L23 Twisted Bilayer Graphene

1. Graphene.

- Textbook band theory.

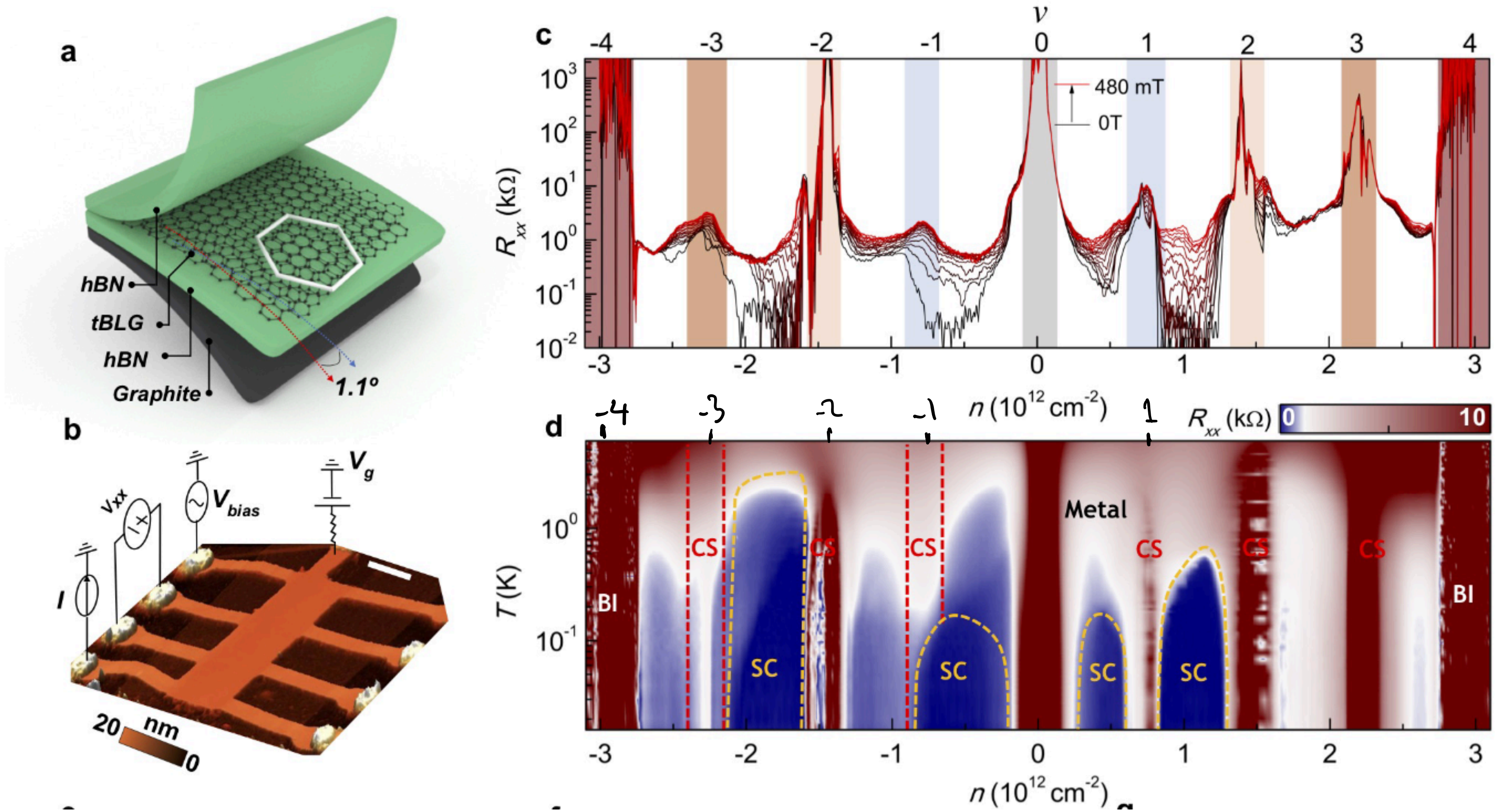
• 2DEG. : all physics of lattice scale embedded in Dirac Disp.

• Weak interactions

Contrast + SCES – strong interactions

2. TBG – reduce $-t \sum (c^{\dagger}_{i\sigma} c_{j\sigma} + h.c)$ $\qquad W_{Graph} \sim 3t \sim 8eV$

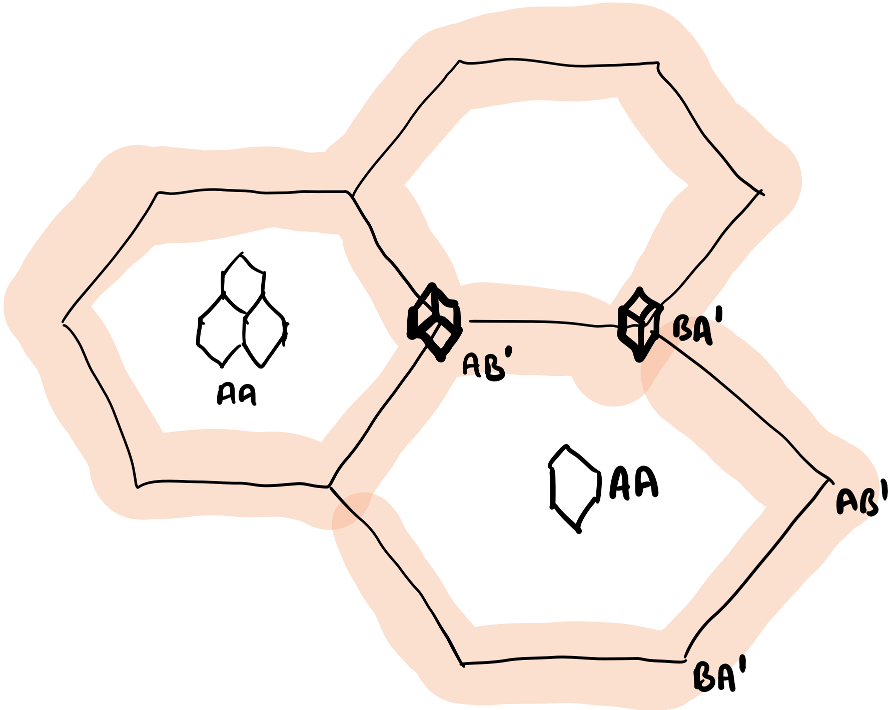
Periodic potential that flattens bands.



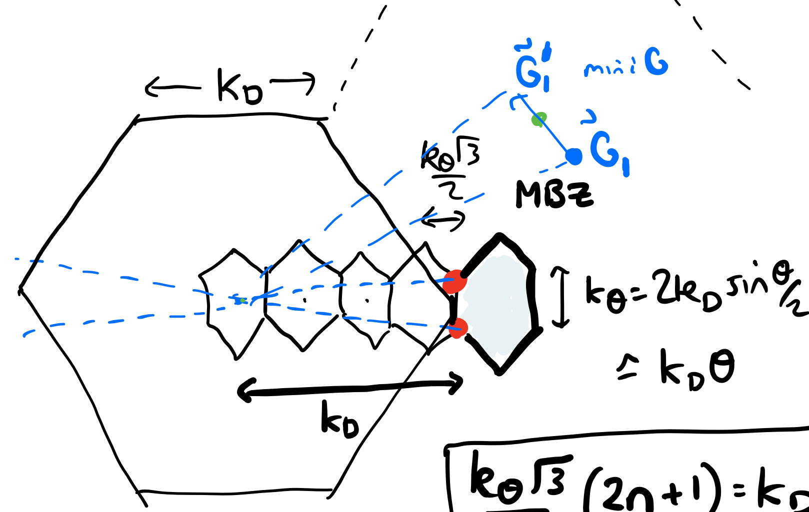
a
hBN
tBLG
hBN
Graphite
1.1°
b
V_g
V_{bias}
V_{xx}
I
20 nm 0
c
ν
-4 -3 -2 -1 0 1 2 3 4
480 mT
0T
R_{xx} (kΩ)
n (10^{12} cm^{-2})
d
-4 -3 -2 -1 1
R_{xx} (kΩ) 0 10
Metal
CS
SC
BI
T(K)
n (10^{12} cm^{-2})

The Bistritzer Macdonald Model

PNAS, 108, 12233 (2011)

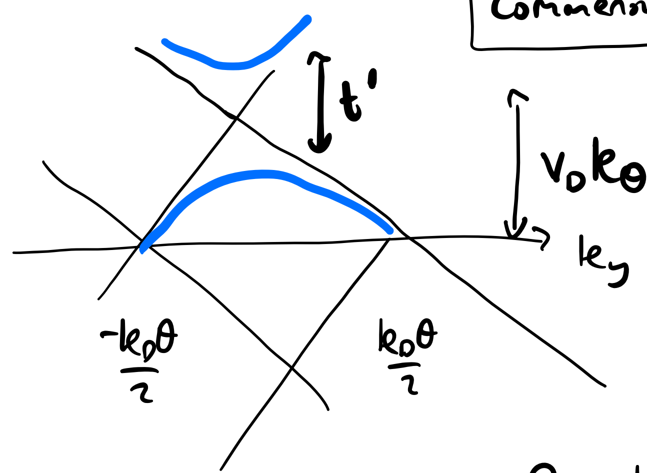


Real space



$$\boxed{\frac{k_\theta\sqrt{3}}{2}(2n+1) = k_D}$$

Commensurate



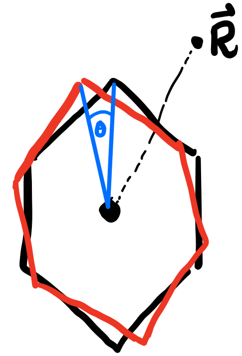
2 spin × 2 dirac cones × 2 valley = 8

$$\vec{R}_\alpha = \vec{R}_{cell} + \vec{\tau}_\alpha$$

$$\vec{\tau}_1 = 0 \quad \tau_2 = (0, a) \equiv (0, 1)$$

When $t'/2 \sim v_0 k_\theta/2$, i.e. $t' \sim v_0 k_\theta$ the velocity goes to zero.

$$\boxed{\text{THE MOIRE PATTERN INDUCES A PERIODIC TUNNELING POTENTIAL}}$$

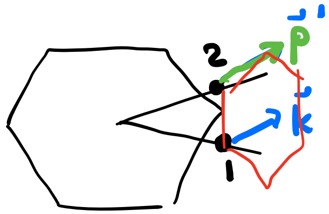


$$\vec{R} = m_1 \hat{n}_1 + m_2 \hat{n}_2$$

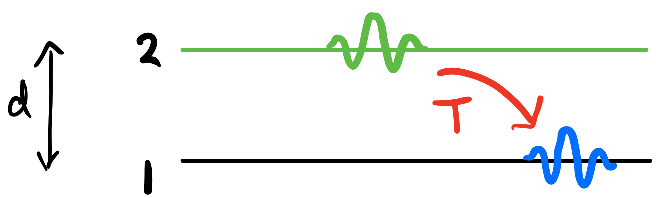
$$\vec{R}' = M(\theta)\vec{R} + \vec{d}$$

translation

we will take $\vec{d}=0$ since the system is incommensurate.



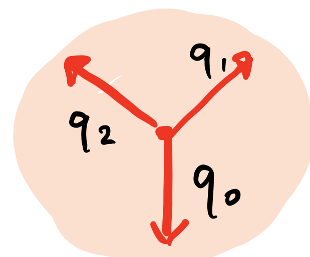
$d \sim 3.4\,\mathring{A}$ $\quad a = 1.4\,\mathring{A}$



$$|2\rangle h(\theta/2)\langle 2|$$

$$|1\rangle h(-\theta/2)\langle 1|$$

$$T^{\alpha\beta}(\vec{r}) = \sum_\ell e^{-i\vec{q}_\ell \vec{r}}\, T^{\alpha\beta}_\ell$$

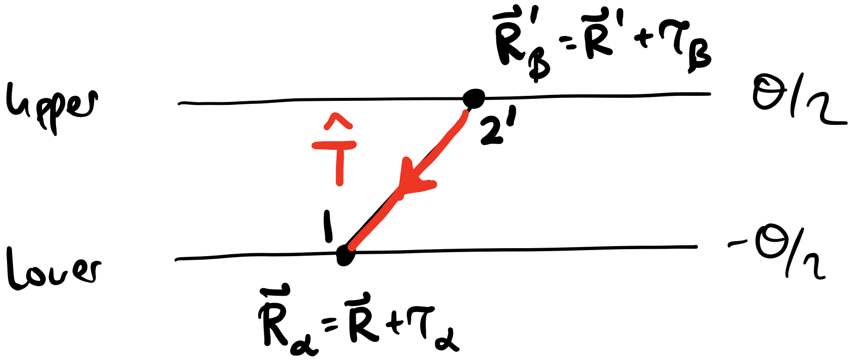


$$q_\ell = -ik_\theta e^{i\ell\phi}$$
$$(\phi = 2\pi/3)$$

$$h(\theta) = vk \begin{bmatrix} 0 & e^{-i(\theta_k - \theta)} \\ e^{i\theta_k - \theta} & 0 \end{bmatrix}$$

$$\boxed{k e^{i\theta_k} = k_x + ik_y}$$

L24: BM Model – Interlayer tunnelling.



$$t\left(\vec{R} + \tau_\alpha - \vec{R}' - \tau'_\beta\right) = \text{tunneling amplitude}$$

$$= f\left(\sqrt{(\vec{R} - \vec{R}' + \tau_\alpha - \tau'_\beta)^2 + d^2}\right)$$

$$\left.\begin{aligned} |\psi^{(1)}_{\vec{k}\alpha}\rangle &= \frac{1}{\sqrt{N_s}} \sum_{\vec{R}} e^{i\vec{k}\cdot(\vec{R} + \vec{\tau}_\alpha)} |R + \tau_\alpha\rangle \\ |\psi^{(2)}_{\vec{p}'\beta}\rangle &= \frac{1}{\sqrt{N_s}} \sum_{\vec{R}'} e^{i\vec{p}\cdot(\vec{R}' + \vec{\tau}'_\beta)} |\vec{R}' + \tau'_\beta\rangle \end{aligned}\right\}$$

Bloch States on the lower + upper Graphene Layers.

(α ↑ sublattice)

$$T^{\alpha\beta}_{kp'} = \langle \psi^{(1)}_{k\alpha} | H_T | \psi^{(2)}_{\vec{p}'\beta} \rangle$$

$$H_T = \sum_{\substack{R\, \tau_\alpha \\ \vec{R}',\, \tau'_\beta}} |R + \tau_\alpha\rangle\, t(\vec{R} + \vec{\tau}_\alpha - \vec{R}' - \vec{\tau}'_\beta)\, \langle \vec{R}' + \tau'_\beta |$$

$$\Rightarrow \boxed{T_{kp}^{\alpha\beta} = \frac{1}{N_s} \sum_{\substack{R\tau_\alpha \\ \vec{R}'\tau'_\beta}} e^{-i\vec{k}(\vec{R}+\vec{\tau}_\alpha)}\, t(R+\tau_\alpha-\vec{R}'-\tau'_\beta)\, e^{i\vec{p}'\cdot(R'+\tau'_\beta)}.}$$

$$t_{\vec{q}} = \int d^2r\, t(r)\, e^{-i\vec{q}\cdot\vec{r}} \iff t(\vec{r}) = \int \frac{d^2q}{(2\pi)^2}\, t_q e^{i\vec{q}\cdot\vec{r}}$$

$$t(\vec{R}) = \frac{1}{V} \sum_q t_q e^{iq\cdot\vec{R}} = \frac{1}{N\Omega} \sum t_q e^{i\vec{q}\cdot\vec{R}}.$$

$$t(\vec{R}_\alpha-\vec{R}'_\beta) = \frac{1}{N\Omega} \sum_q t_{\vec{q}}\, e^{i\vec{q}(\vec{R}_\alpha-\vec{R}'_\beta)}$$

$$\Rightarrow T_{kp'}^{\alpha\beta} = \frac{1}{N^2\Omega} \sum_{\substack{\vec{R}_1\tau_\alpha \\ \vec{R}'_2\tau'_\beta \\ \vec{q}}} e^{-i(k-\vec{q})(R_1+\tau_\beta)}\, e^{i(\vec{p}'-q)(R'_2+\tau'_\beta)}\, t_{\vec{q}}$$

$$\vec{R}'_2+\tau'_\beta = M(\vec{R}_2+\tau_\beta)+d$$

TAKE $d=0$