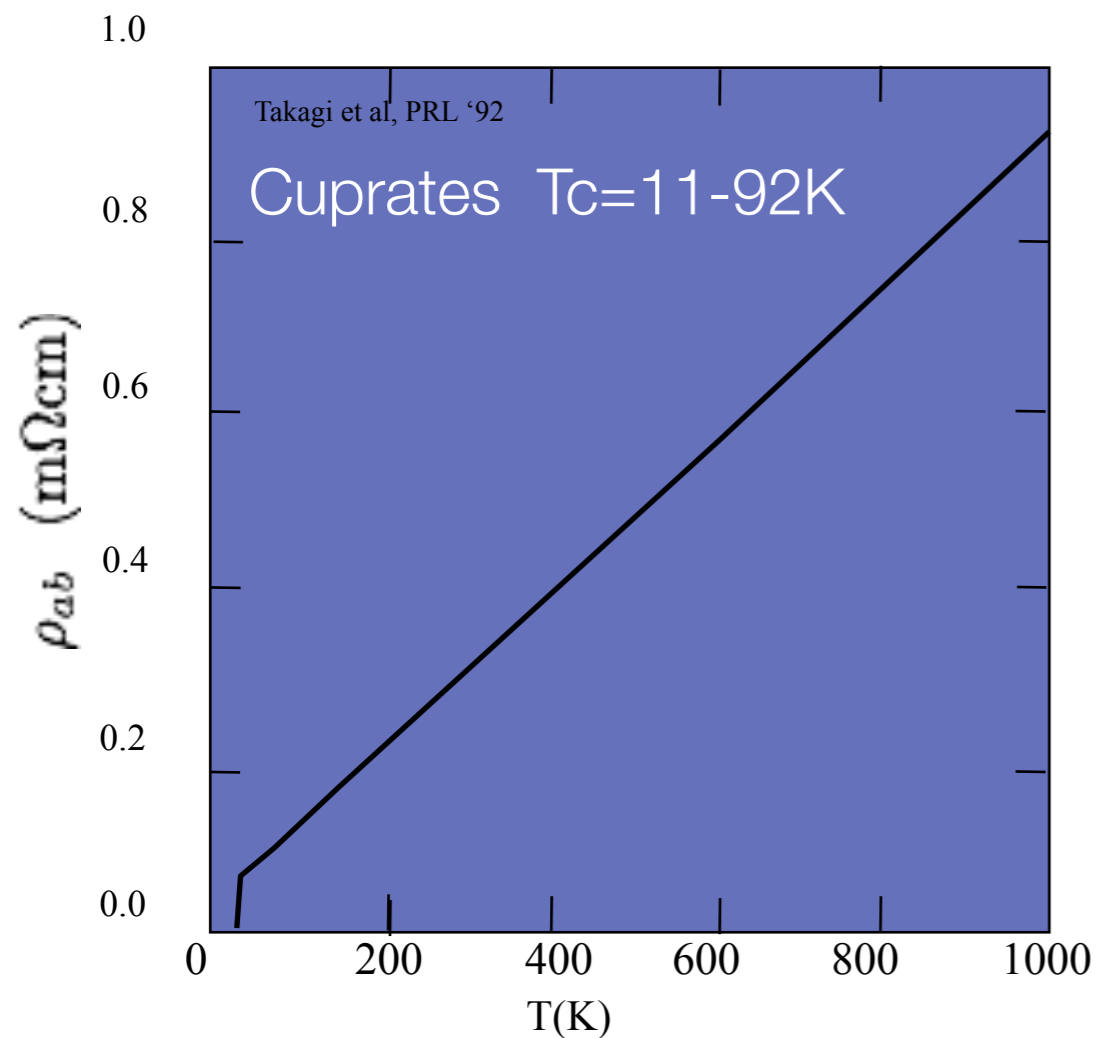
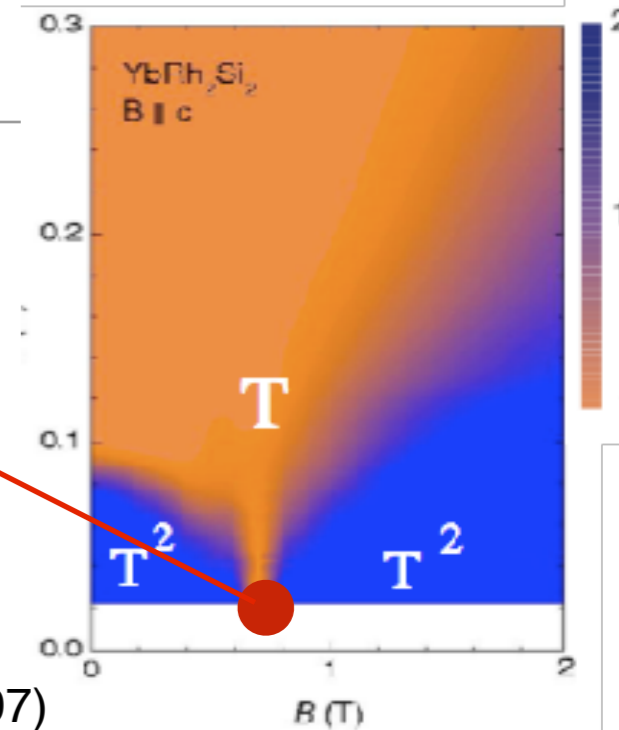
Dark Matter Challenges of the Solid State.

- Linear resistivity in strange metals



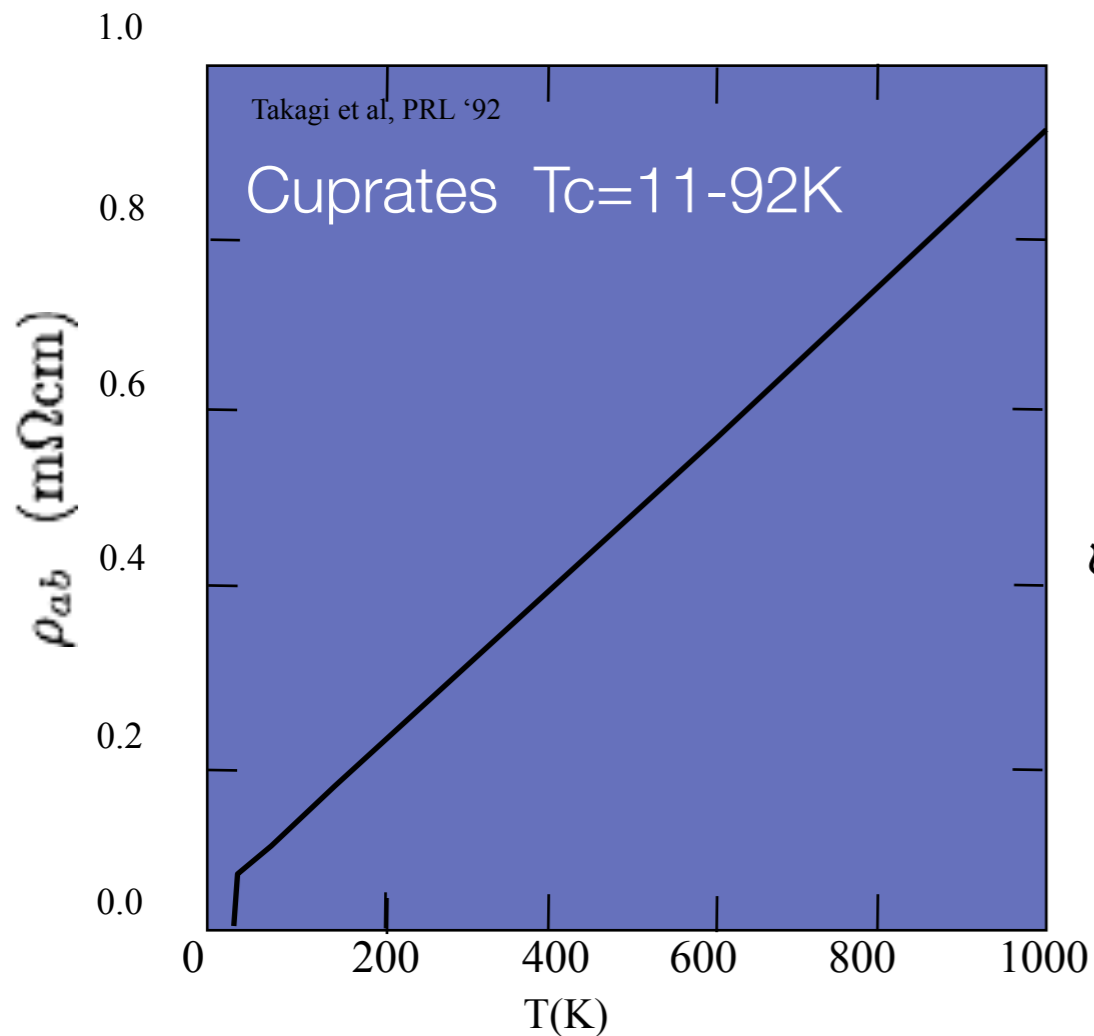
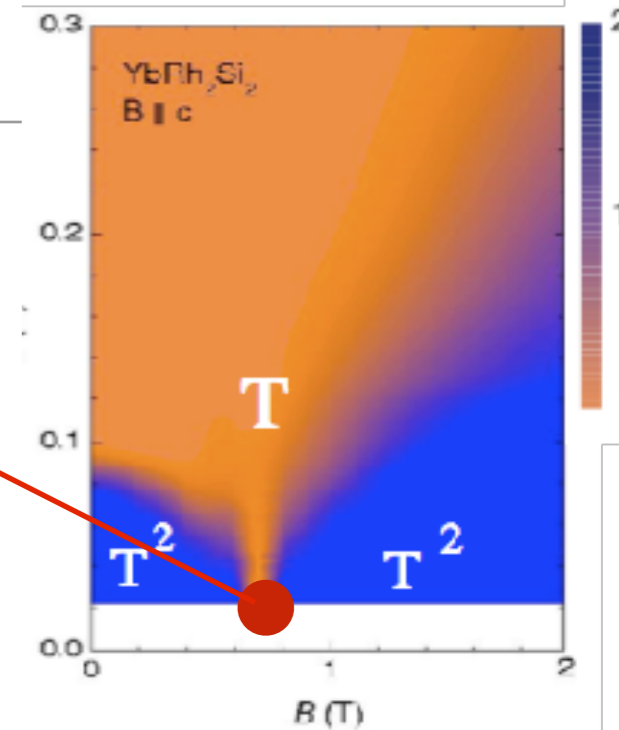
Dark Matter Challenges of the Solid State.

- Linear resistivity in strange metals
- Link with quantum criticality and break-down of Fermi Liquid

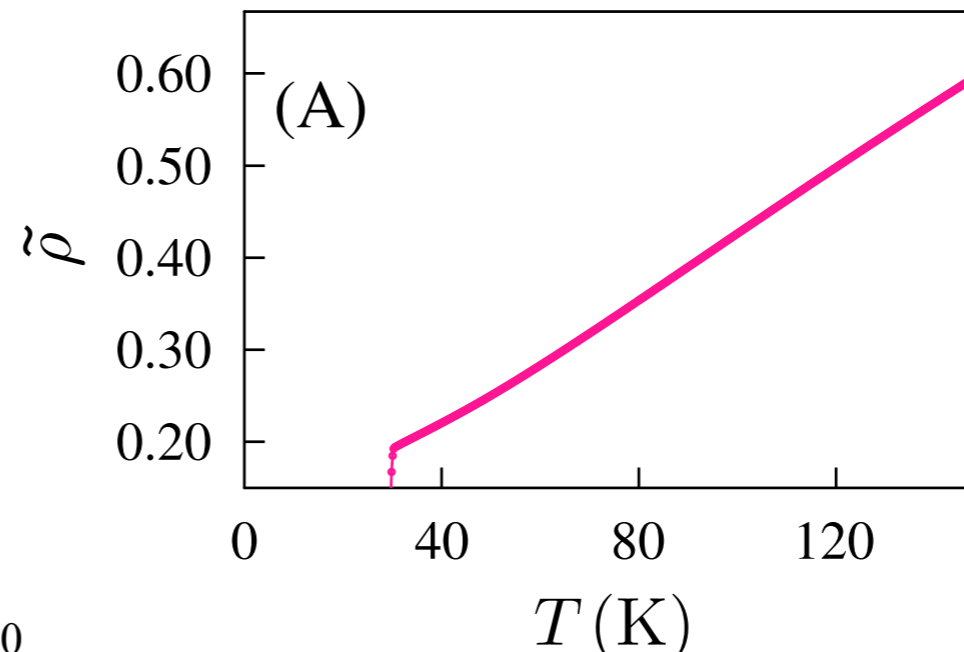


Dark Matter Challenges of the Solid State.

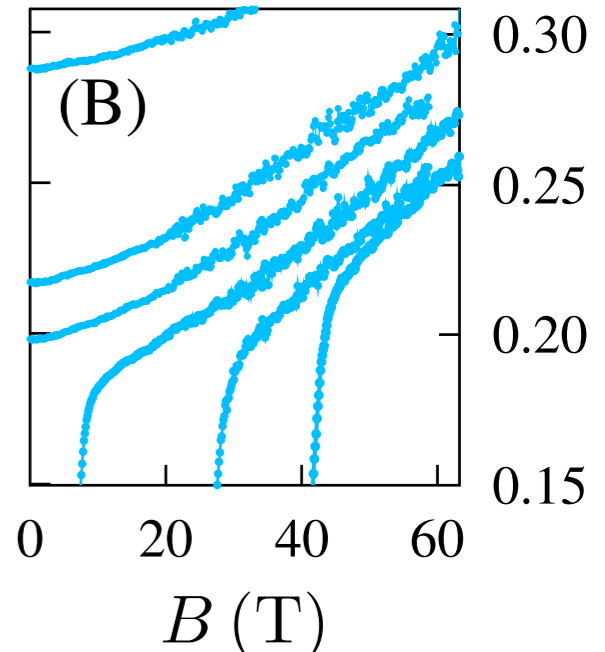
- Linear resistivity in strange metals
- Link with quantum criticality and break-down of Fermi Liquid



BaFe₂As_{2-x}P_x (x=0.31)

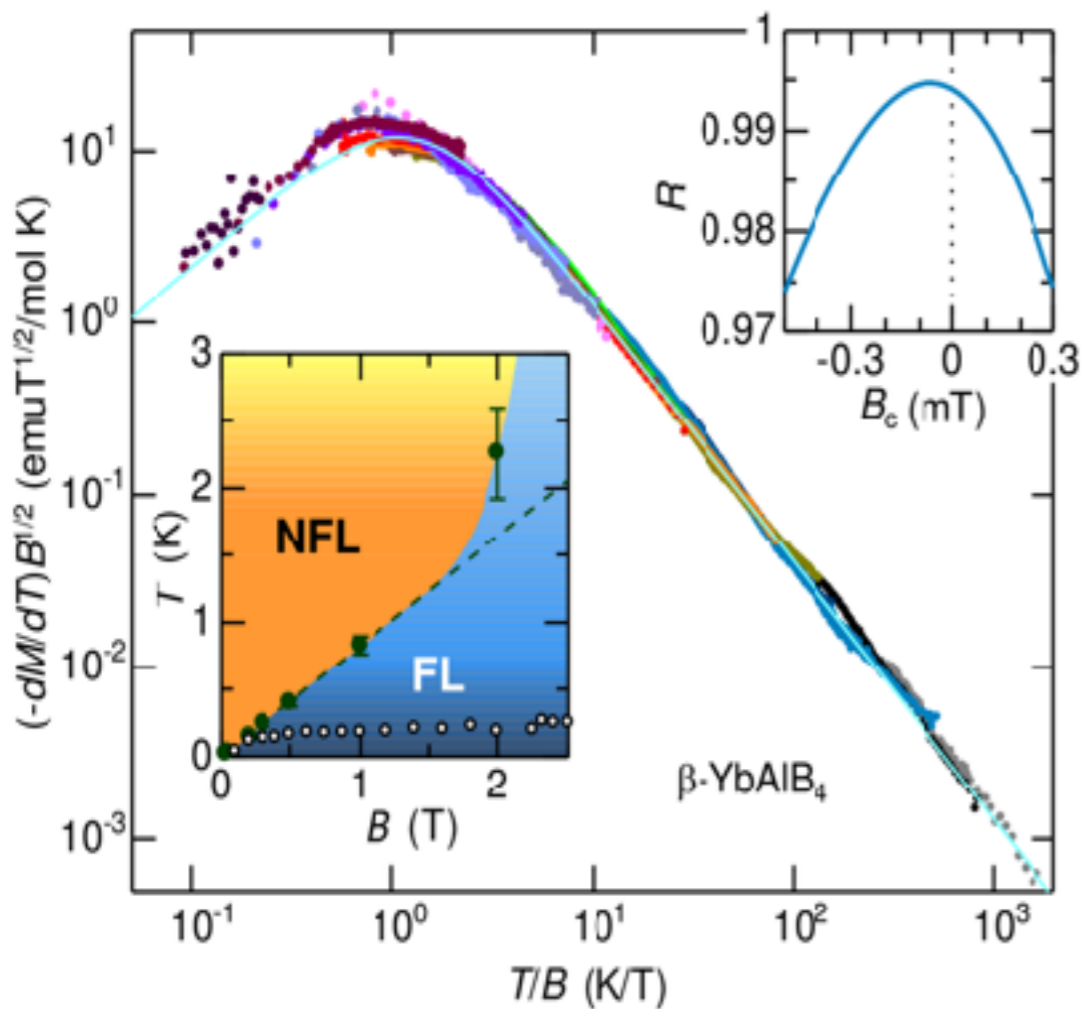
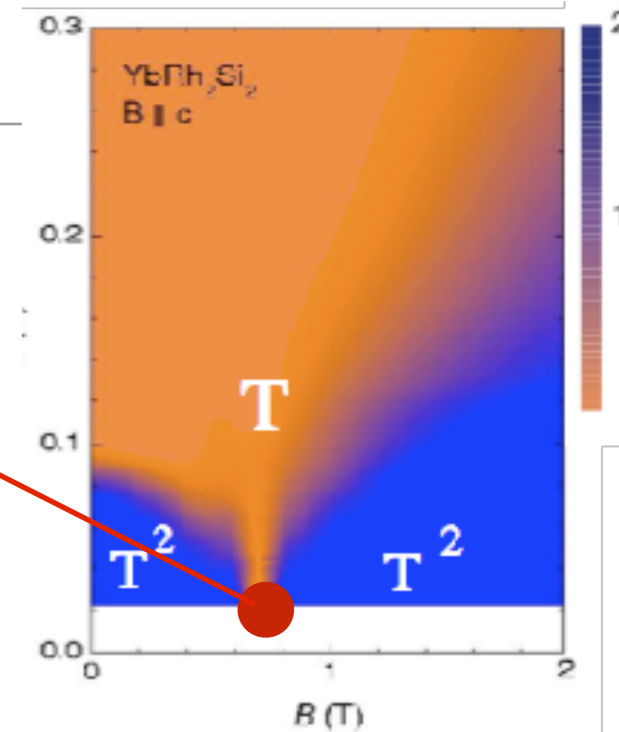


Hayes et al (2015)

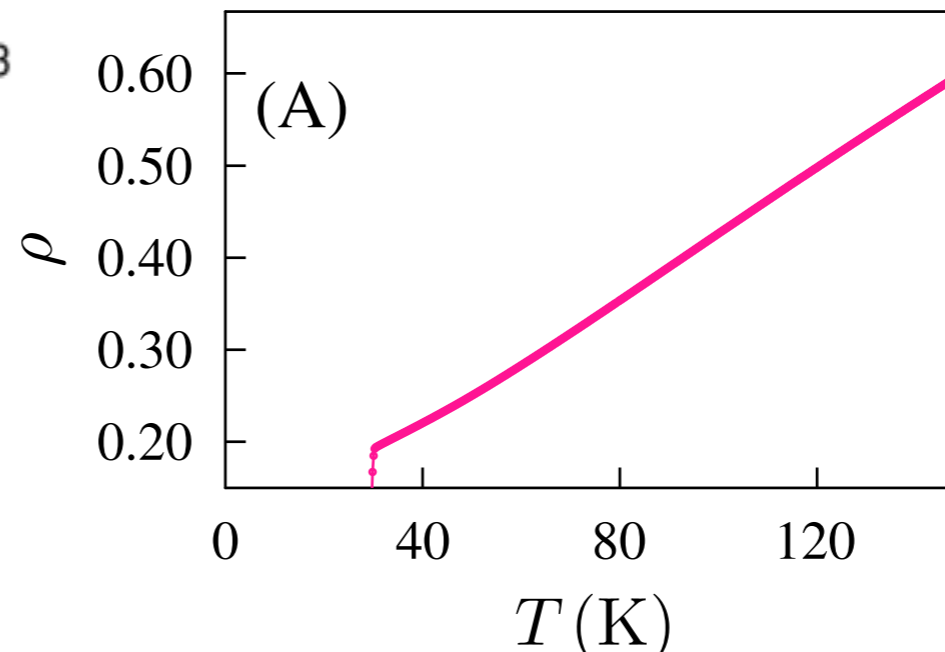


Dark Matter Challenges of the Solid State.

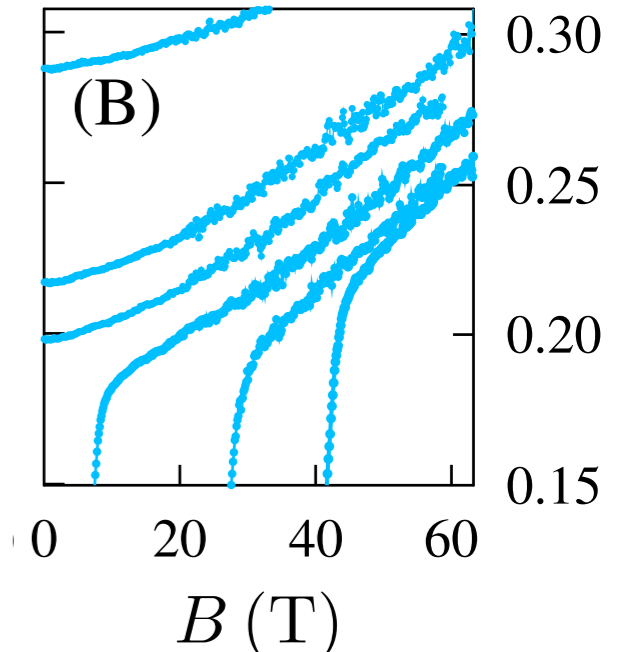
- Linear resistivity in strange metals
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BaFe₂As_{2-x}P_x (x=0.31)

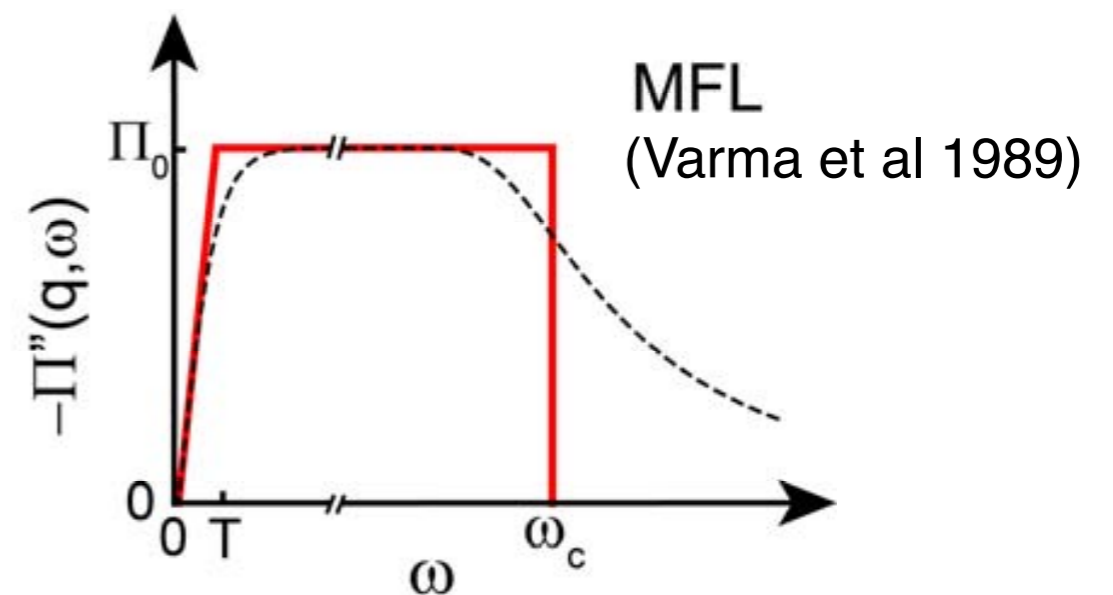
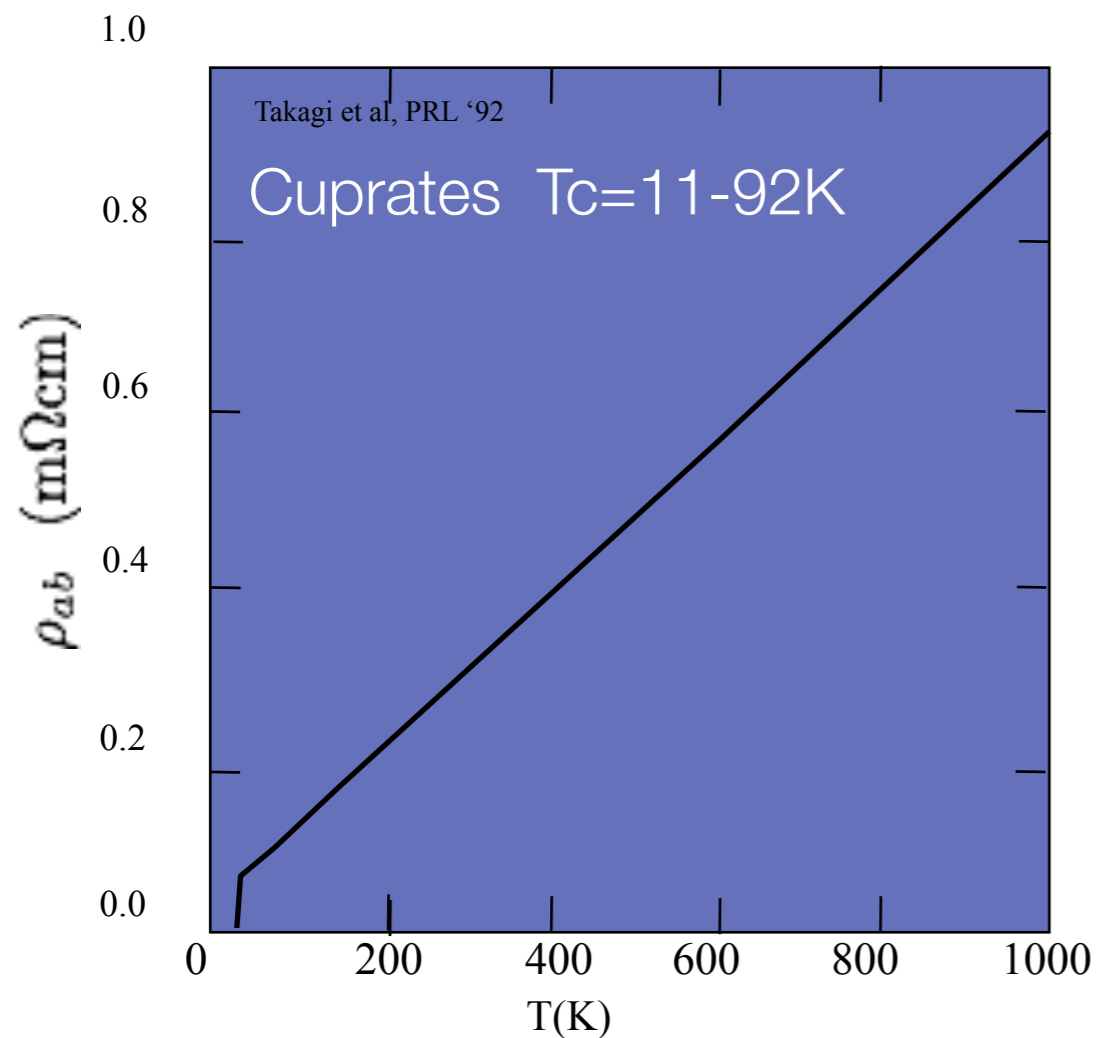


Hayes et al (2015)



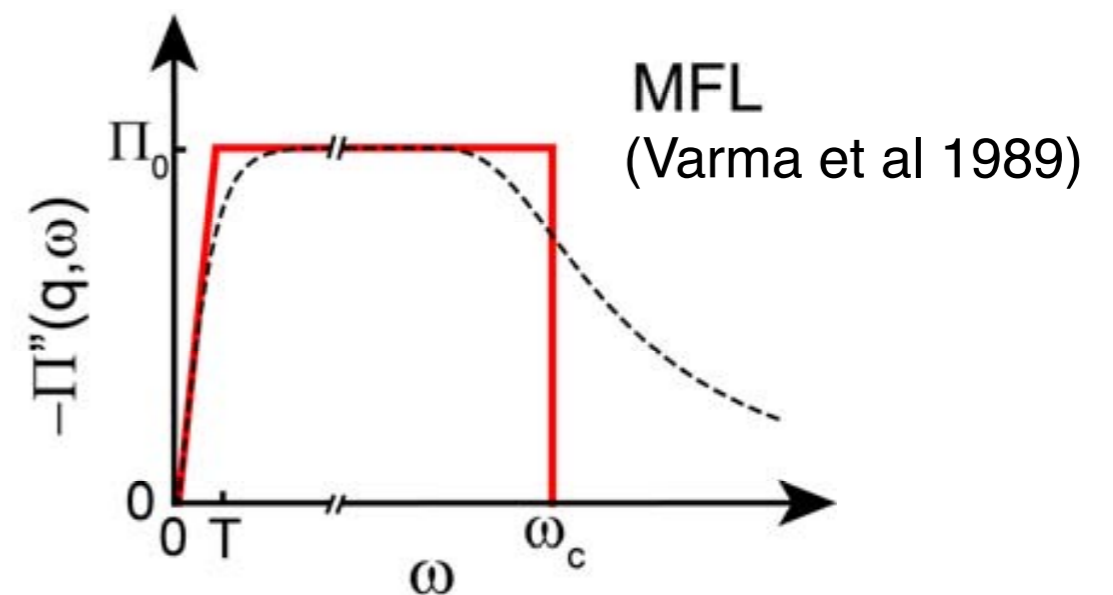
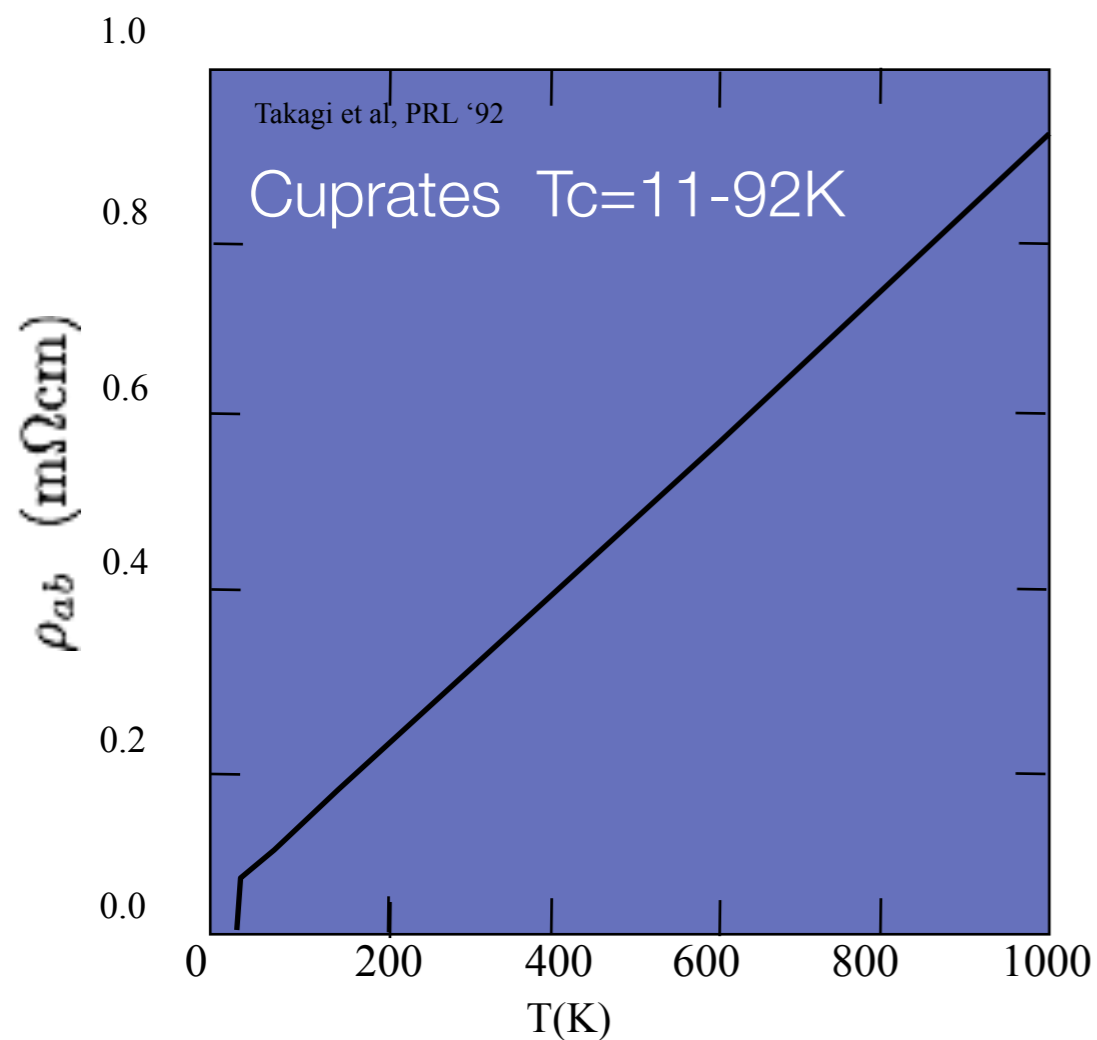
Dark Matter Challenges of the Solid State.

- Linear resistivity in strange metals $\Gamma_{tr} \sim k_B T$
- Link with quantum criticality and break-down of Fermi Liquid



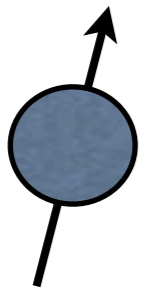
Dark Matter Challenges of the Solid State.

- Linear resistivity in strange metals $\Gamma_{tr} \sim k_B T$
- Link with quantum criticality and break-down of Fermi Liquid



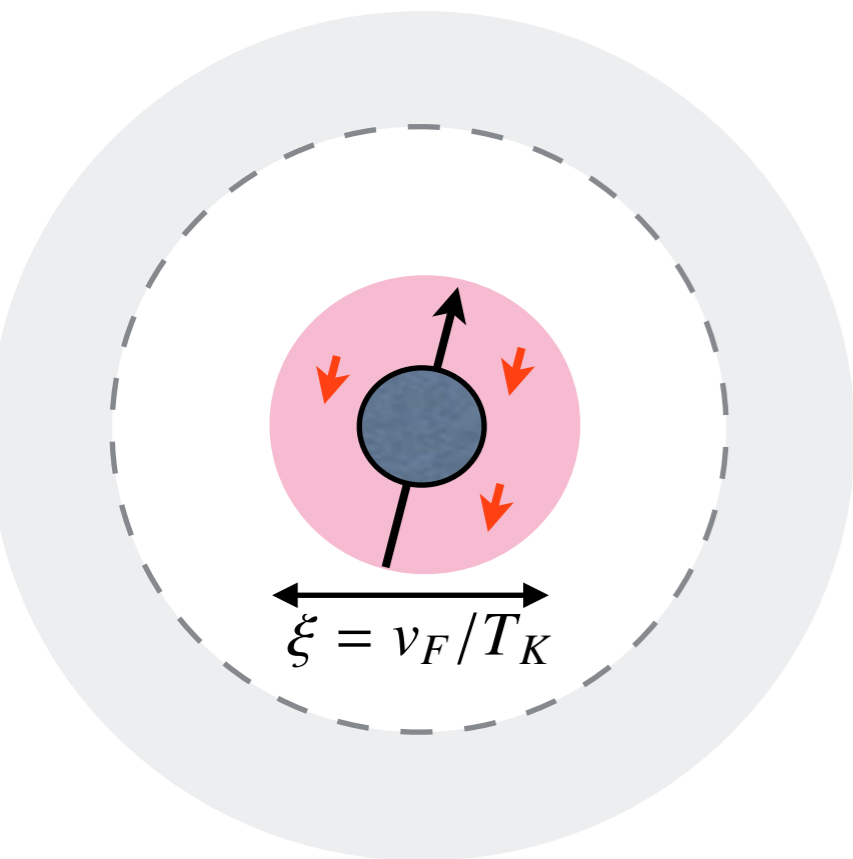
Unidentified (local) critical scattering.
Abbamonte: EELS

Kondo effect: Iconic example of Entanglement



$$\xi = v_F / T_K$$

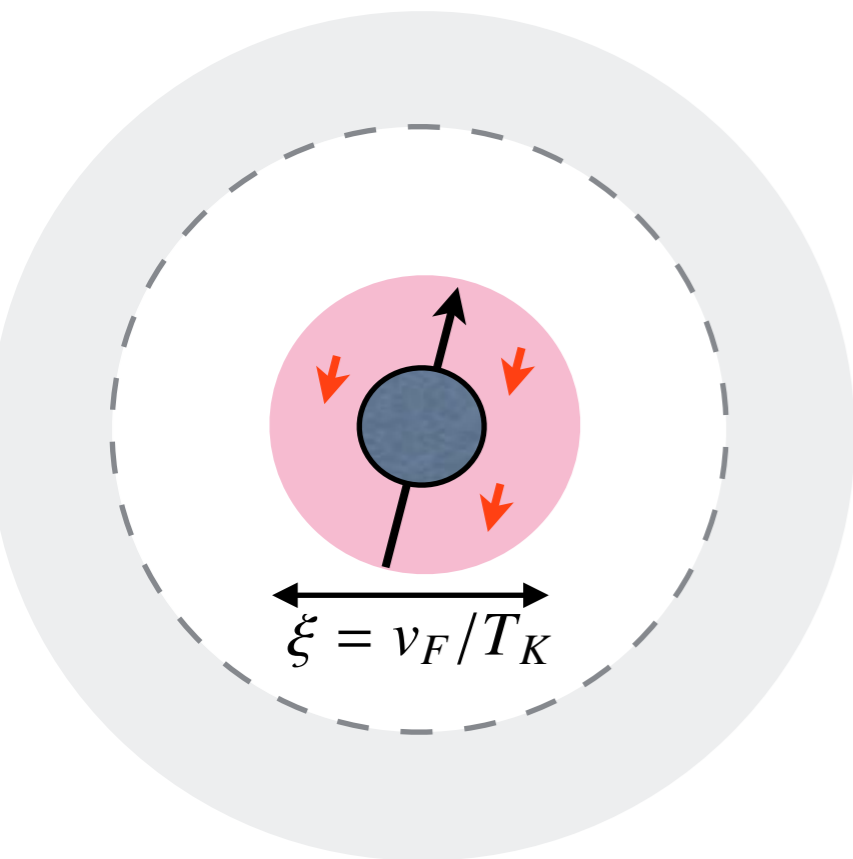
Kondo effect: Iconic example of Entanglement



Spin screened by
conduction
electrons: entangled

↑ ↓ - ↓ ↑

Kondo effect: Iconic example of Entanglement



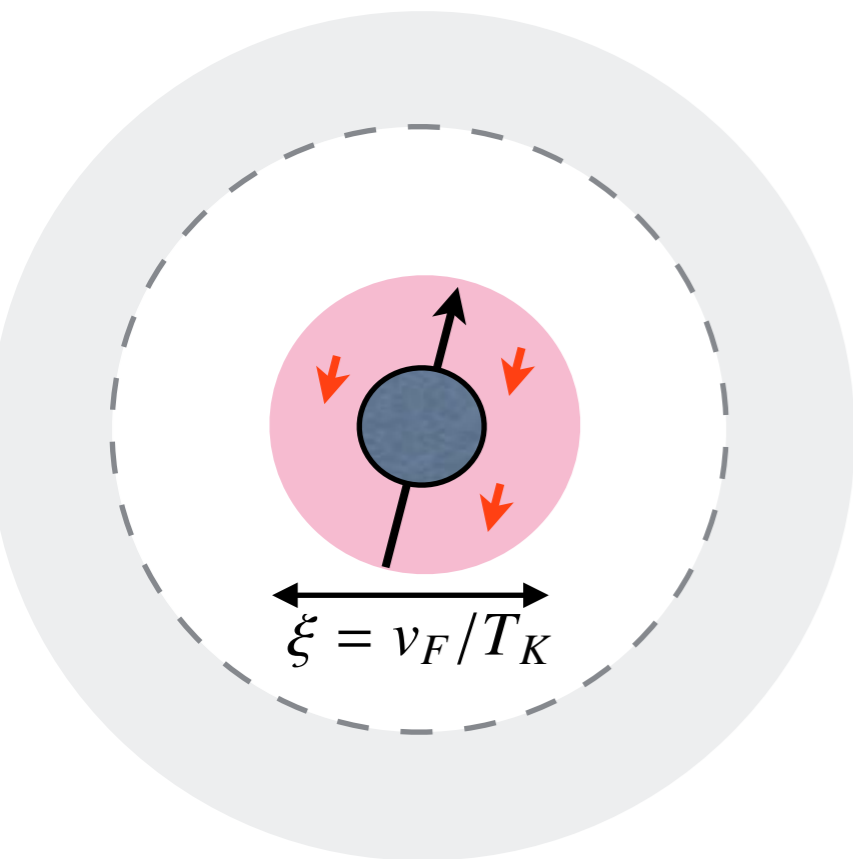
Spin screened by
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electrons: entangled

$\uparrow \downarrow - \downarrow \uparrow$

$$S(T) = \int_0^T \frac{C_V}{T'} dT'$$

“Spin entanglement entropy”

Kondo effect: Iconic example of Entanglement

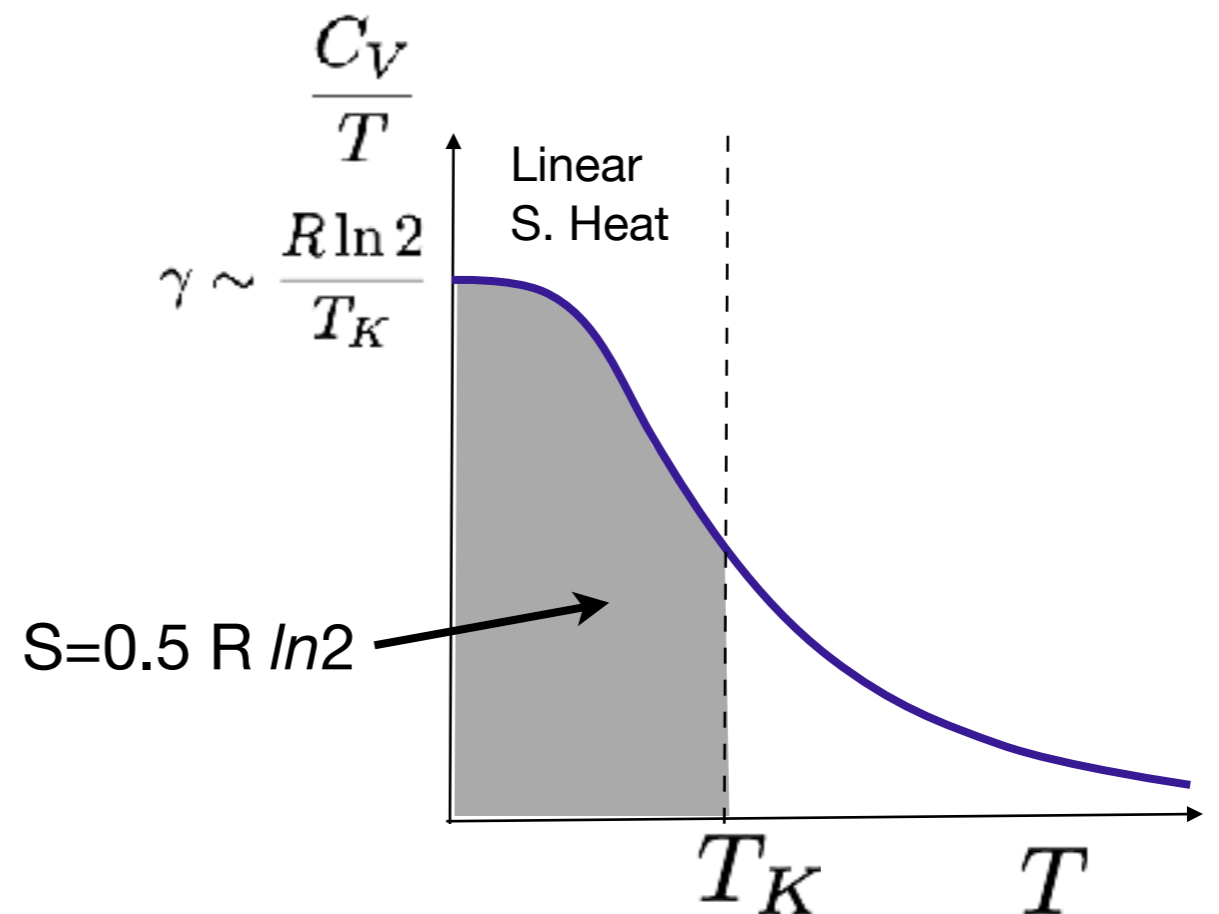


Spin screened by
conduction
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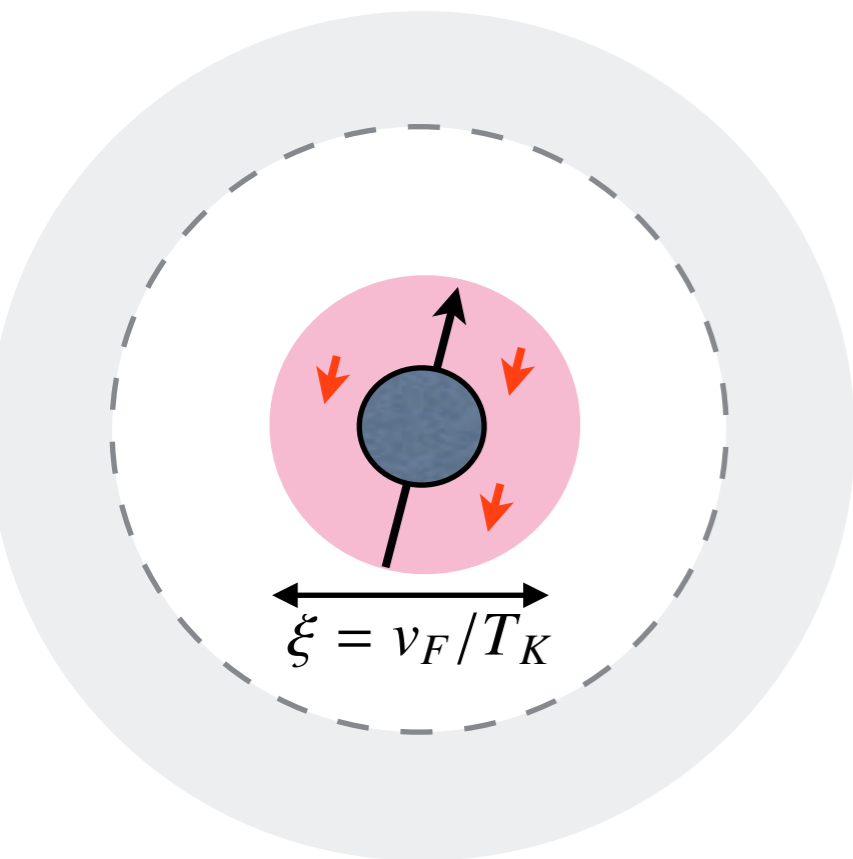
$\uparrow \downarrow - \downarrow \uparrow$

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“Spin entanglement entropy”



Kondo effect: K

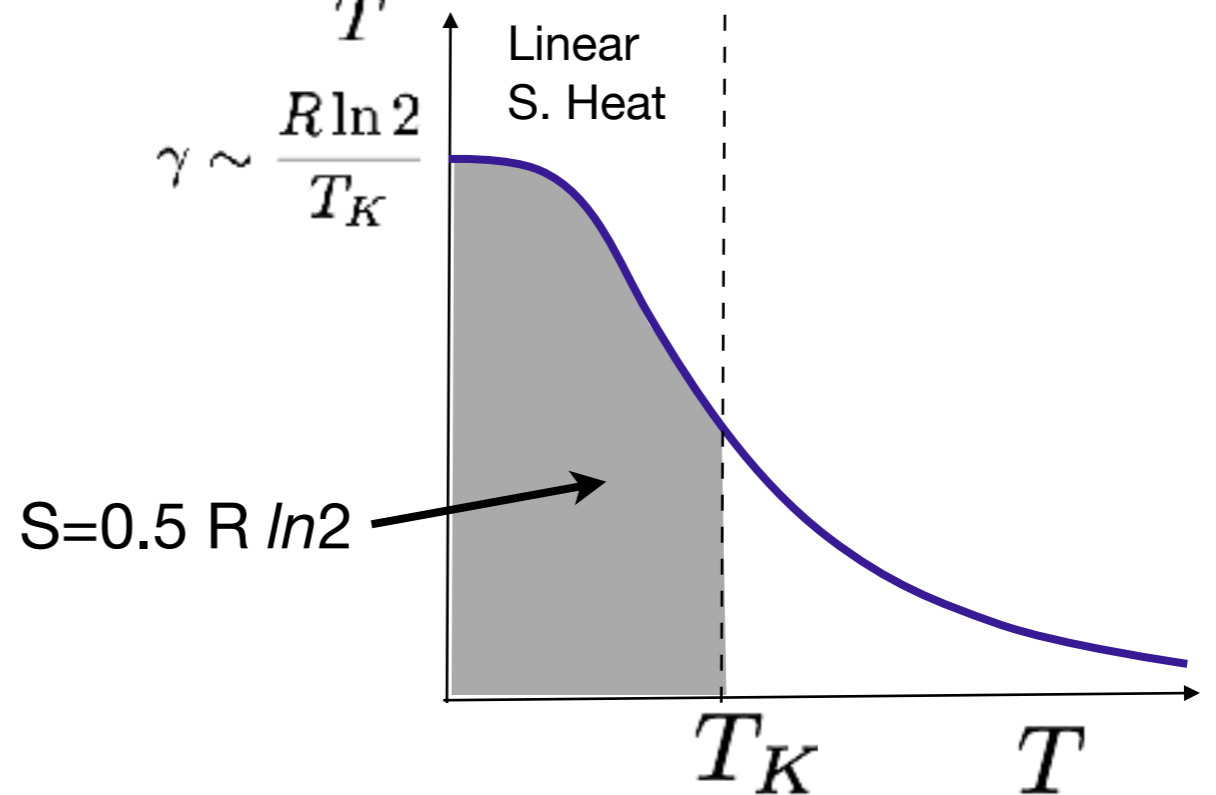
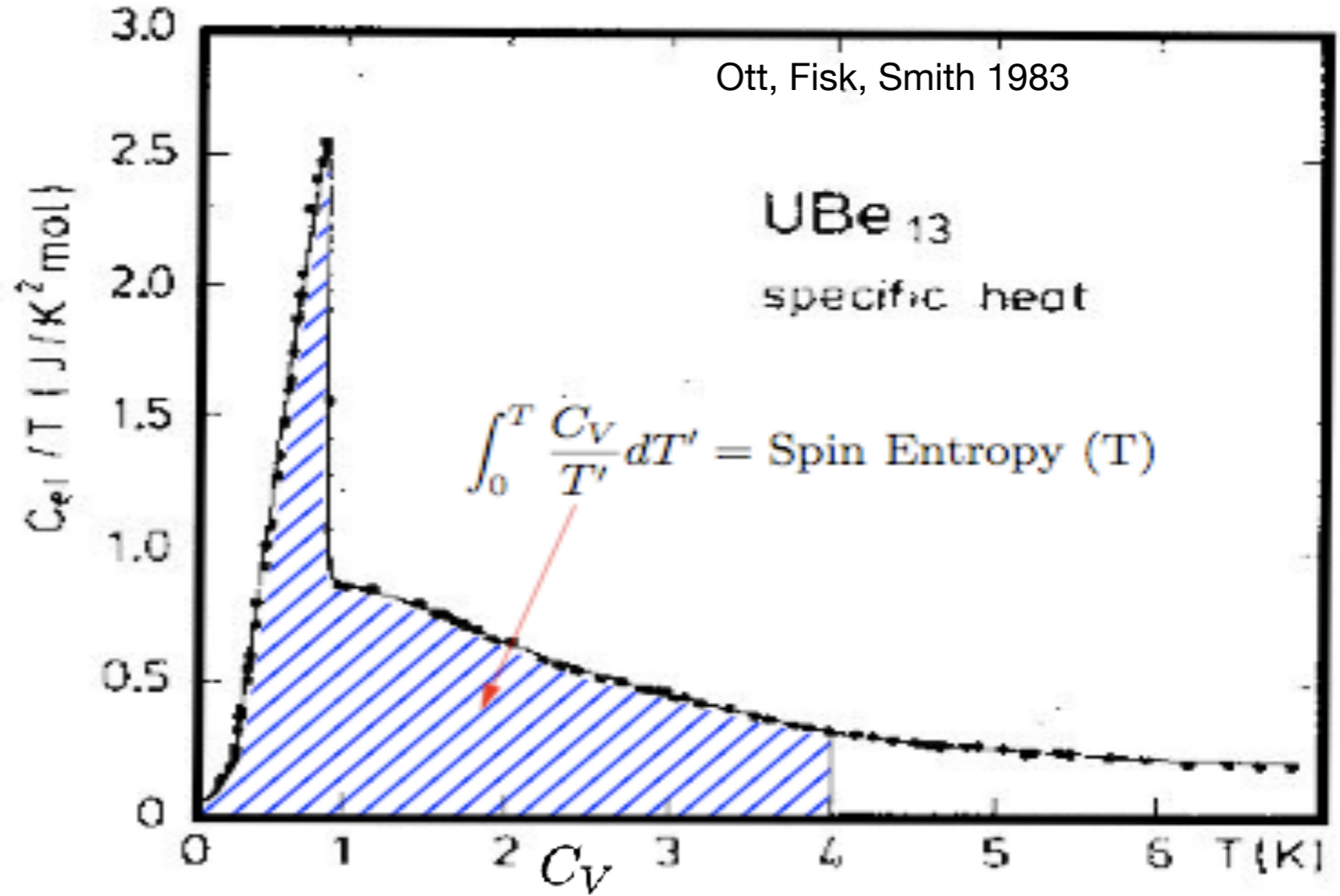


Spin screened by conduction electrons: entangled

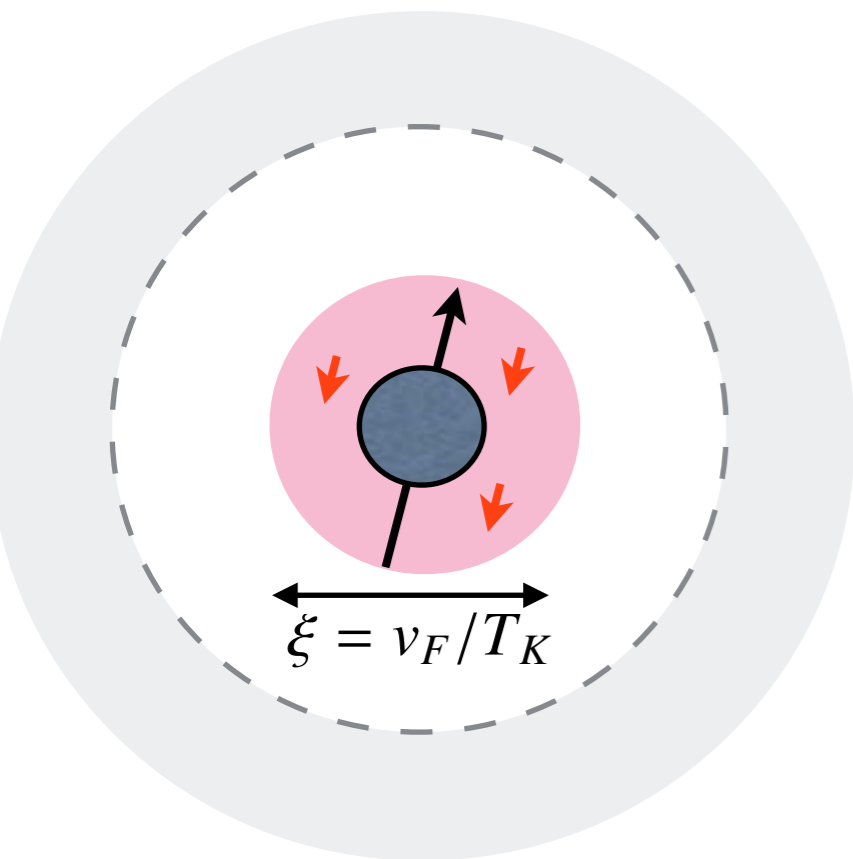
$\uparrow \downarrow - \downarrow \uparrow$

$$S(T) = \int_0^T \frac{C_V}{T'} dT'$$

“Spin entanglement entropy”



Kondo effect: $k_B T_K$

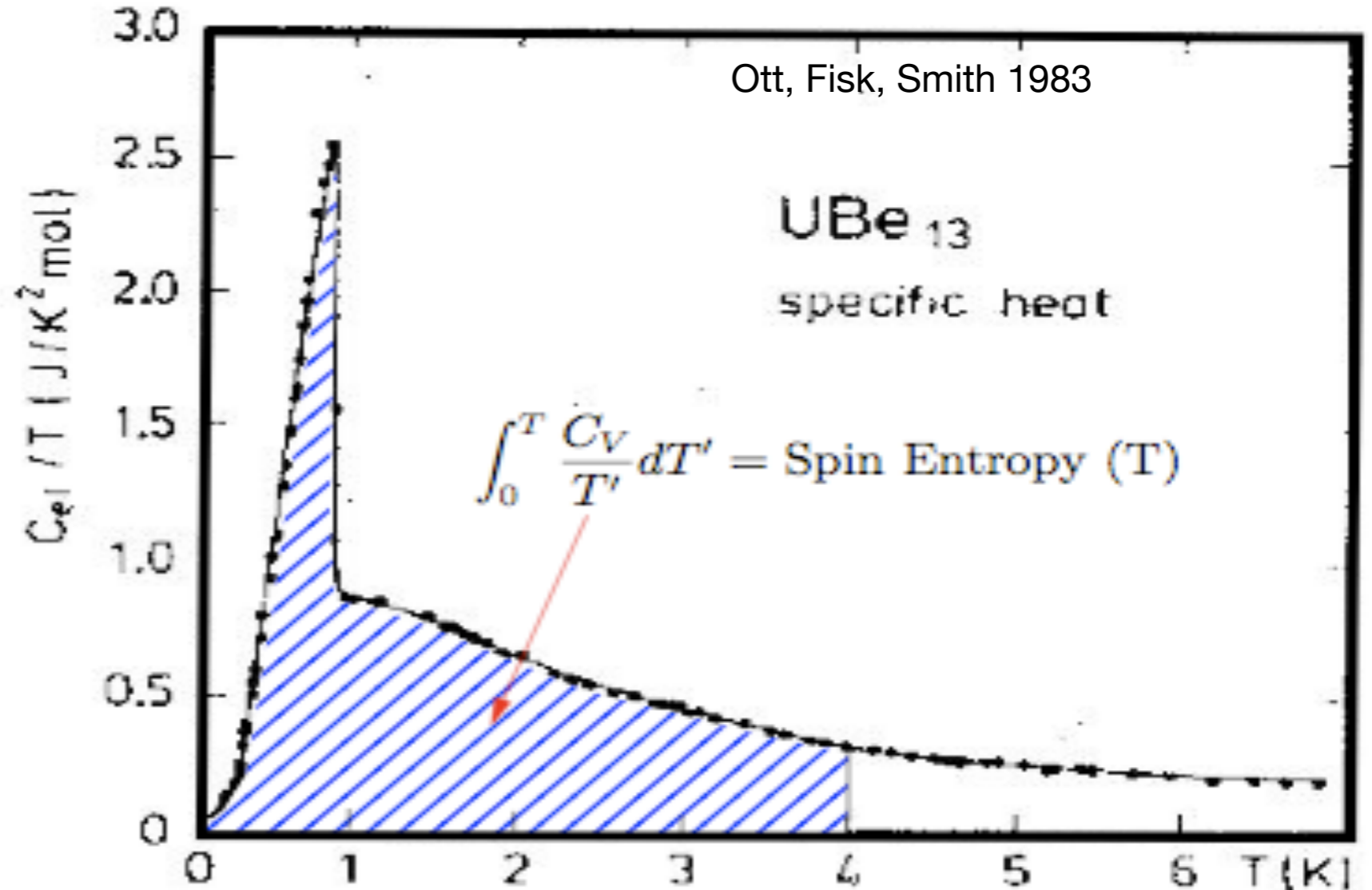


Spin screened by
conduction
electrons: entangled

$\uparrow \downarrow - \downarrow \uparrow$

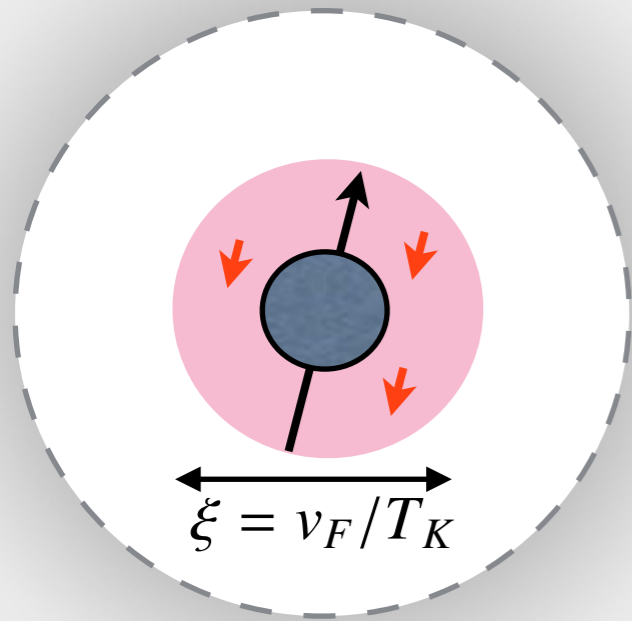
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“Spin entanglement entropy”



SCES: What new forms of entanglement are possible?

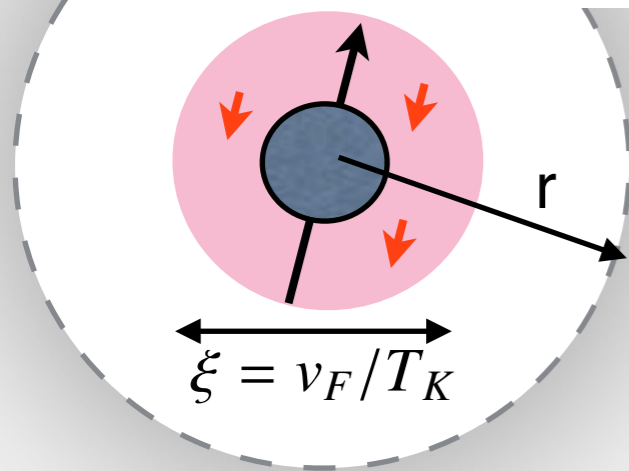
Kondo effect: QI perspective



Kondo effect: QI perspective

Density Matrix

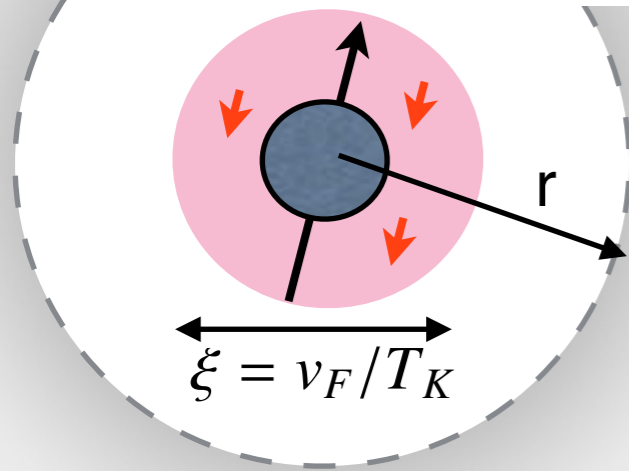
$$\hat{\rho}(r) = \text{Tr}_{r' > r} [|\psi\rangle\langle\psi|]$$



Kondo effect: QI perspective

Density Matrix

$$\hat{\rho}(r) = \text{Tr}_{r' > r} [|\psi\rangle\langle\psi|]$$



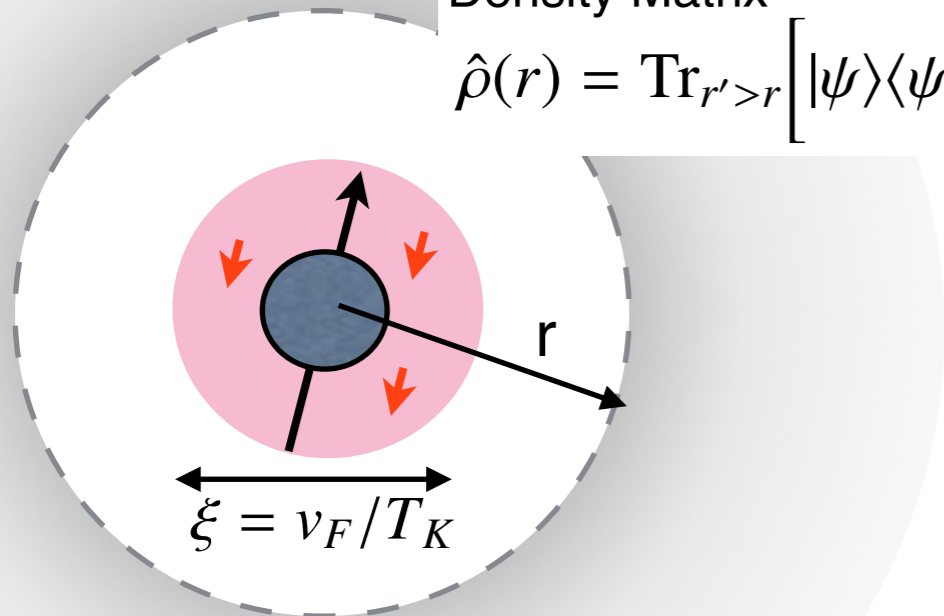
$$S(r) = -\text{Tr}[\rho(r) \ln \rho(r)]$$

Entanglement Entropy

Kondo effect: QI perspective

Density Matrix

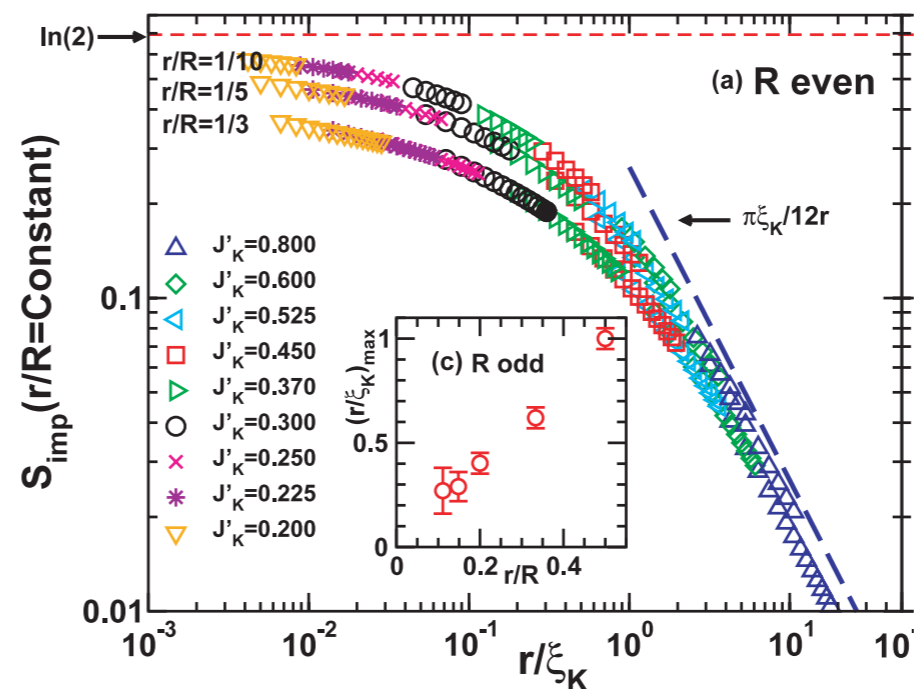
$$\hat{\rho}(r) = \text{Tr}_{r' > r} [|\psi\rangle\langle\psi|]$$



$$S(r) = -\text{Tr}[\rho(r) \ln \rho(r)]$$

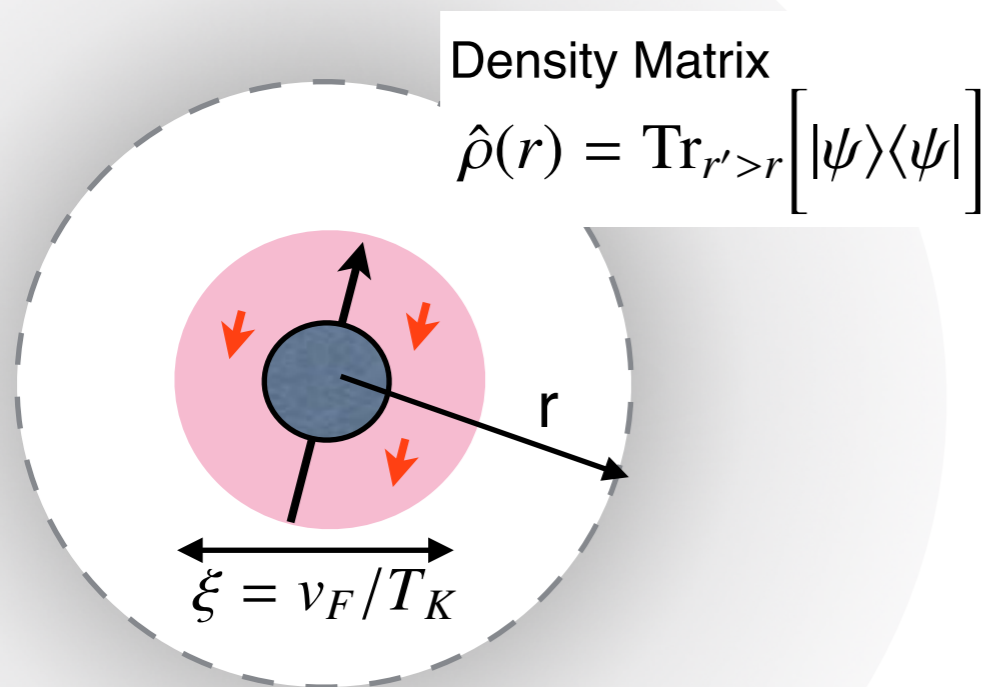
Entanglement Entropy

$$S_i(r) = S(r) - S_{BULK}(r)$$



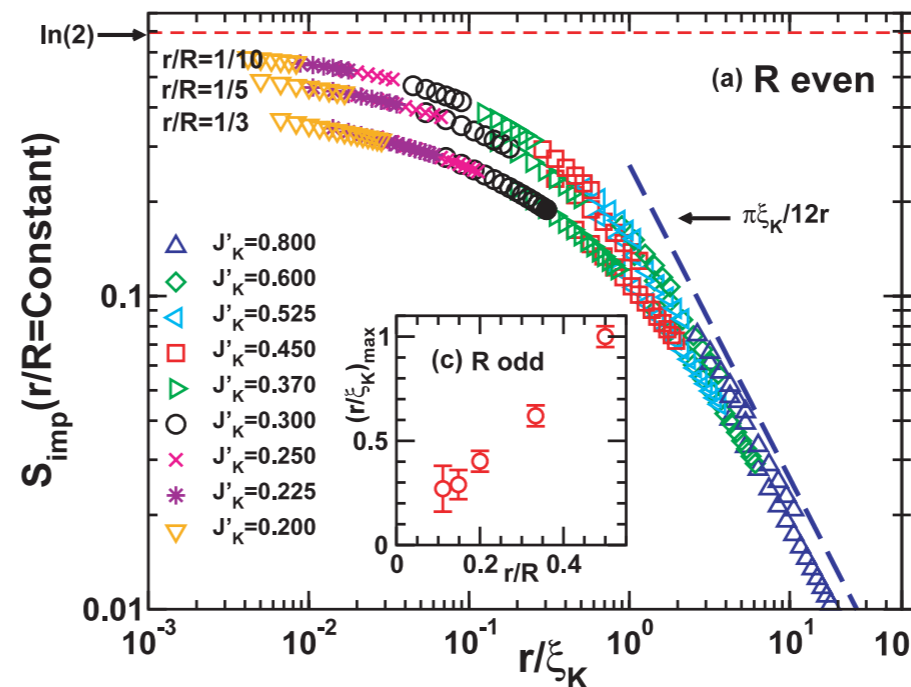
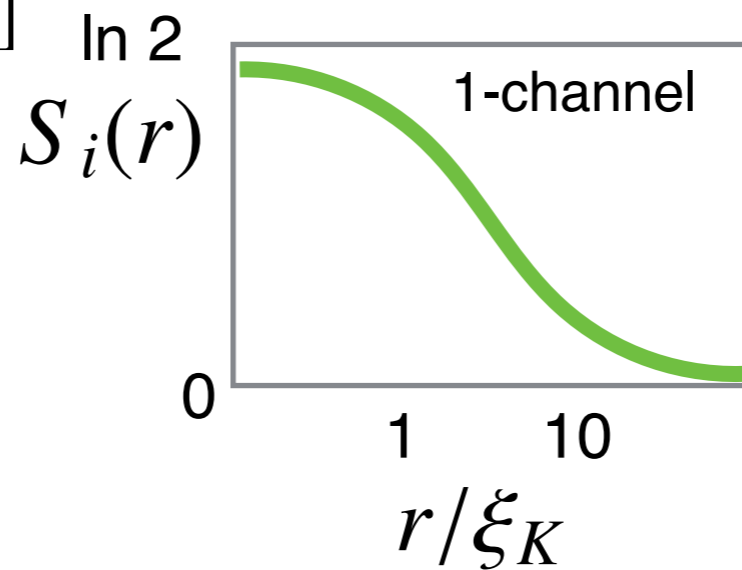
Sorenson, Chang, LaFlorescie & Affleck, JSM (2007)

Kondo effect: QI perspective



$S(r) = -\text{Tr}[\rho(r) \ln \rho(r)]$
 Entanglement Entropy

$S_i(r) = S(r) - S_{BULK}(r)$

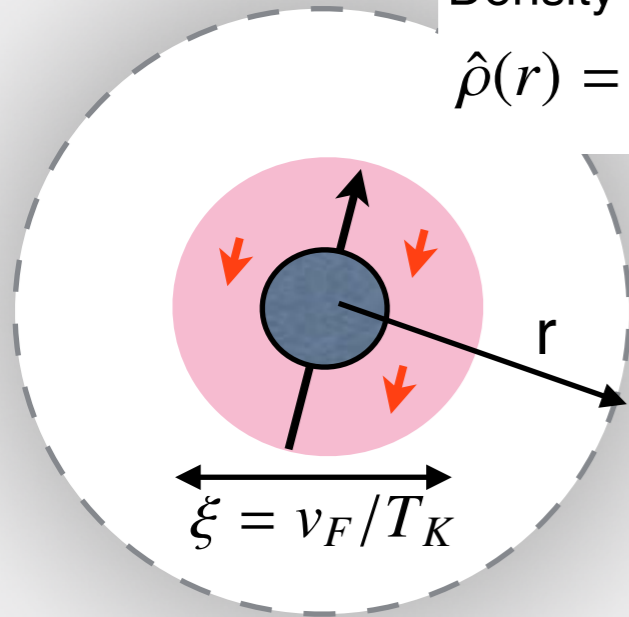


Sorenson, Chang, LaFlorescic & Affleck, JSM (2007)

Kondo effect: QI perspective

Density Matrix

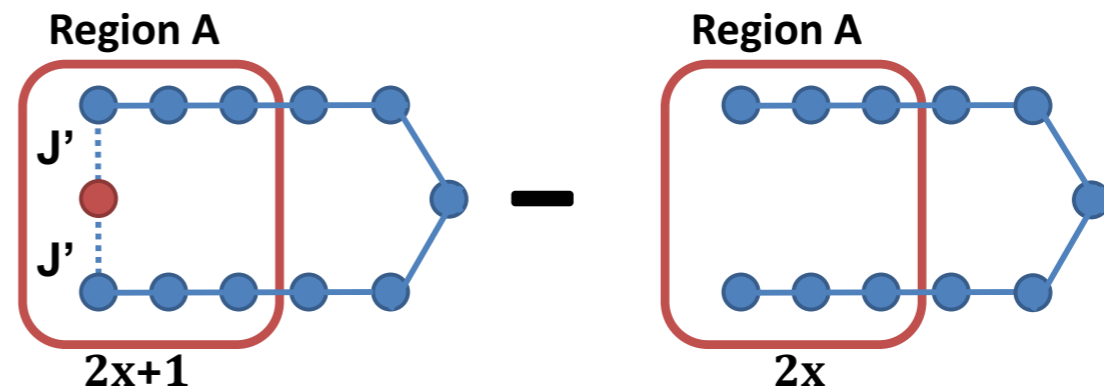
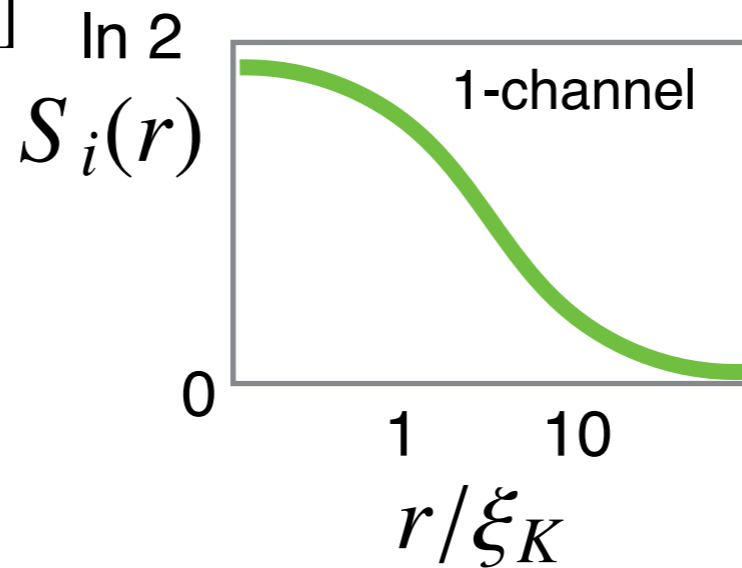
$$\hat{\rho}(r) = \text{Tr}_{r' > r} [|\psi\rangle\langle\psi|]$$



$$S(r) = -\text{Tr}[\rho(r) \ln \rho(r)]$$

Entanglement Entropy

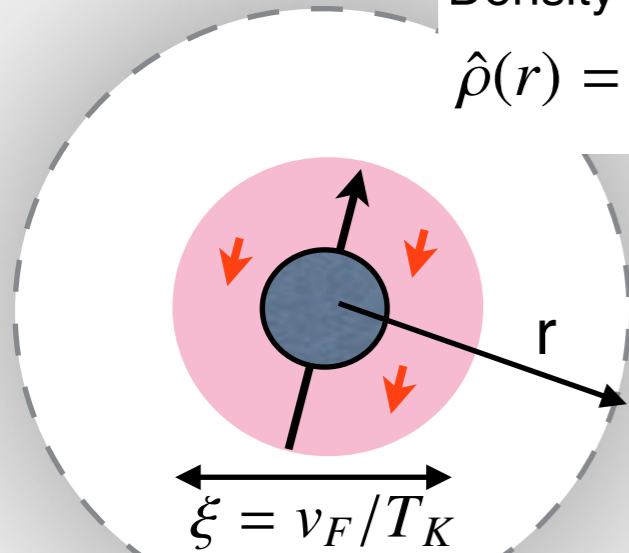
$$S_i(r) = S(r) - S_{BULK}(r)$$



2-channel: Quantum Critical

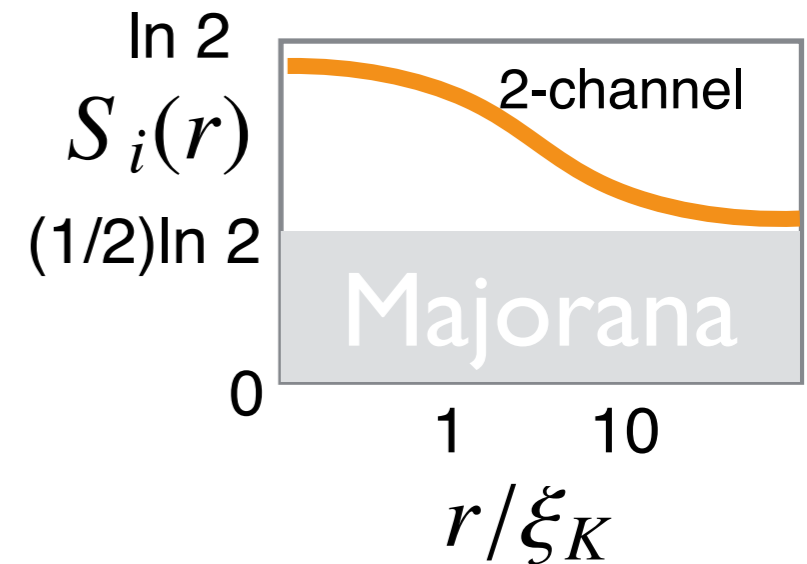
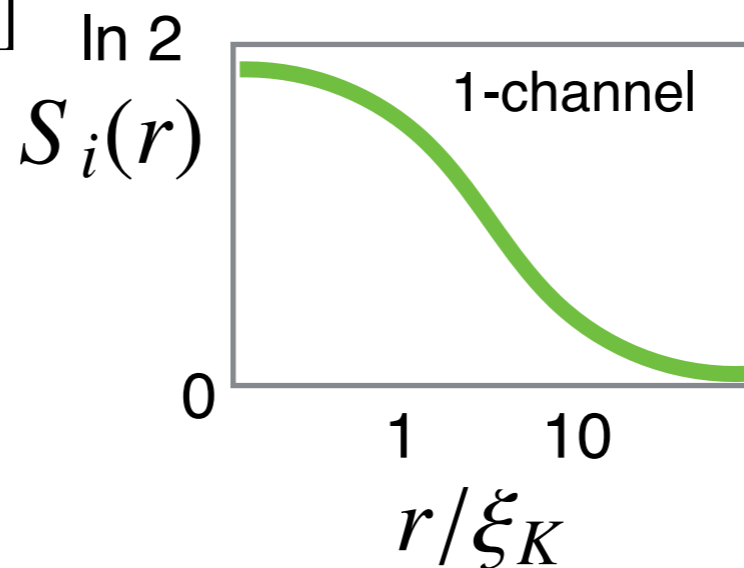
Kondo effect: QI perspective

Density Matrix
 $\hat{\rho}(r) = \text{Tr}_{r' > r} [|\psi\rangle\langle\psi|]$



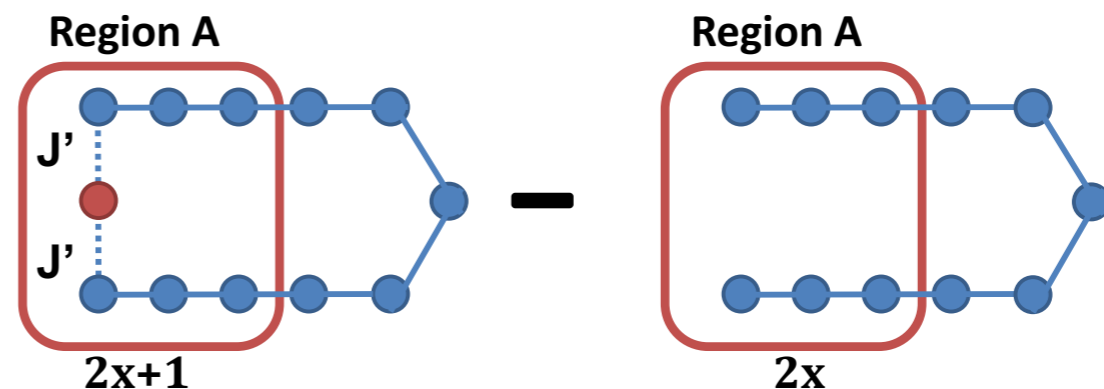
$S(r) = -\text{Tr}[\rho(r) \ln \rho(r)]$
 Entanglement Entropy

$S_i(r) = S(r) - S_{BULK}(r)$



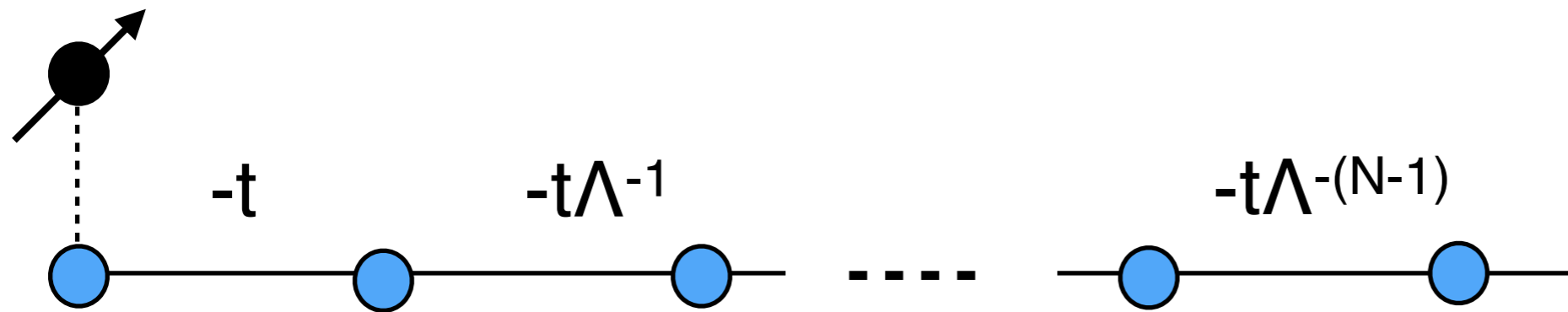
Alkurtass et al, PRB 93,081106R (2016)

Screening cloud of 2-channel Kondo model is infinitely large



2-channel: Quantum Critical

From Wilsonian NRG to Tensor Networks

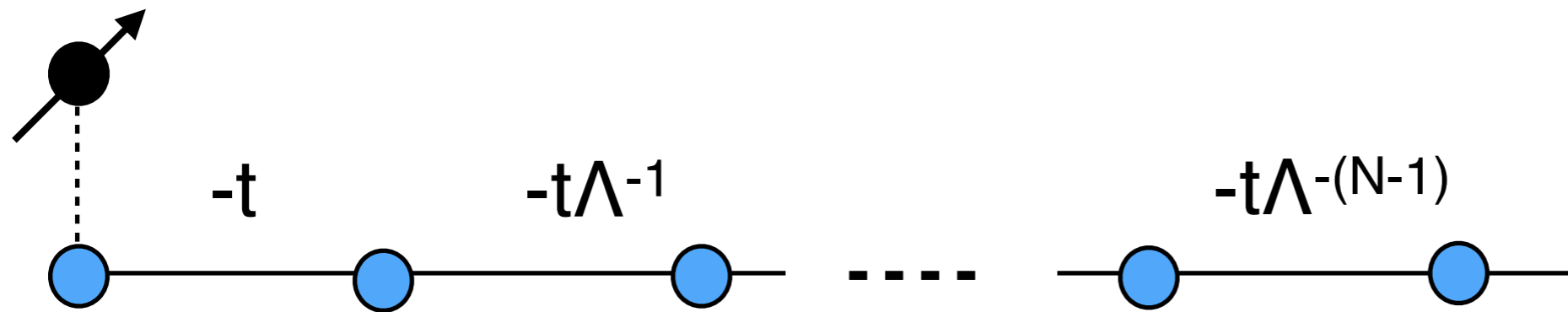


NRG Wilson 1973
DMRG White 1992

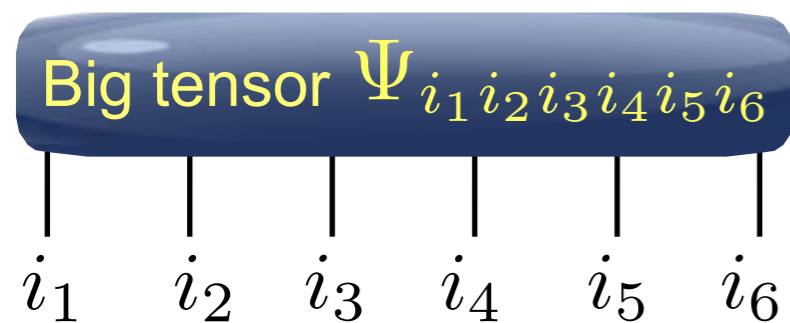
From Wilsonian NRG to Tensor Networks



NRG Wilson 1973
DMRG White 1992



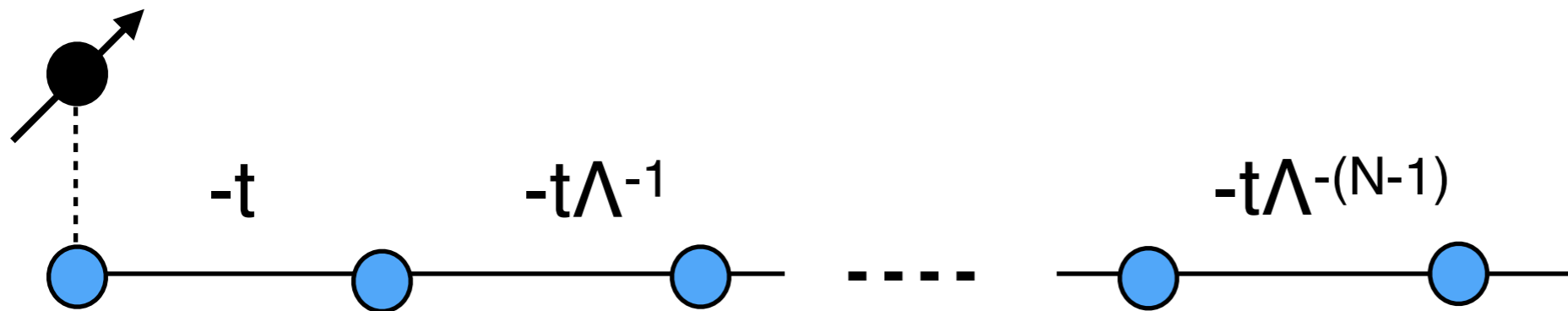
Wavefunction



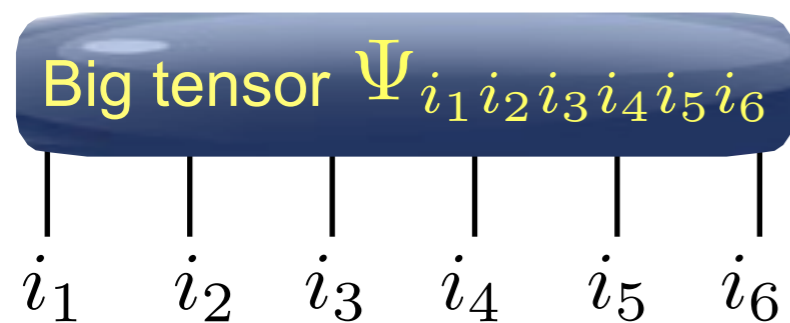
From Wilsonian NRG to Tensor Networks



NRG Wilson 1973
DMRG White 1992



Wavefunction

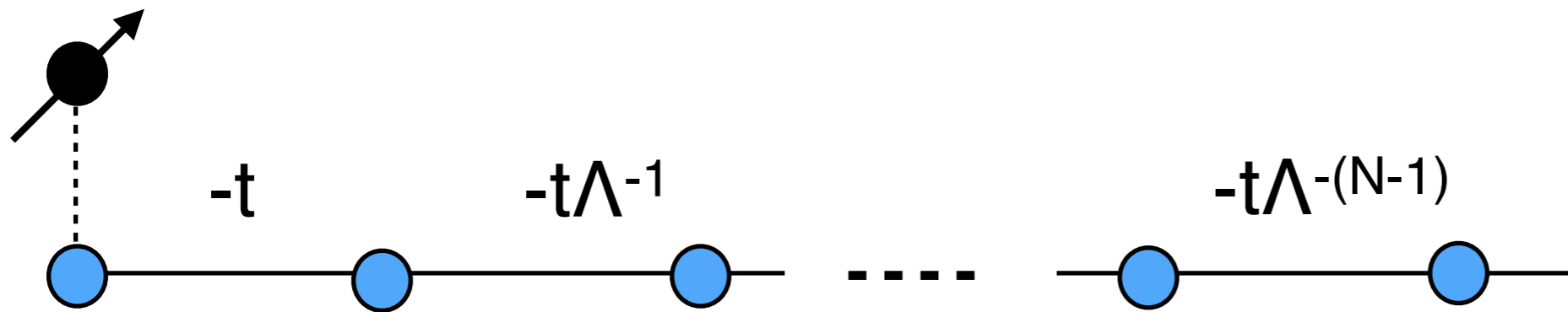


$$\sim e^N$$

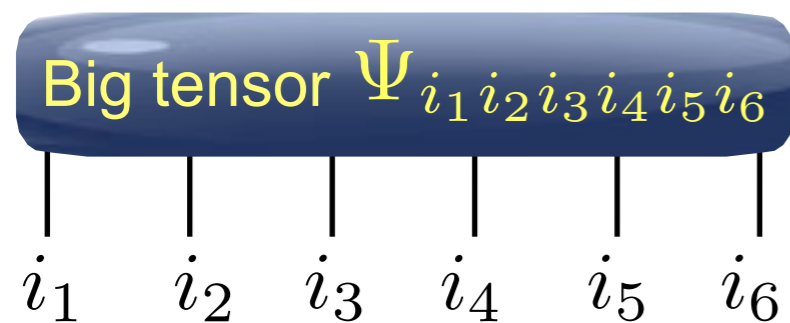
From Wilsonian NRG to Tensor Networks



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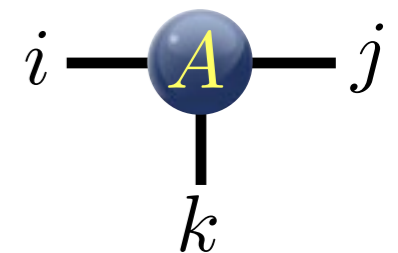
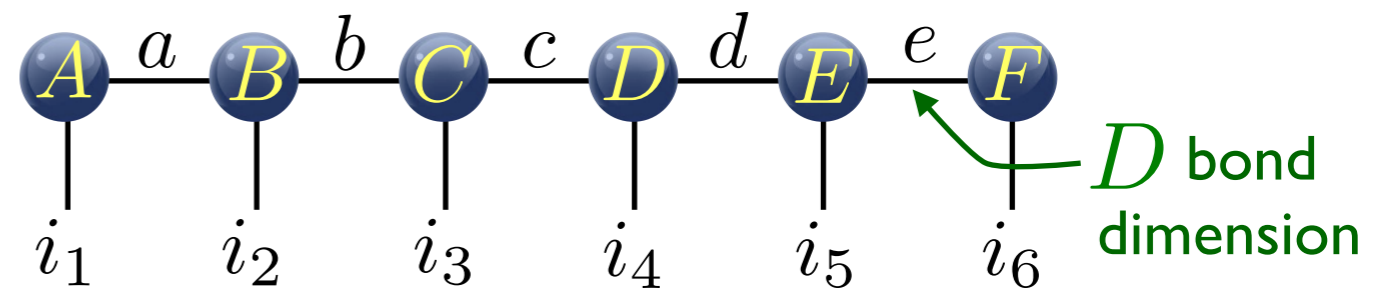


Wavefunction



$\sim e^N$

Tensor network: matrix product state (**MPS**)



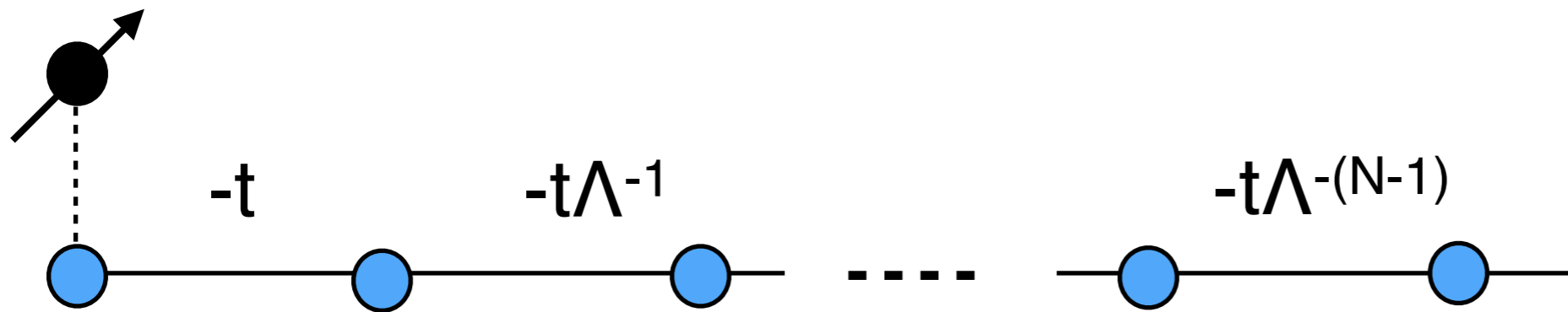
A_{ijk}

rank-3 tensor

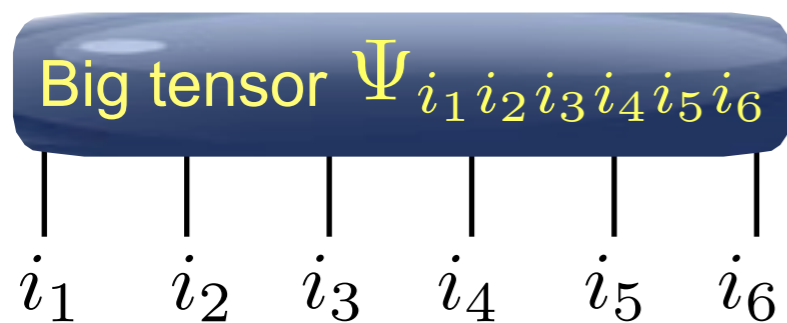
From Wilsonian NRG to Tensor Networks



NRG Wilson 1973
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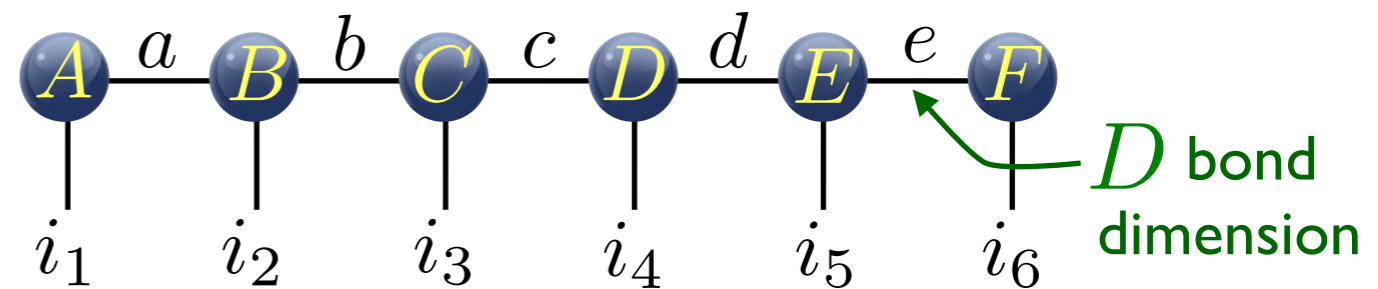


Wavefunction

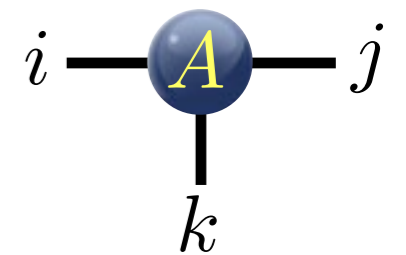


$\sim e^N$

Tensor network: matrix product state (**MPS**)



Polynomial in N



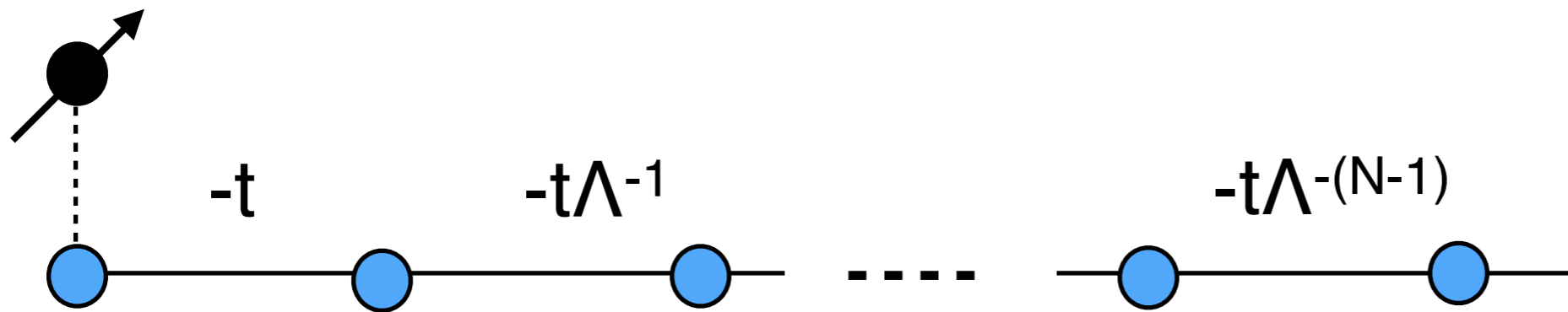
A_{ijk}

rank-3 tensor

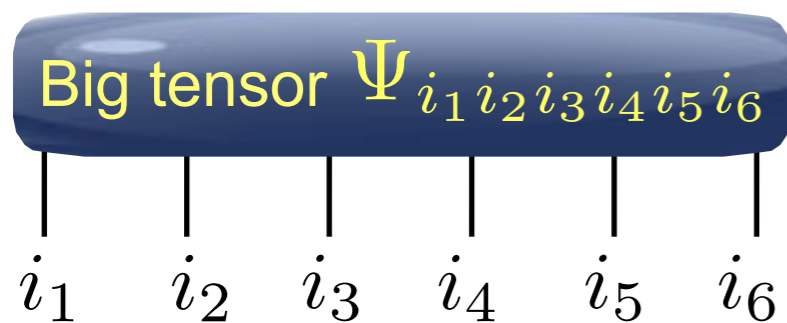
From Wilsonian NRG to Tensor Networks



NRG Wilson 1973
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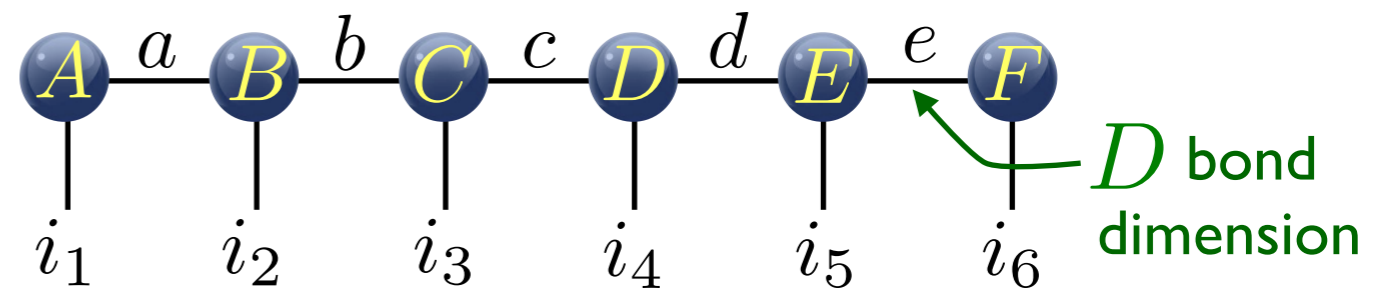


Wavefunction

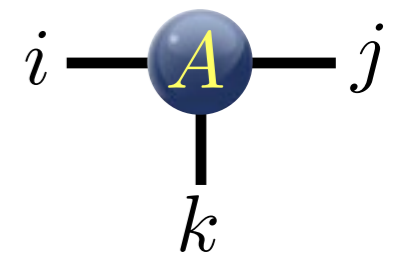


$$\sim e^N$$

Tensor network: matrix product state (MPS)



Polynomial in N



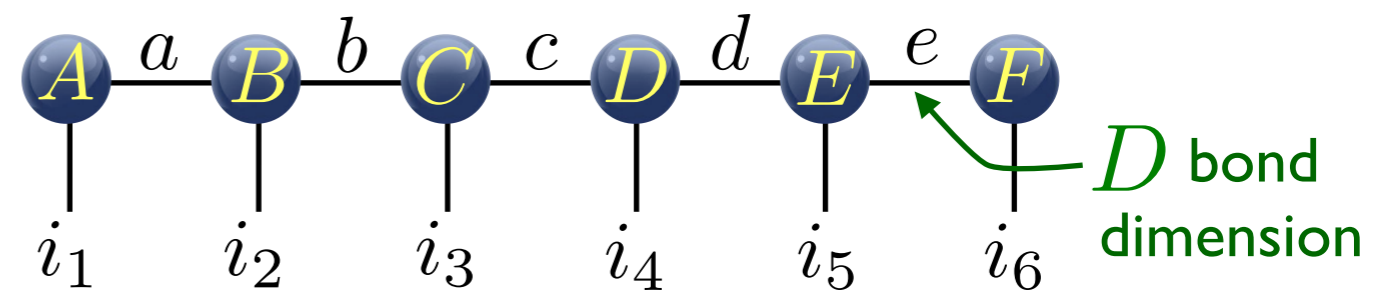
$$A_{ijk}$$

rank-3 tensor

Variational MPS is equivalent to Density Matrix Renormalization Group

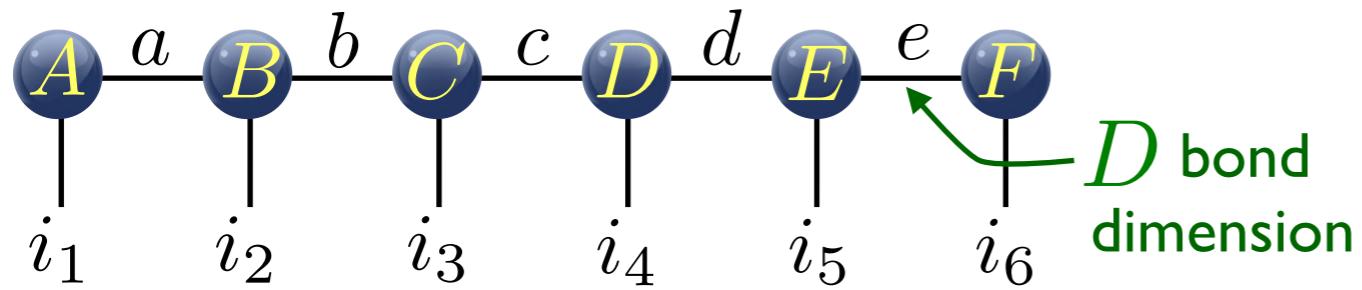
From Wilsonian NRG to Tensor Networks

Tensor network: matrix product state (**MPS**)



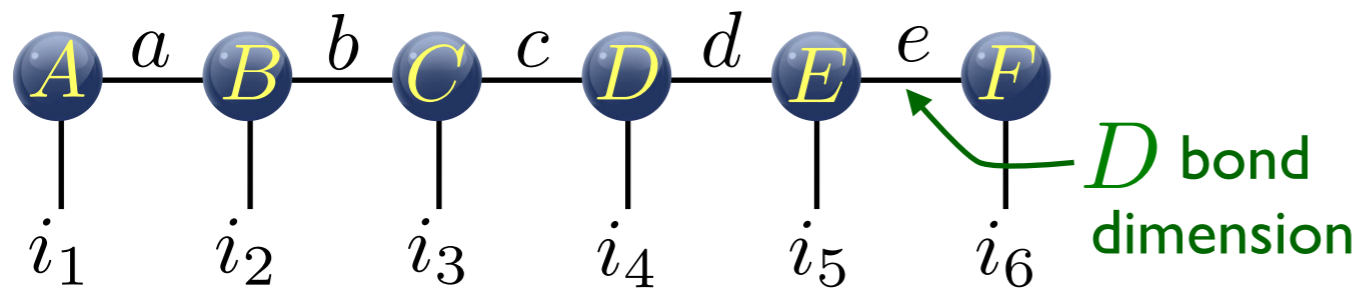
From Wilsonian NRG to Tensor Networks

Tensor network: matrix product state (**MPS**)



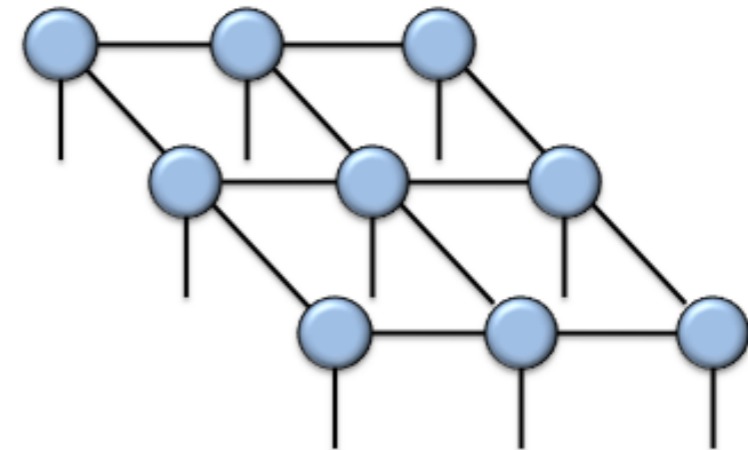
From Wilsonian NRG to Tensor Networks

Tensor network: matrix product state (**MPS**)



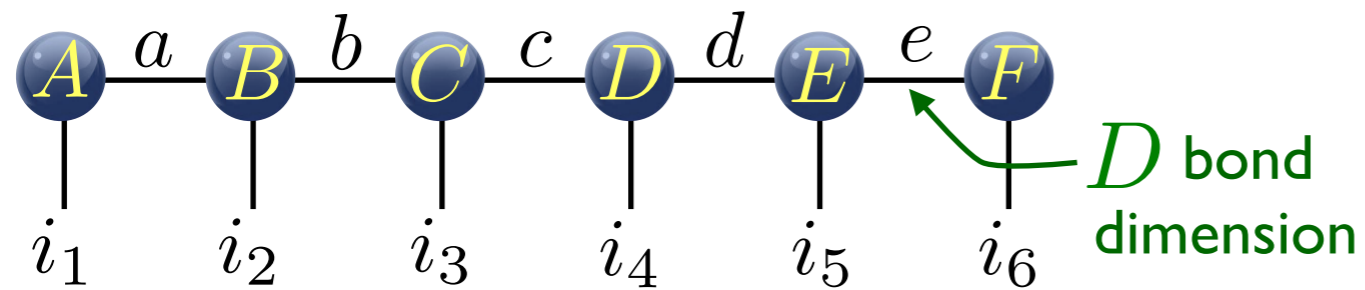
PEPS (2D)

projected entangled-pair state



From Wilsonian NRG to Tensor Networks

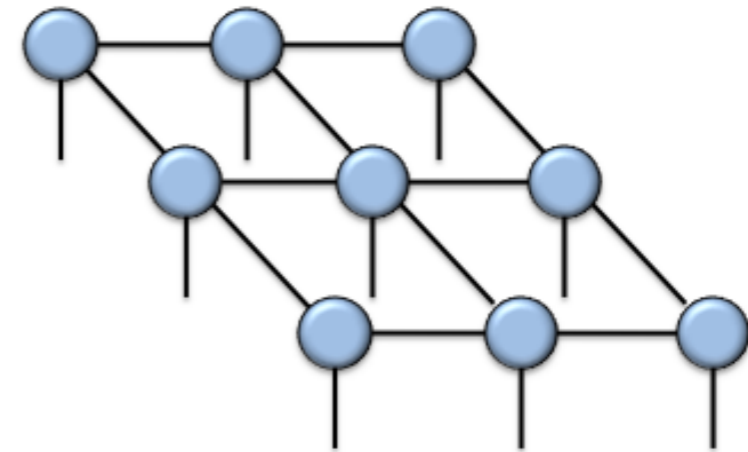
Tensor network: matrix product state (**MPS**)



Capability for detailed study of spectral functions in impurity, 1 & 2D.

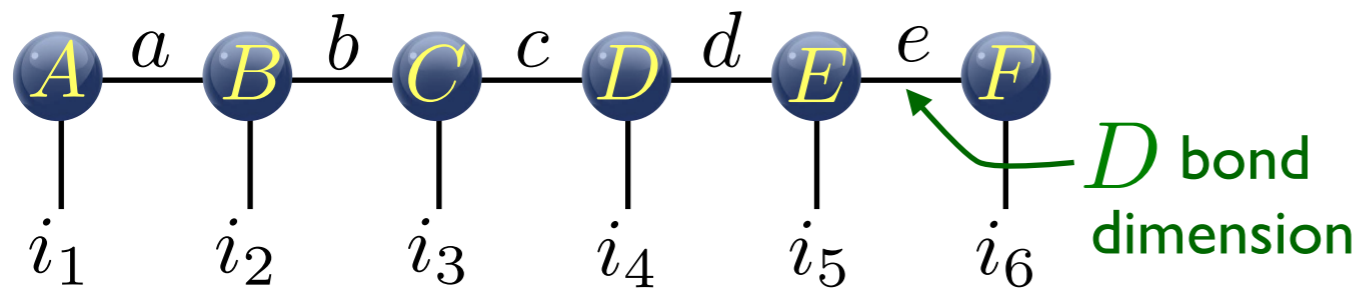
PEPS (2D)

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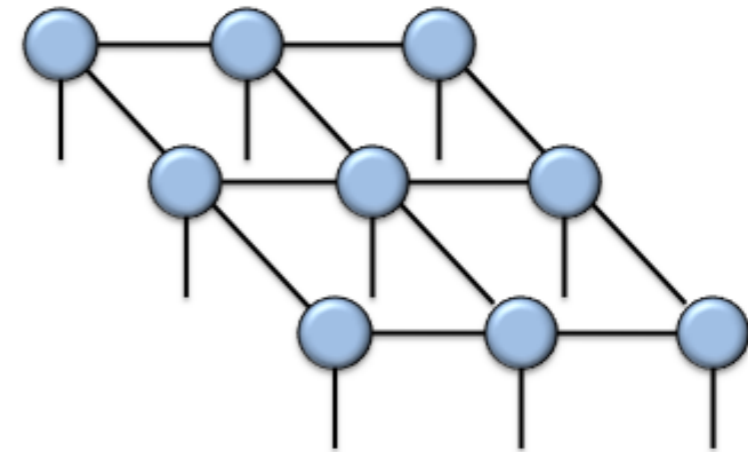
From Wilsonian NRG to Tensor Networks

Tensor network: matrix product state (**MPS**)

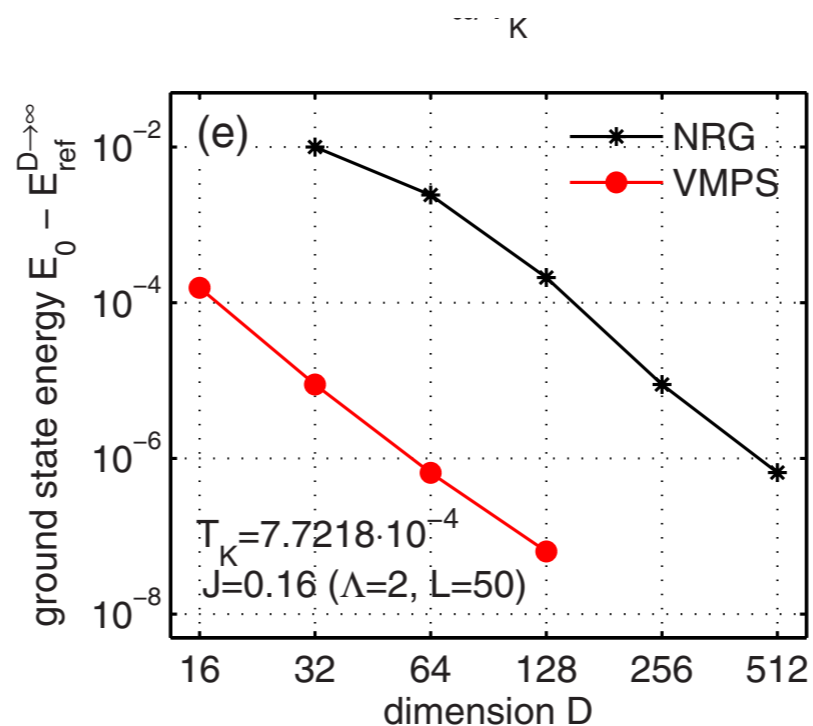
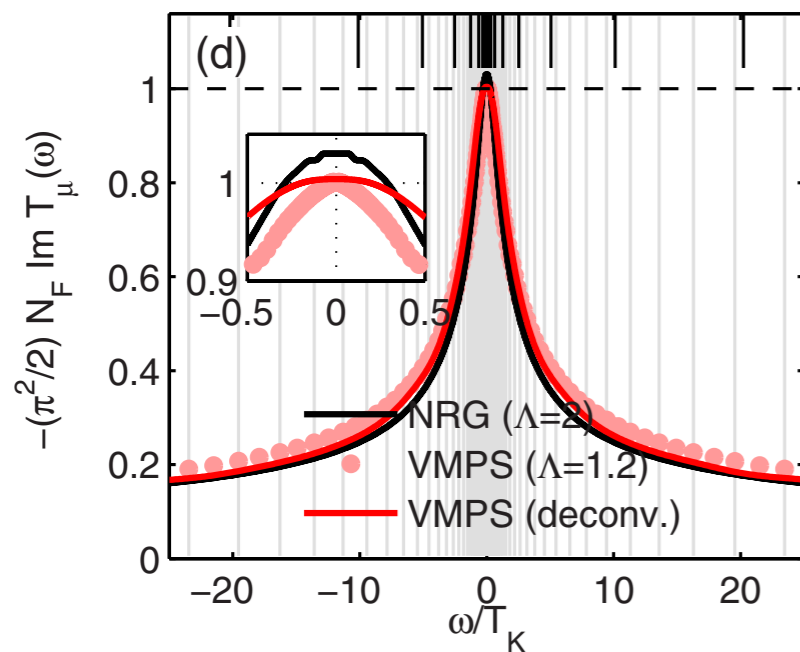


PEPS (2D)

projected entangled-pair state



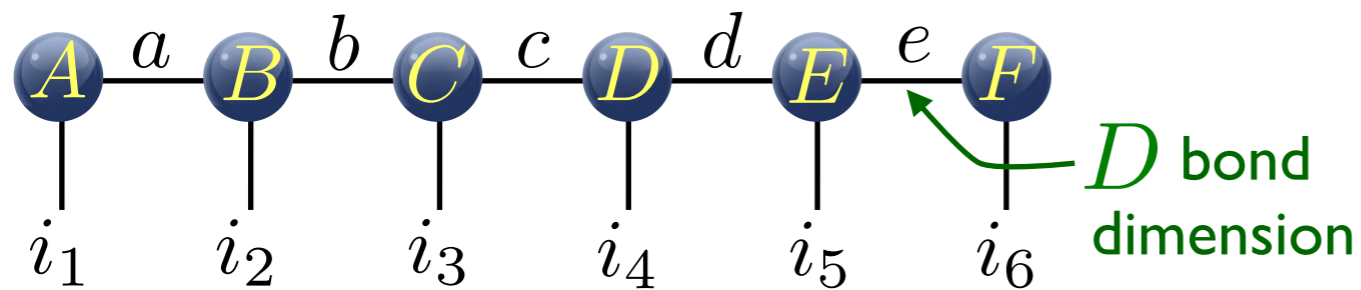
Capability for detailed study of spectral functions in impurity, 1 & 2D.



Weichelsbaum et al (2009)

From Wilsonian NRG to Tensor Networks

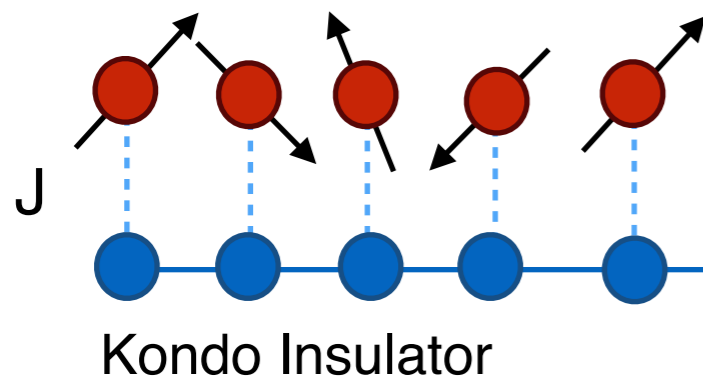
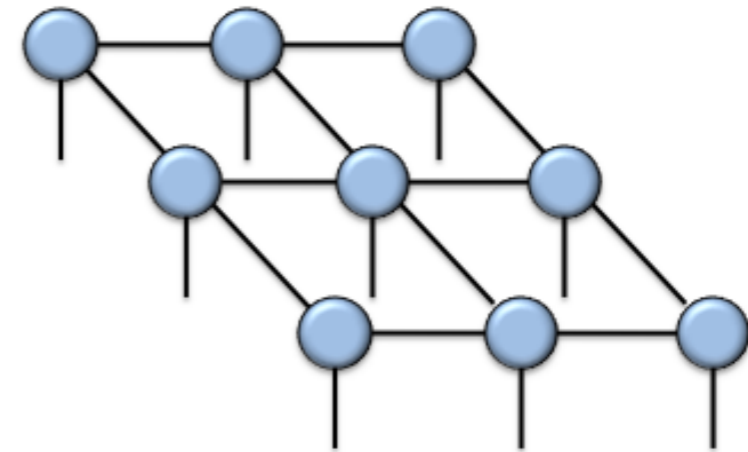
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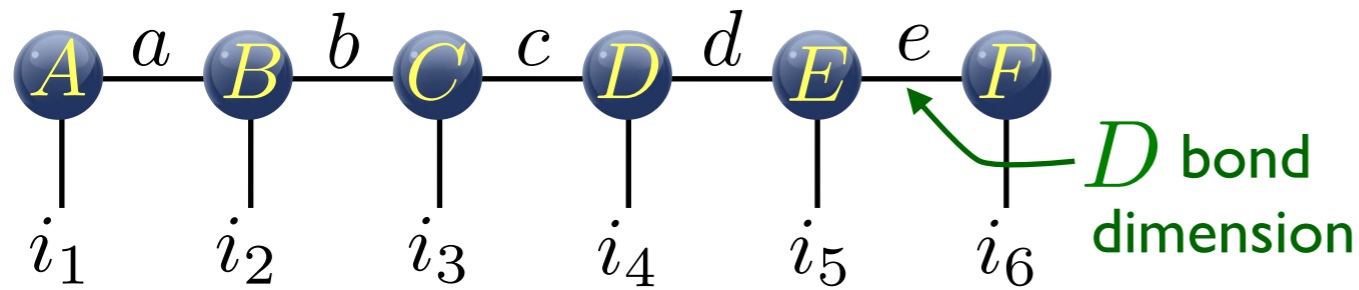
PEPS (2D)

projected entangled-pair state

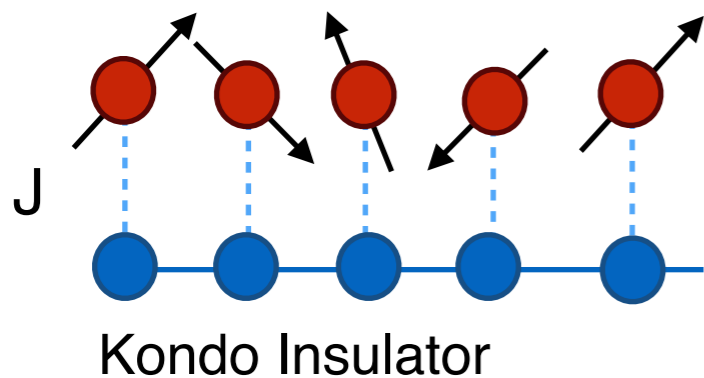


From Wilsonian NRG to Tensor Networks

Tensor network: matrix product state (**MPS**)

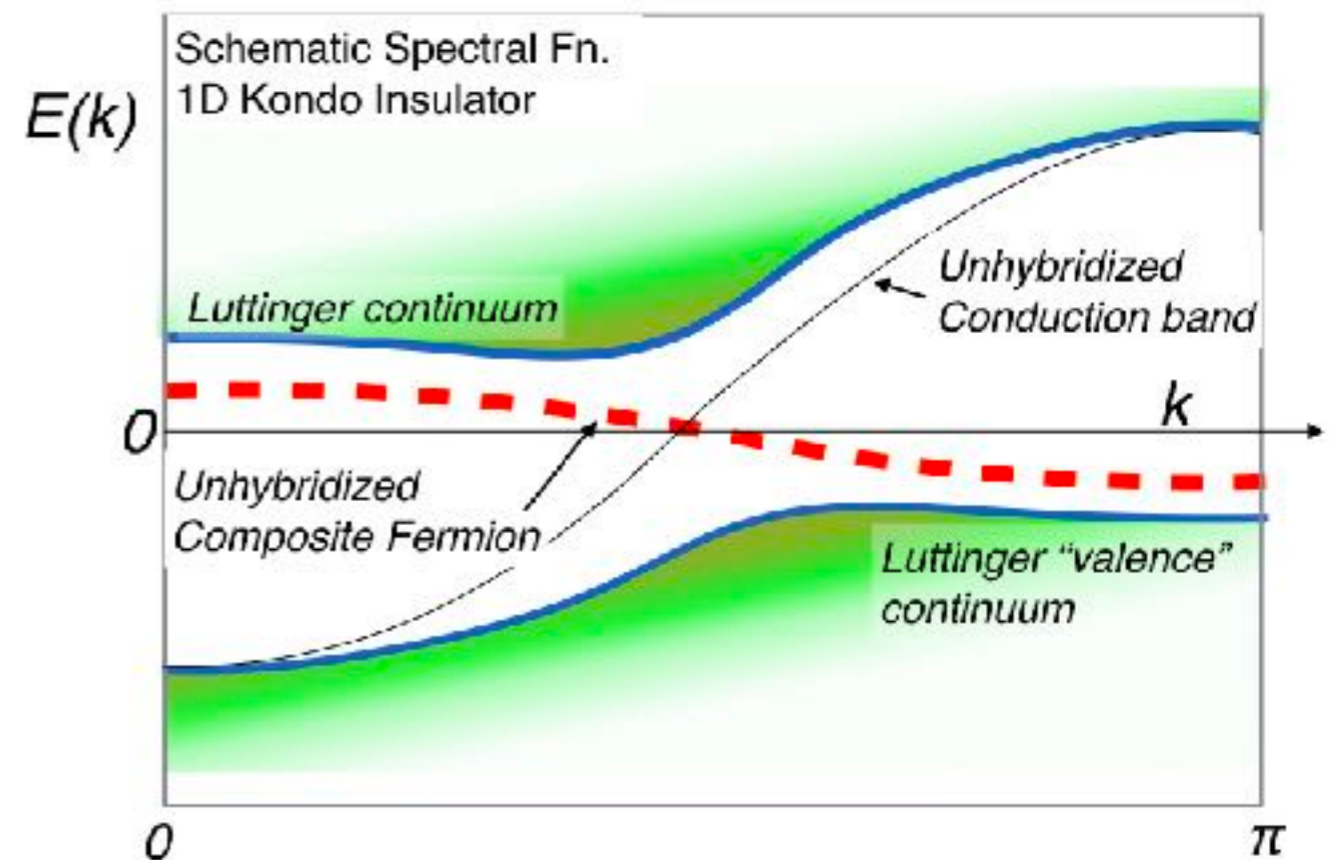
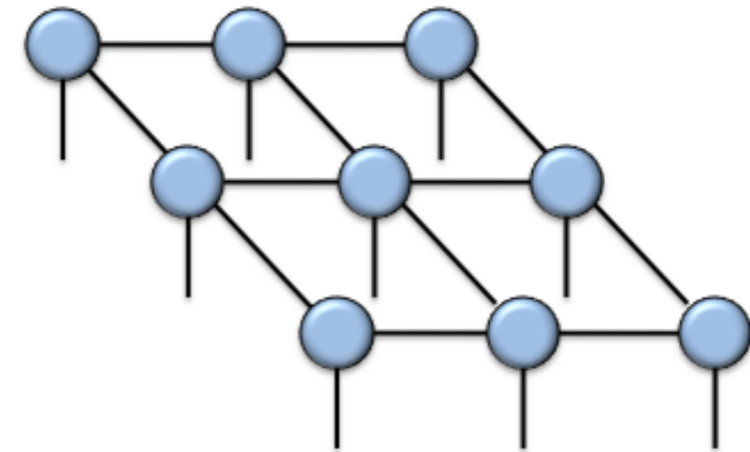


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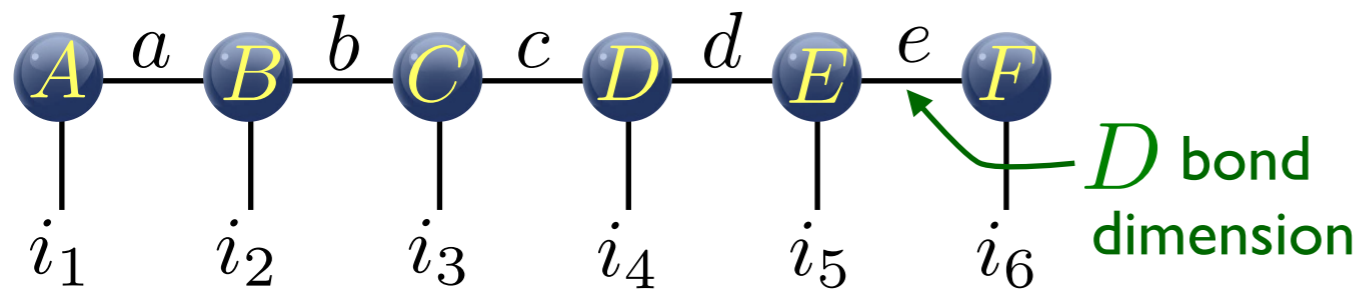
PEPS (2D)

projected entangled-pair state

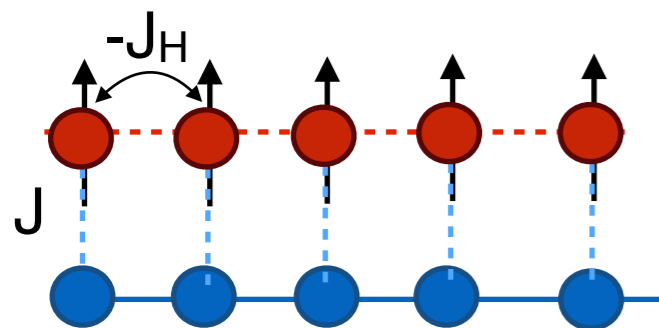


From Wilsonian NRG to Tensor Networks

Tensor network: matrix product state (**MPS**)



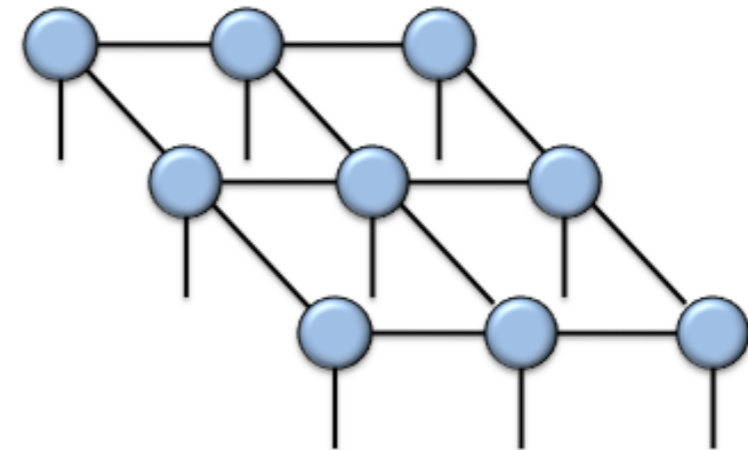
Capability for detailed study of spectral functions in impurity, 1D & 2D



FM Kondo-Heisenberg

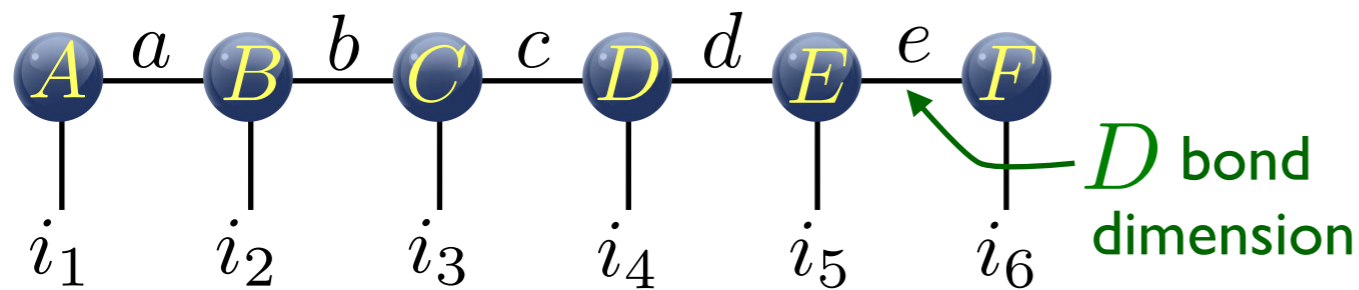
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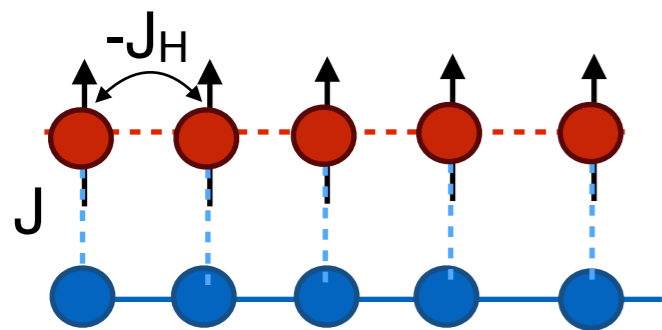


From Wilsonian NRG to Tensor Networks

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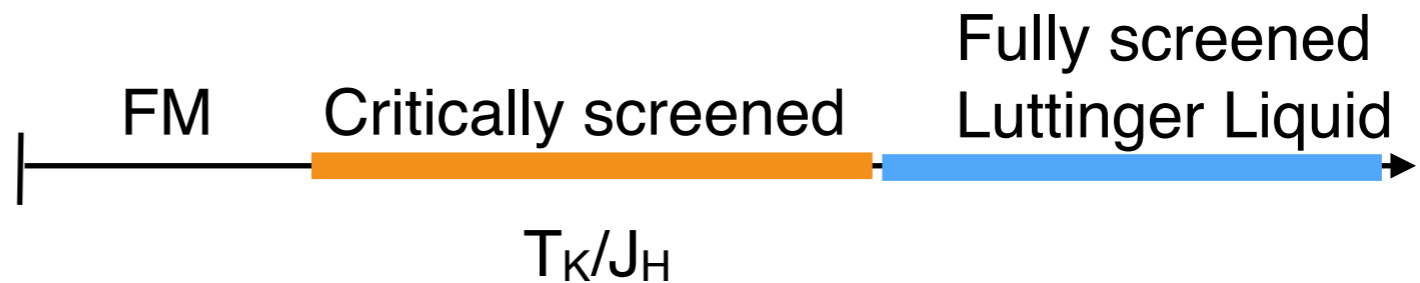
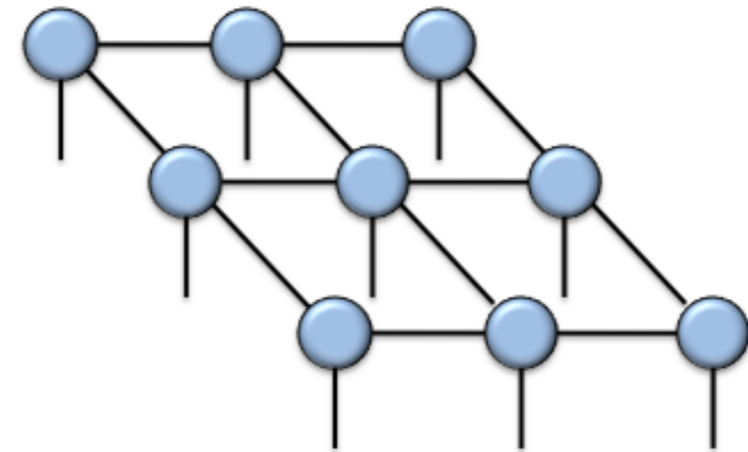
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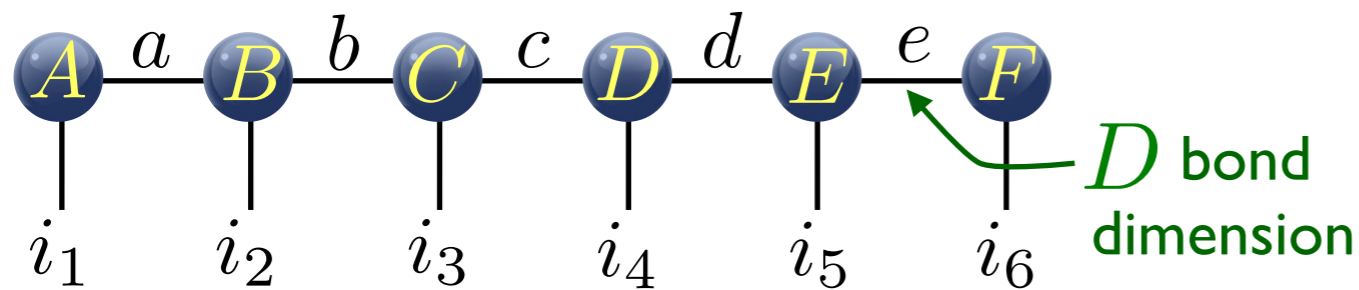
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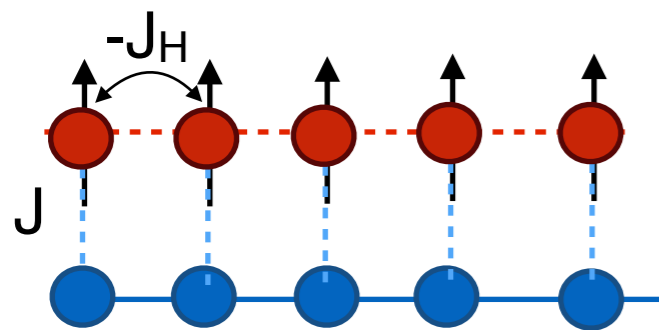
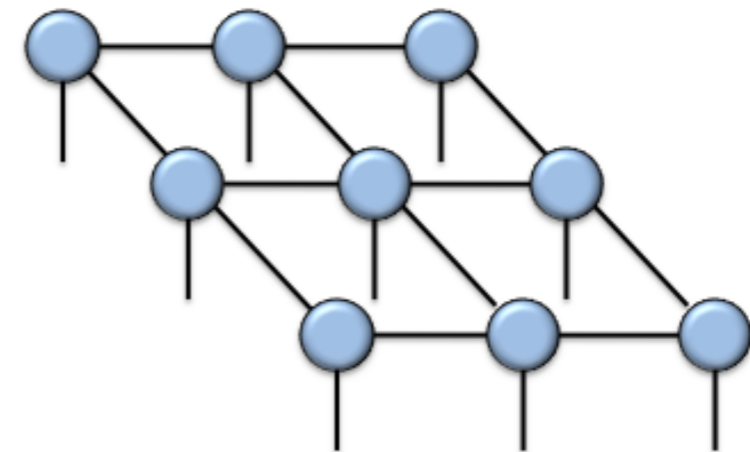
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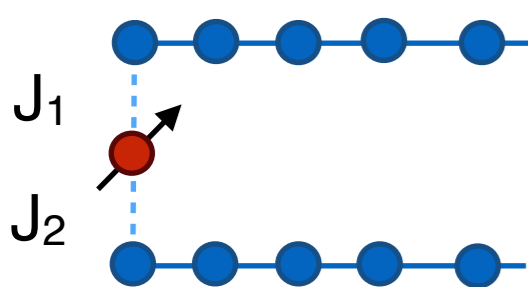
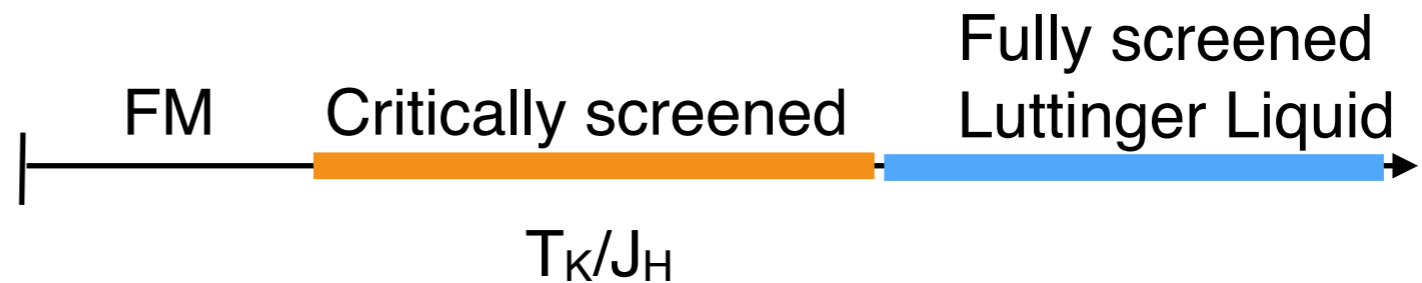
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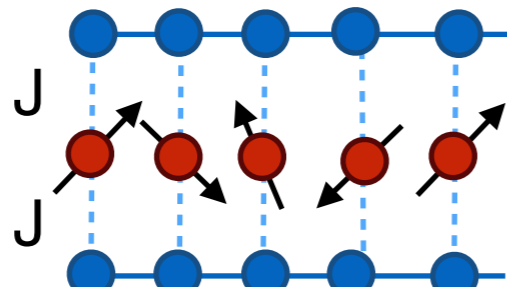
projected entangled-pair state



FM Kondo-Heisenberg

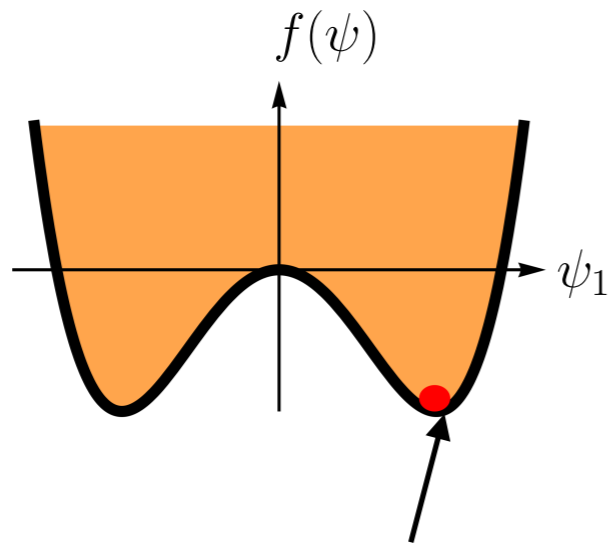


2-channel Kondo



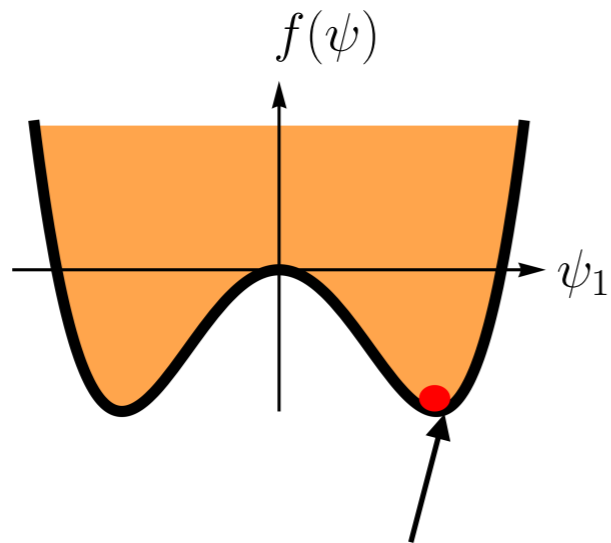
2-channel Kondo Lattice

Beyond Hartree Fock/BCS



Order parameter ψ

Beyond Hartree Fock/BCS



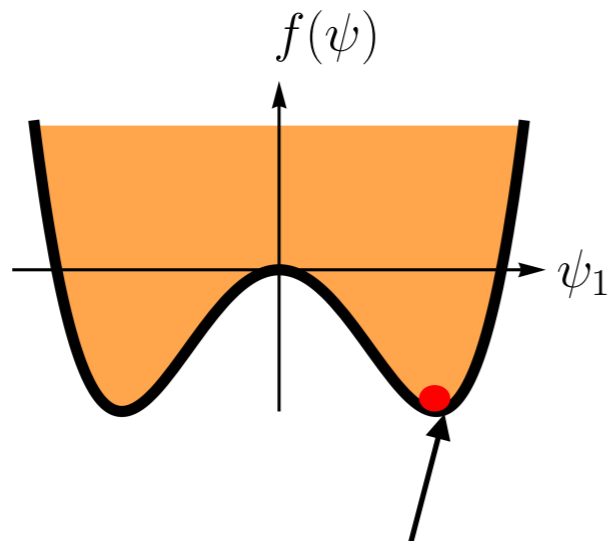
Order parameter ψ

- Order Parameters are bosons, and must contain an even number of fermions: even charge, integer spin.

$$\Psi = \langle \hat{\psi}_{\uparrow} \hat{\psi}_{\downarrow} \rangle \quad \vec{M} = \langle \psi^{\dagger} \vec{\sigma} \psi \rangle$$

cf BCS theory, Stoner Magnetism.

Beyond Hartree Fock/BCS



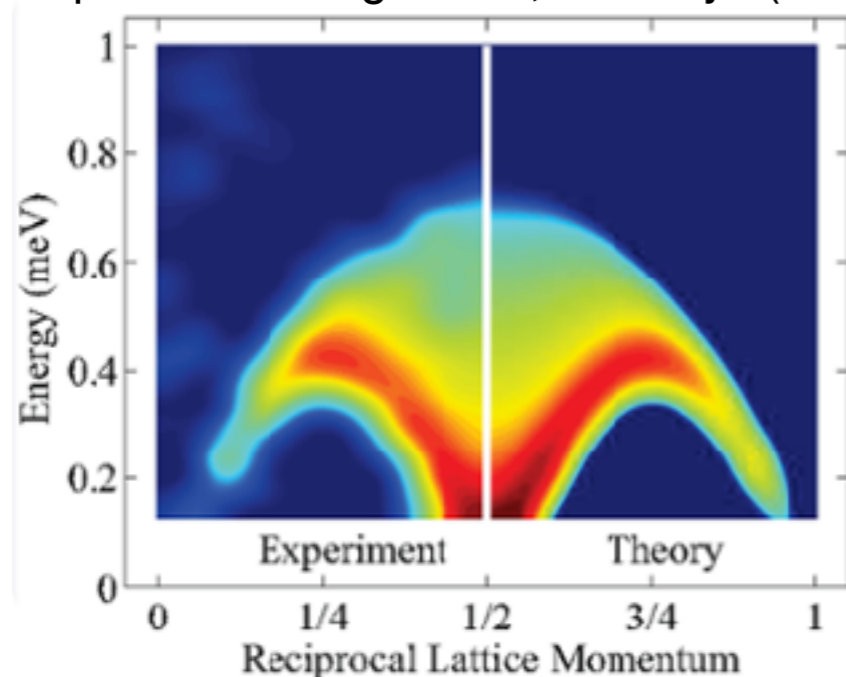
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Spinons: Mourigal et al, Nat Phys (2013)

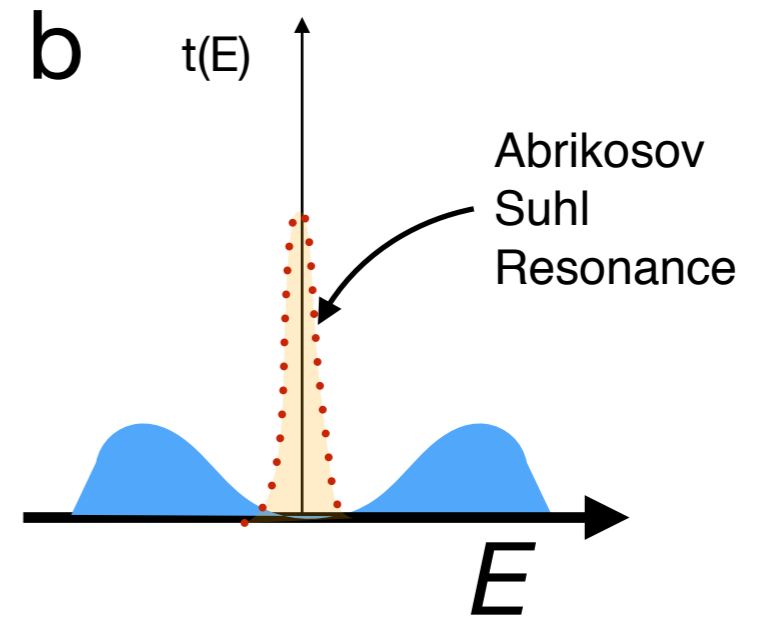
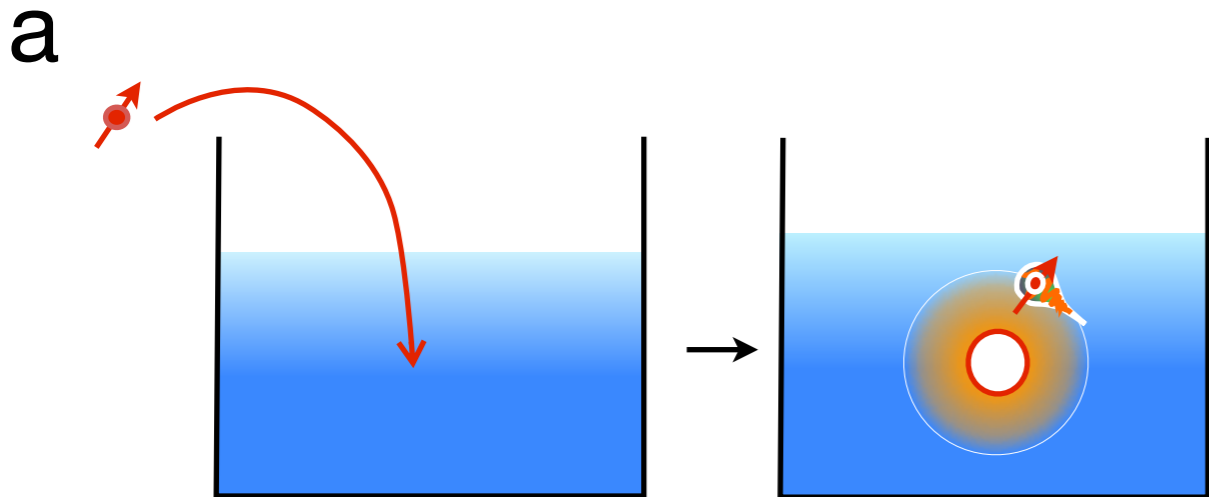


- But can order parameters, like excitations, fractionalize?

$$\psi = \sqrt{\text{Multipole}} = \text{Spinorial OP}$$

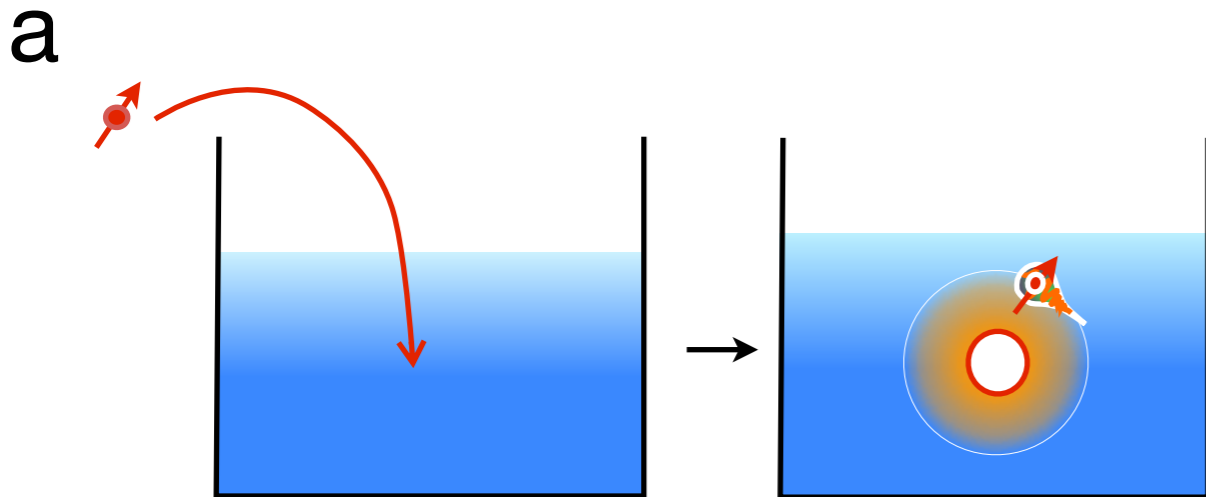
P. Chandra, P. Coleman, R. Flint, Nature (2013)

Beyond Hartree Fock/BCS: 3 body Bound States

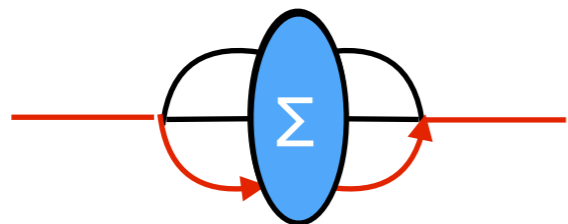


$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} + J \psi_0^{\dagger} \vec{\sigma} \psi_0 \cdot \vec{S}_0$$

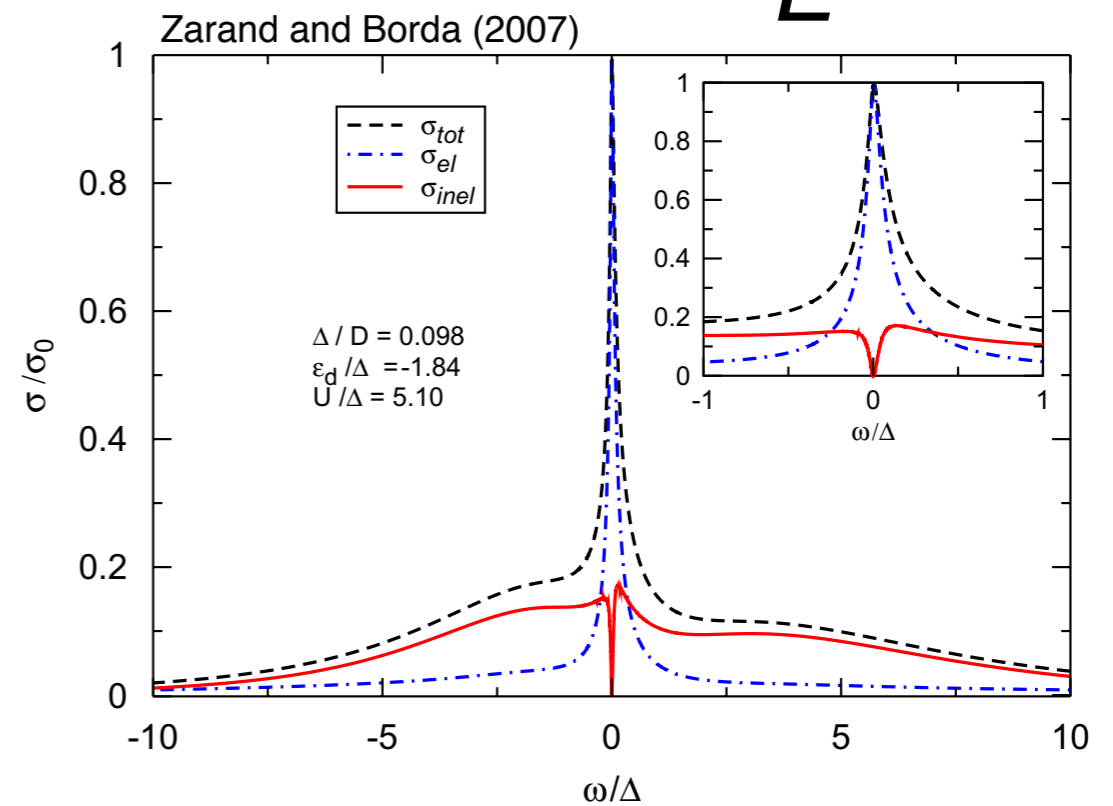
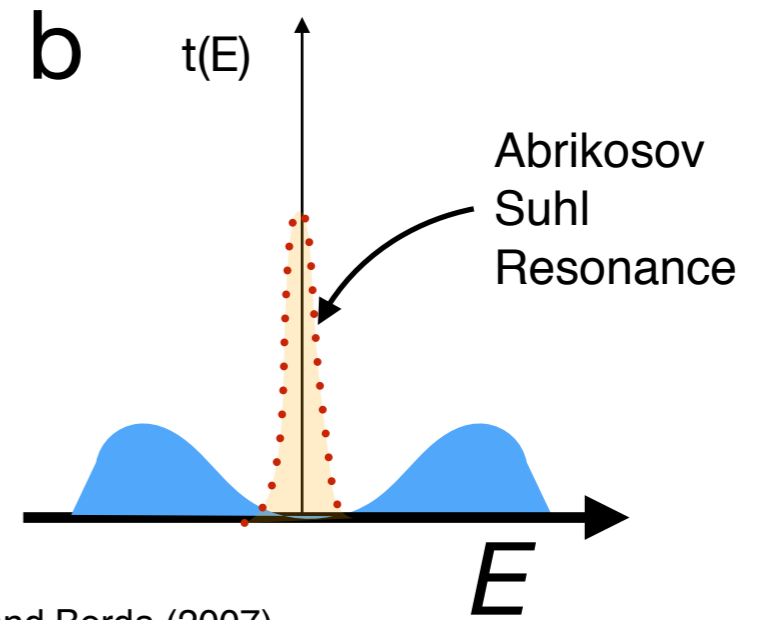
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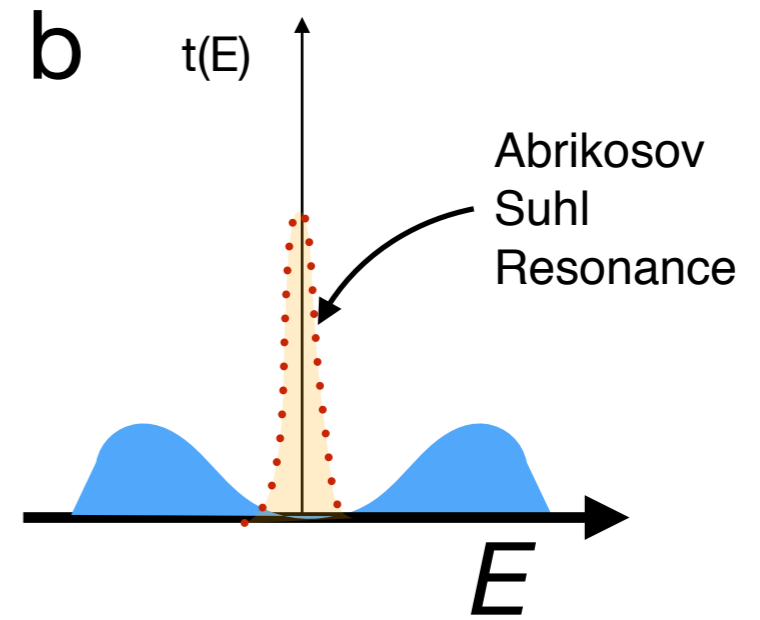
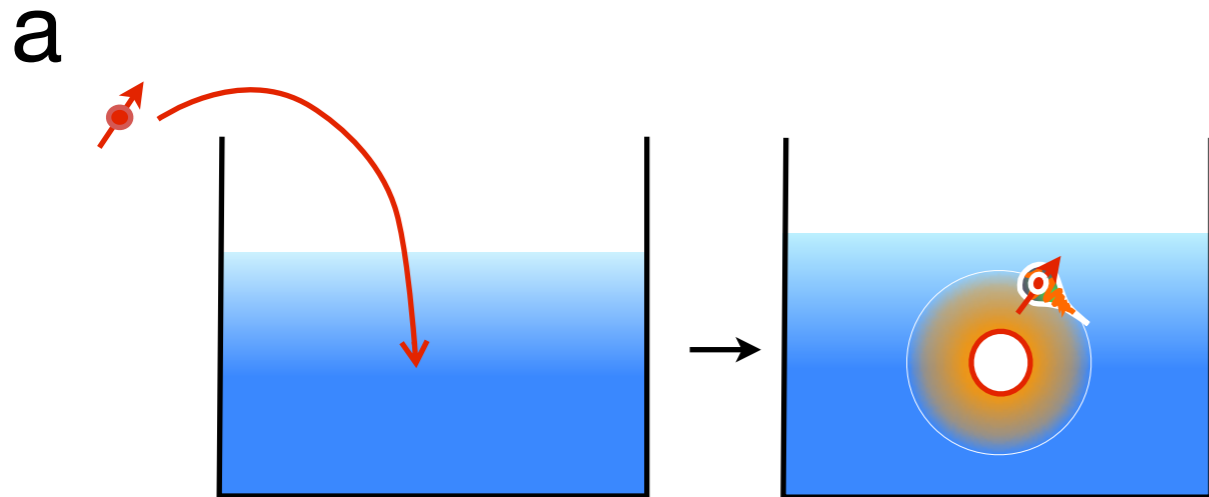
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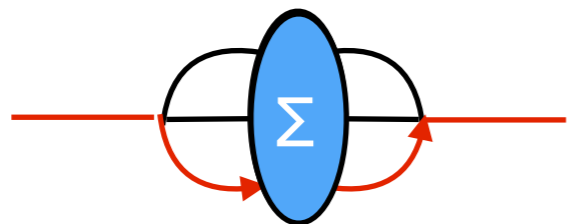
Irreducible t-matrix
 $t(E) = \Sigma(E) + t(E)G_0(E)\Sigma(E)$



Beyond Hartree Fock/BCS: 3 body Bound States



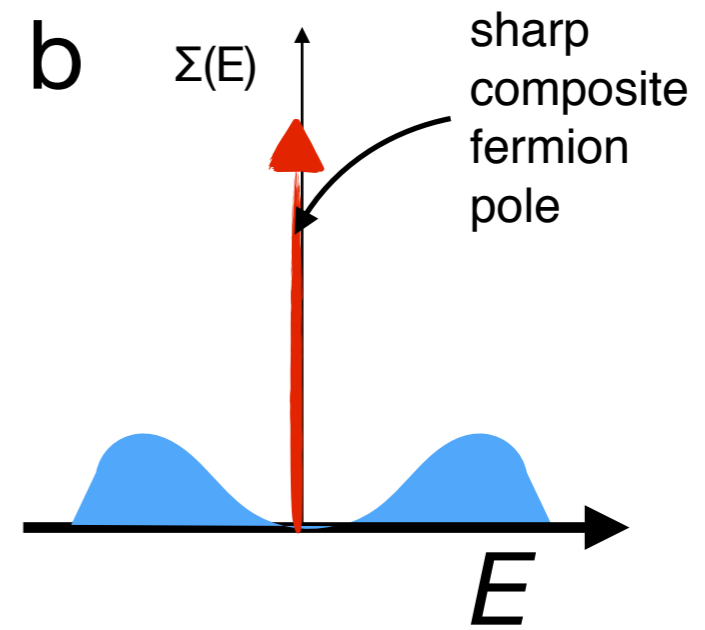
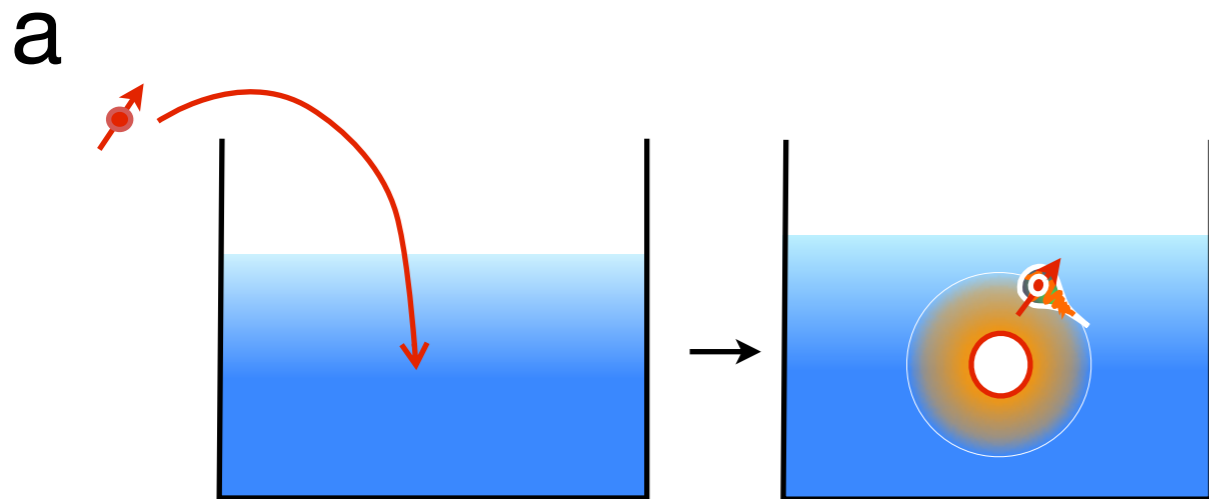
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$$\Sigma(\omega) \sim \frac{V^2}{\omega}$$

Beyond Hartree Fock/BCS: 3 body Bound States



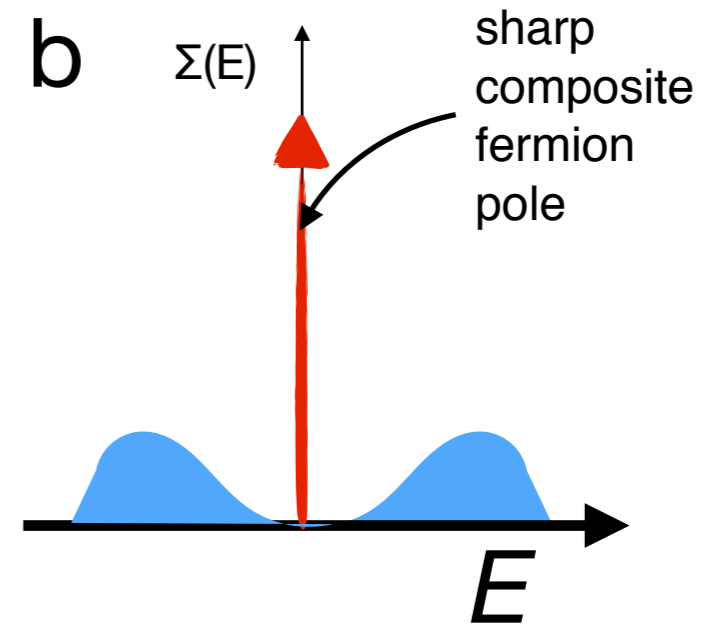
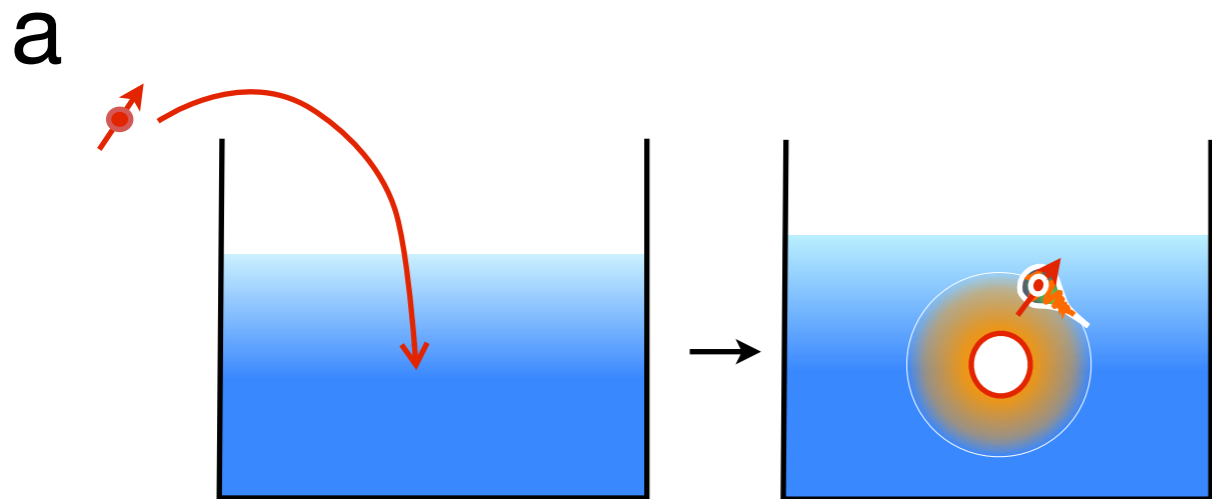
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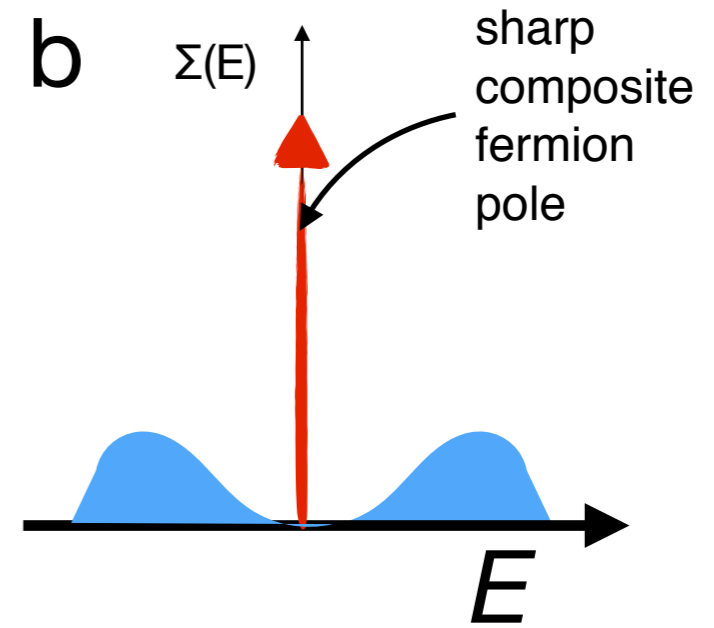
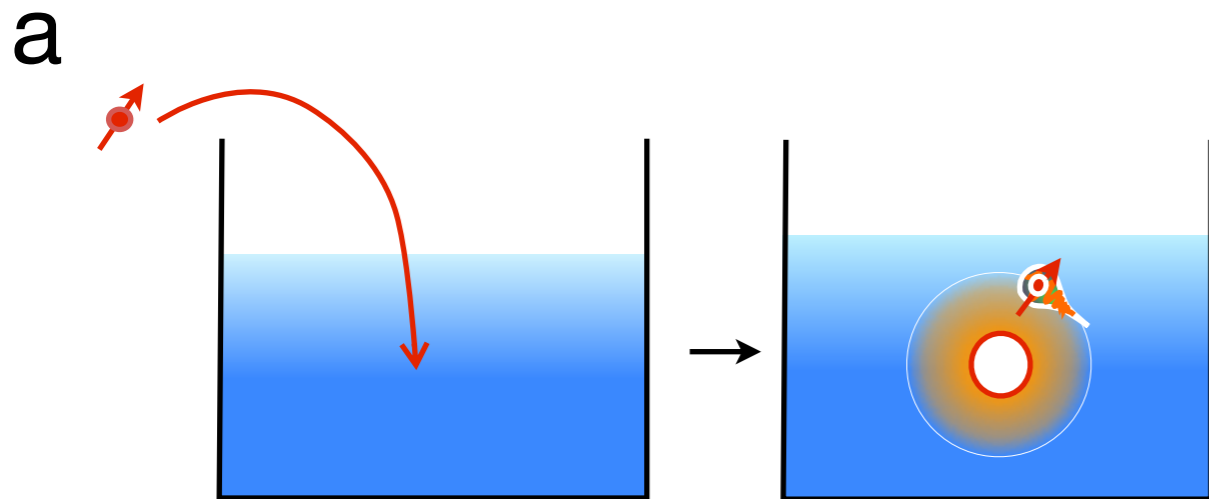


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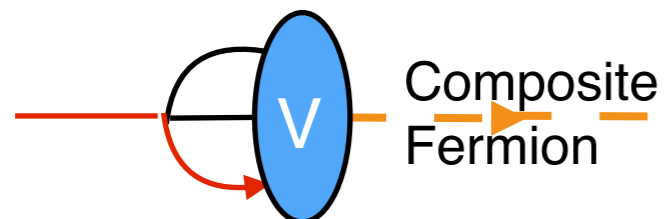
Beyond Hartree Fock/BCS: 3 body Bound States



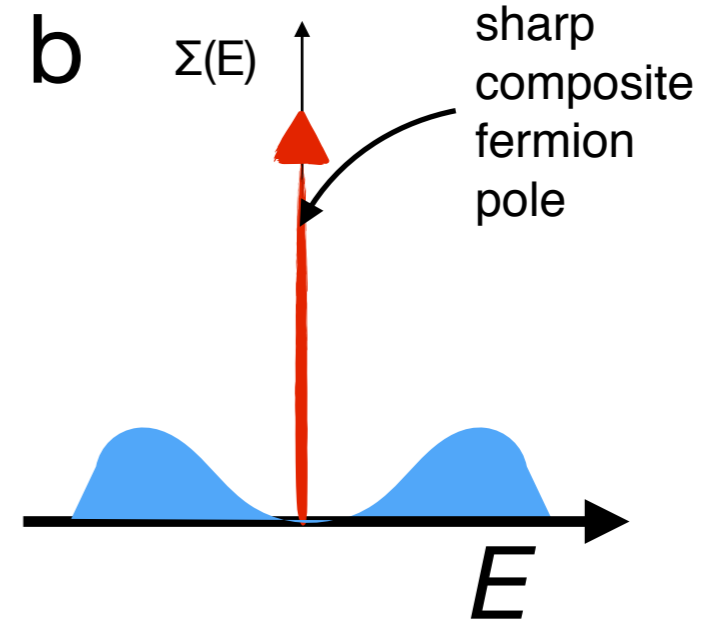
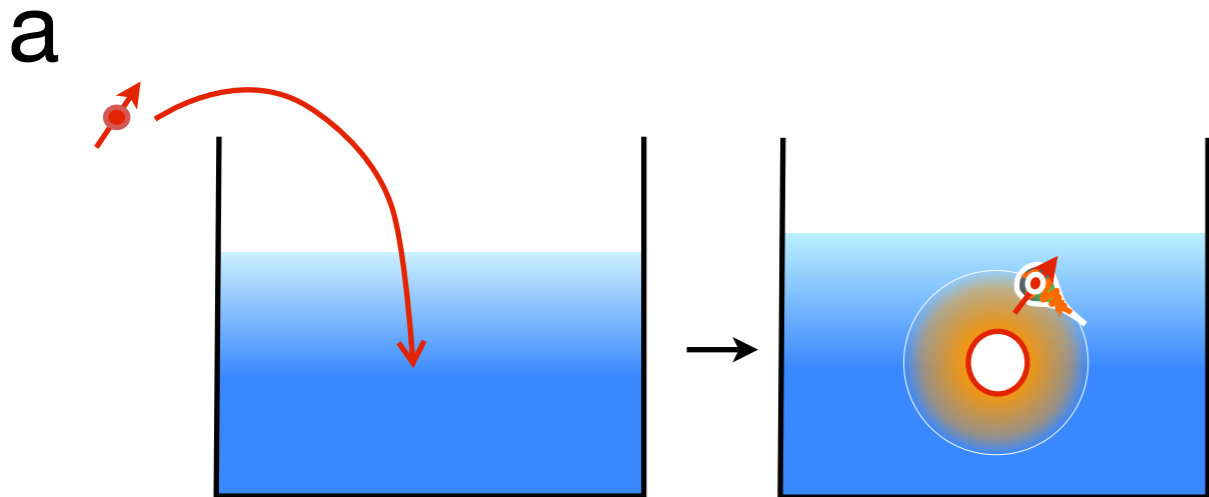
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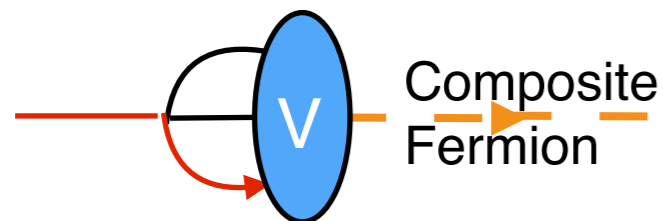
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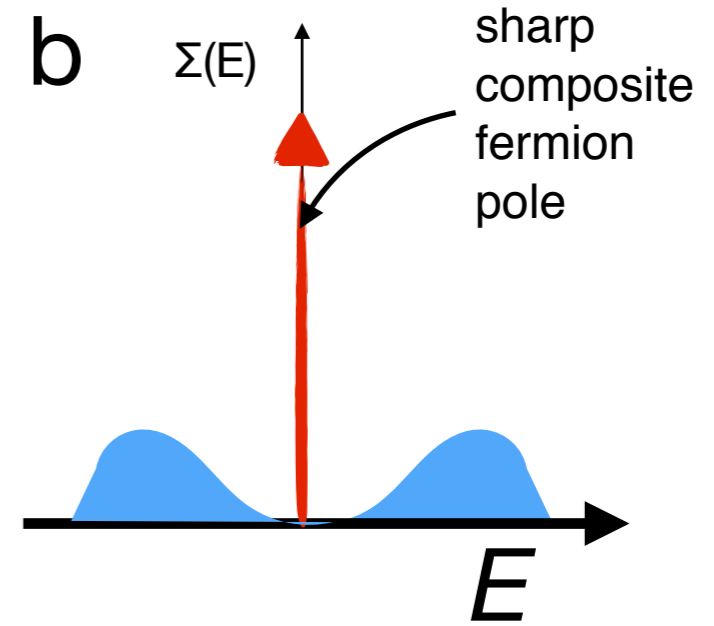
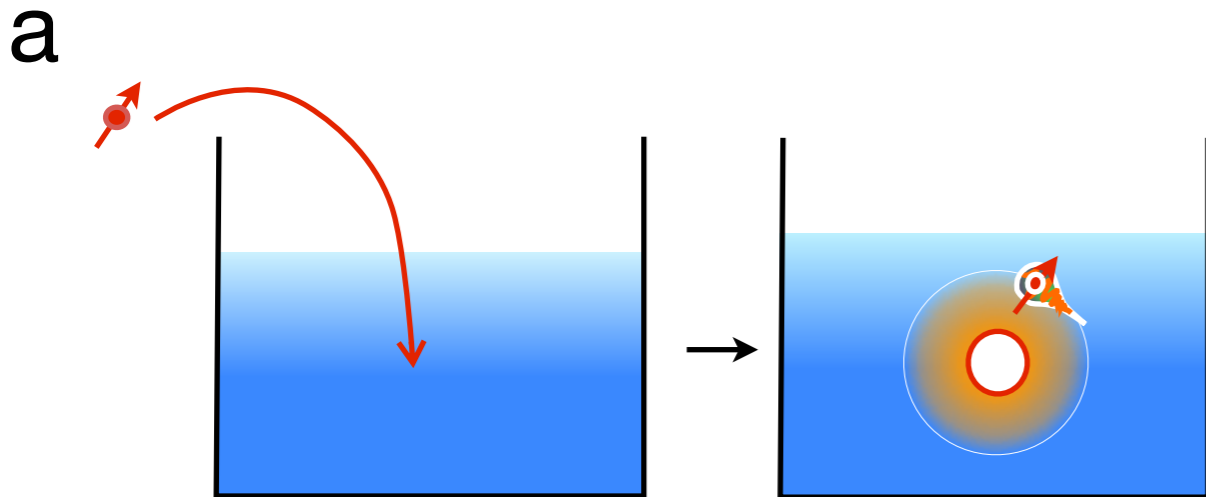
$$\Sigma(\omega) \sim \frac{V^2}{\omega}$$



$$\mathcal{F}_\alpha = J(\vec{\sigma} \cdot \vec{S}_0) \psi_{0\alpha} \rightarrow V f_\alpha(0)$$

Read and Newns, 1983
Three-body Bound State

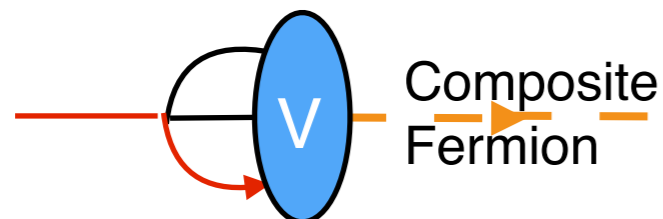
Beyond Hartree Fock/BCS: 3 body Bound States



$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} + V(\psi_j^\dagger f_j + \text{H.c.})$$



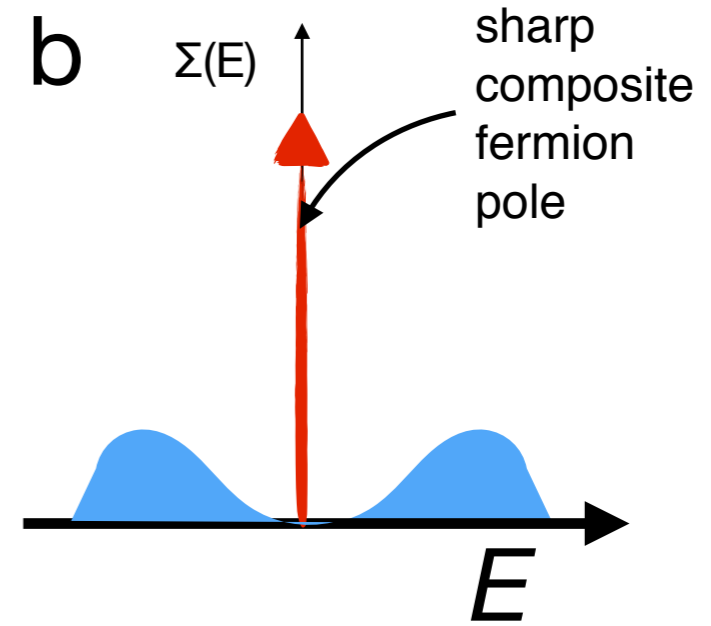
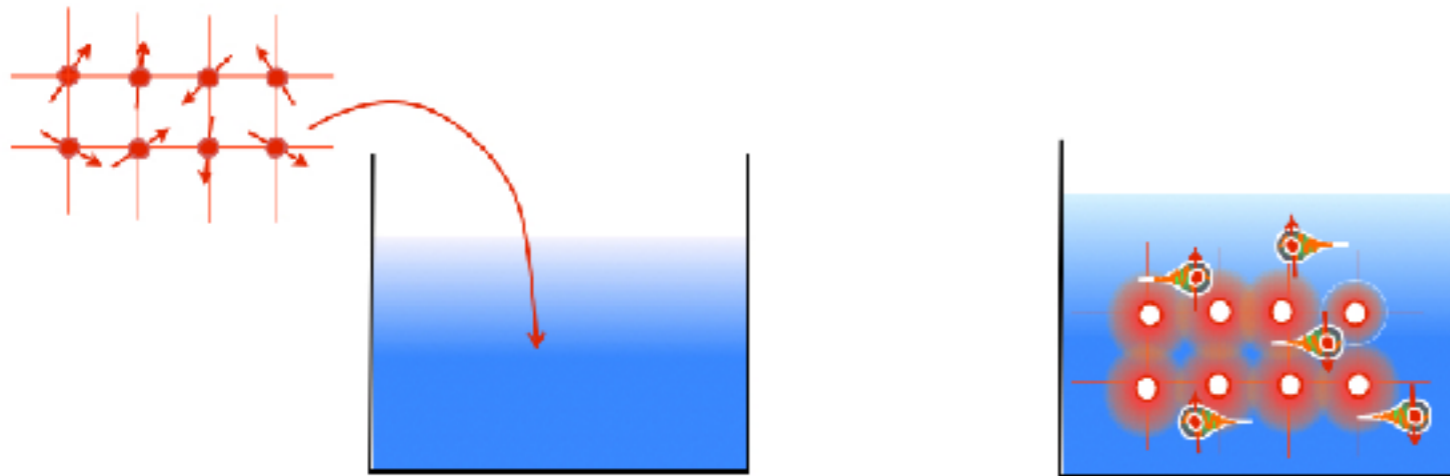
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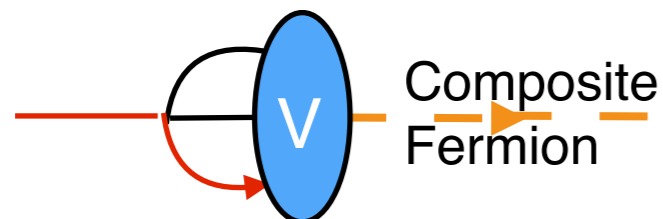
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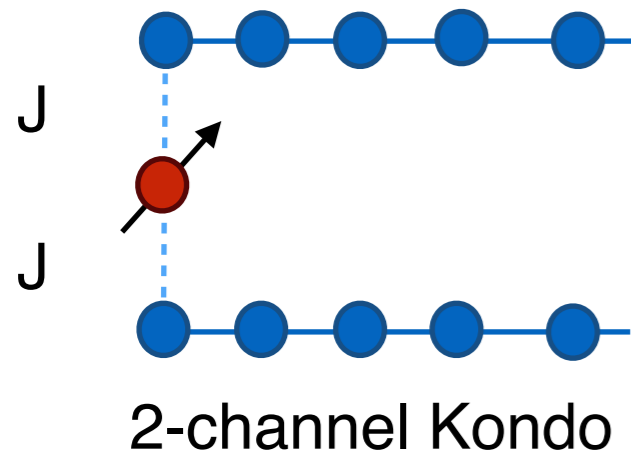


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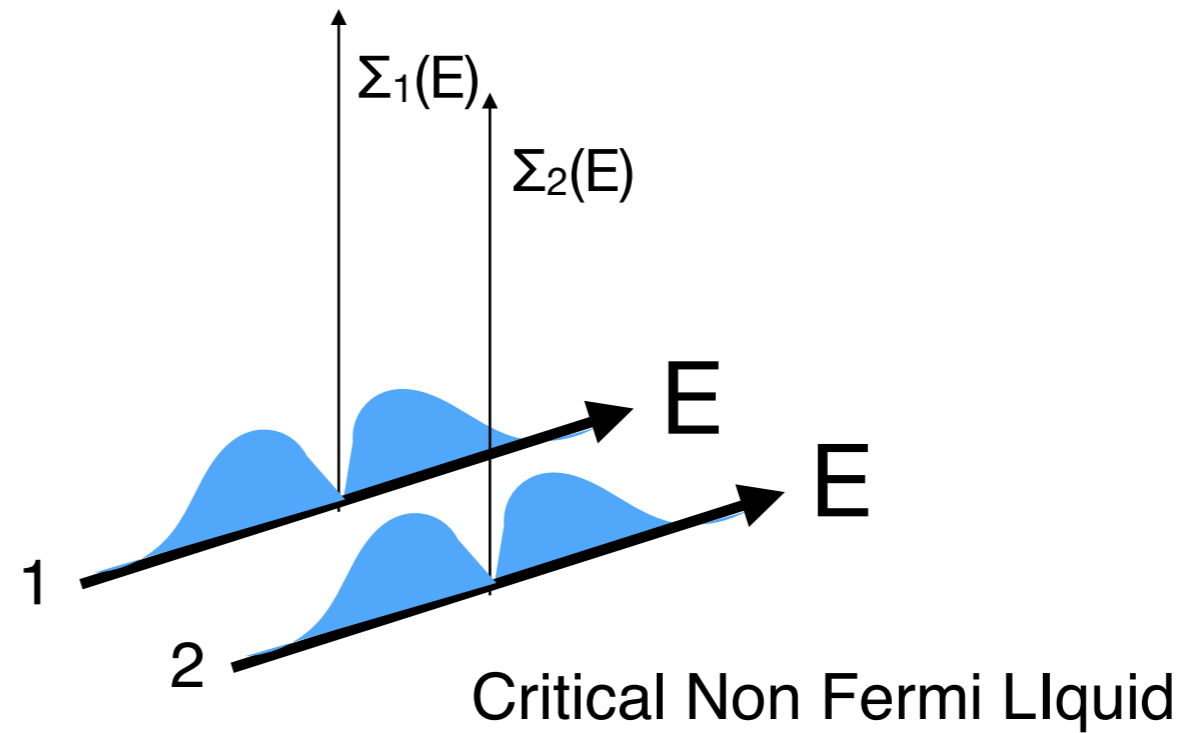
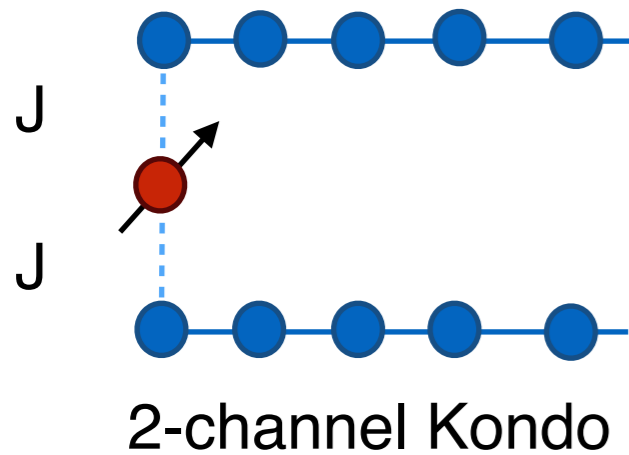
Order Parameter Fractionalization Hypothesis

P. Chandra, P. Coleman, Y. Komijani



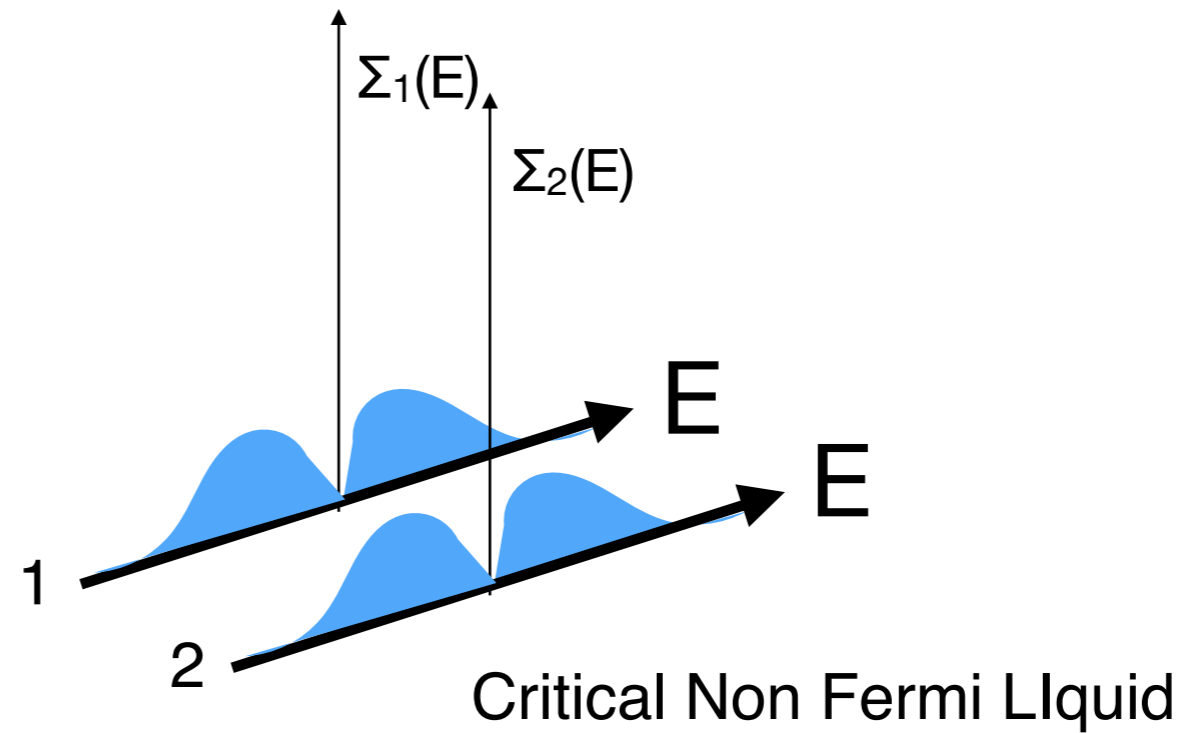
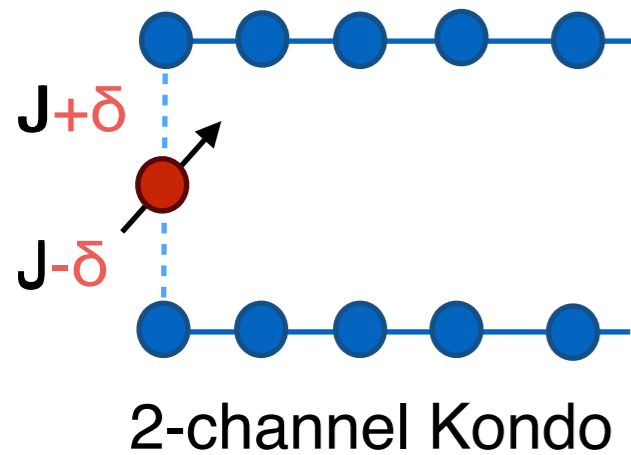
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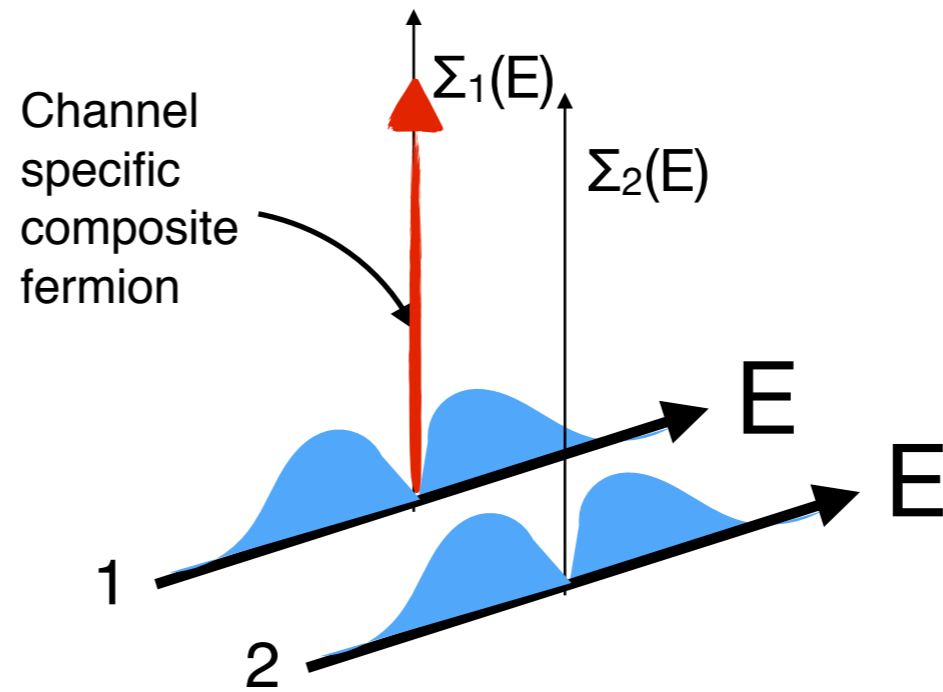
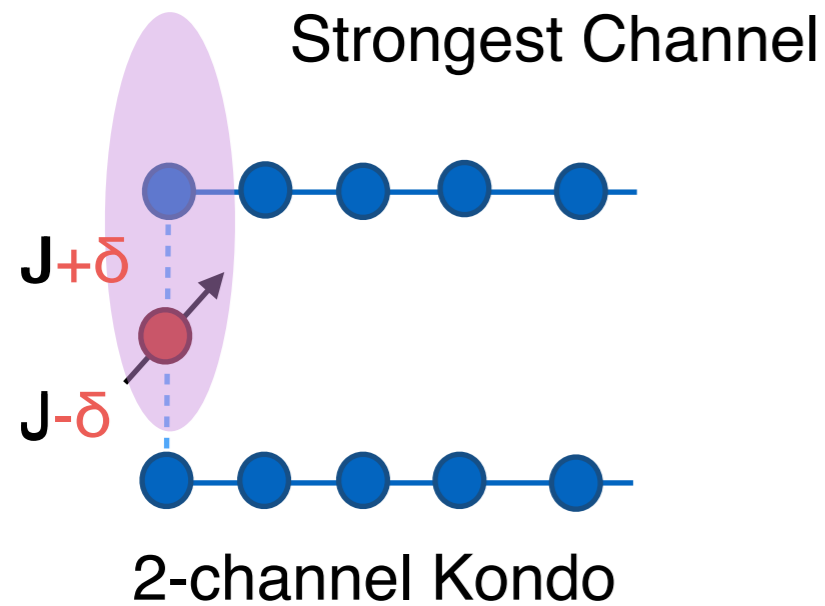
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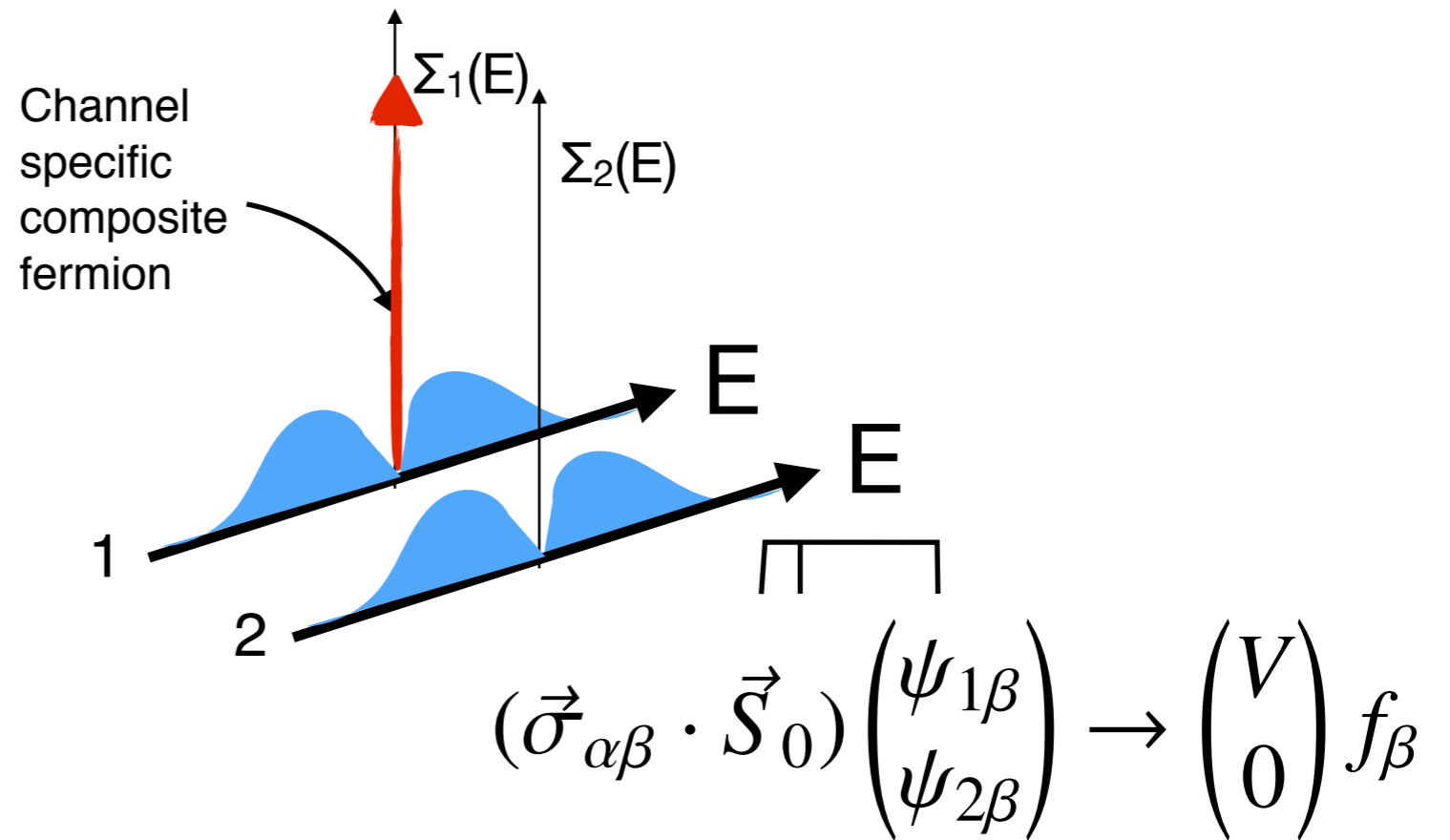
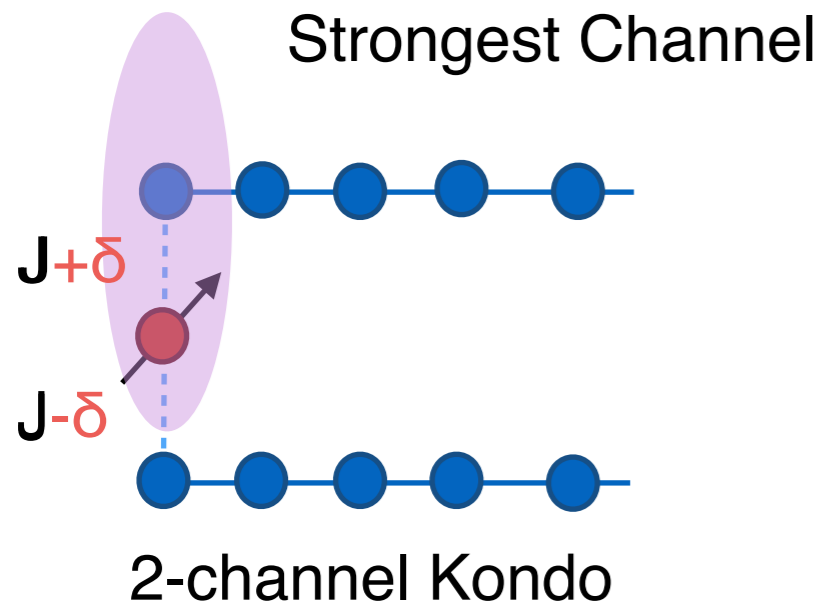
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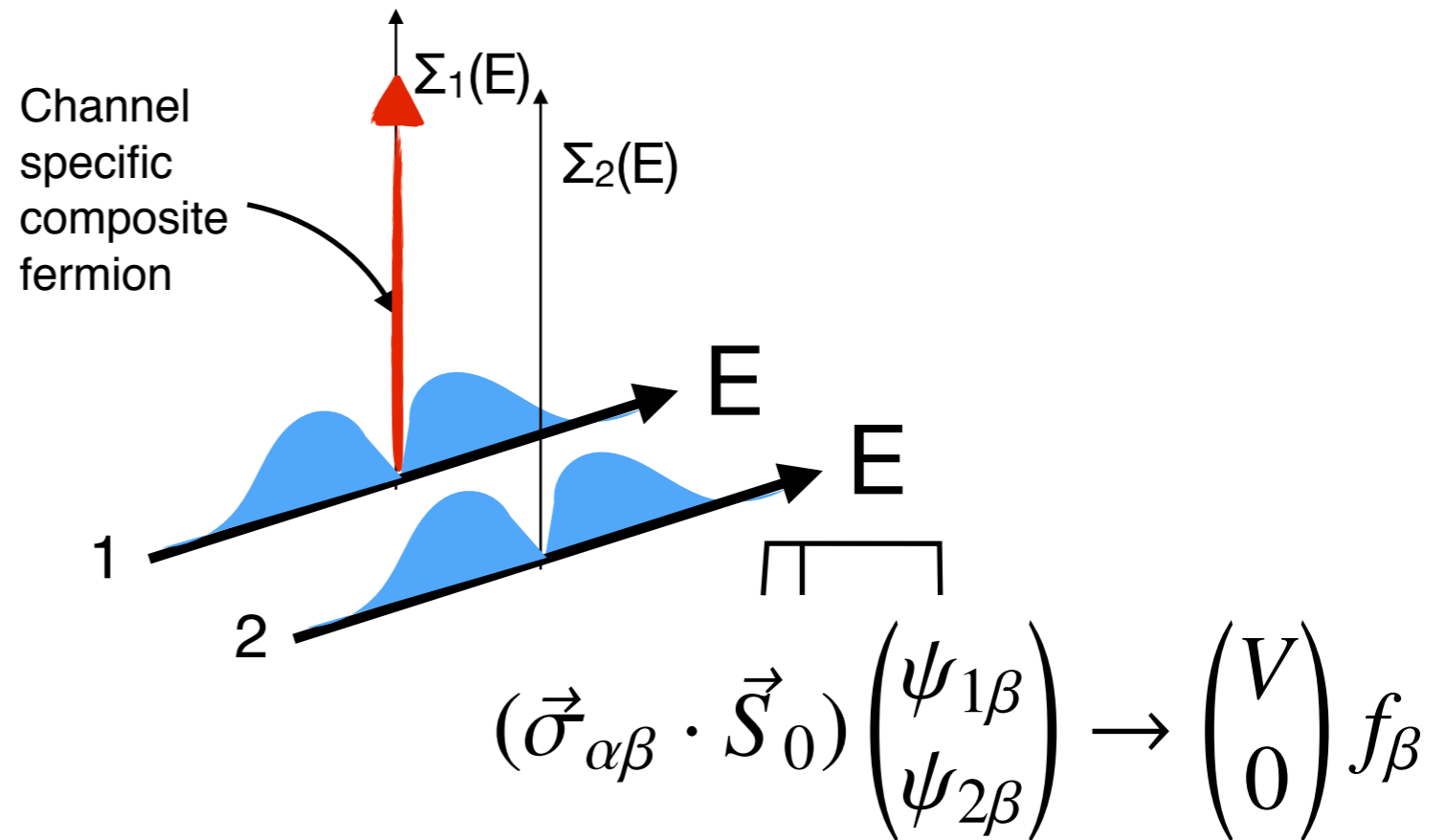
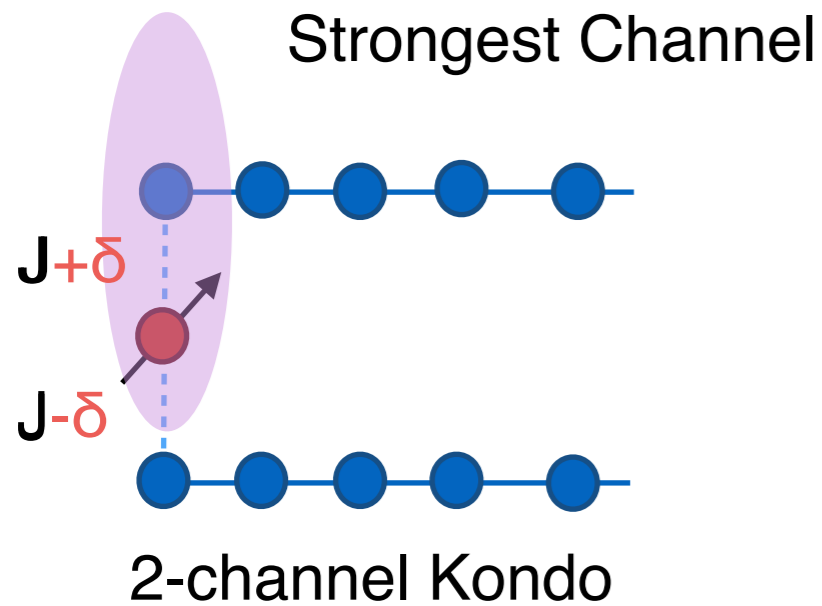
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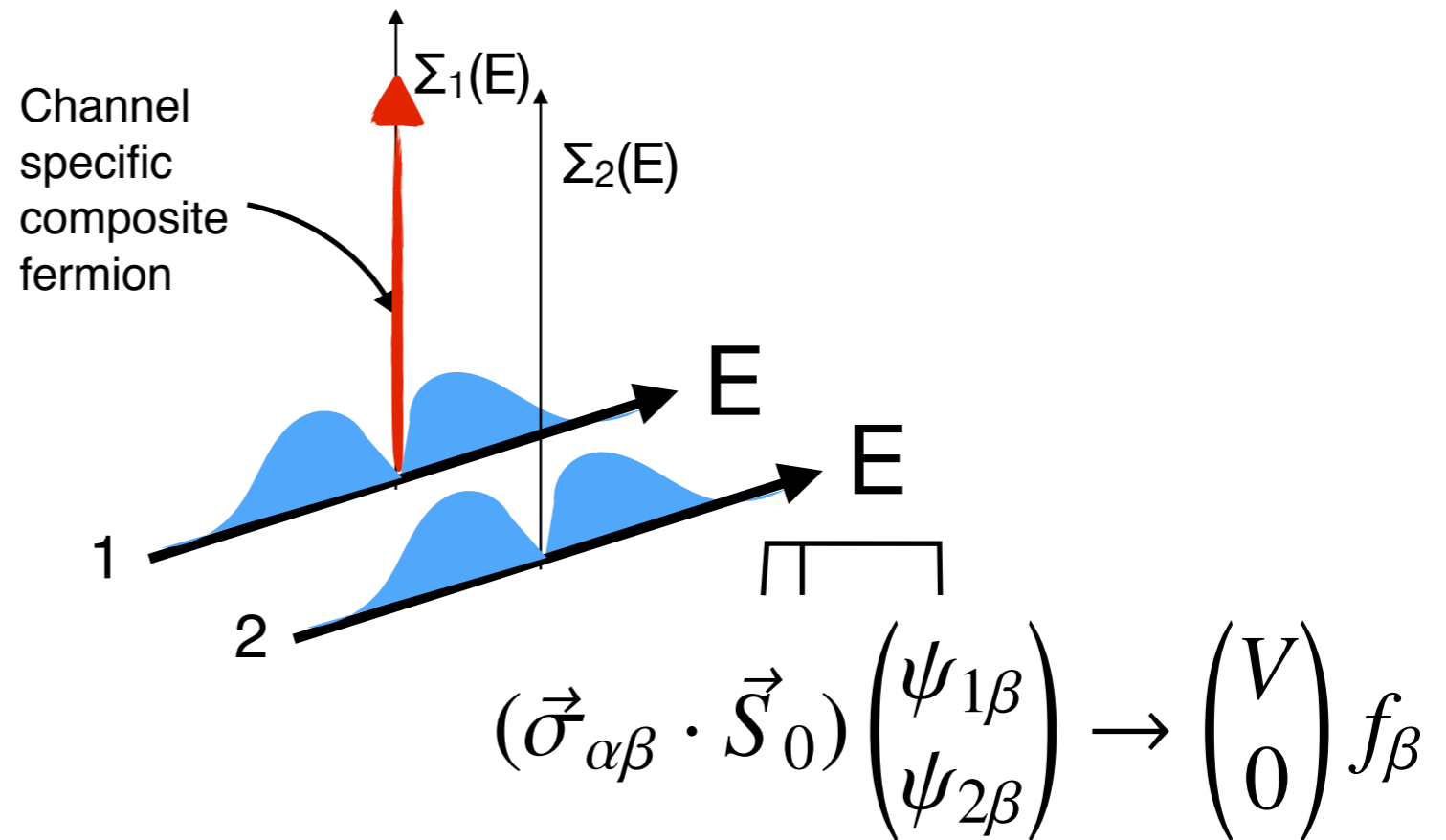
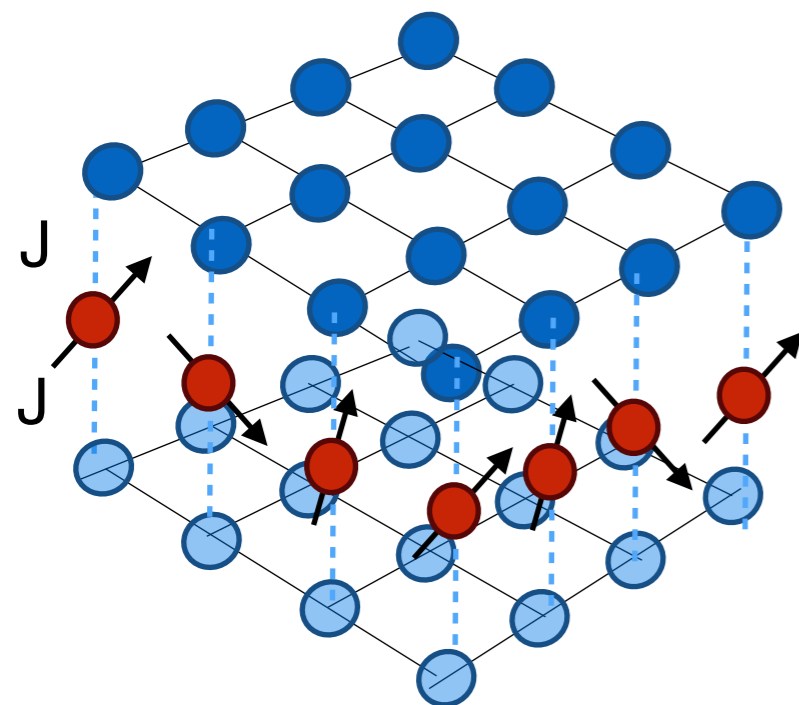
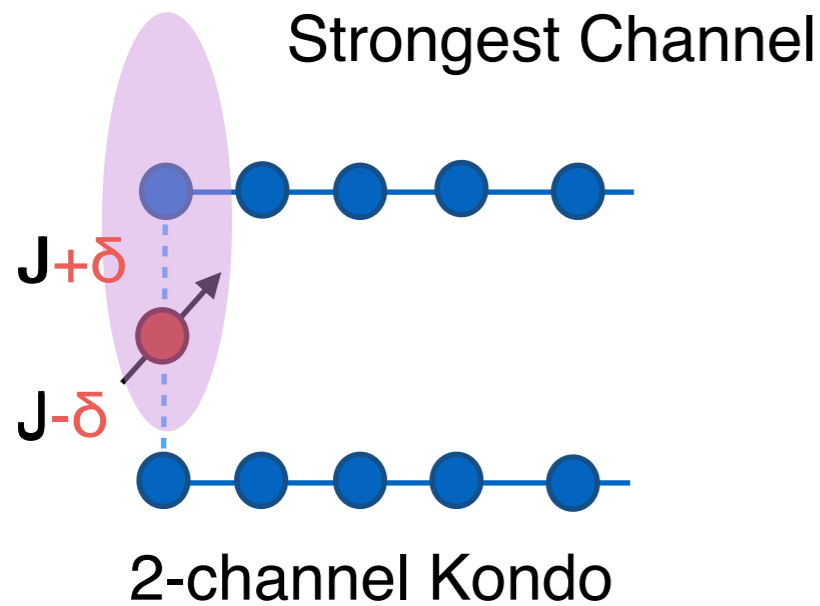
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Hybridization Weiss Field

Order Parameter Fractionalization Hypothesis

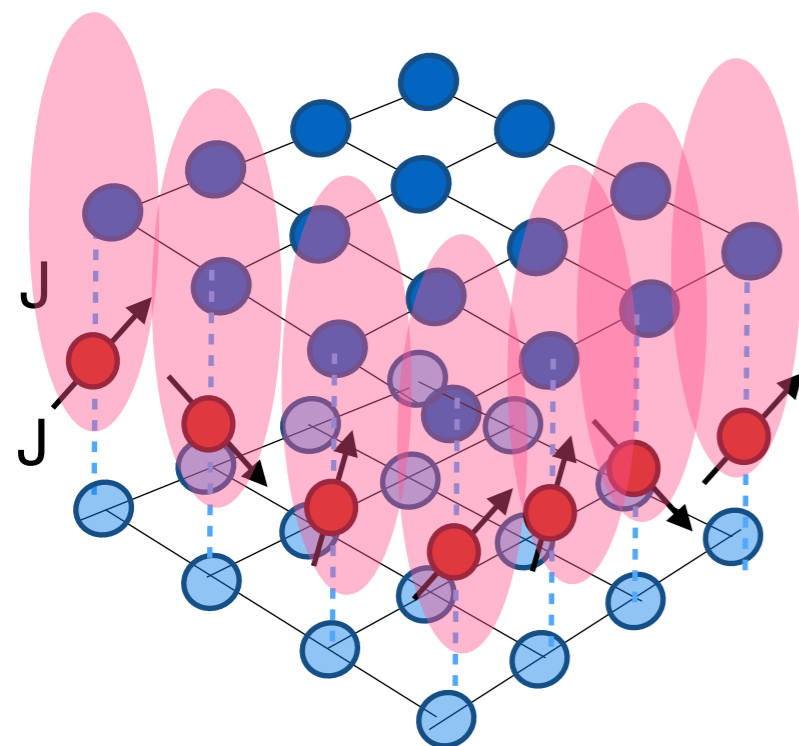
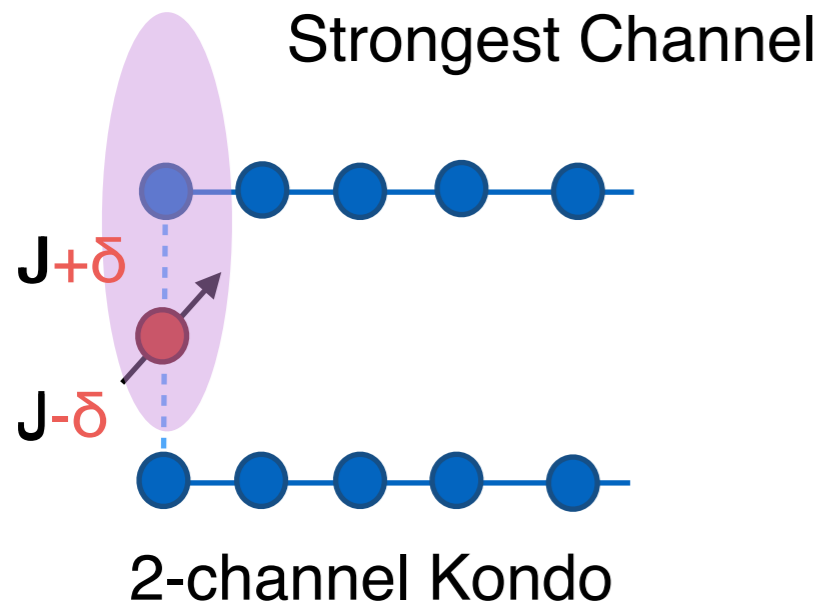
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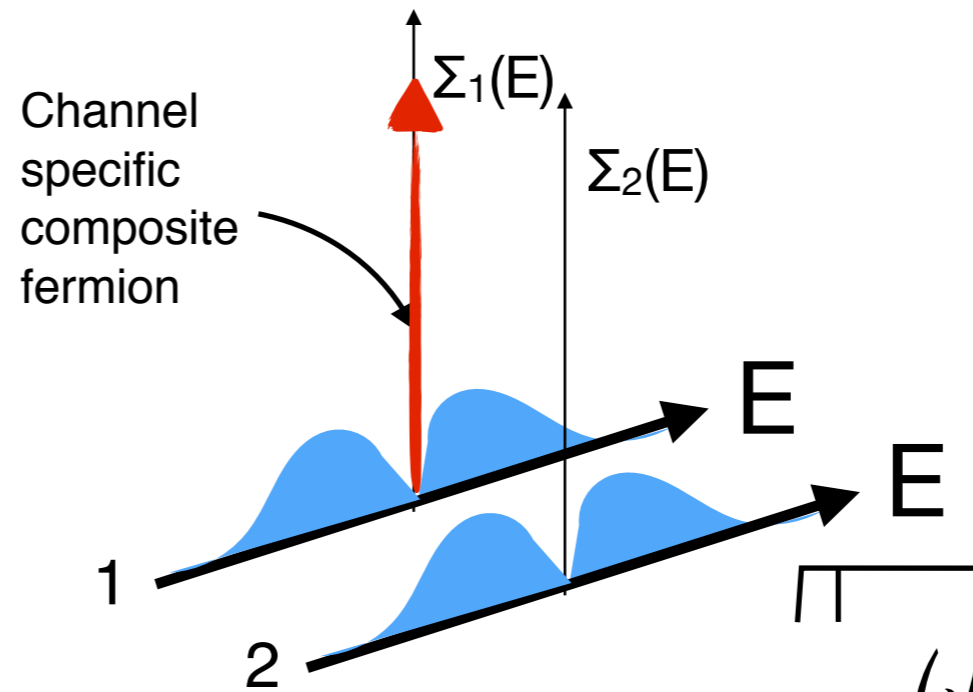
Hybridization Weiss Field

Order Parameter Fractionalization Hypothesis

P. Chandra, P. Coleman, Y. Komijani



2-channel Kondo Lattice



$$(\vec{\sigma}_{\alpha\beta} \cdot \vec{S}_0) \begin{pmatrix} \psi_{1\beta} \\ \psi_{2\beta} \end{pmatrix} \rightarrow \begin{pmatrix} V \\ 0 \end{pmatrix} f_{\beta}$$

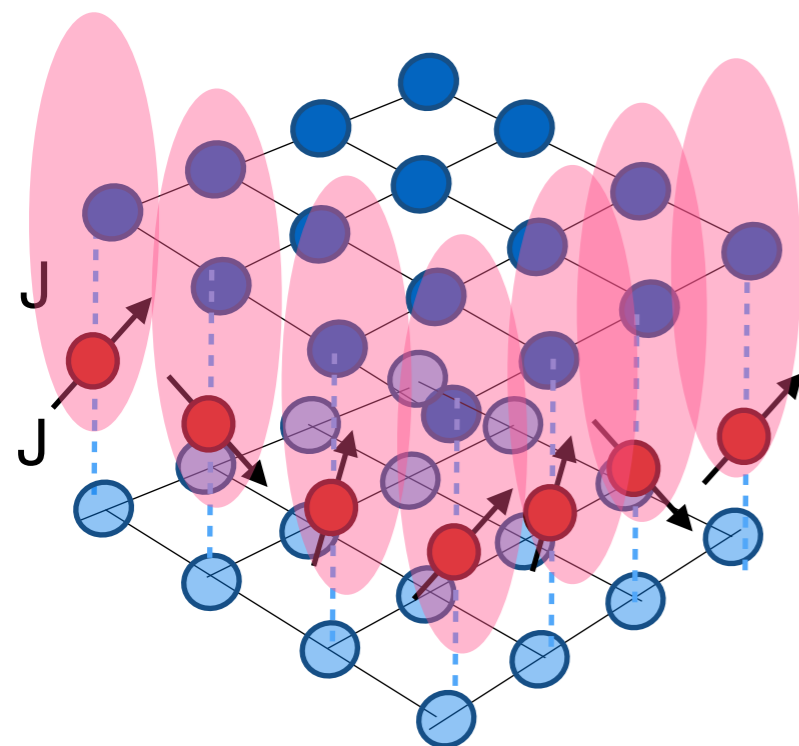
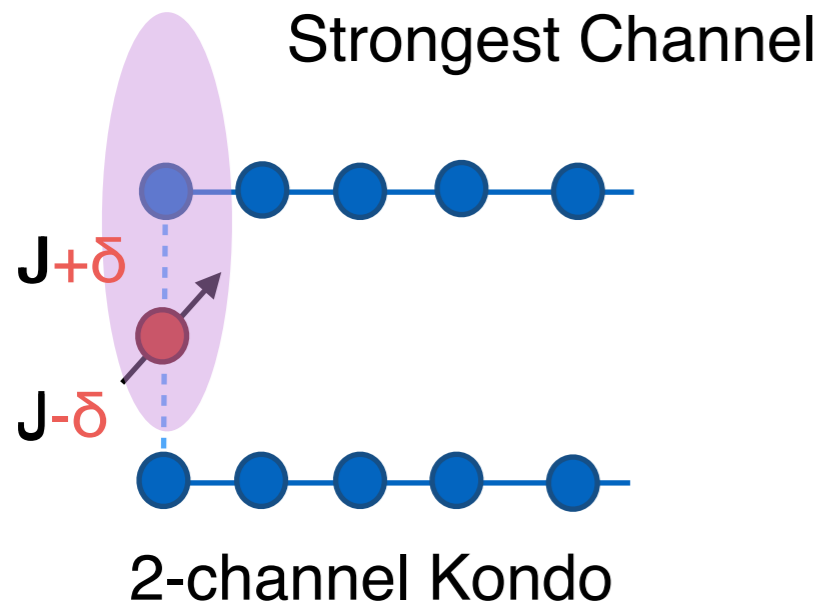
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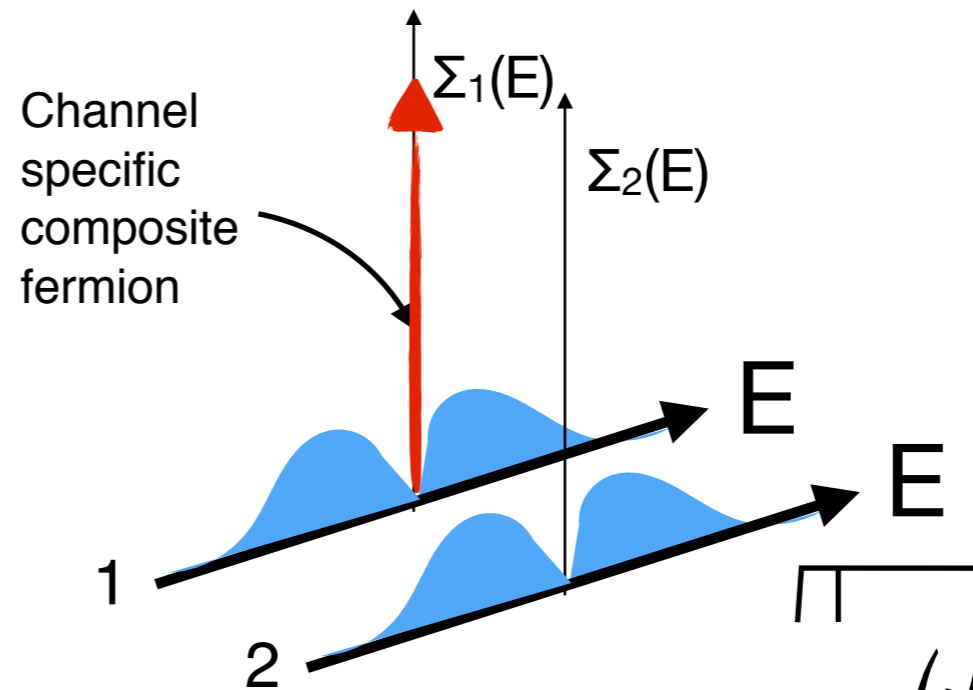
Spinor OP Forms Spontaneously

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2-channel Kondo Lattice



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Fractionalization of Bound-State into Fermion+OP

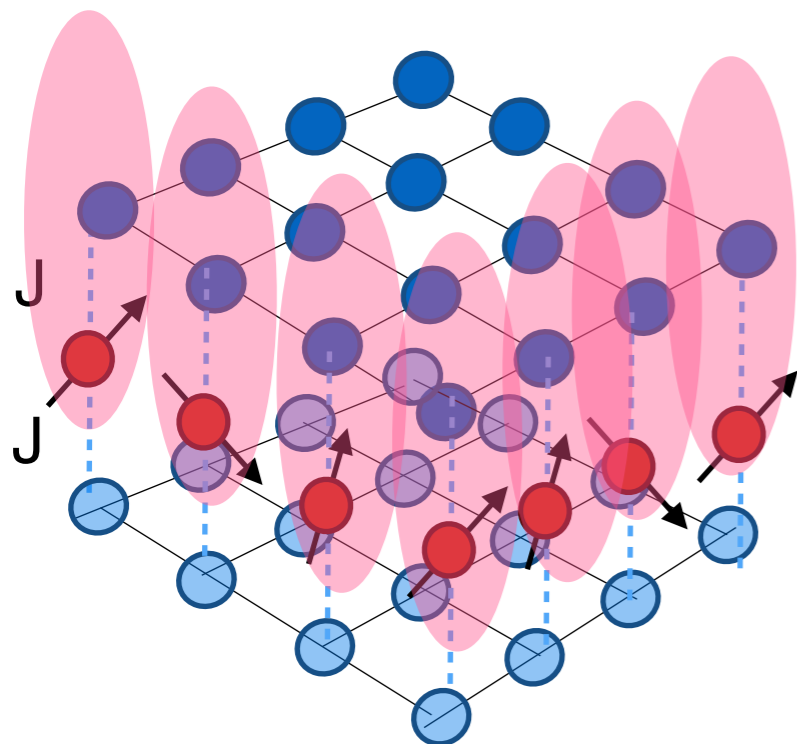
Order Parameter Fractionalization Hypothesis

P. Chandra, P. Coleman, Y. Komijani

Composite order

$$\Psi = \langle (\psi_1^\dagger \vec{\sigma} \psi_1 - \psi_2^\dagger \vec{\sigma} \psi_2) \cdot \vec{S} \rangle$$
$$\propto |V_1|^2 - |V_2|^2$$

Emery and Kivelson 1993



2-channel Kondo Lattice

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Spinor OP Forms Spontaneously

Fractionalization of Bound-State into Fermion+OP

Order Parameter Fractionalization Hypothesis

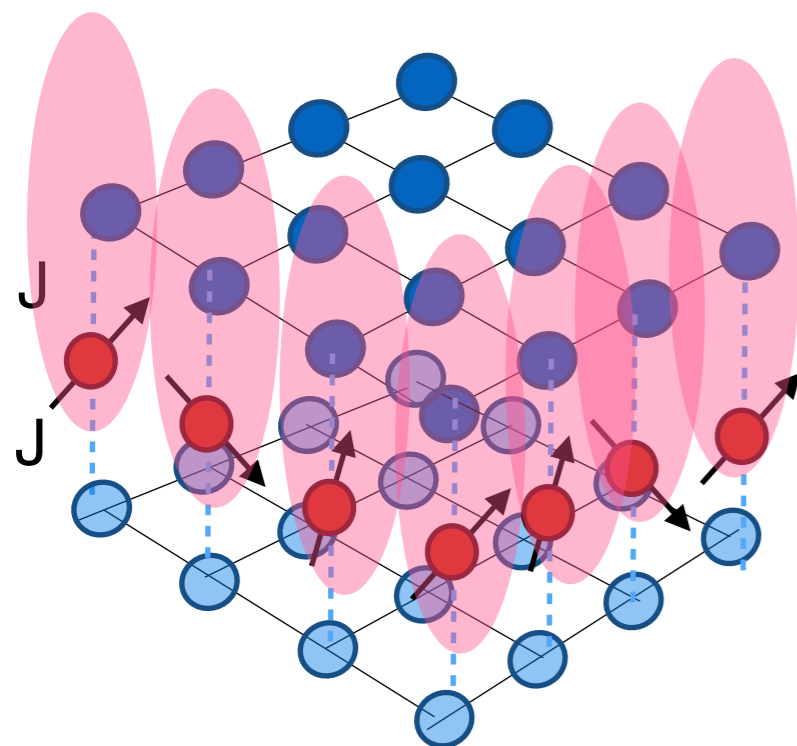
P. Chandra, P. Coleman, Y. Komijani

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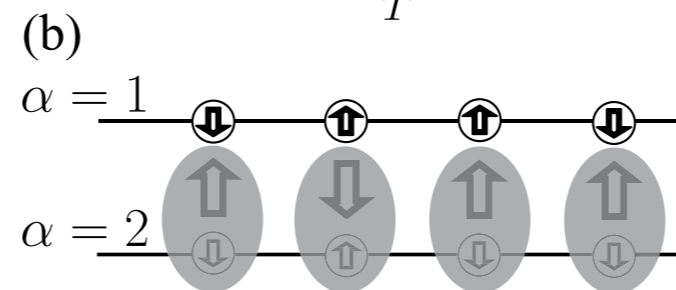
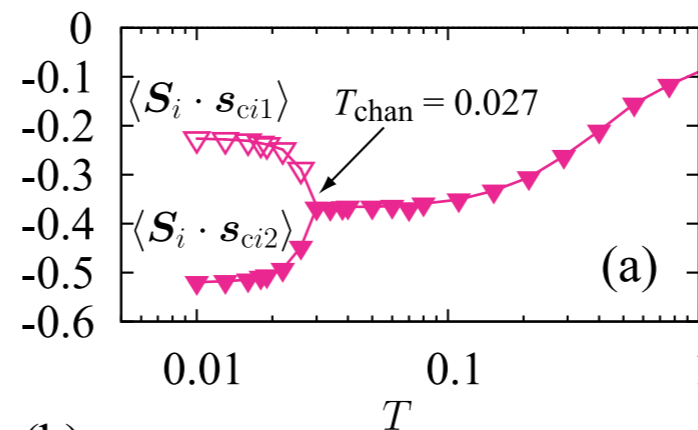
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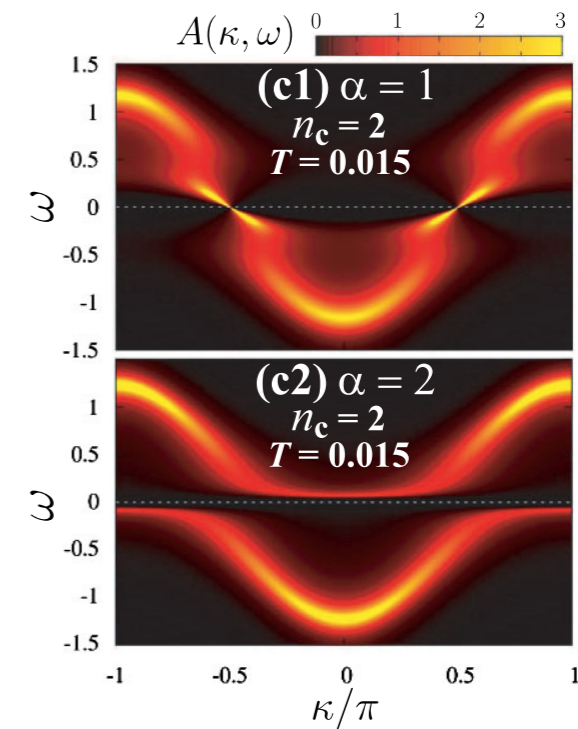
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2-channel Kondo Lattice



Hisono, Otsuki & Kuromoto, PRL 107, 247202 (2011)



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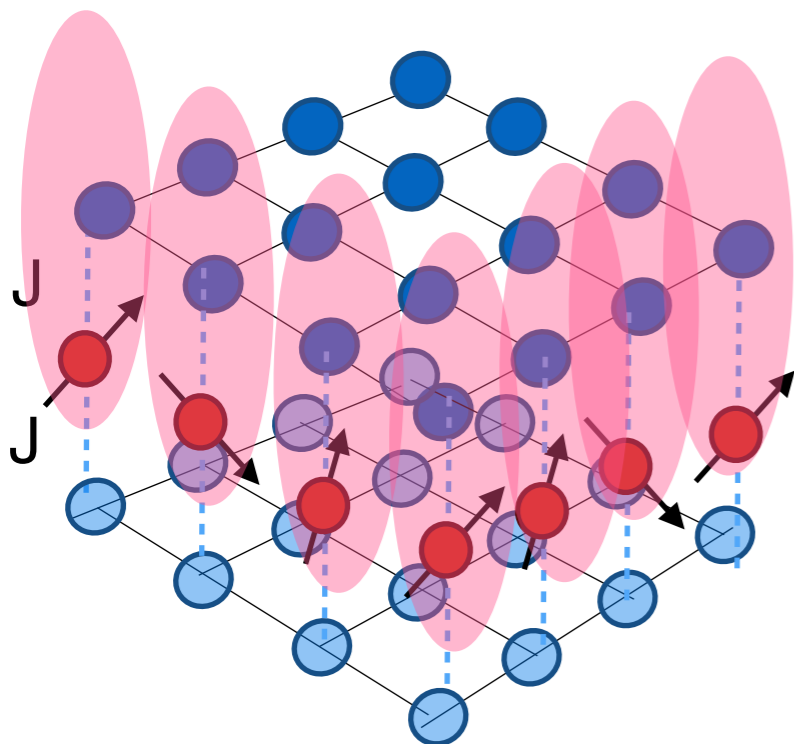
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Order Parameter Fractionalization Hypothesis

P. Chandra, PC, Y. Komijani

Composite Order

Kondo Majorana	$(\vec{S} \cdot \vec{\sigma})_{\alpha\beta} \psi_{\beta}$	$V f_{\alpha}$	HF	$\langle \psi^{\dagger} (\vec{\sigma} \cdot \vec{S}) \psi \rangle \propto V ^2$
		$(\vec{\sigma} \cdot \vec{\eta})_{\alpha\beta} \mathcal{V}_{\beta}$	Odd-w triplet/ Skyrme Insulator	$\langle \psi_{\uparrow} \psi_{\downarrow} \vec{S} \rangle \propto \mathcal{V}^T \vec{\sigma} \sigma_2 \mathcal{V}$
2-channel	$(\vec{S} \cdot \vec{\sigma})_{\alpha\beta} \psi_{\lambda\beta}$	$V_{\lambda} f_{\alpha}$	Composite Multipole	$\langle (\psi_1^{\dagger} \vec{\sigma} \psi_1 - \psi_2^{\dagger} \vec{\sigma} \psi_2) \cdot \vec{S} \rangle$
		$V_{\lambda} f_{\alpha} + \Delta_{\lambda} \bar{a} f_{-\alpha}^{\dagger}$	Composite Pair	$\langle (\psi_1 \vec{\sigma} \sigma_2 \psi_2) \cdot \vec{S} \rangle$
		$\Psi_{\alpha} \hat{\chi}_{\lambda}$	Hastatic	$\langle c^{\dagger} \vec{\sigma} (\vec{I} \cdot \vec{\tau}) c \rangle \propto \Psi^{\dagger} \vec{\sigma} \Psi$



2-channel Kondo Lattice

Emery and Kivelson 1993

$$(\vec{\sigma}_{\alpha\beta} \cdot \vec{S}_j) \psi_{\lambda\beta} \rightarrow V_{\lambda} f_{\alpha}(0)$$

Spinor OP Forms Spontaneously

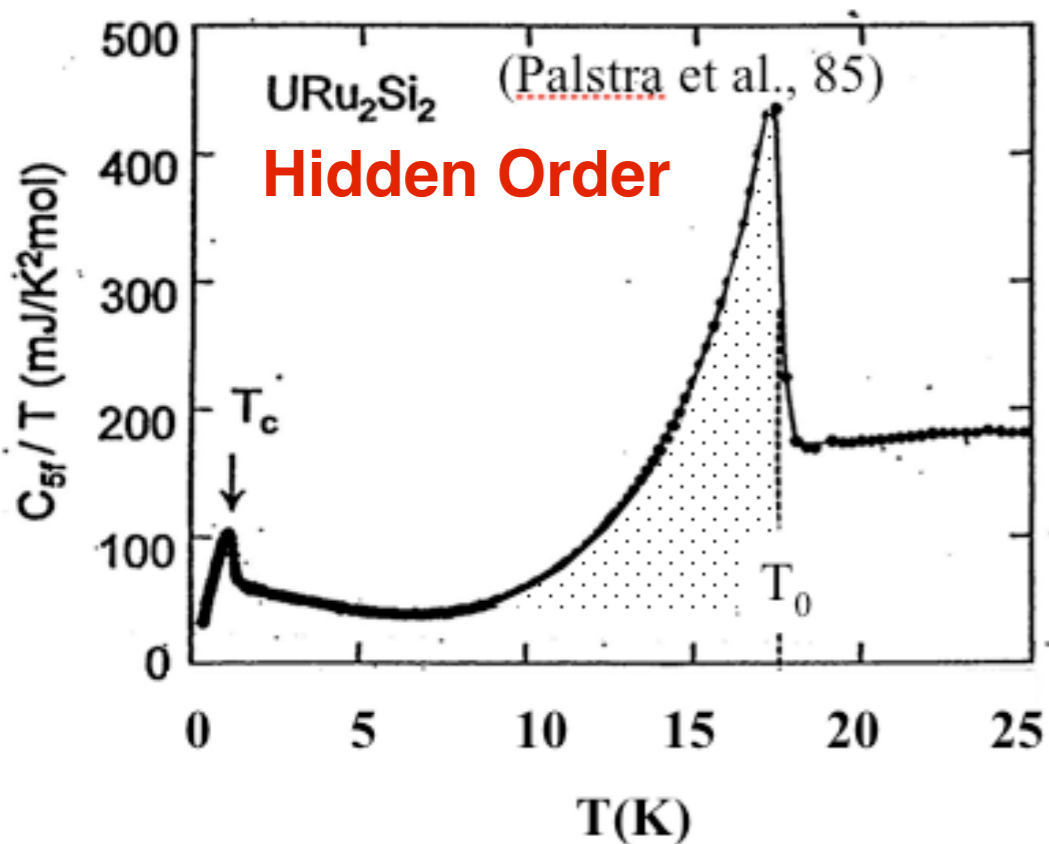
Fractionalization of Bound-State into Fermion+OP

Order Parameter Fractionalization Hypothesis

P. Chandra, PC, Y. Komijani

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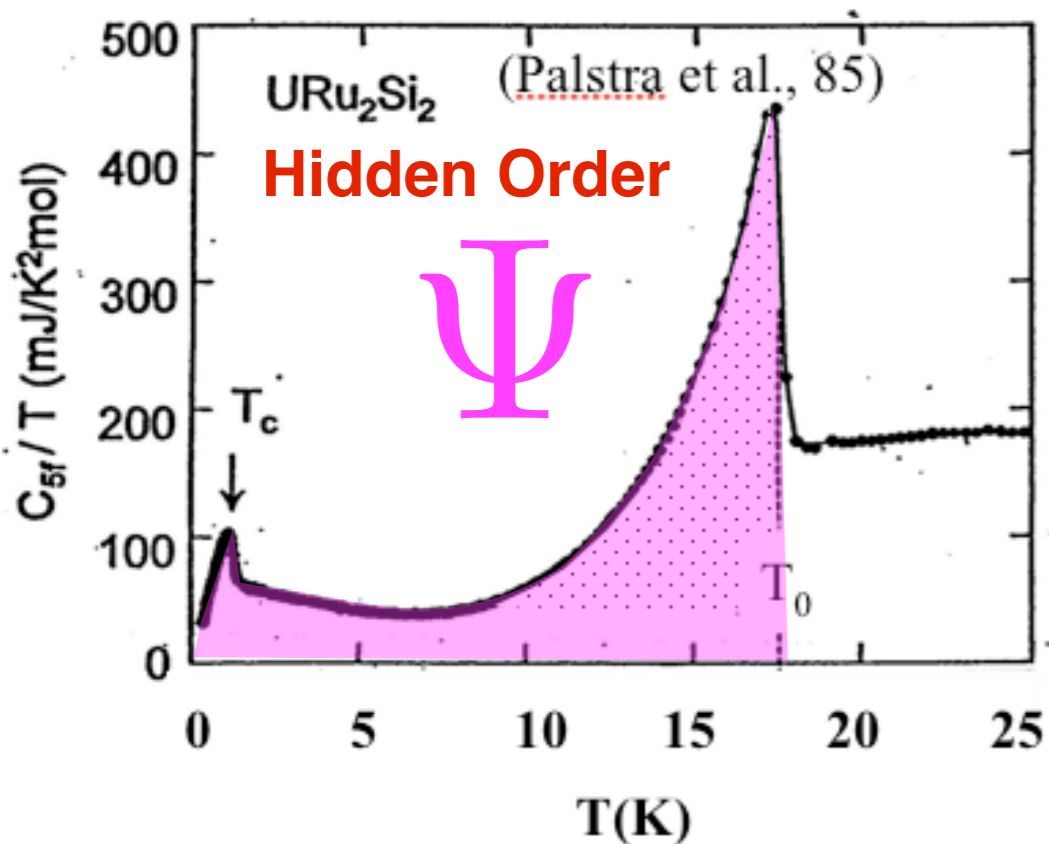


Order Parameter Fractionalization Hypothesis

P. Chandra, PC, Y. Komijani

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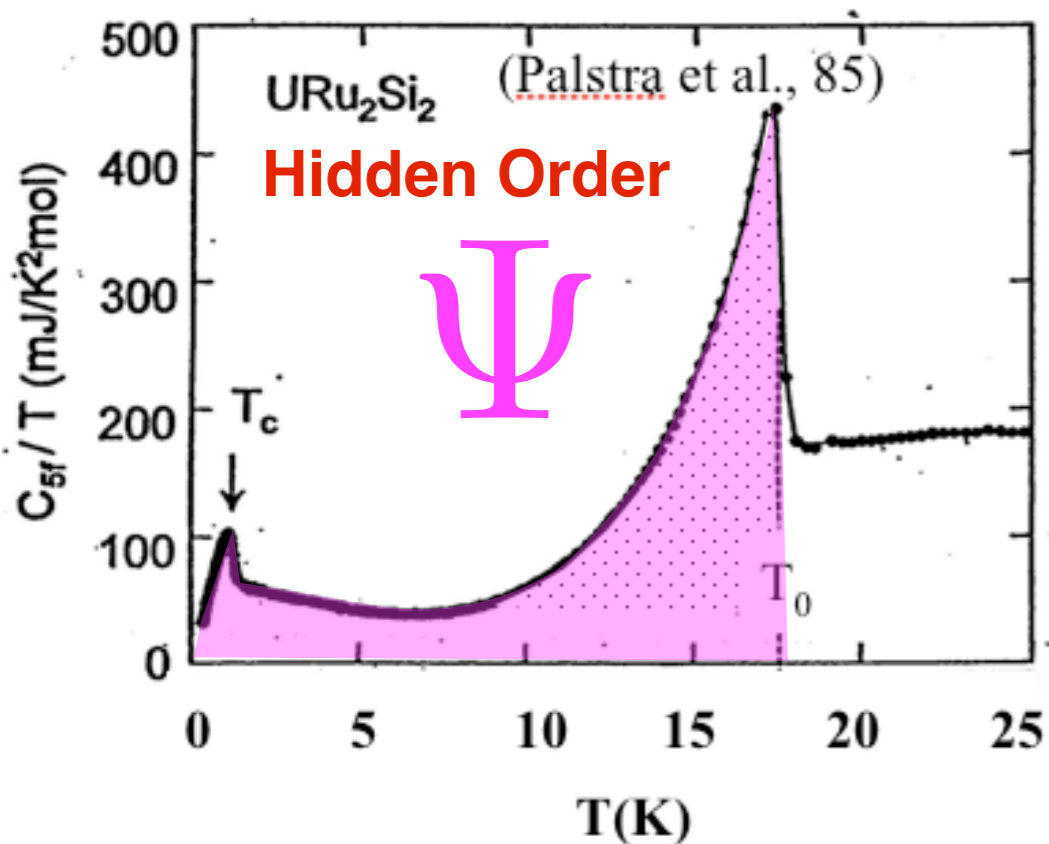


Order Parameter Fractionalization Hypothesis

P. Chandra, PC, Y. Komijani

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See eg:

J=3 Kiss & Fazekas, Phys Rev B, (2005)

J=4 Kotliar & Haule, Nat Phys, (2009)

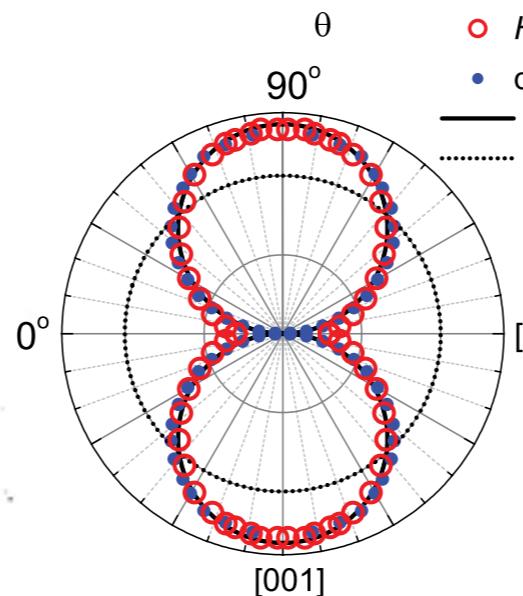
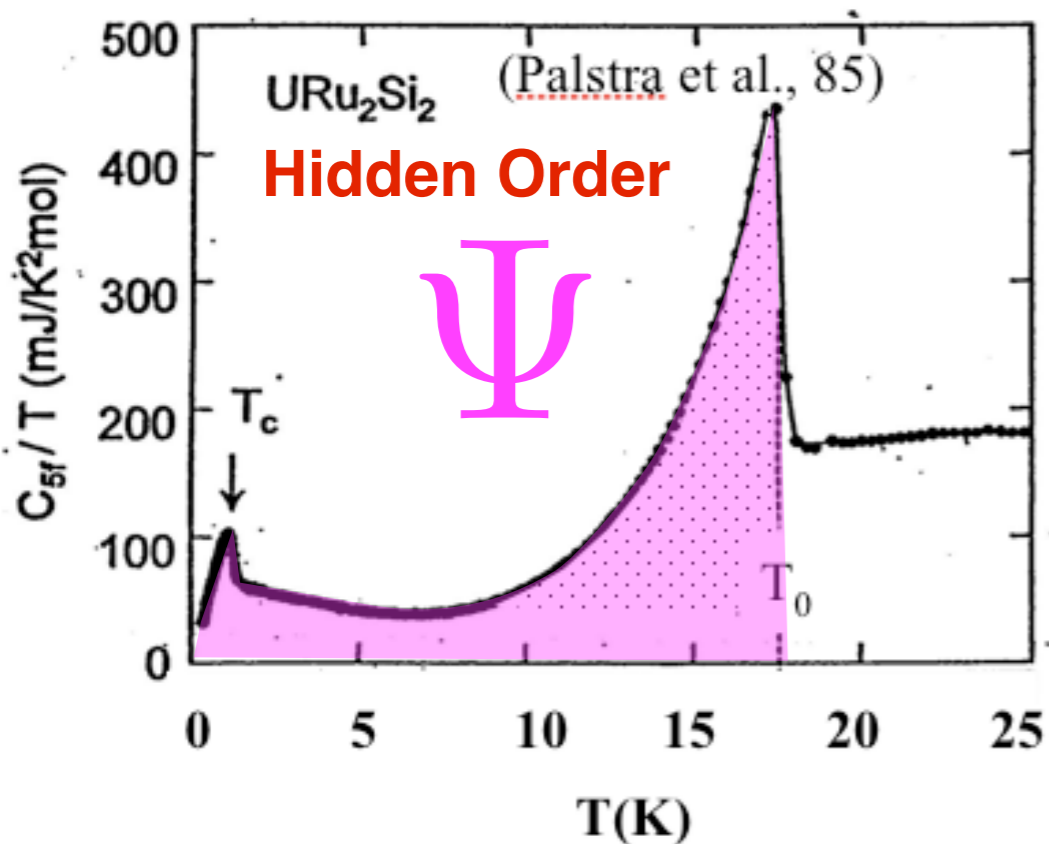
J=5 Ikeda et al, Nat. Phys (2012)

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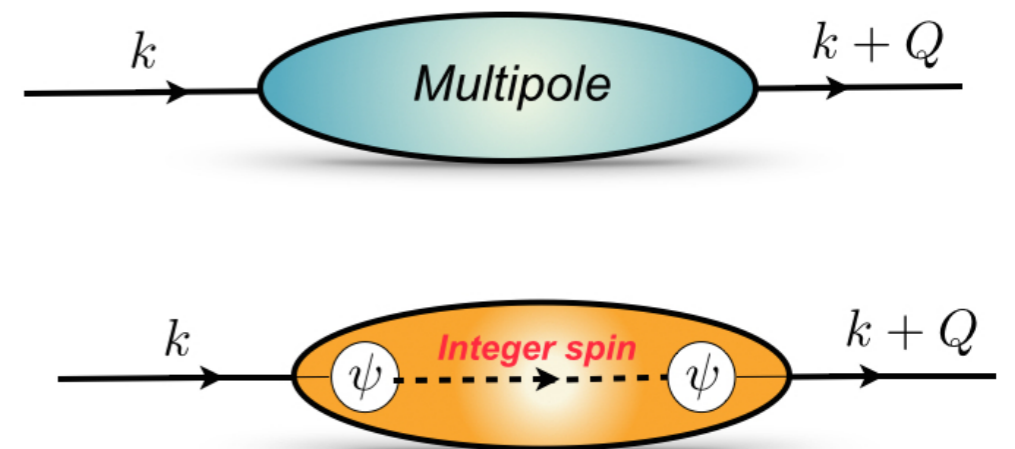
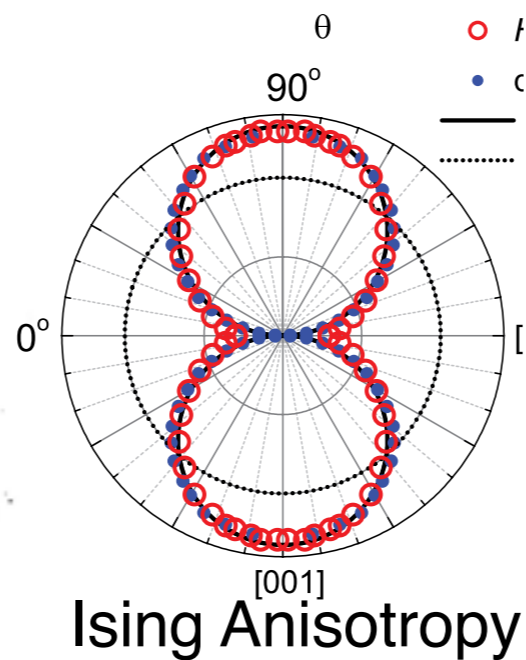
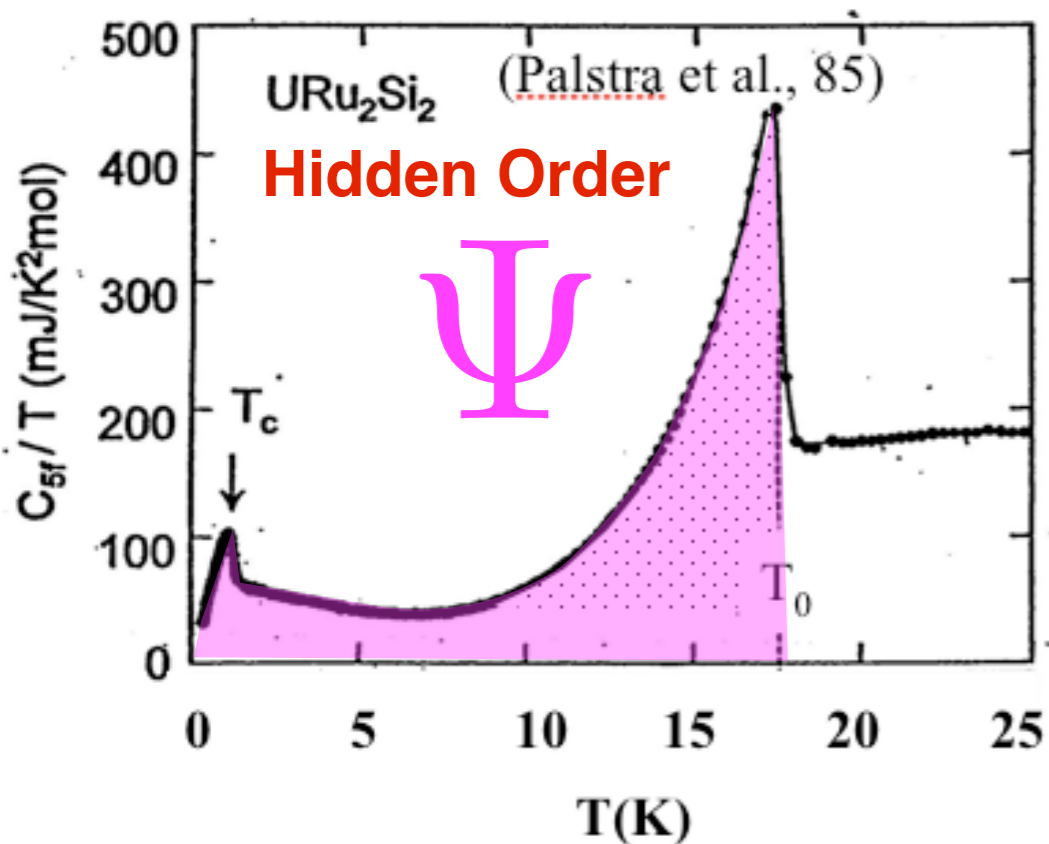
J=5 Ikeda et al, Nat. Phys (2012)

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P. Chandra, PC, Y. Komijani

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$$\psi = \sqrt{\text{Multipole}} = \text{Spinorial OP}$$

Chandra, Coleman, Flint, Nature (2013)

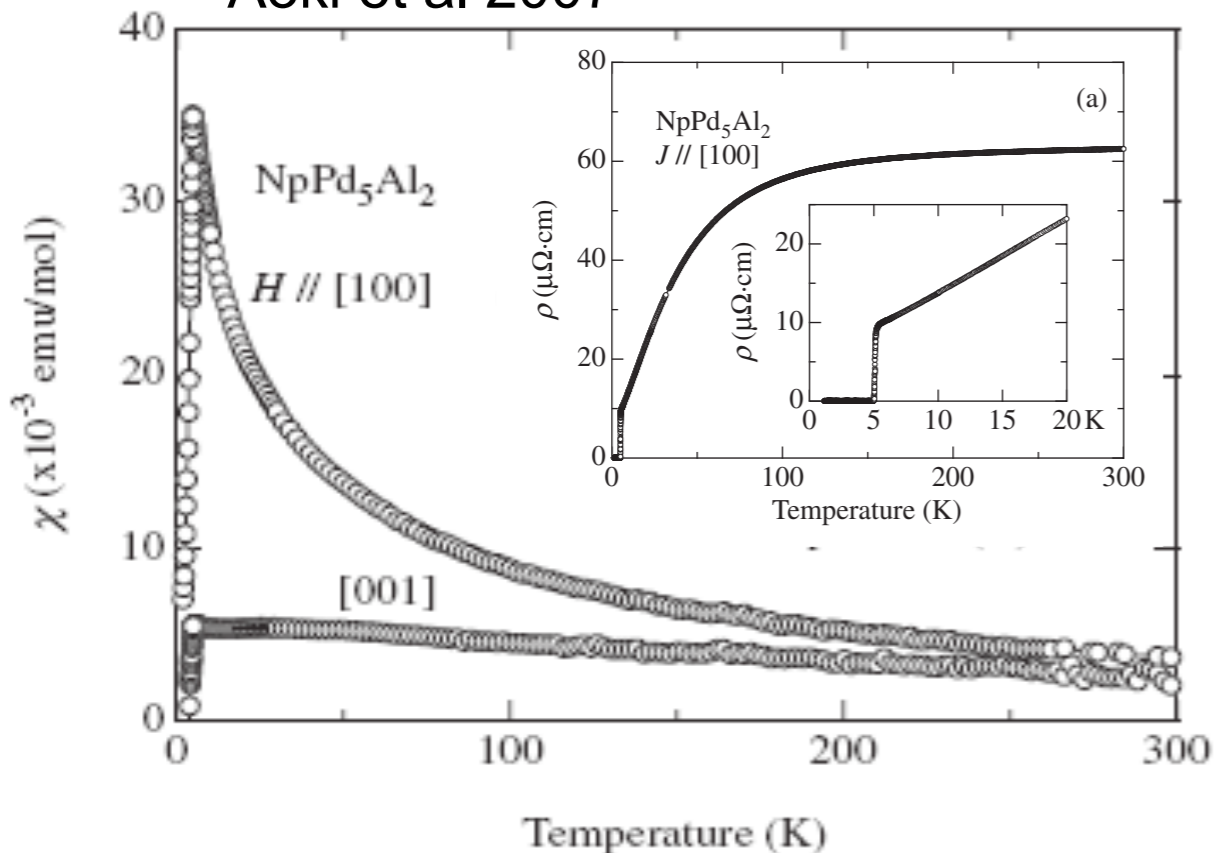
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P. Chandra, PC, Y. Komijani

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Aoki et al 2007



See: Flint, Dzero, PC, Nat Phys. (2008)

NpPd_5Al_2 $T_C = 4.5\text{K}$
Curie Law SC

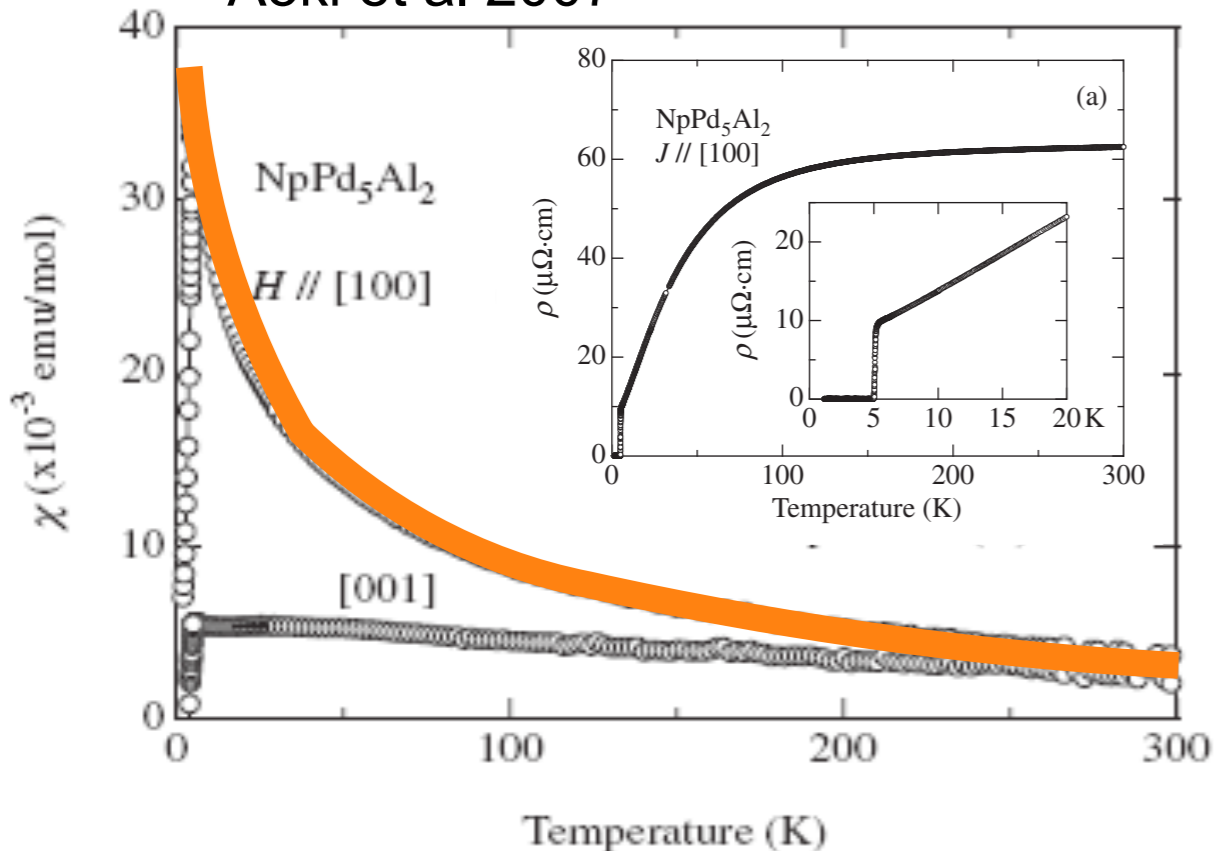
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P. Chandra, PC, Y. Komijani

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Aoki et al 2007



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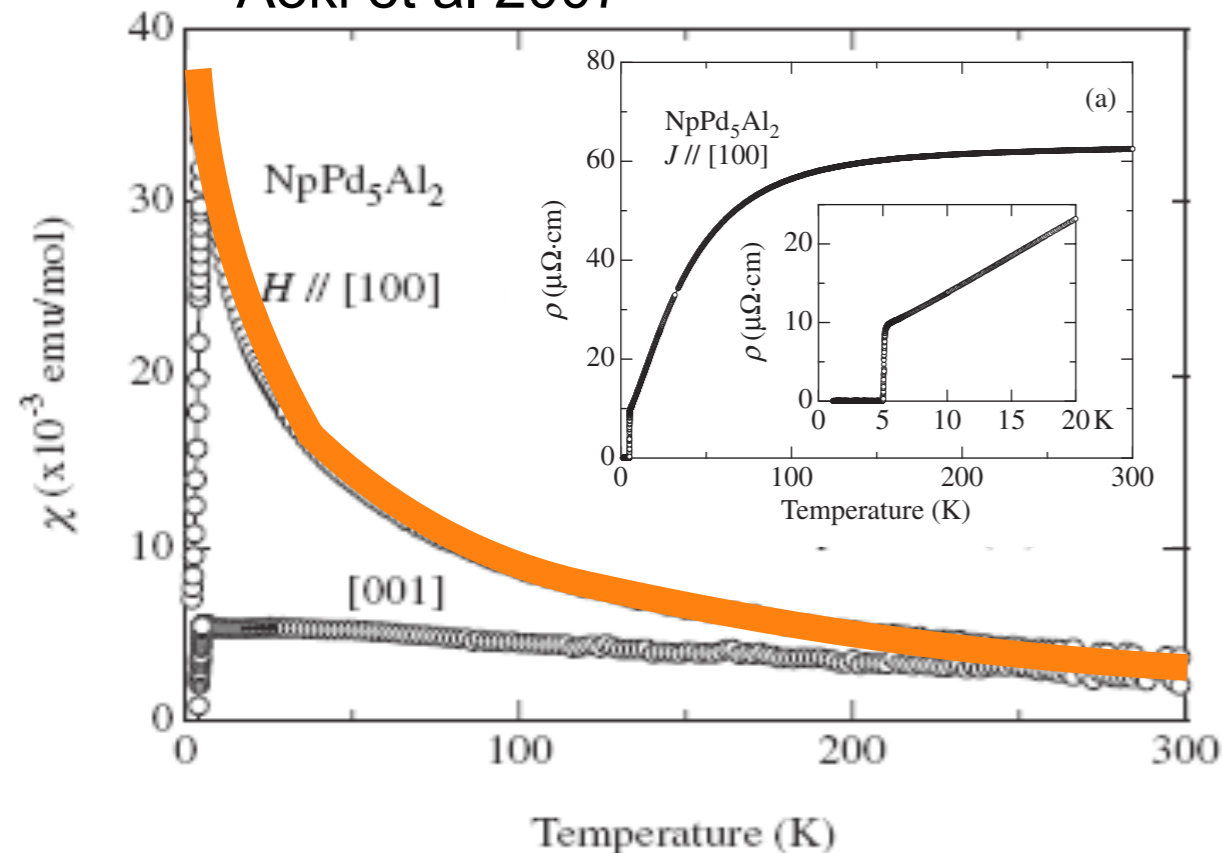
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Aoki et al 2007



Composite order

$$\langle (\psi_1 \vec{\sigma} \sigma_2 \psi_2) \cdot \vec{S} \rangle \propto (V_1 \Delta_2 - V_2 \Delta_1)$$

See: Flint, Dzero, PC, Nat Phys. (2008)

NpPd_5Al_2 $T_C = 4.5\text{K}$
Curie Law SC

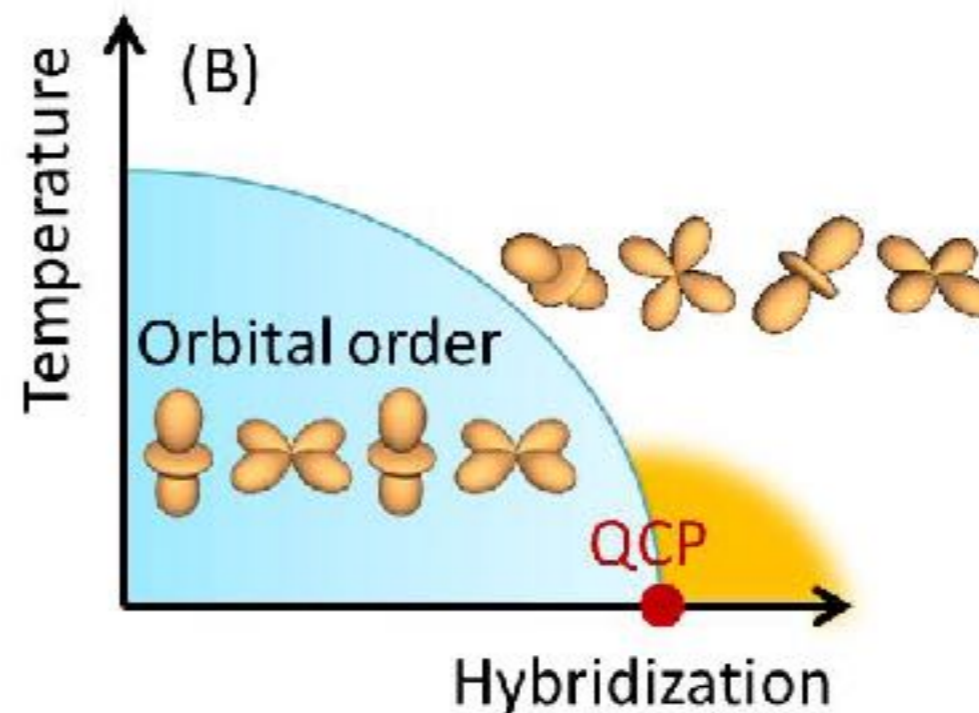
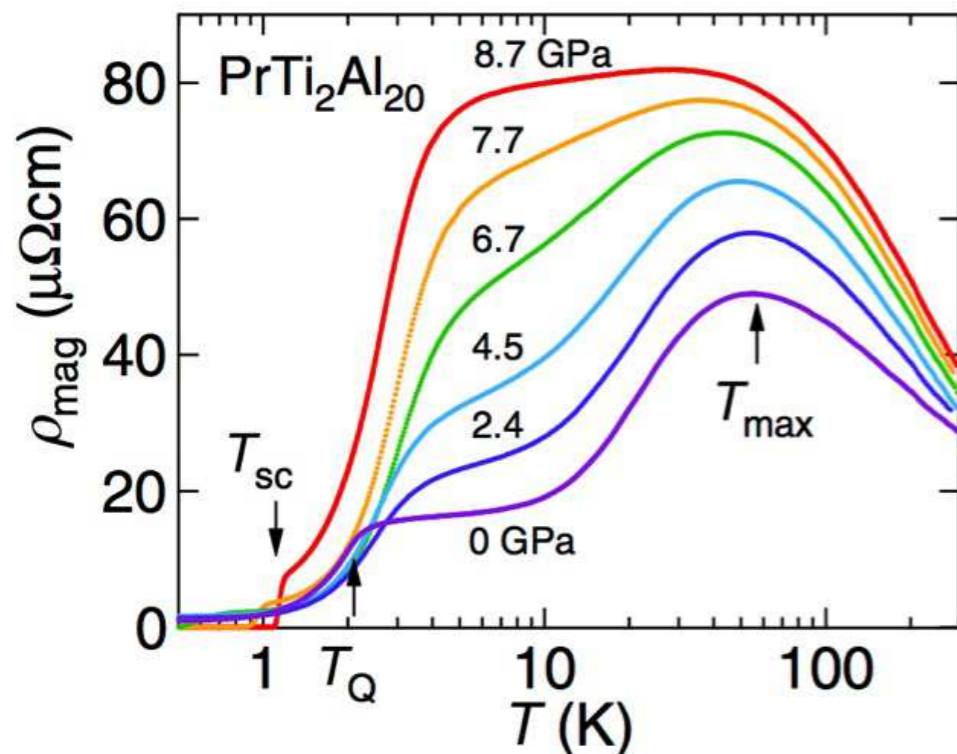
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A. Sakai, K. Kuga, and S. Nakatsuji, J. Phys. Soc. Jpn. **81**, 083702 (2012).

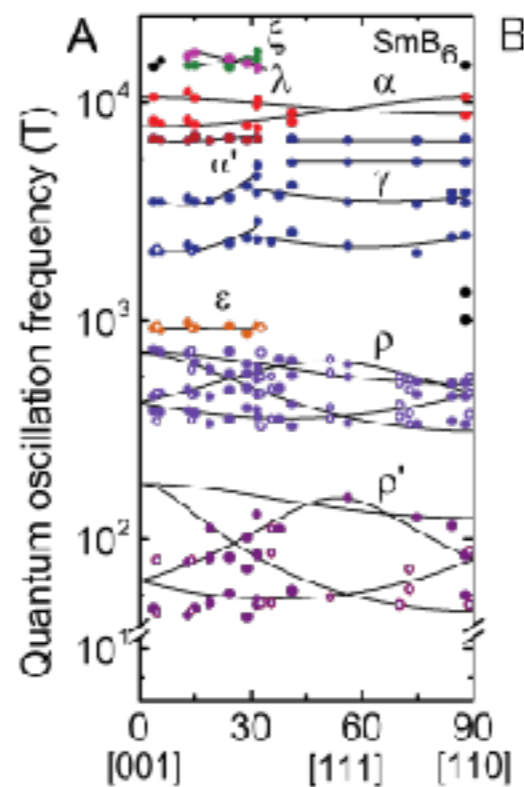
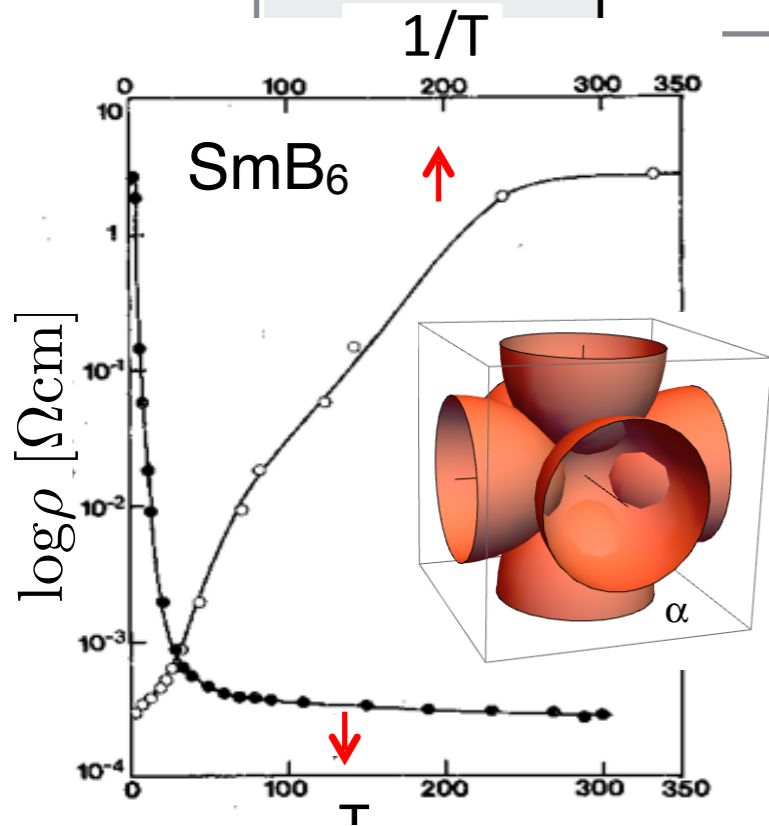


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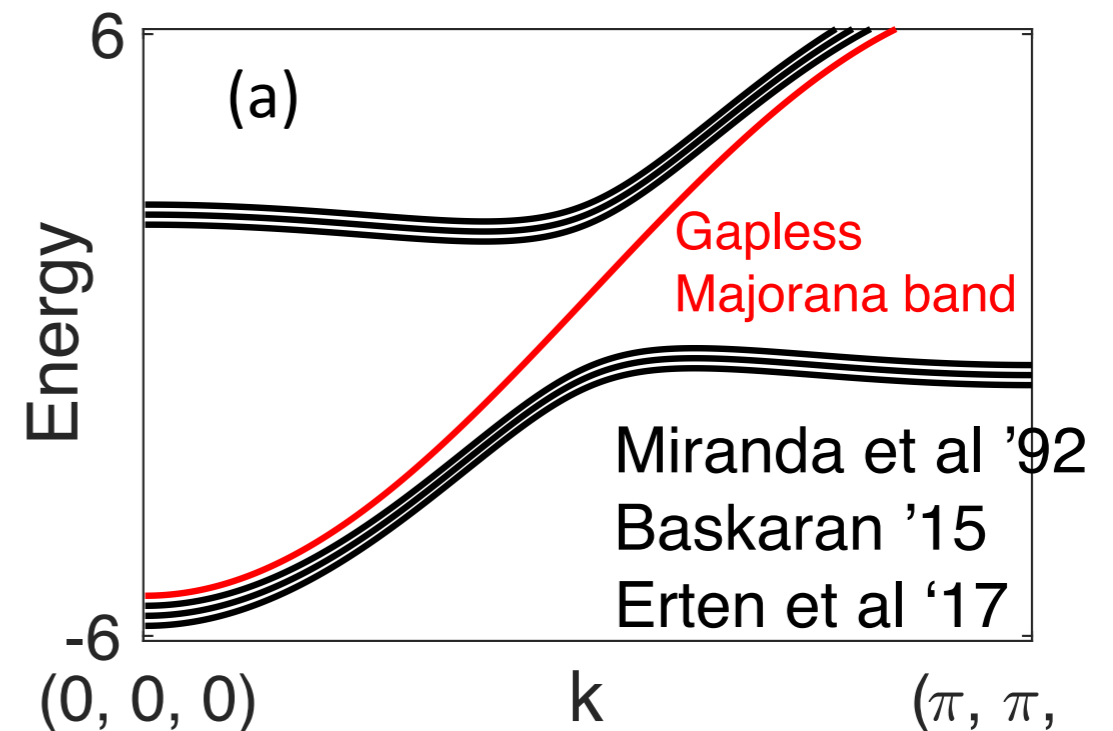
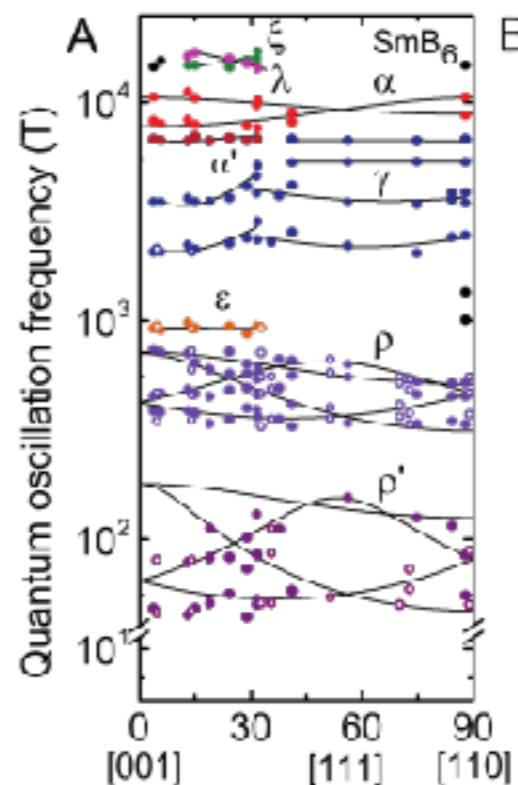
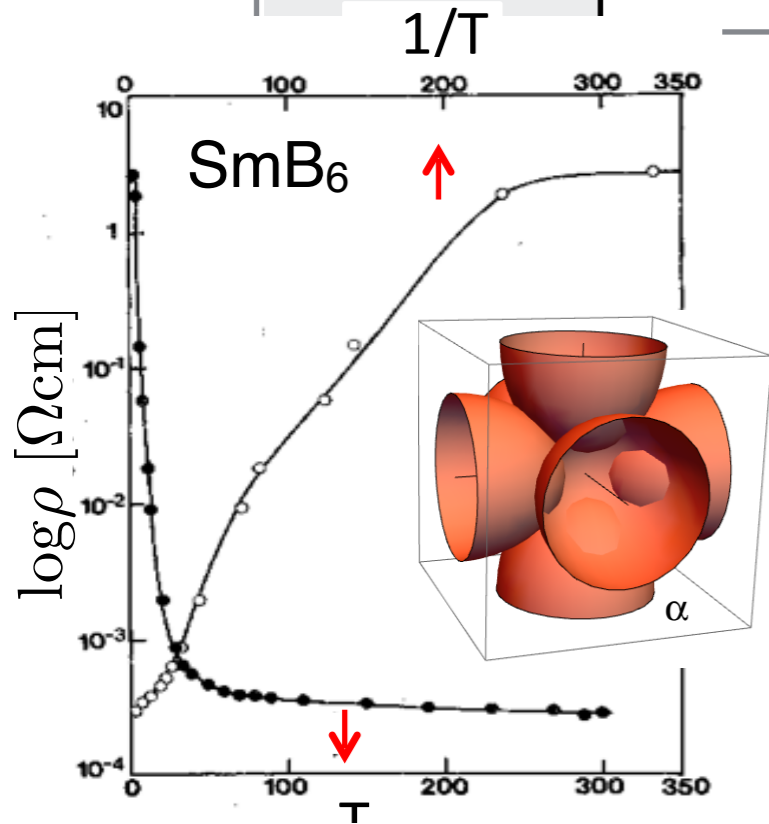


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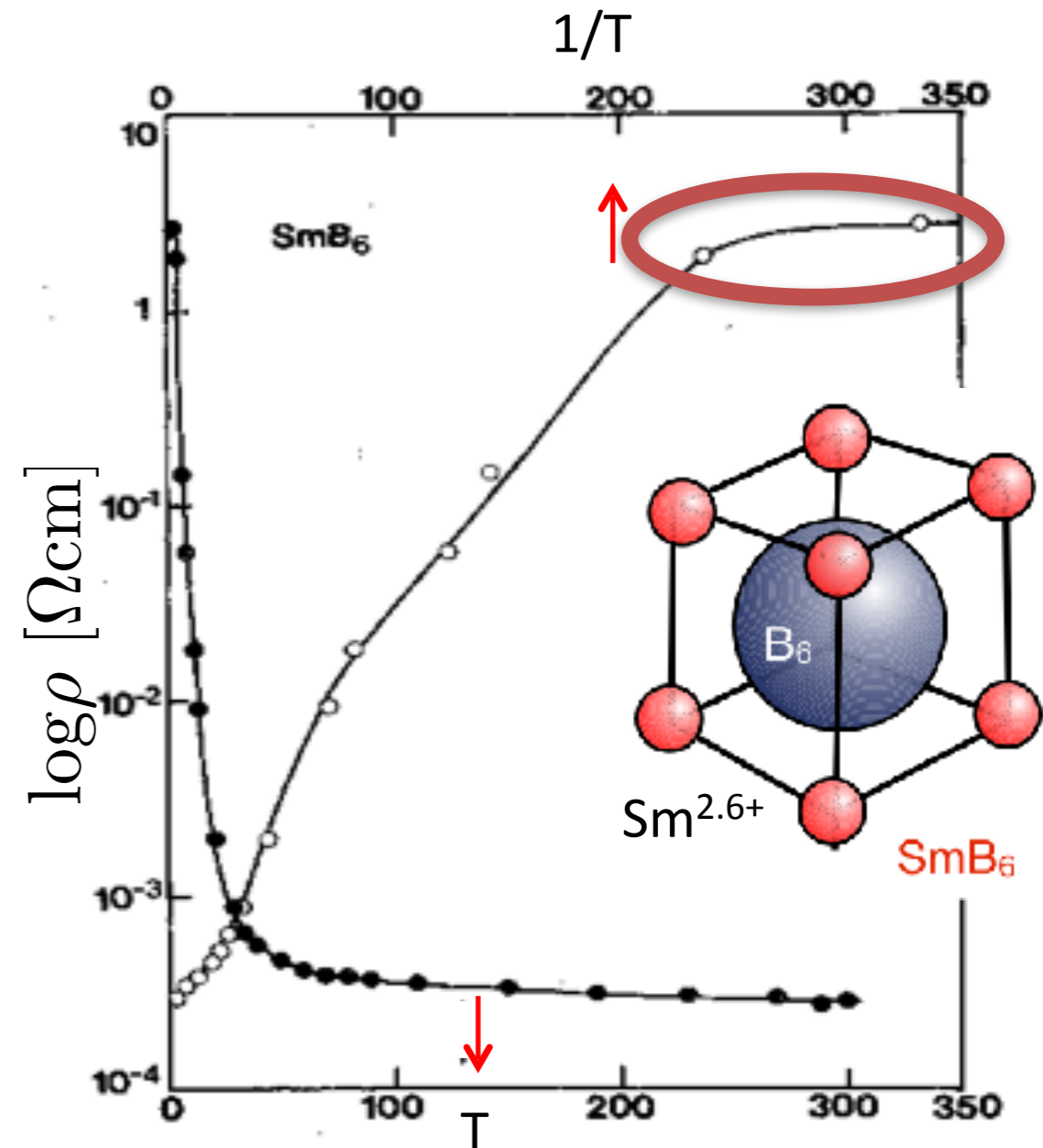
SmB₆ : Strange Insulator

A possible Topological Insulator?

- Insulating gap opens at $T_K \sim 50\text{K}$
- Resistivity plateau below $T \sim 3\text{K}$
- Topological surface states?

Dzero, Sun, Galitski, Coleman, PRL **104**, 106408 (2010)

Takimoto, J. Phys. Soc. Jpn. **80**, 123710 (2011)



Menth, Buehler, Geballe PRL **22**, 295 (1969)

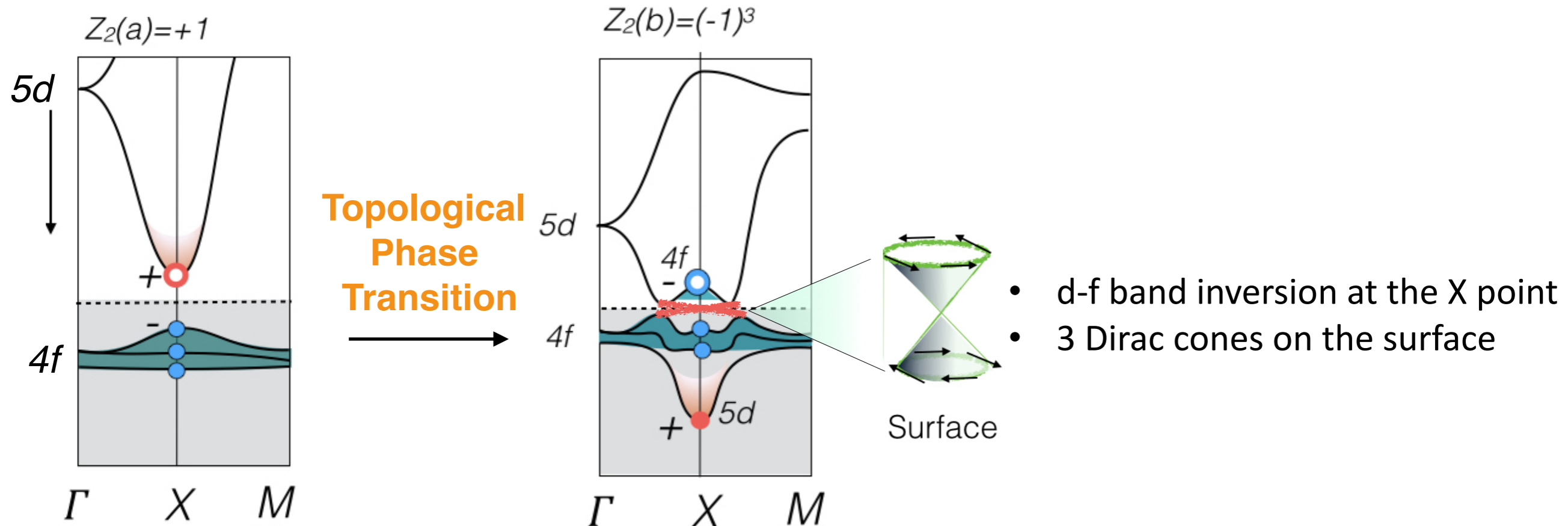
Allen, Batlogg, Watcher PRB **20** 4807 (1979)

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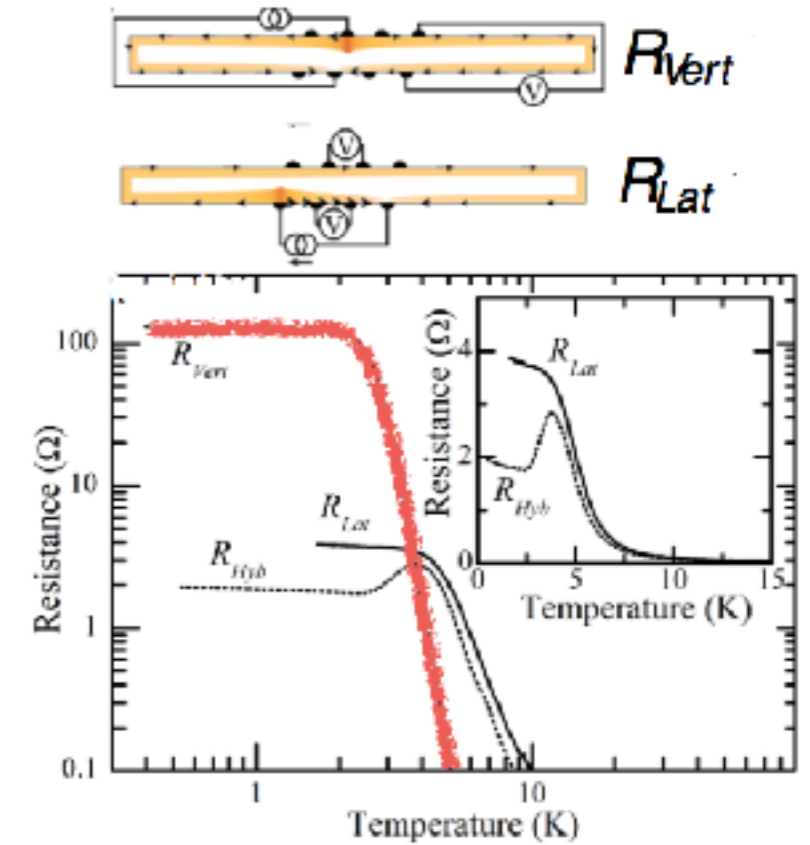
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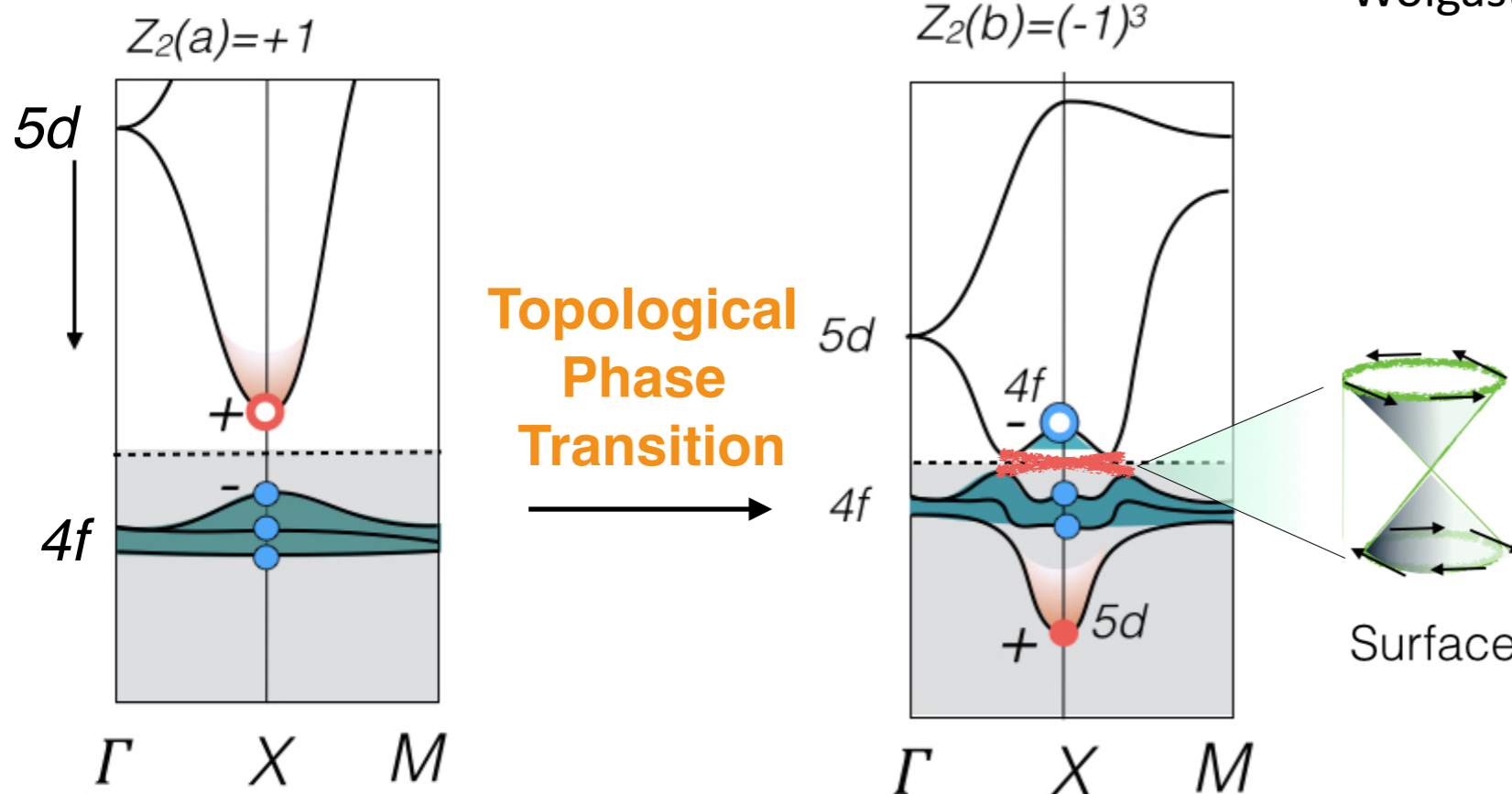
Dzero, Sun, Galitski, Coleman, PRL **104**, 106408 (2010)

Takimoto, J. Phys. Soc. Jpn. **80**, 123710 (2011)

Nonlocal transport \rightarrow surface cond.



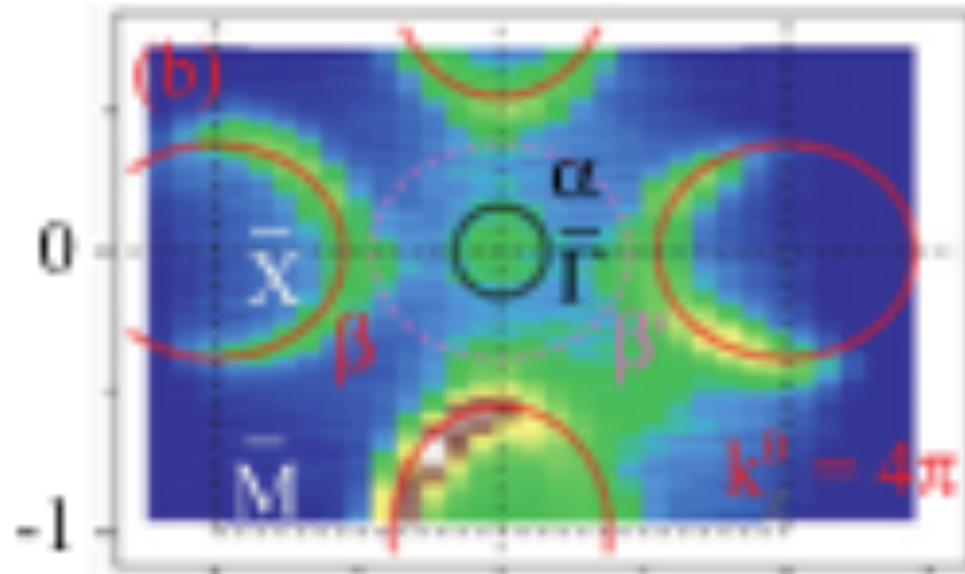
Wolgast et al, Phys Rev B, 88, 180405 (2013)



- d-f band inversion at the X point
- 3 Dirac cones on the surface

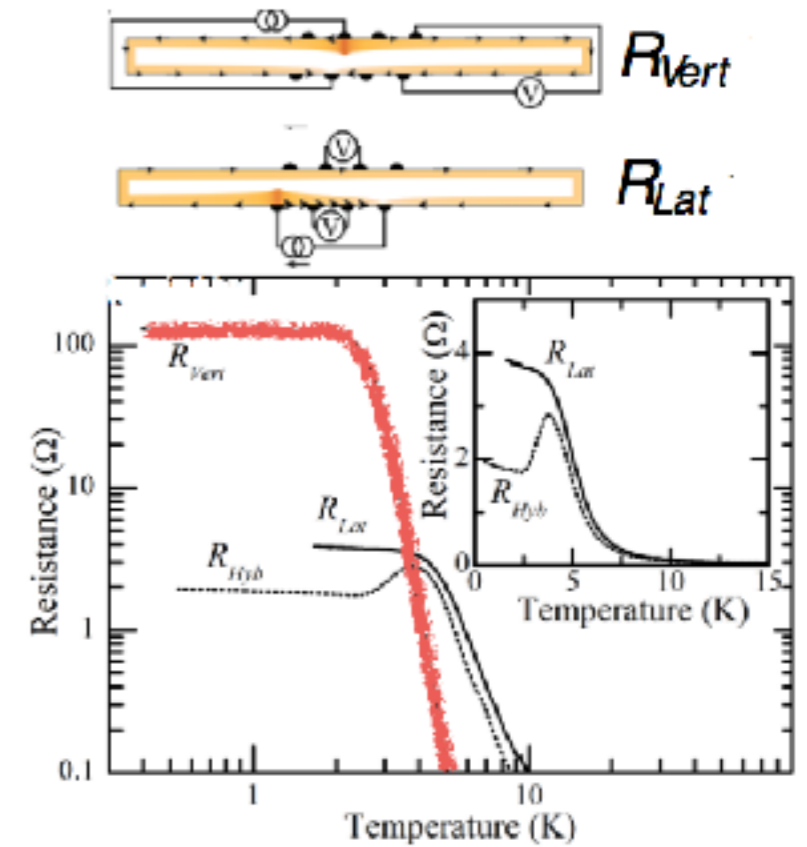
A possible Topological Insulator?

ARPES \rightarrow Surface states

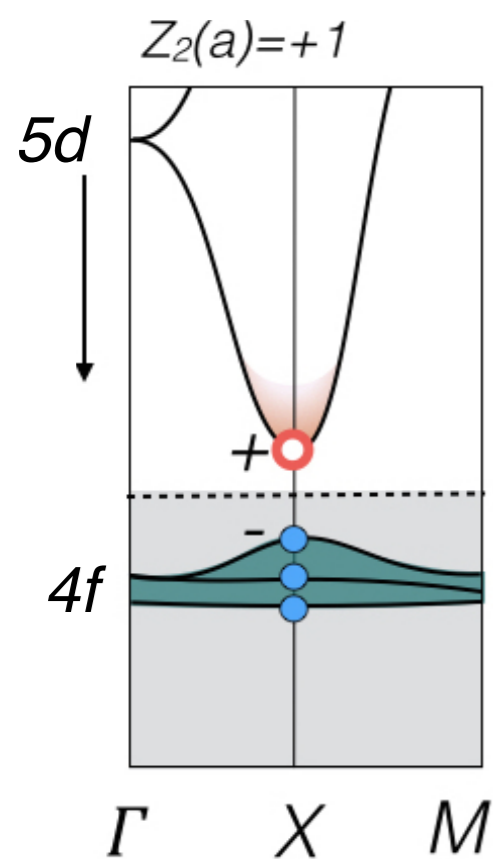


Xu et. al PRB **88**, 121102(R) (2013)

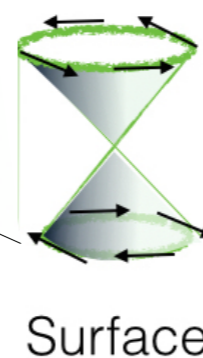
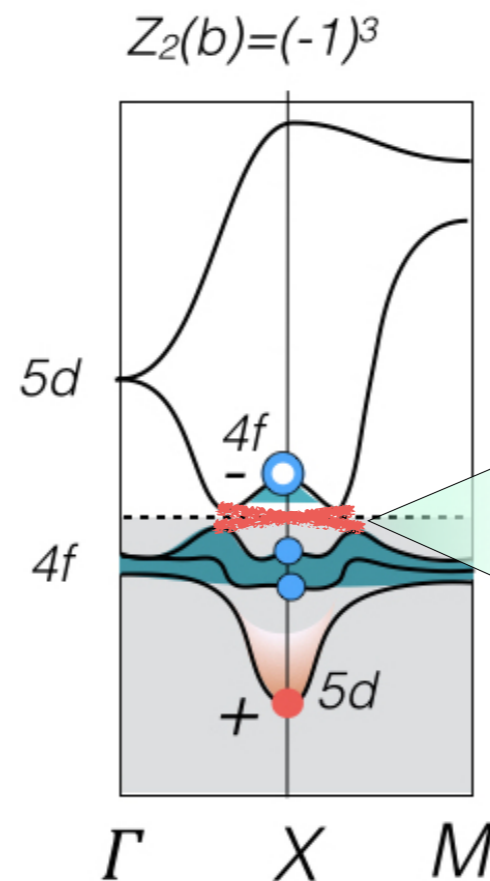
Nonlocal transport \rightarrow surface cond.



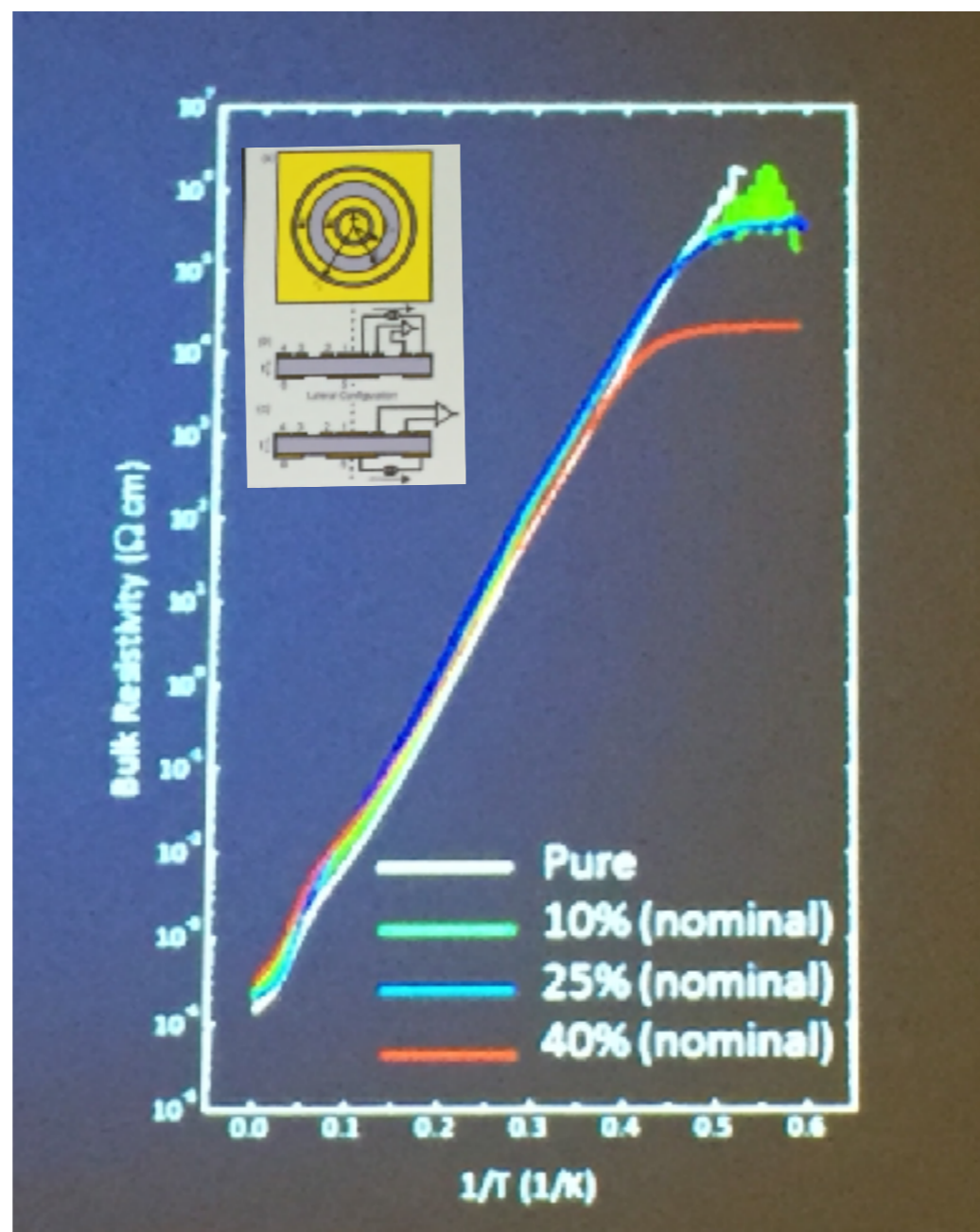
Wolgast et al, Phys Rev B, **88**, 180405 (2013)



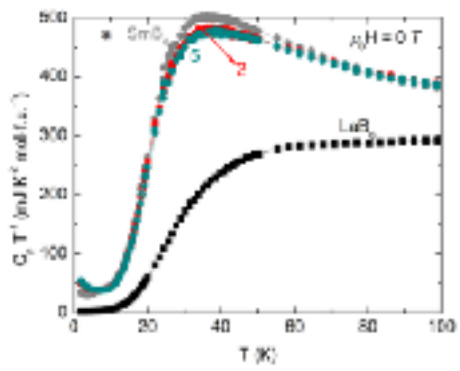
Topological Phase Transition



- d-f band inversion at the X point
- 3 Dirac cones on the surface

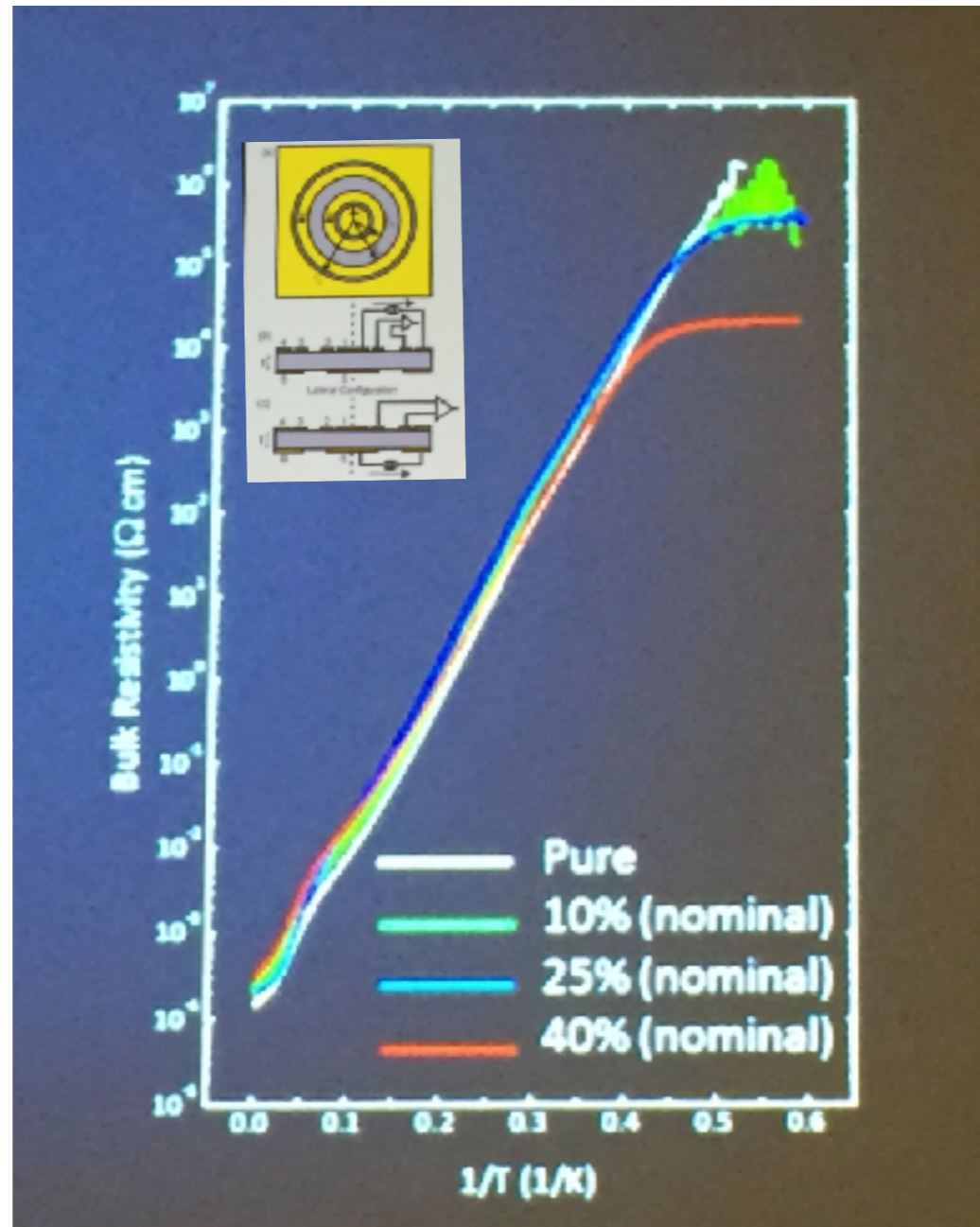


Robust bulk insulator
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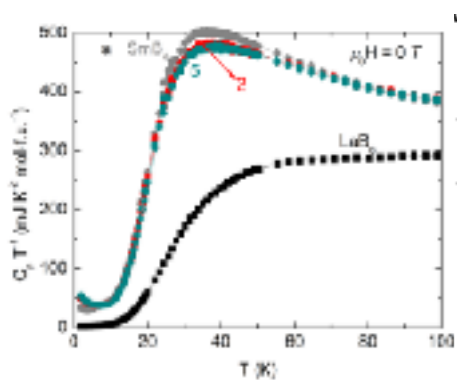
Linear Sheat

Phalen, PRX 4 031012 (2014)



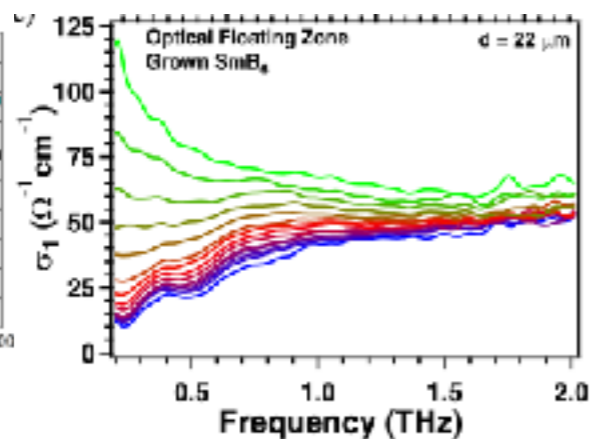
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- Bulk Linear SHeat **10x** LaB_6



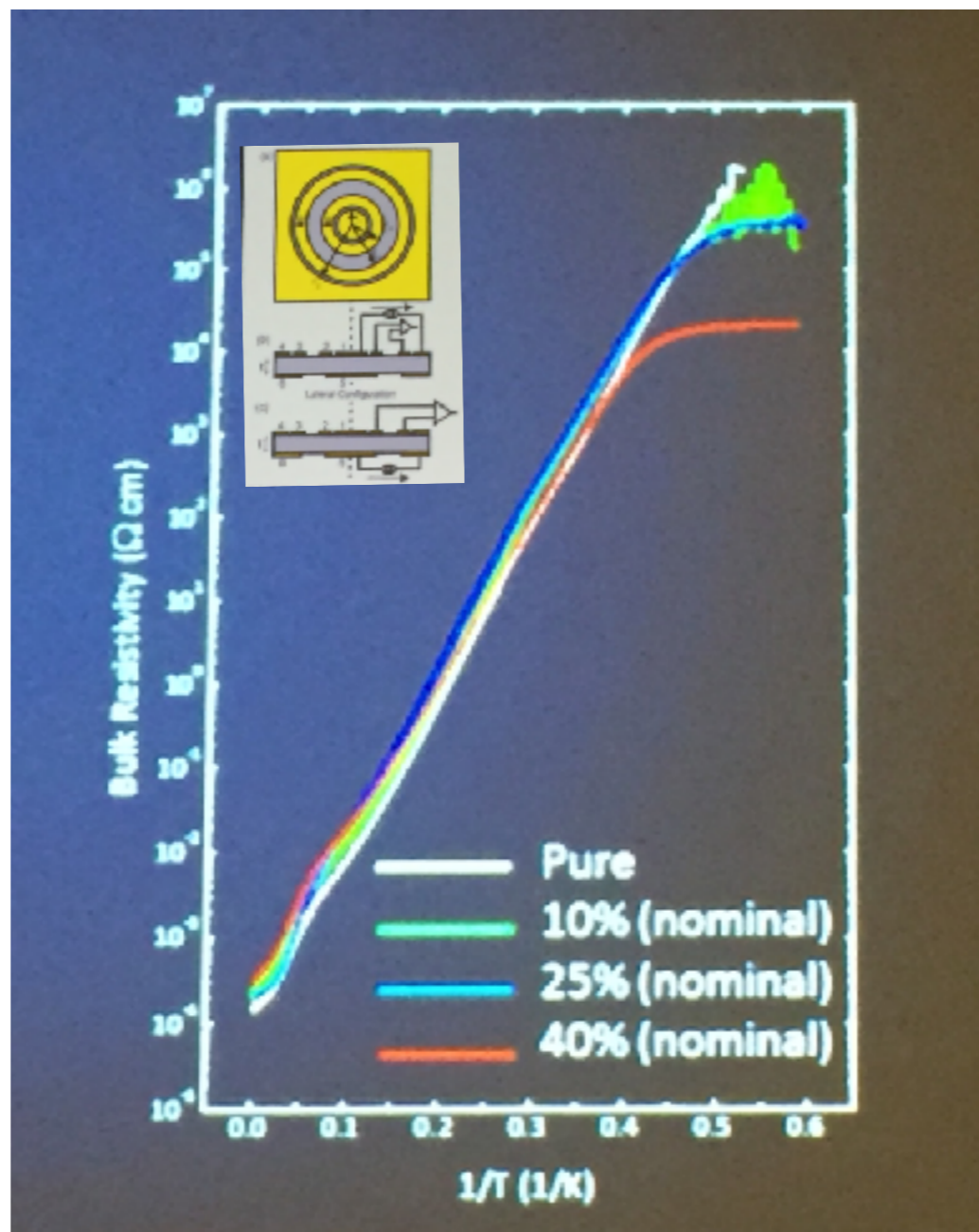
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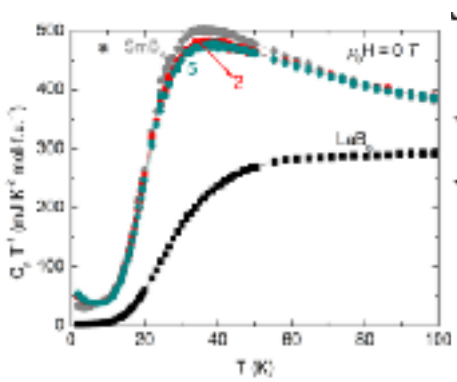
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Laurita, *et al.* PRB 94, 165154 (2016).



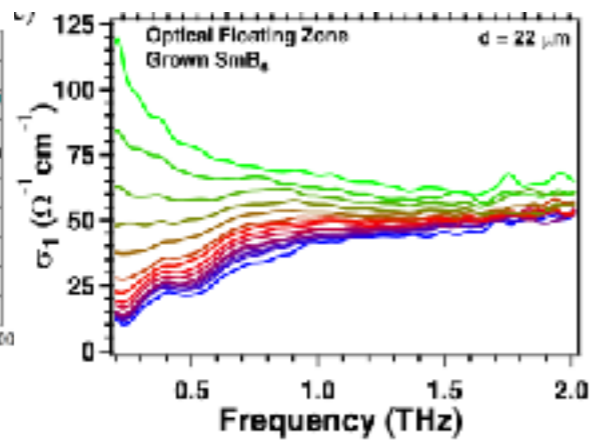
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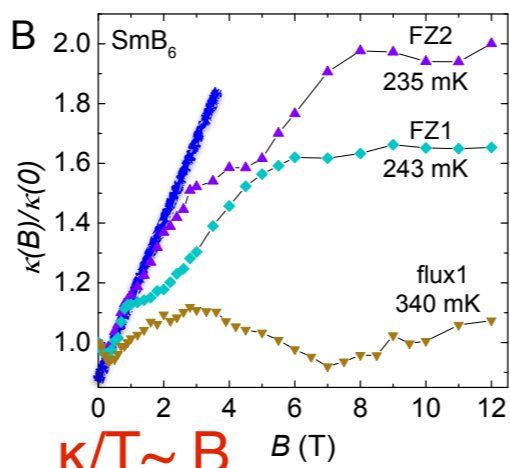
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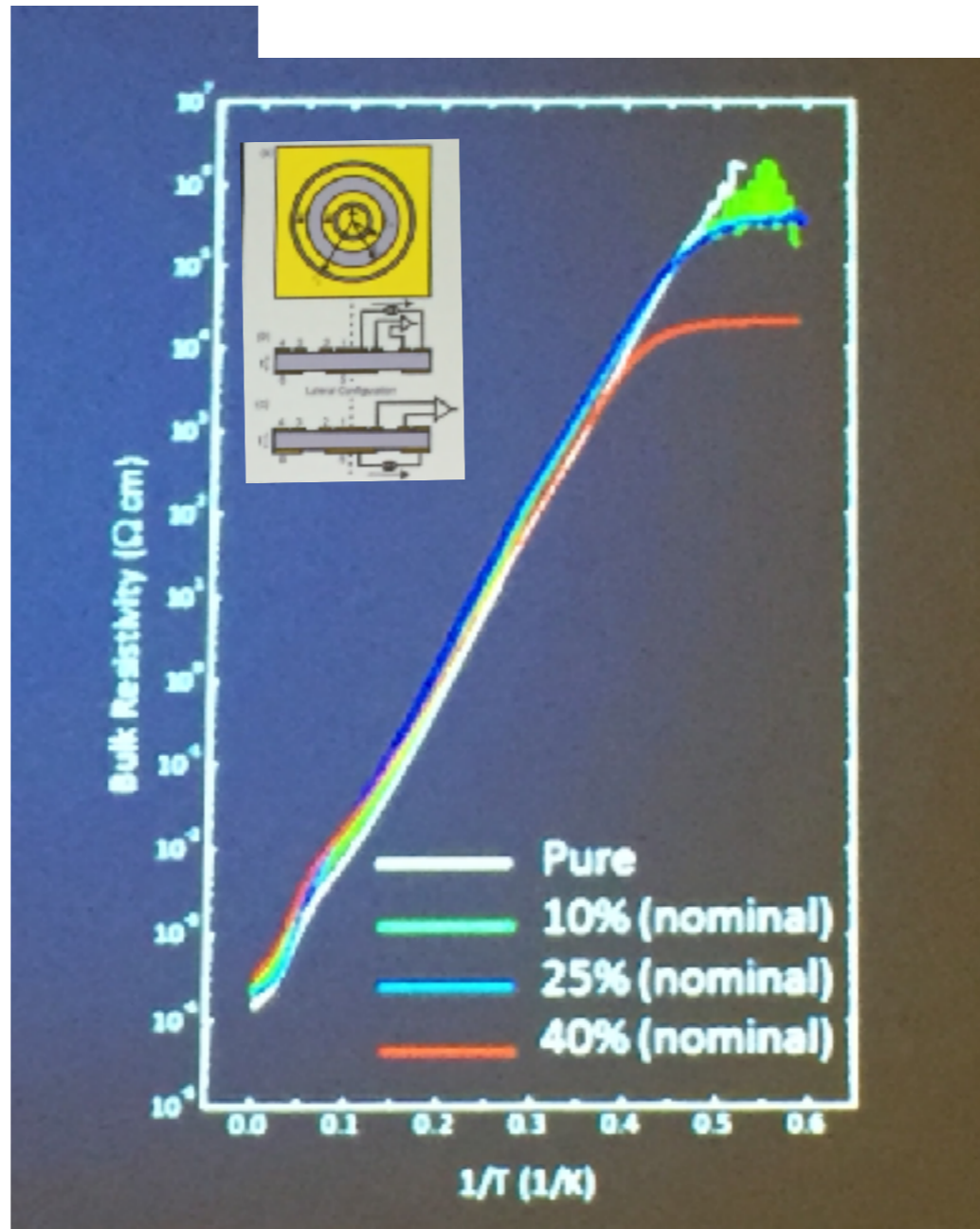
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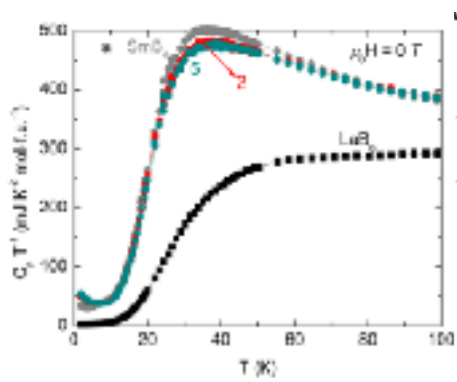
$$\kappa/T \sim B$$

M. Hartstein, M Sutherland
S. Sebastian *et al.* (Preprint)



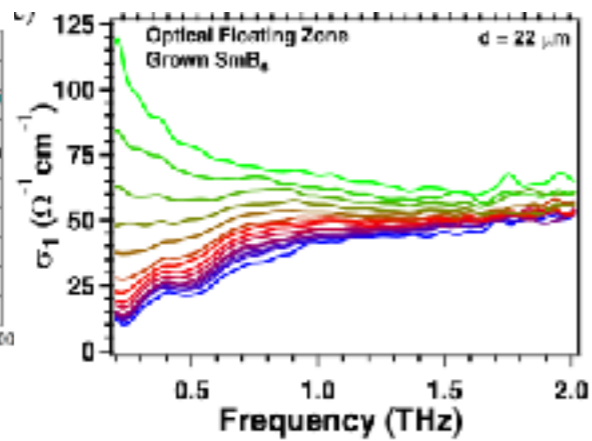
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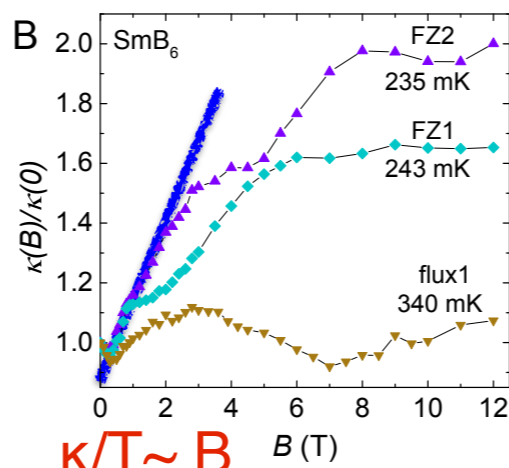
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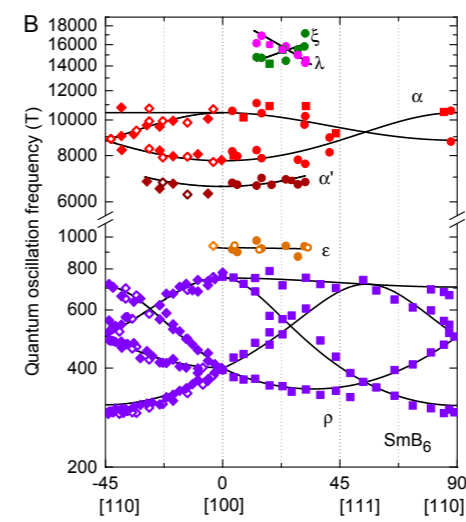
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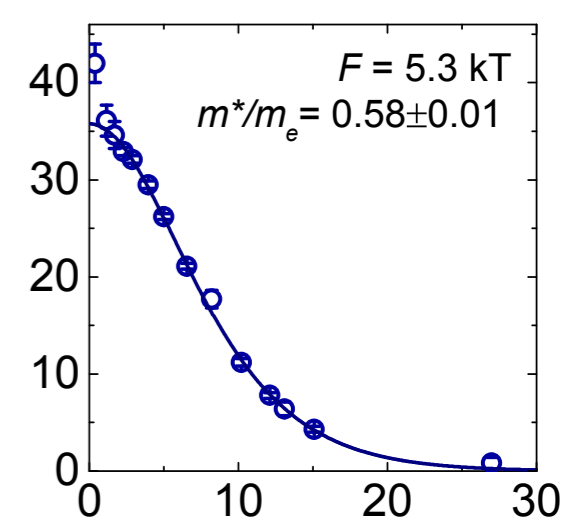


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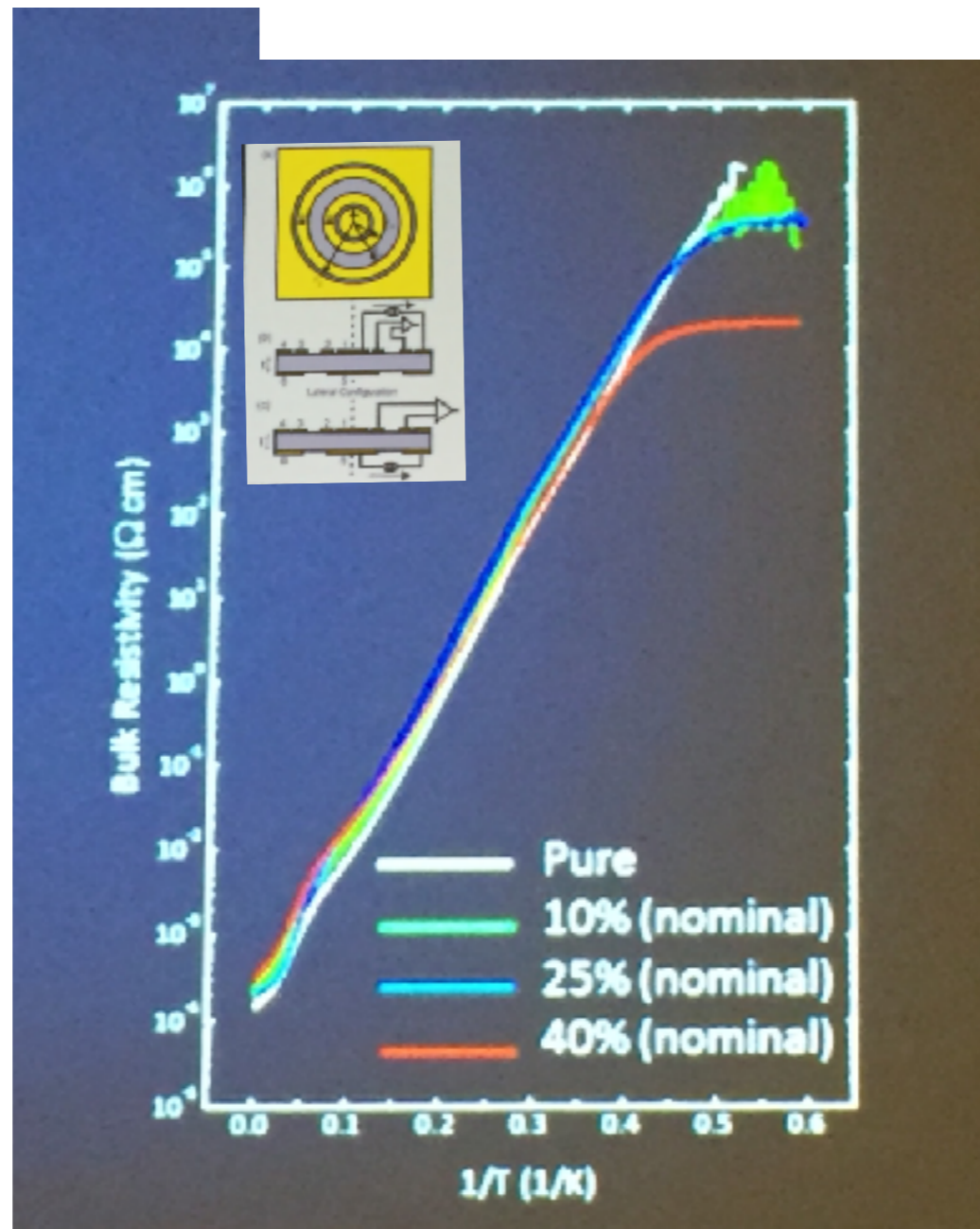


Bulk dHvA



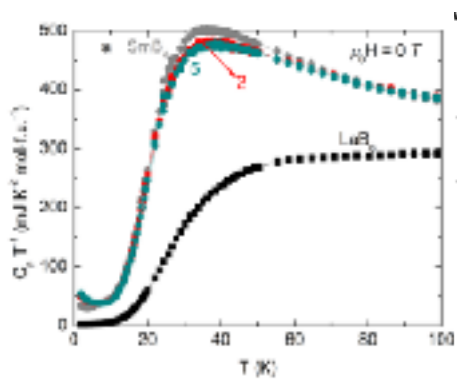
Lifschitz Kosevich=
FT of Fermi Function.

The SmB₆ Conundrum



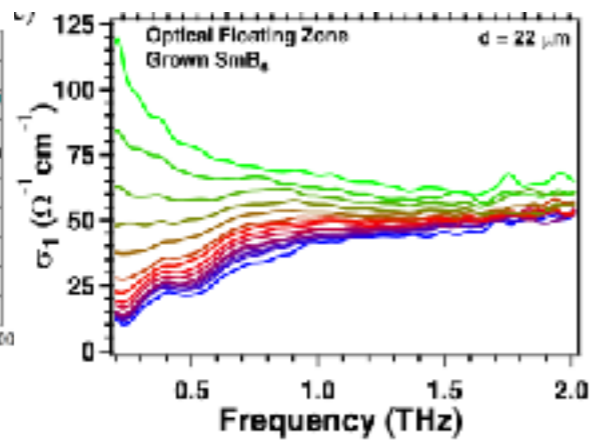
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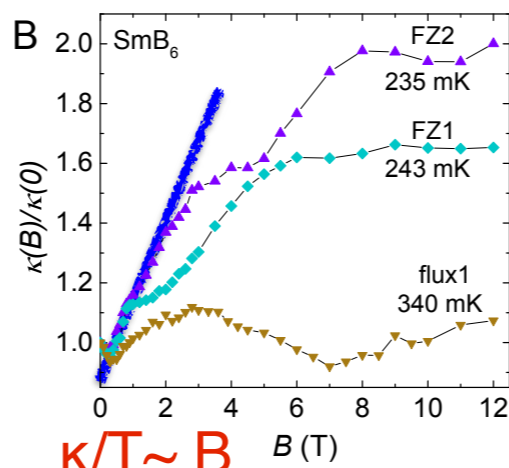
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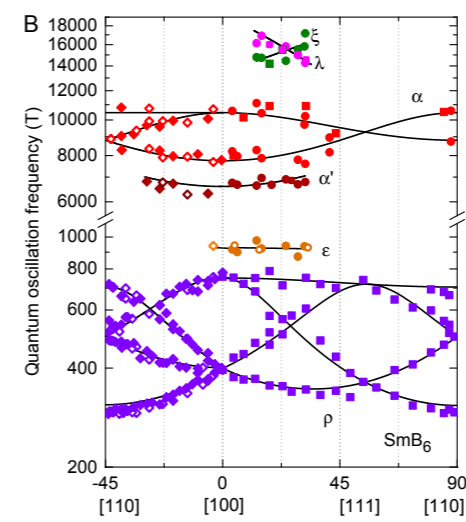
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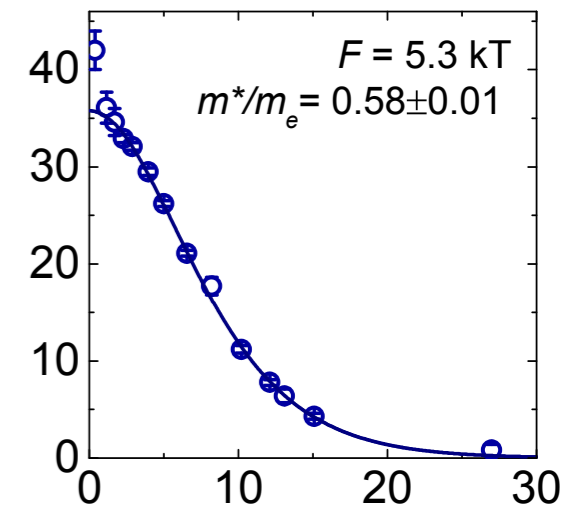


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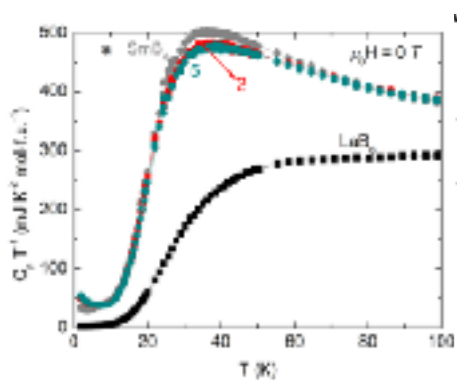


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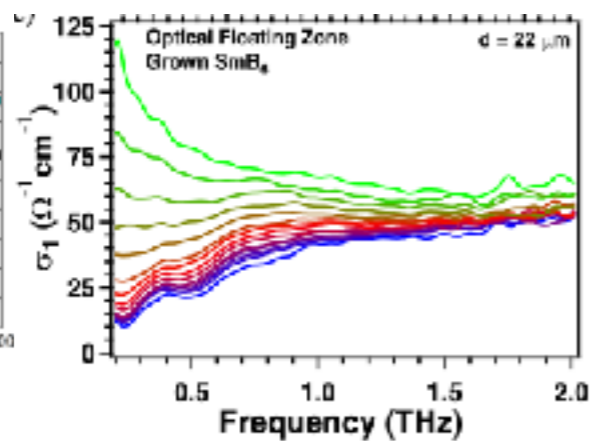
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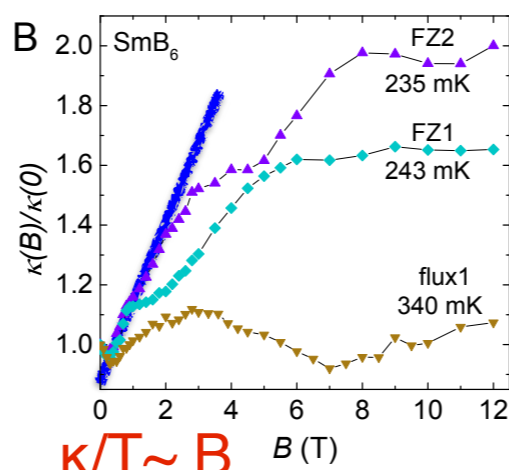
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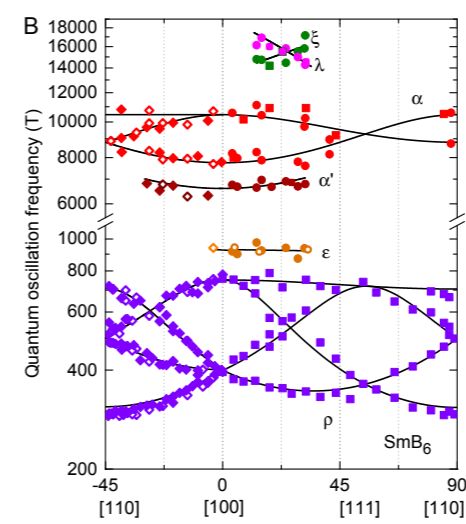
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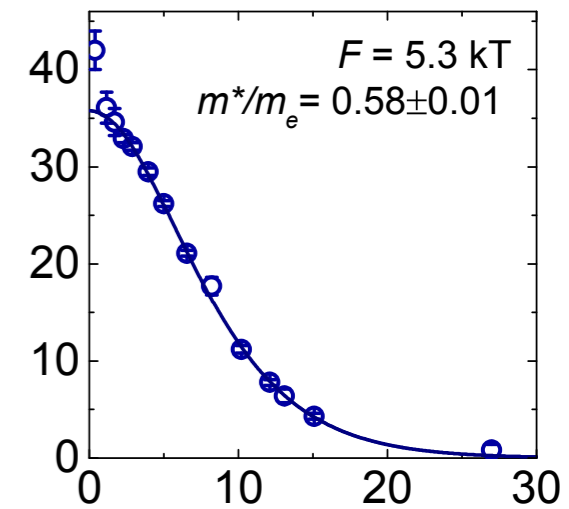


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Excitonic/Breakdown Theory

J. Knolle and N. Cooper PRL 118, 096604 (2017)

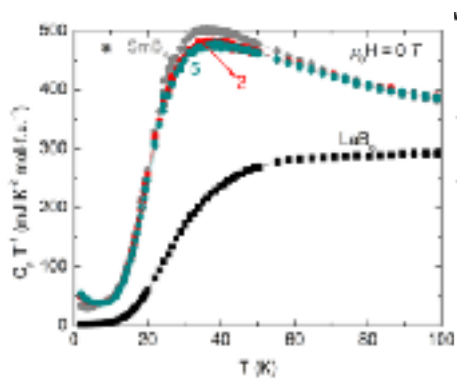
Fractionalization

P. Coleman, E. Miranda, and A. Tsvelik, *Physica B: Condensed Matter* 186-188, 362 (1993).

G. Baskaran, ArXiv e-prints (2015), [arXiv:1507.03477](https://arxiv.org/abs/1507.03477).

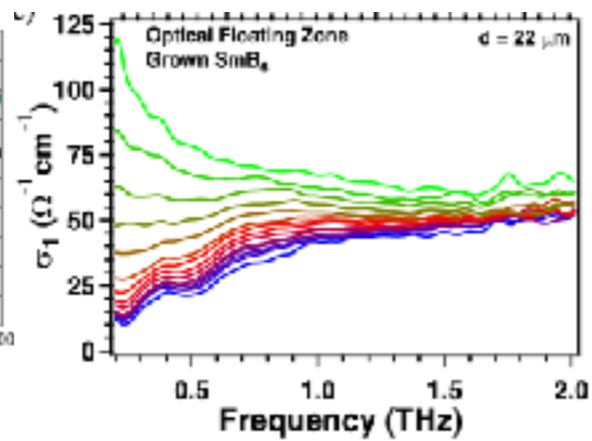
T. Senthil *et al.*, [arXiv:1707](https://arxiv.org/abs/1707)

O. Erten, P-Y Chang, Coleman and A. Tsvelik, [arXiv:1701.06582](https://arxiv.org/abs/1701.06582), PRL in press.



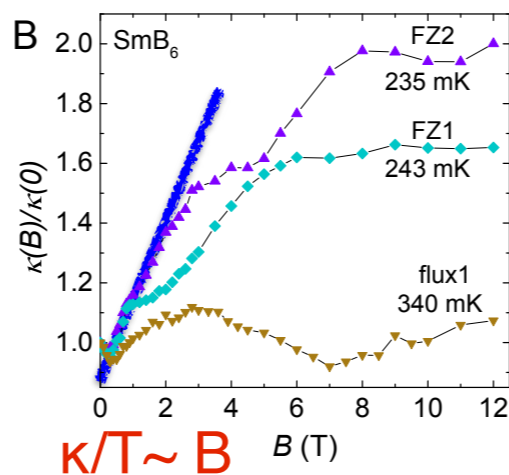
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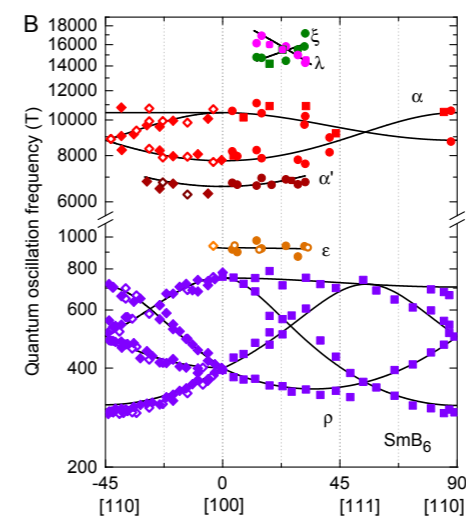
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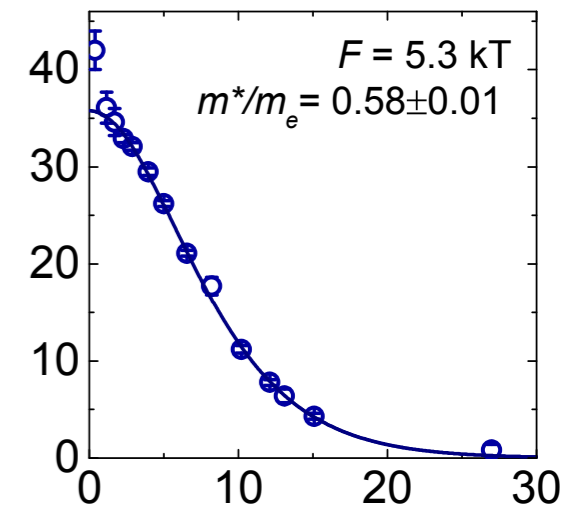


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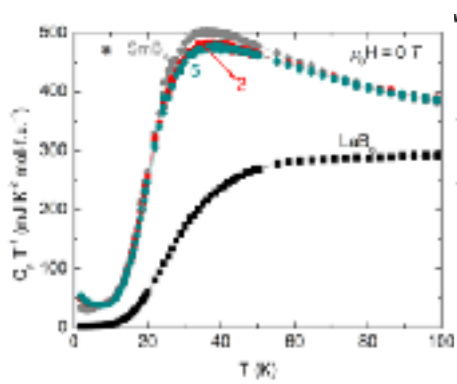
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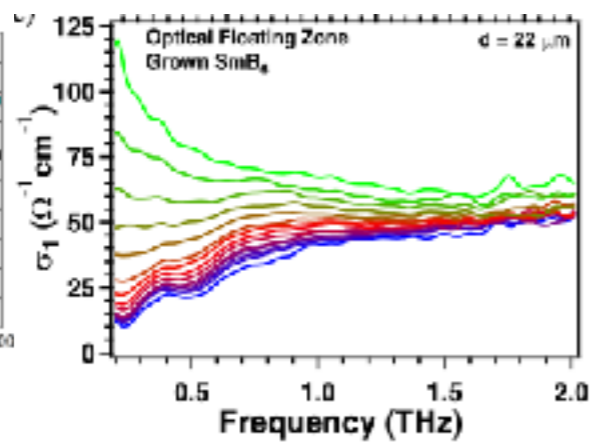
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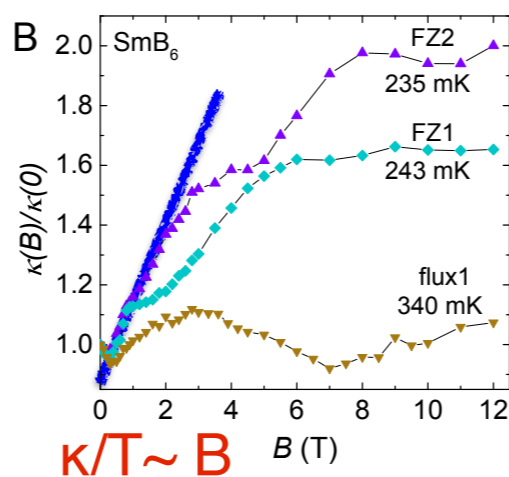
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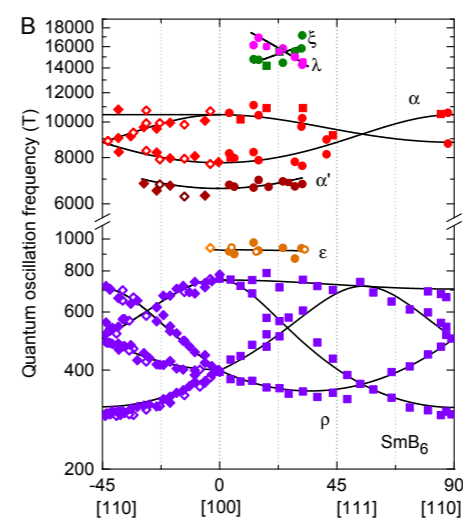
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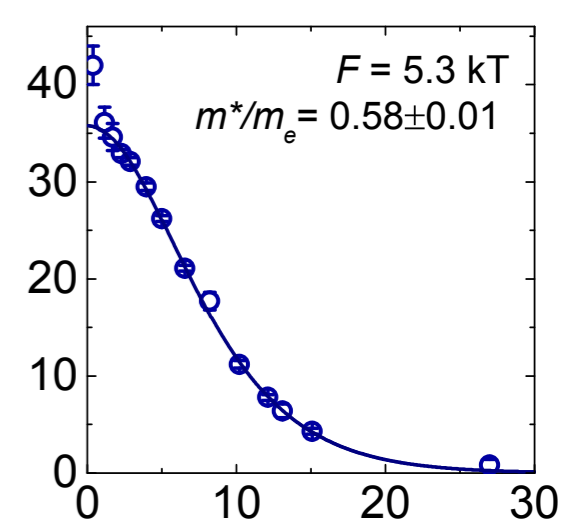


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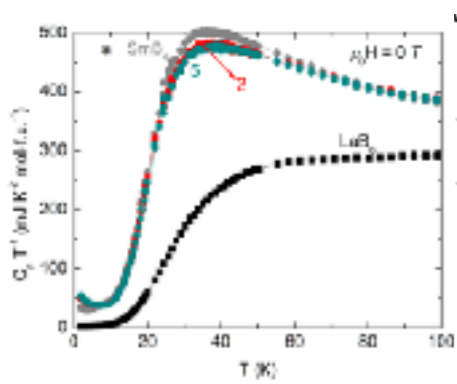
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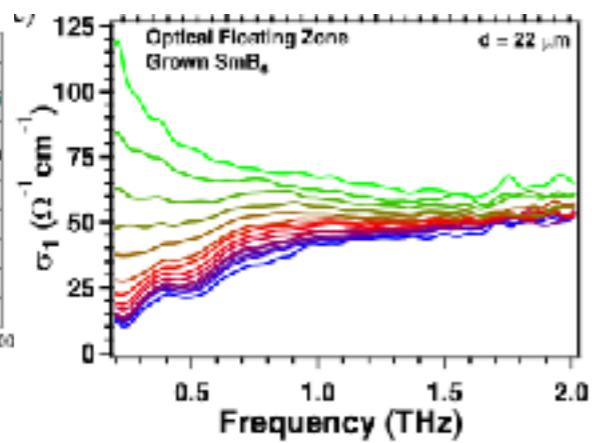
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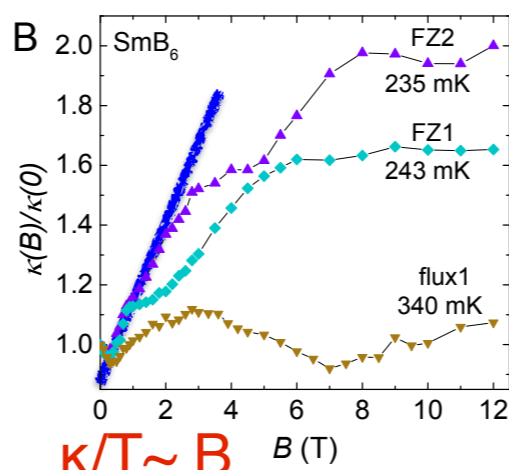
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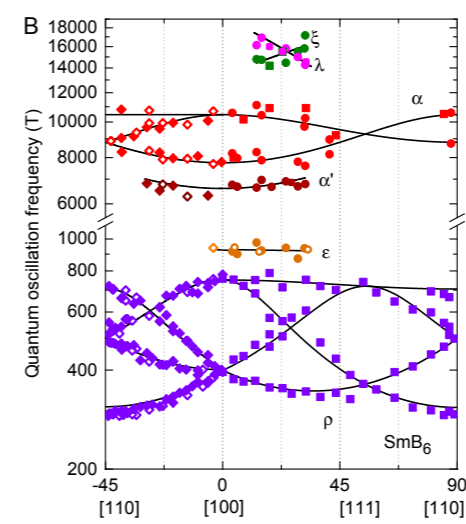
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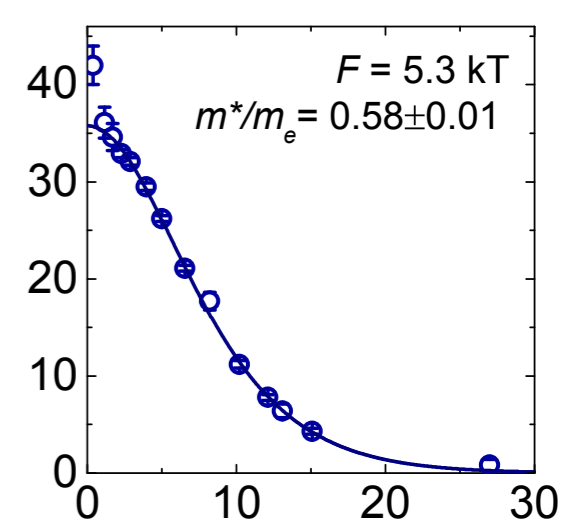


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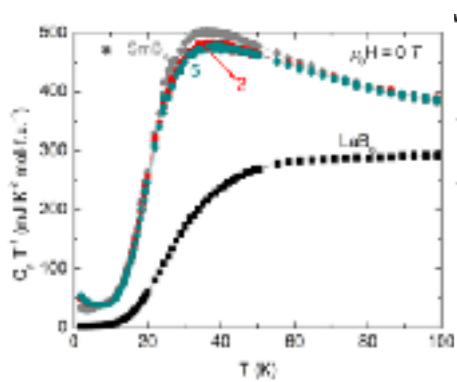
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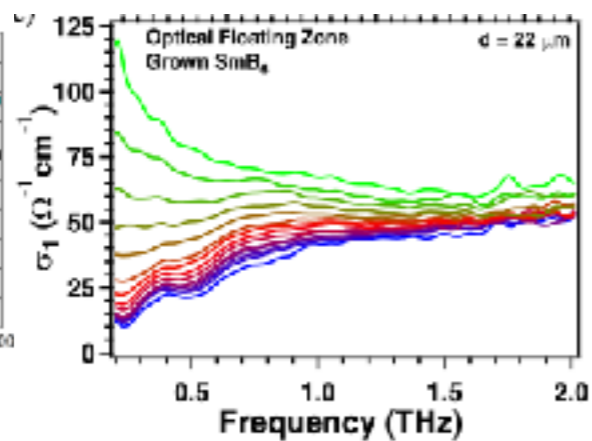
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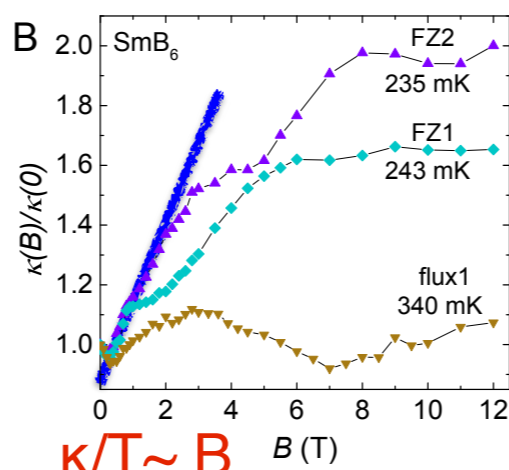
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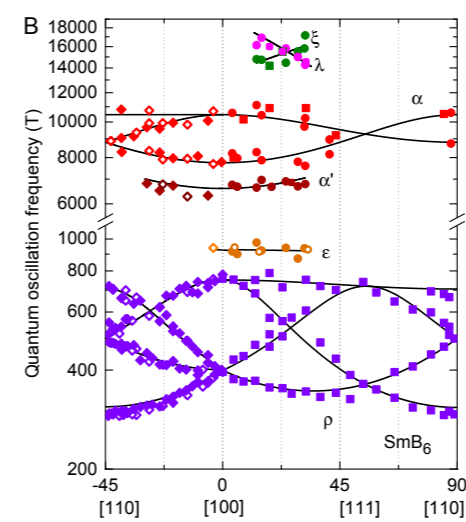
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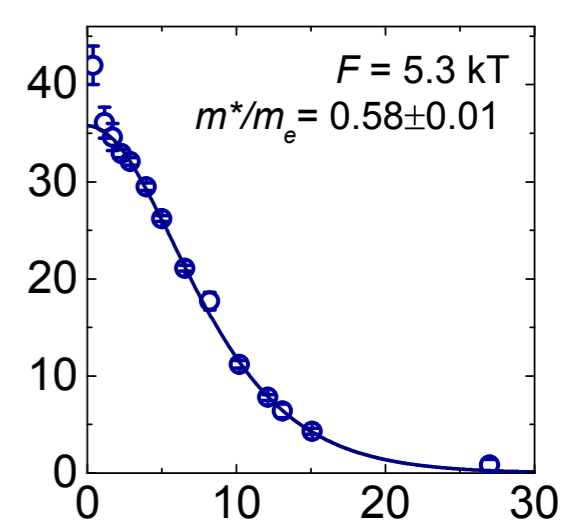


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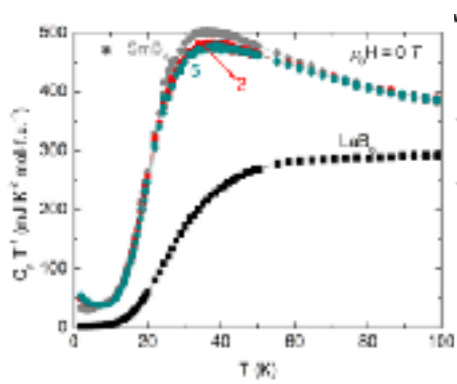
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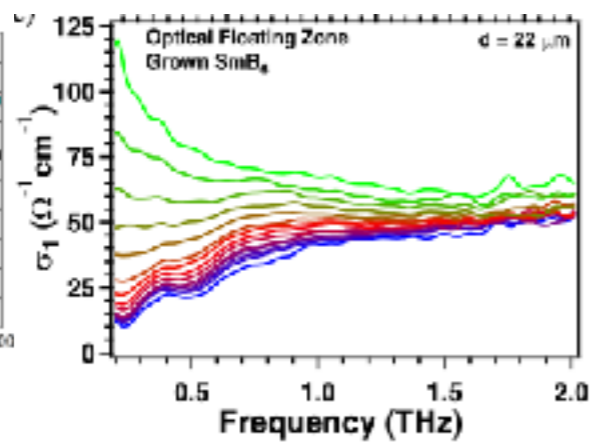
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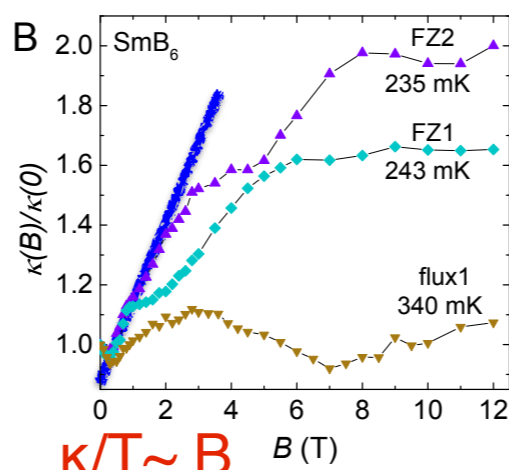
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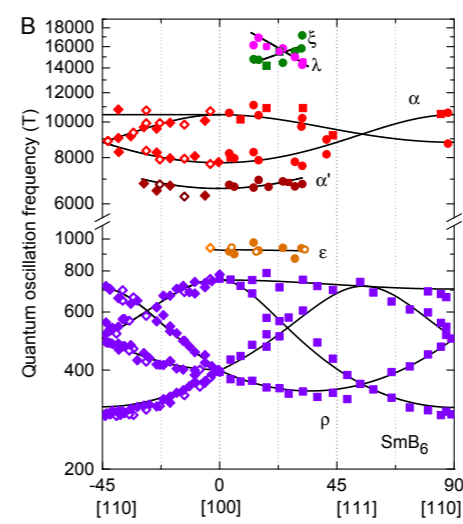
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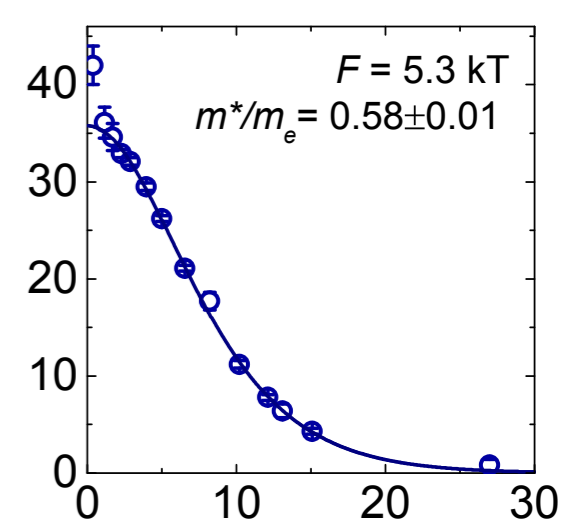


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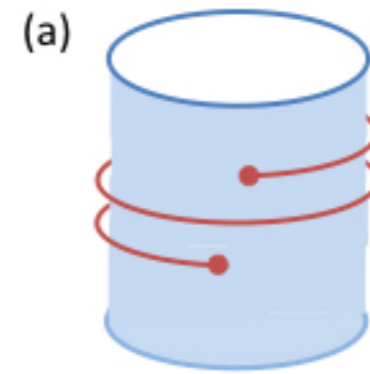
→ Can we have broken gauge invariance without superconductivity?

SmB₆: a Skyrme Dielectric?

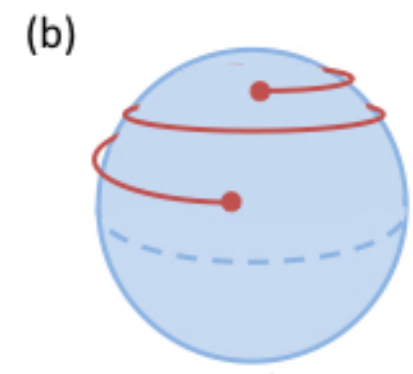
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Onur Erten, Po-Yao Chang, Piers Coleman and Alexei Tsvetik **arXiv:1701.06582**

Superconductivity: requires
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$$\pi_1(S^1) = \mathbb{Z}$$



$$\pi_1(S^2) = 0$$

Hanson, Oganesyan and Sondhi,
Annals Of Physics vol. 313, 497 (2004)

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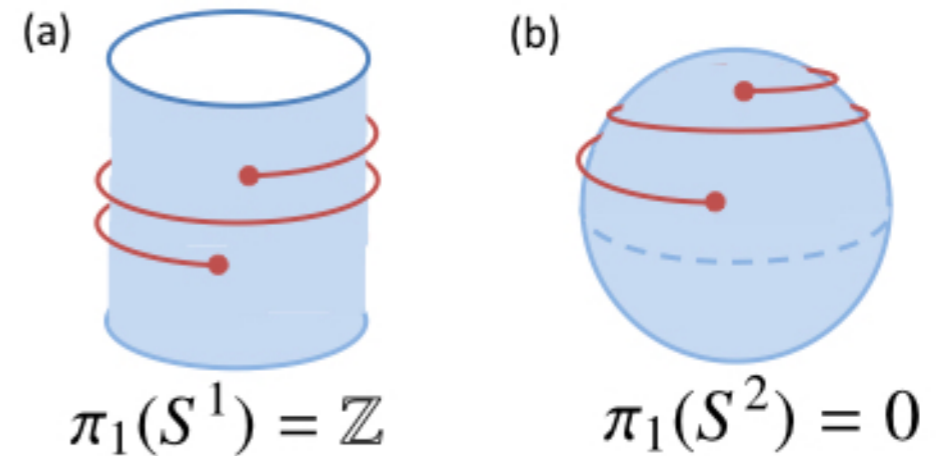
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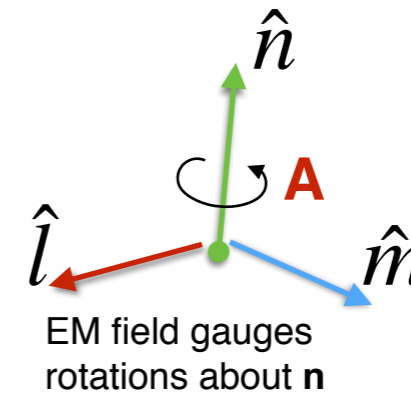
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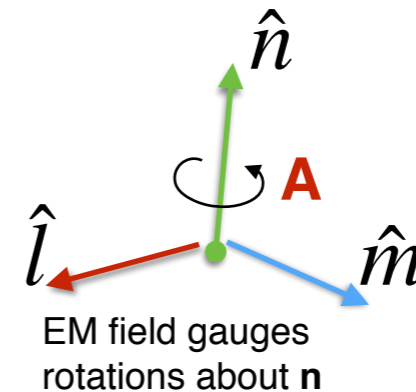
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$$\nabla^2 \mathbf{B} = \frac{1}{\lambda_L^2} (\mathbf{B} - \phi_0 \boldsymbol{\rho}(x))$$

London equation

$$\boldsymbol{\rho}(x) = \mathbf{B} / \phi_0$$

Skyrmion density



**Fluid of Coreless skyrmions of
the n field allow field penetration.**

SmB₆: a Skyrme Dielectric?

Coleman, Miranda, Tselik, 1993, Baskaran 2015

Onur Erten, Po-Yao Chang, Piers Coleman and Alexei Tselik **arXiv:1701.06582**

Superconductivity: requires
Meissner *and* topological rigidity.

s-wave odd-frequency triplet.

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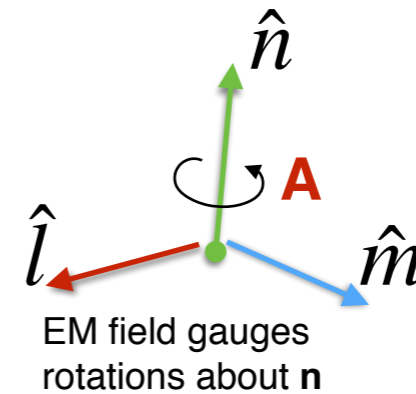
Skyrmion density

$$\kappa \sim B$$

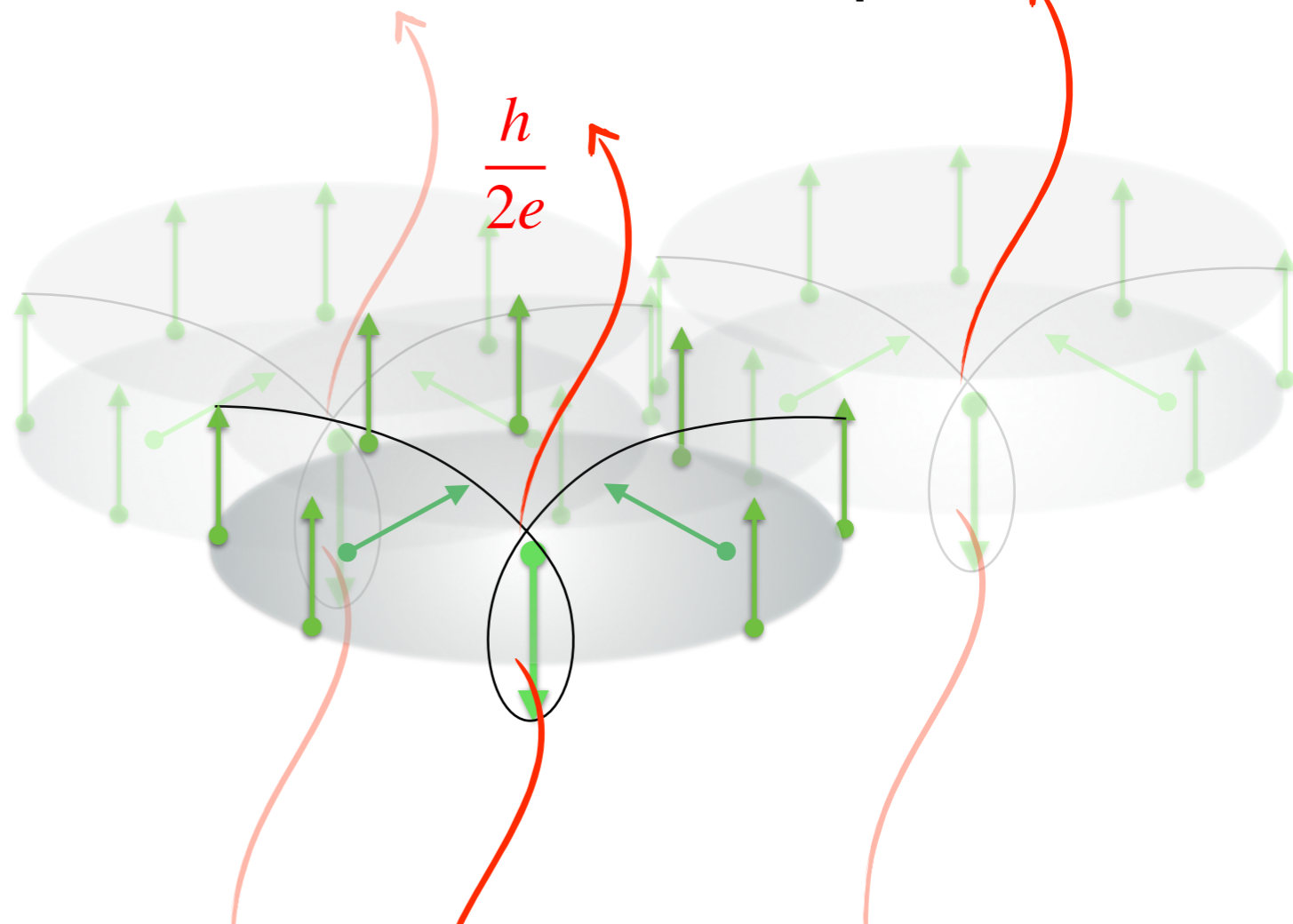
Skyrmion fluid

coreless, mobile

$$n_S \sim B \rightarrow \kappa \sim B$$



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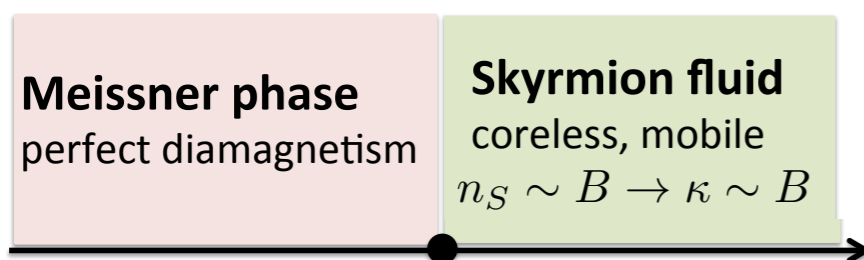
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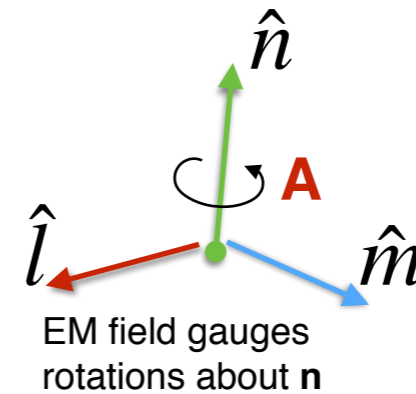
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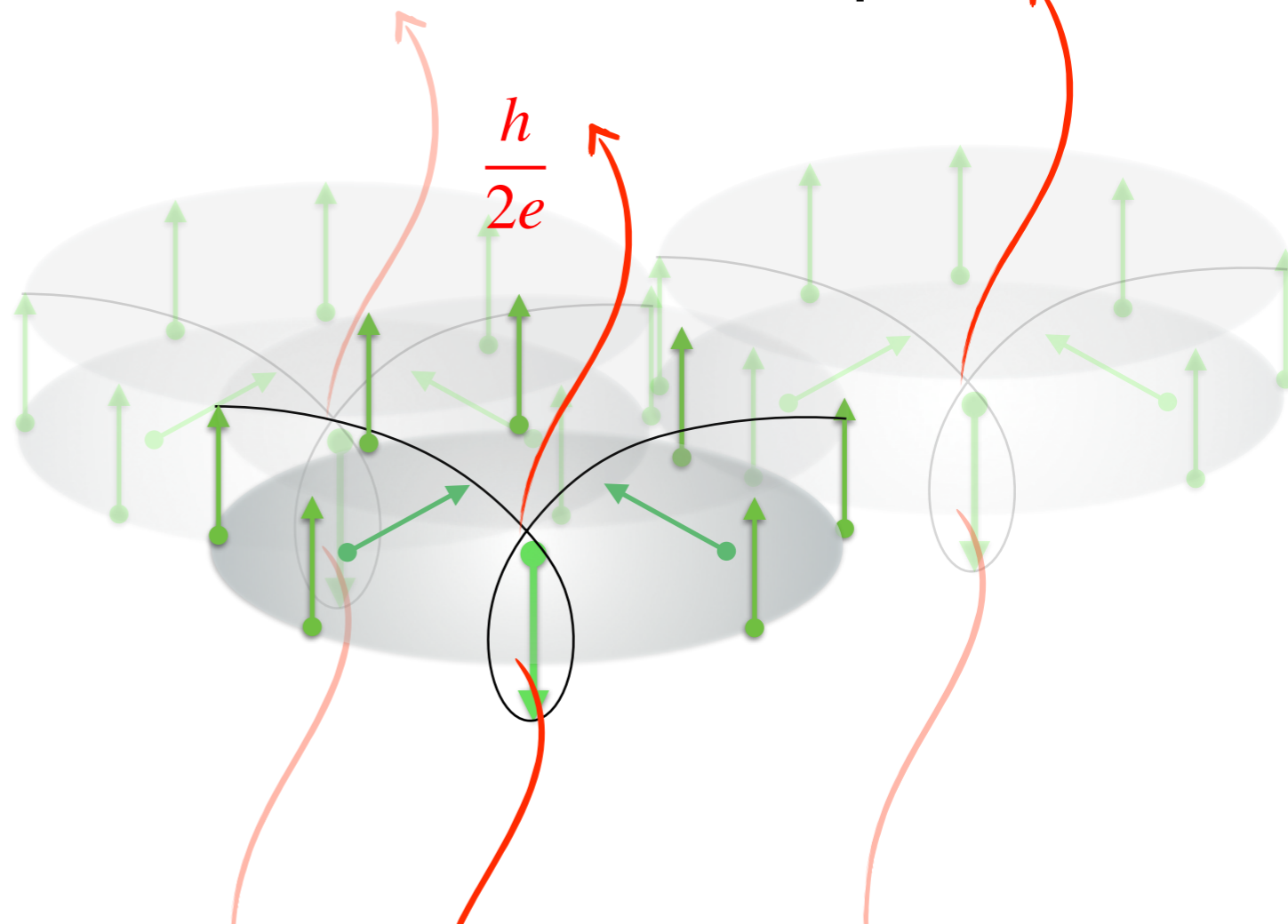
(b)



$$H_{c1} \sim \frac{1}{137c} \left(\frac{V_K}{a} \right) \sim 1 \text{ Gauss}$$



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$$\vec{S}_j = -\left(\frac{i}{2}\right) \vec{\eta}_j \times \vec{\eta}_j$$

Majorana spin
representation

(b)

$$\kappa \sim B$$

Meissner phase
perfect diamagnetism

Skyrmion fluid
coreless, mobile
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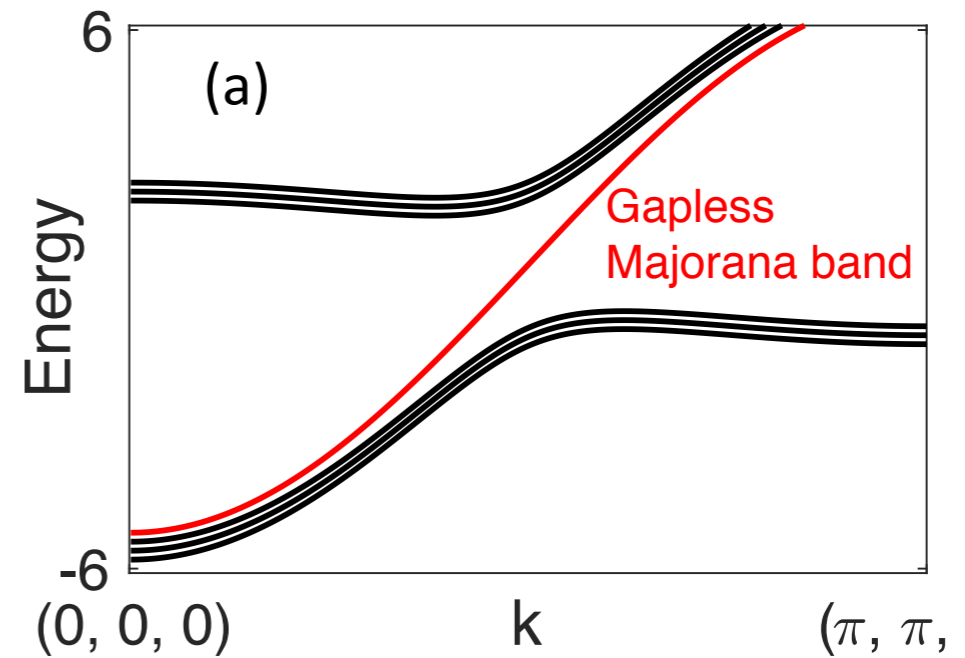
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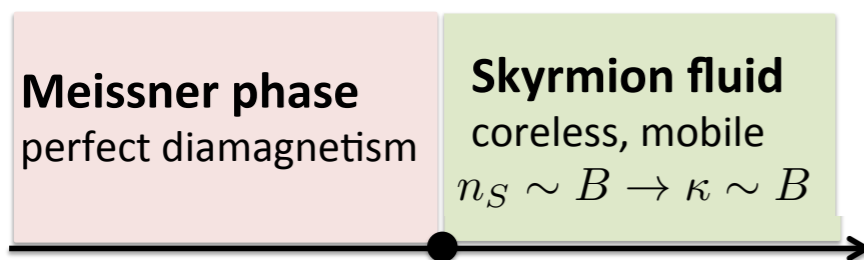
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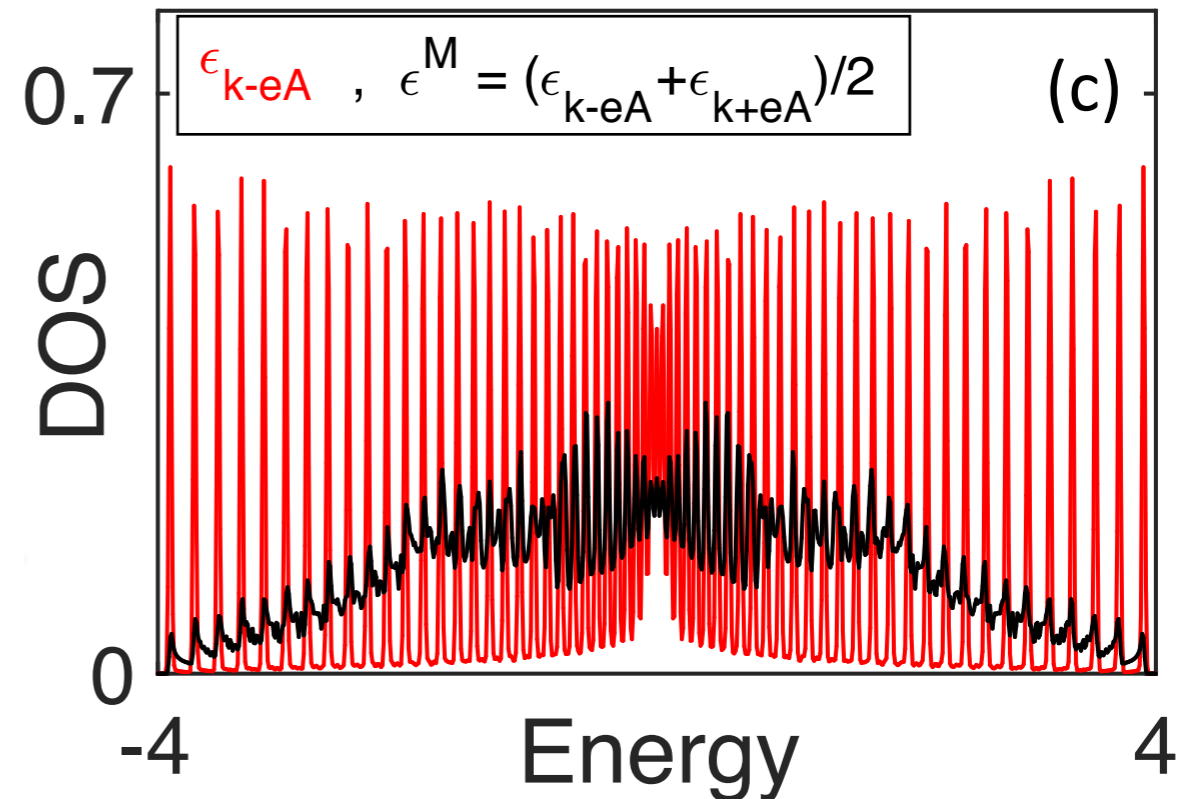
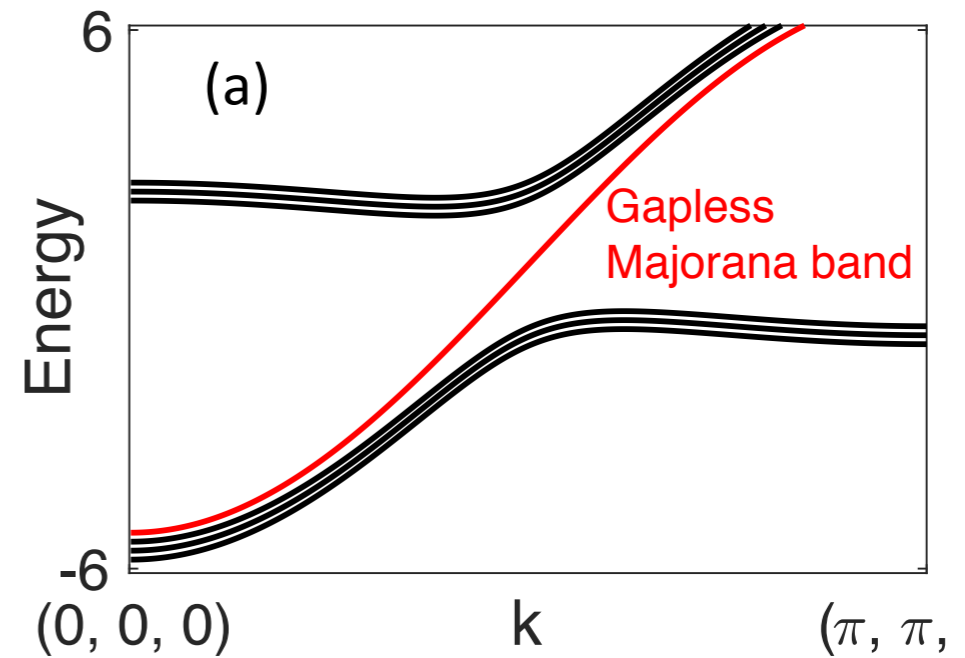
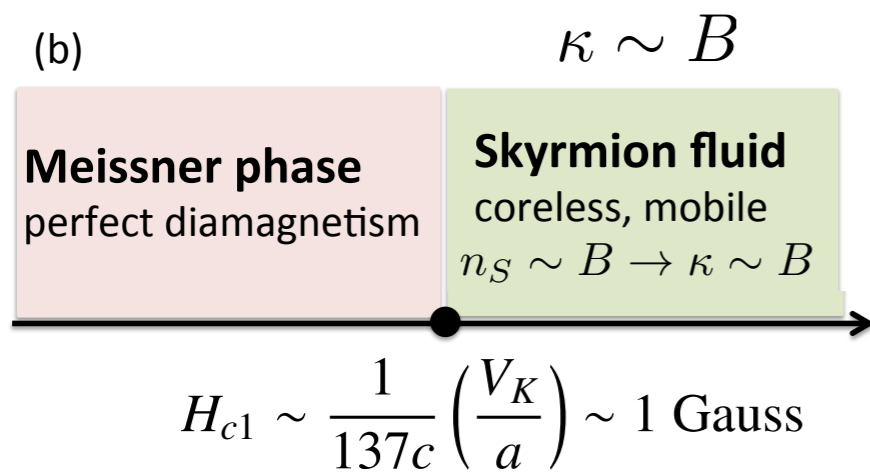
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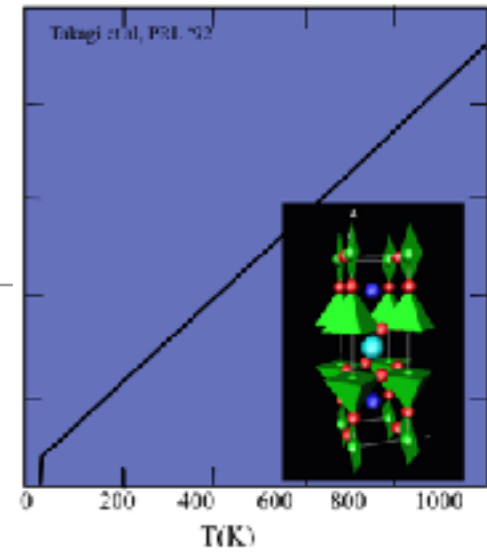
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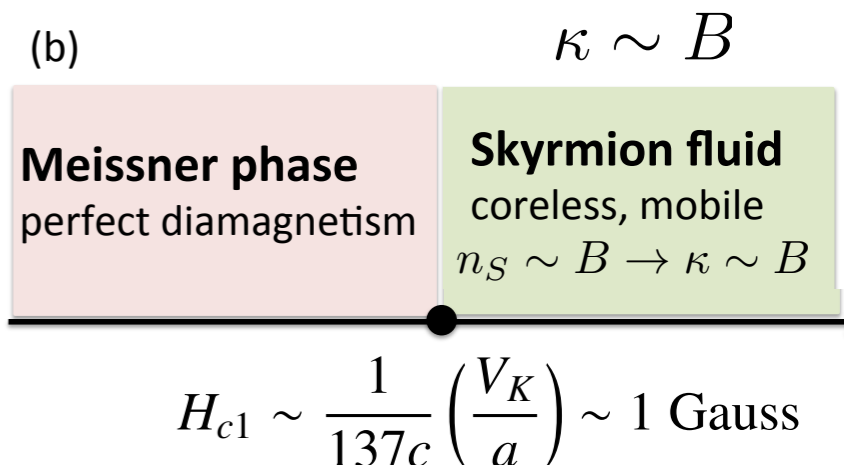
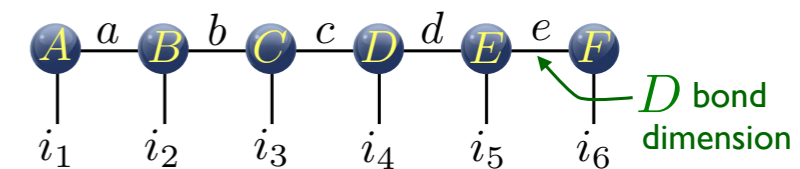


Conclusions

- 50 Years of Novel Phases. Discovery often awaits new concepts.
- Dark Matter Challenges of the Solid State. Potential for qualitatively new advances in our understanding of quantum matter.
- Quantum I: MPS, PEPS. Tools to manipulate and explore new mechanisms of entanglement.
- Beyond Hartree Fock/BCS: Order parameter fractionalization hypothesis.
- Fermi surface in an insulator. Skyrme Dielectric?



Tensor network: matrix product state (**MPS**)



Thank you!