621 Many Body Physics II

2022

Meets	Weds	12.10 - 1.30	Serin 287
	Fri	2 - 3.20	" "
Extra Class	Monday ?	WHAT TIME ?	

Piers Coleman
Serin 268
pcoleman@physics

- First class Jan 19^{th}
- Week of Jan 24th — No classes

Special Challenge! Intro to 620 was not held in Fall 2021.
Class with a broad range of backgrounds.

Purpose of today: to discuss the scope of the course
to arrange catch-up sessions for those who need them.

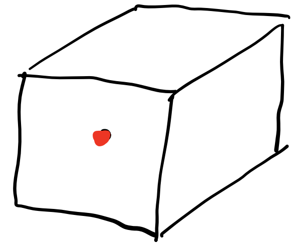
MANY BODY PHYSICS : What is it ?

Mathematical machinery for studying the EMERGENT PHYSICS of macroscopically large ensembles of particles. Modern context (outside astrophysics) involves Quantum Mechanics. The field exists at the intersection of Statistical + Quantum Mechanics, Quantum Field Theory, Condensed Matter Physics + Cold Atoms.

Our Goals This Semester

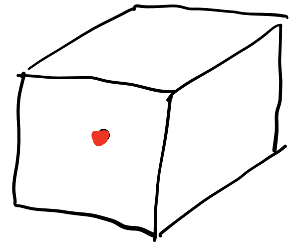
- Finite temperature, imaginary time + response functions
- Transport theory $V = IR$
- Broken symmetry + Ginzburg Landau
- Path integrals
- Superconductivity
- Heavy Fermion Physics.

- INTRODUCTIONS IN CLASS
- MANY PARTICLES + SECOND QUANTIZATION
- WHY IMAGINARY TIME ?
- PATH INTEGRALS : WHAT + WHY ?
- WHAT IS A SUPERCONDUCTOR ?



$$\psi^{+}(x_1)|0\rangle = |x_1\rangle$$

$$\psi(x) = \langle x|\psi\rangle$$



$$\psi^{\dagger}(x_1)|0\rangle = |x_1\rangle$$

$$\psi(x) = \langle x|\psi\rangle$$

- Many Particles + Second Quantization ✓
- Why Imaginary Time? ✓
- Path Integrals: What + Why? ✓
- What is a Superconductor?

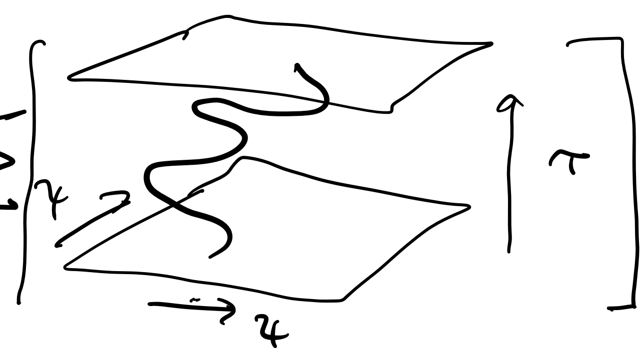
Many Particles + 2nd Quantization

BROKEN SYMM

$$Z = \sum e^{-\beta(E_j - \mu N_j)} = \int e^{-S[\bar{\psi}, \psi]}$$

configs in SPACE TIME

$$Z = \sum e^{-\beta(E_j - \mu N_j)} \equiv \sum$$



PATH INTEGRALS

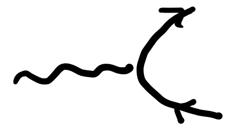
"coherent states"

EIGENSTATES OF FIELD OPERATOR

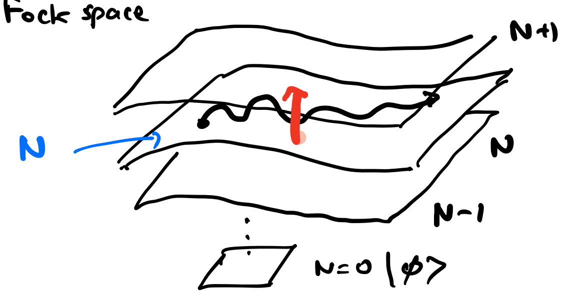
$$\boxed{\hat{\psi}(x)|\psi\rangle = \psi(x)|\psi\rangle}$$

$\langle \hat{\psi}(x) \rangle = \psi'(x)$ — DRAMATIC MANIFESTATION OF FOCK SPACE

$$\langle N|\, \psi^\dagger(y)\, \psi(x)\, |N\rangle = \rho(y,x) \xrightarrow{|x-y|\to\infty} \psi^*(y)\, \psi(x)$$



"Fock space



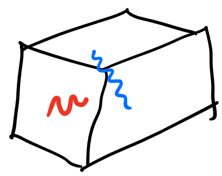
$$-i\frac{\partial}{\partial t}\psi = h\psi$$

$$\hat{\rho}(x) = \sum_{j=1}^{N} \delta^3(\vec{x}-x_j) \qquad \langle \hat{\rho}(x)\rangle = \int |\psi|^2 \sum \delta^3(x-x_i)\, d^3x_i$$

$$= |\psi|^2 \longrightarrow \hat{\rho}(x) = \psi^\dagger(x)\,\psi(x)$$

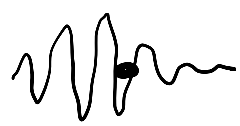
2nd Quantization

Many particles



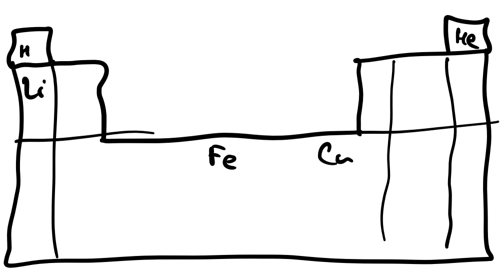
$$\psi(1,2,\ldots N,t) = (\pm 1)^P\, \psi(P_1,P_2,\ldots P_N,t)$$

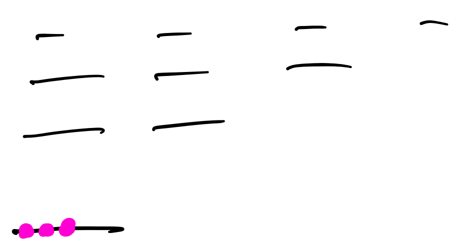
One particle physics



$\psi(x,t)$

$P(x,t) = |\psi|^2$





WHAT IS IMAGINARY TIME

Schrödinger wavefn.

$$\psi(x_1, \dots x_N) = \langle x_1, x_2 \dots x_N | \psi \rangle$$

$$\hbar i \frac{\partial|\psi\rangle}{\partial t} = \hat{H}|\psi\rangle \implies \boxed{|\psi(t)\rangle = e^{-\frac{iHt}{\hbar}}|\psi(0)\rangle}$$

Stat mech

$$Z = \sum e^{-\beta(E_j - \mu N_j)} = e^{-\beta F[\mu, T]}$$

$$= \sum_j \langle j| e^{-\beta(\hat{H} - \mu\hat{N})} |j\rangle = \mathrm{Tr}\, \underbrace{e^{-\beta[\hat{H} - \mu\hat{N}]}}$$

$$\sum_j |j\rangle\langle j| = 1 \qquad \text{(completeness)}$$

$$e^{-\frac{\hat{H}}{k_B T}} \longleftrightarrow e^{-\frac{itH}{\hbar}}$$

$E_k \to \epsilon_k = (E_k - \mu)$

$H - \mu N = H'$

$$Z = \mathrm{Tr}[e^{-H'/k_B T}] \implies \boxed{\text{All of Stat Mech}}$$

$$t = \left(\frac{\hbar}{k_B T}\right) \times -i \quad = \quad -i\,\tau_P$$

$$\boxed{\tau_{PLANCK} = \hbar / k_B T}$$

$$Z = \mathrm{Tr}\left[U(-i\,\tau_{PLANCK})\right]$$

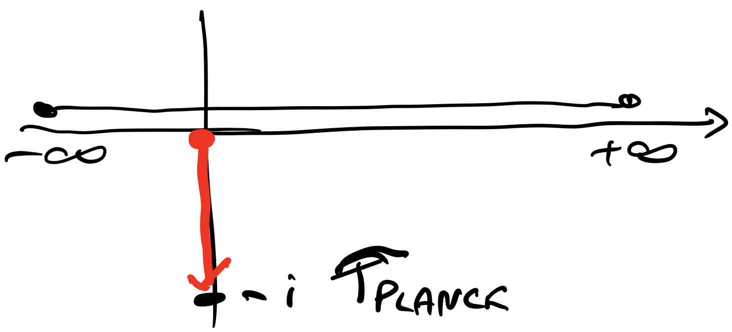
Bloch 1930s

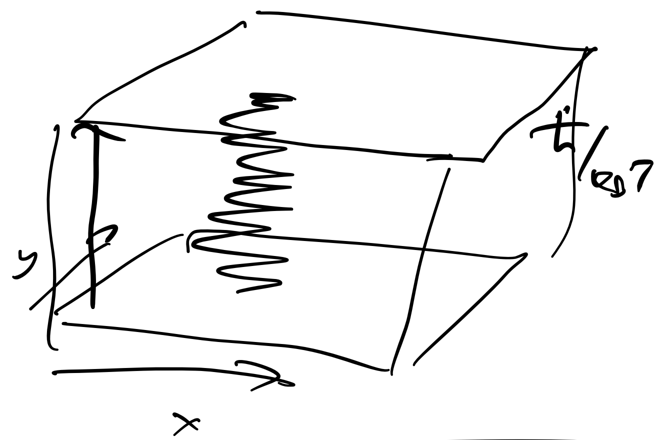
Japanese

Matsubara Kubo

1950s

Imaginary time QM





Correlation fns in Imag time

FLUCTUATIONS

$$\langle A(t)\,A(0)\rangle_T$$

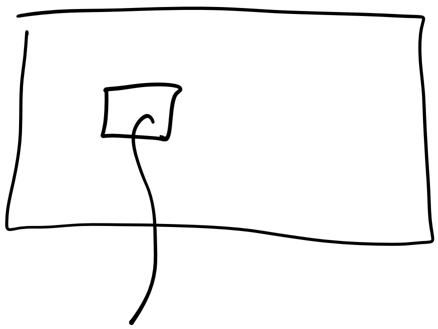
$$\langle A(\tau)\,A(0)\rangle \longleftrightarrow -i\,\langle [A(t), A(0)]\rangle$$

$$= Q(t)$$

$$\langle A(t)\rangle = \int Q(t-t')\,B(t')$$

DISSIPATION

RESPONSE in Real time



Subsystem at definite μ

$$\mathrm{Tr}\left[e^{-\beta(\hat{H}-\mu\hat{N})}\right]$$

$$\downarrow$$

$$\sum_{N_0}\left(\mathrm{Tr}_{\hat{N}=N_0}\left(e^{-\beta\hat{H}}\right)e^{\beta\mu N_0}\right)$$