$$U_0 = m^2 \sum_{i>j} -\ln|z_i - z_j|$$

Coulomb Interaction Energy

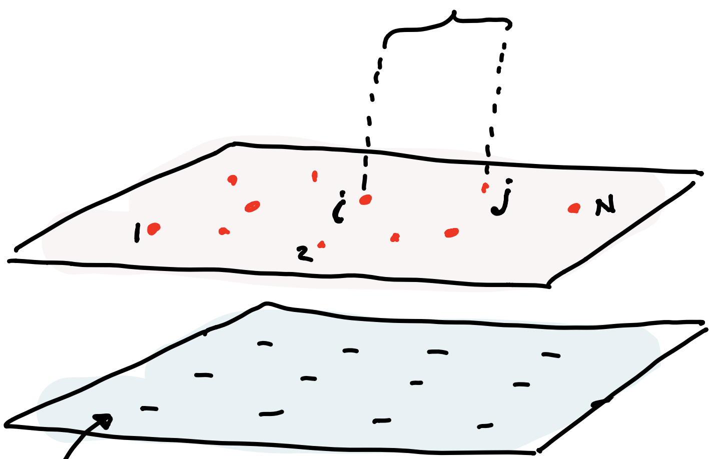
$$m\left(\frac{1}{4}\sum_j |z_j|^2\right) = m\sum_j \phi(z_j)$$

Potential from constant background charge $\rho_B = -\frac{1}{2\pi e^2}$

$$-\nabla^2\left(\frac{1}{4}|z_j|^2\right) = -\left(\nabla_x^2 + \nabla_y^2\right)\left(\frac{1}{4}\frac{(x^2+y^2)}{e^2}\right) = -\frac{1}{e^2} = 2\pi\rho_B$$

$$\Rightarrow \rho_B = -\frac{1}{2\pi e^2}$$

$-m^2 \ln|z_i - z_j|$



$\rho_B = -\frac{1}{2\pi e^2}$

"FAKE PLASMA"

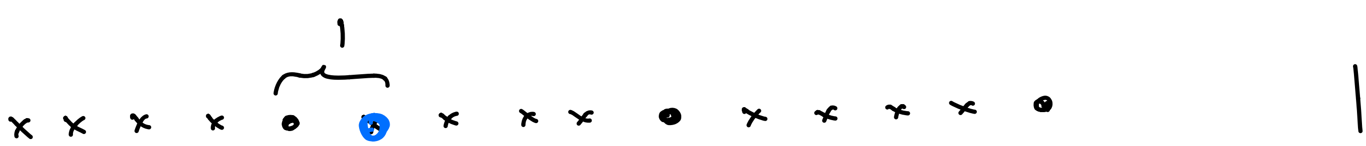
$$\boxed{\begin{array}{l}\text{Neutrality} \Leftrightarrow \text{Largest } |\Psi|^2 \\ nm + \rho_B = 0\end{array}}$$

$$\Rightarrow n = \frac{1}{m}\left(\frac{1}{2\pi e^2}\right) = \frac{1}{m}\left(\frac{N_\phi}{A}\right)$$

$\equiv \frac{1}{m}$ th filled L.L.

$$2\pi e^2 B = \phi_0$$

$$\Rightarrow \frac{1}{2\pi e^2} = \frac{B}{\phi_0} = \frac{\phi}{\phi_0 A} = \frac{N_\phi}{A}$$



$m = 5$

$$\boxed{\Psi_m = \prod_{i<j} (z_i - z_j)^m \, e^{-\sum |z_j|^2/4}}$$

$z_i = \left(\frac{x_i + i y_i}{\ell}\right)$

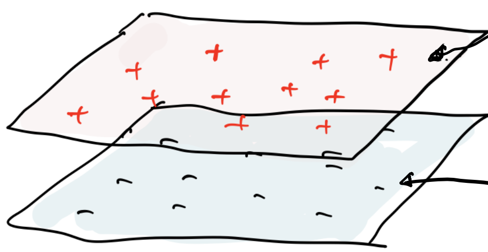
$m=1$ Filled LLL.

$m = 3, 5 \ldots$

LAUGHLIN STATES

$$|\Psi_m|^2 = e^{-\frac{2}{m} V_{cl}[z]}$$

$$V_{cl} = m^2 \sum -\ln|z_i - z_j| + m \sum \frac{|z_j|^2}{4}$$



$\rho_e = \frac{1}{m}\left(\frac{N\Phi}{A}\right)$ $\frac{1}{m}$ th filled LL

$-\frac{1}{2\pi \ell^2} = -\frac{N\Phi}{A}$

$$\Psi_{hole} = \prod (z_j - Z) \; \Psi_m$$

"Anyon"

Fractionalized particle

- $e^* = \frac{e}{3}$
- Fractional statistics

Hole in the LLL

Remove e^- from $m=0$ state at origin

$$f_{2\,\text{hole}} = \begin{vmatrix} z_1 & z_2 \\ z_1^2 & z_2^2 \end{vmatrix} = z_1 z_2^2 - z_2 z_1^2 = \underbrace{z_1 z_2}_{\substack{\text{zeroes at} \\ \text{the origin}}} (z_2 - z_1)$$

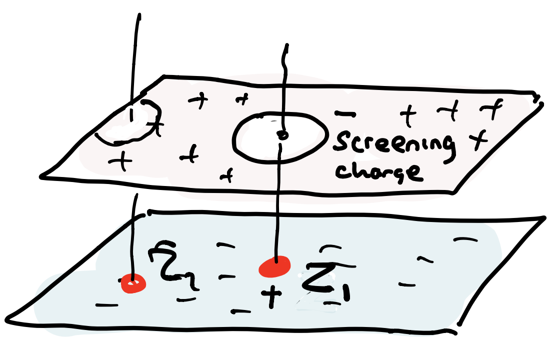
$$\Psi_{\text{hole}}[z] = \left(\prod_i z_i\right) \Psi_1[z]$$

location of hole

$$\Psi_{\text{hole}}[Z, z] = \prod_i (z_i - Z)\, \Psi_1(z)$$

$$|\Psi_{\text{hole}}|^2 = e^{-\beta U'_{\text{classical}}} \qquad \beta = \frac{2}{m} \qquad U'_c = -\frac{m}{2} \ln|\Psi|^2$$

$$U'_{\text{class}} = \sum_{i>j} -\ln|z_i - z_j| + \frac{1}{4}\sum_j |z_j|^2 + \sum_j -\ln|z_j - Z|$$



Perfect screening in Plasma

$\Rightarrow$ Precisely one electron Removed from vicinity of zero.

$\Rightarrow$ HOLE CHARGE $\underline{\underline{+e}}$

$$\Psi_{\text{Many holes}} = \prod_{k=1}^{M} \prod_i (z_i - Z_k)\, \Psi_1(z)$$

5.5 LAUGHLIN'S WAVEFN.

$$\Psi = f_N(z)\, e^{-\sum |z|_j^2 / 4}$$

$$\boxed{f_N[z] = \prod_{i>j}^{N} (z_i - z_j)^m .}$$

- Each particle sees an m-fold zero ("m fluxes") at every other particle $\Rightarrow$ good correlations, smaller Coulomb energy.
- For antisymmetry m must be <u>odd</u>.
- Plasma analogy tells us $n = \frac{1}{m} \frac{1}{2\pi \ell^2} = \frac{1}{m} \times$ density of mag flux $= \frac{1}{m}\left(\frac{N\Phi}{A}\right)$
- LL filling is $\nu = \frac{1}{m} = \frac{1}{3}, \frac{1}{5}, \frac{1}{7} \, \ldots\ldots$

No adjustable parameters (other than m), yet almost exact for almost any realistic repulsion.

Consider

Projects pairs with $L_{rel} = m'$

$$V = \sum_{m'=0}^{\infty} \sum_{i>j} V_{m'} P_{m'}(i\,j)$$

But $P_{m'}(i,j)$ & $P_{m''}(k,\ell)$ don't commute.

Hard core potential : $V_{m'} = 0$ for all $m' \geq m$, then since

$$P_{m'}(i,j)\Psi_m = 0 \qquad (m' < m)$$

$$\Rightarrow \quad V\Psi_m(z) = 0 \qquad \text{e.g. } m=3 \quad V = V_0 P_0 + V_1 P_1 + V_2 P_2$$

EXACT EIGENSTATE

Since the angular momenta of pairs can only change discretely, the Laughlin ground state has a gap.

E.g $m=3$, any excitation <u>weakens</u> the correlations by forcing at least one pair of particles to have relative angular momentum 1, rather than 3. $\Delta E \sim V_1$.

$\boxed{\text{INCOMPRESSIBLE}}$ $\Rightarrow$ no density fluctuations.

$$U(r) = -m^2 \ln |r|/r_0$$

$$\nabla^2 U = -2\pi m^2 \delta(r) \quad \Rightarrow \quad U(q) = \frac{2\pi m^2}{q^2}$$

$$-q^2 U_q = -2\pi m^2$$

$$U_{class} = \frac{1}{2L^2} \sum_{q \neq 0} \rho_q \frac{2\pi m^2}{q^2} \rho_{-q}$$

$$\Rightarrow \quad \langle |\rho_q|^2 \rangle = L^2 \frac{q^2}{4\pi} \qquad \text{Severely suppressed at long wavelengths.}$$

$q^2/2\pi$?

Short distances, convenient to discuss

$$g(z) = \frac{(L^2)^2}{\langle\psi|\psi\rangle} \int d^2z_3 \dots d^2z_N \, |\psi(0, z, z_3 \dots z_N)|^2$$

$$\lim g(z \to \infty) = 1.$$

Note

$$g(z) = \frac{1}{n^2} \langle \psi_g | \psi^\dagger(0) \psi^\dagger(z) \psi(z) \psi(0) | \psi_g \rangle$$

$$m=1 \qquad g(z) = \frac{1}{n^2}\left[\overbrace{\langle \psi^\dagger \psi^\dagger \psi \psi \rangle}^{n^2} + \langle \psi^\dagger \psi^\dagger \psi \psi \rangle\right]$$

$$= \frac{1}{n^2}\left[\overbrace{\langle \psi^\dagger(x) \psi(x) \rangle^2}^{n^2} - \underbrace{\langle \psi^\dagger(0) \psi(z) \rangle}\langle \psi^\dagger(z) \psi(0) \rangle\right]$$

$\psi(z) = \varphi_\ell(z) c_\ell$

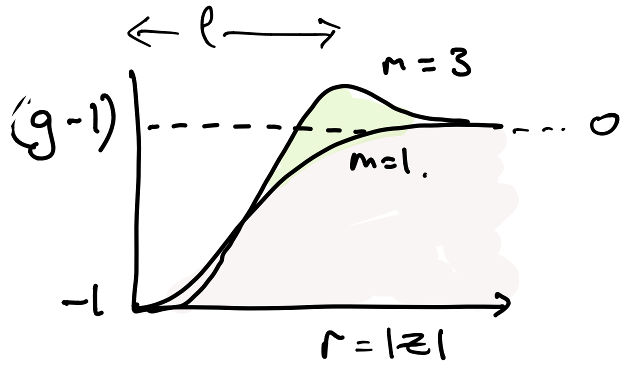
$\langle \psi^\dagger(0) \psi(z) \rangle = \sum_\ell \varphi_\ell(z) \varphi_\ell^*(0)$

$\varphi_\ell(z) = \frac{z^\ell}{\sqrt{2^\ell \ell!}} e^{-\bar{z}z/4} \times n$

$$= 1 - \frac{1}{n^2} \left| \sum_\ell \varphi_\ell(z) \varphi_\ell^*(0) \right|^2$$

$$= 1 - e^{-\frac{1}{2}|z|^2}$$

$$m>1 \qquad g(z) \sim |z|^{2m}.$$



BACKGROUND

$$\frac{\langle \Psi | V | \Psi \rangle}{\langle \Psi | \Psi \rangle} = \frac{nN}{2} \int d^2z \, \frac{e^2}{\epsilon |z|} \left(g(z) - 1 \right)$$

$\boxed{\text{Correlation energy.}}$

$$V = \frac{1}{2} \int \frac{e^2}{\epsilon |\vec{r}_1 - \vec{r}_2|} :(\hat{\rho}_e - \rho_B)_{\vec{r}} (\hat{\rho}_e - \rho_B)_{\vec{r}'}:$$

$$\frac{V_3}{N} = \left(-0.4100 \pm 0.0001 \right) \frac{e^2}{\epsilon \ell}$$

$$\frac{V_5}{N} = \left(-0.3277 \pm 0.0002 \right) \frac{e^2}{\epsilon \ell}$$

16.6 QUASIPARTICLES OF THE LAUGHLIN STATE

ANYONS

$$\Psi_Z^{(+)}(z) = \left[\prod_{j=1}^{N} (z - Z) \right] \Psi_m(z) \qquad + e/m$$

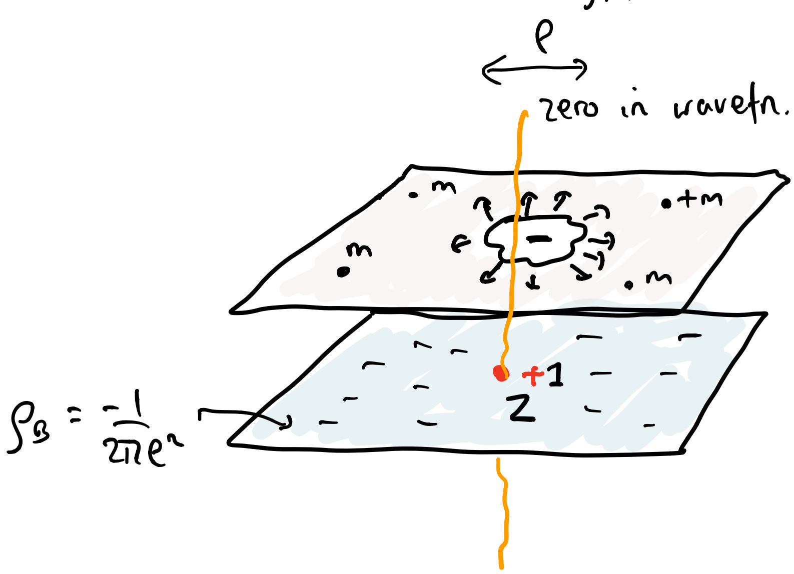
$$\Psi_{\bar{Z}}^{-}(z) = \left[\prod_j \left(2\frac{\partial}{\partial z_j} - \bar{Z} \right) \right] \Psi_m(z) \qquad - e/m$$

only on polynomial part $\left(2\frac{\partial}{\partial z_j} + \bar{z}_j/2 \right)$

$$e^{2\ln|z_j - z|}$$

$$|\Psi_z^+|^2 = e^{-\beta U_{ce}} \, e^{-\beta V} \qquad \beta = \frac{2}{m}$$

$$V = m \sum_{j=1}^{N} \left(-\ln|z_j - z|\right)$$



$\frac{1}{m}$ th particle excluded in plasma.

But this means $\frac{1}{m}$ th of an electron is physically repelled from the hole $\Rightarrow$

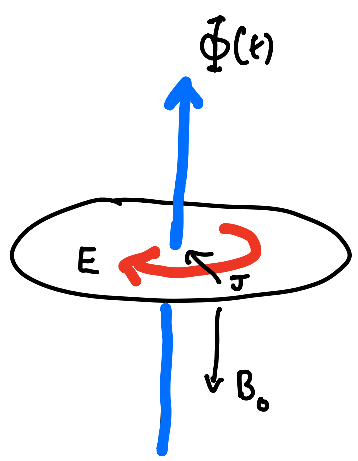
$$\boxed{e^* = \frac{e}{m}}$$

positive fractional charge!

Does the charge e^* depend on details of the wavefunction? The value of e^* is tied to the value of $\sigma_{xy} = (e^2/h)\, 1/m$.

$$\psi(x) \to \psi(x)\, e^{i 2\pi \left(\frac{x}{L}\right)}$$

Suppose we adiabatically insert a flux at position z, ramping up the flux to $\Phi_0 = h/e$. After this process the final Hamiltonian is equivalent to the starting H,



since a unit flux can be gauged away. But, the system does not return to its ground state, instead an anyon is created!

$$\oint_{\text{clockwise}} \vec{E}\cdot d\vec{\ell} = -\frac{\partial \Phi}{\partial t}$$

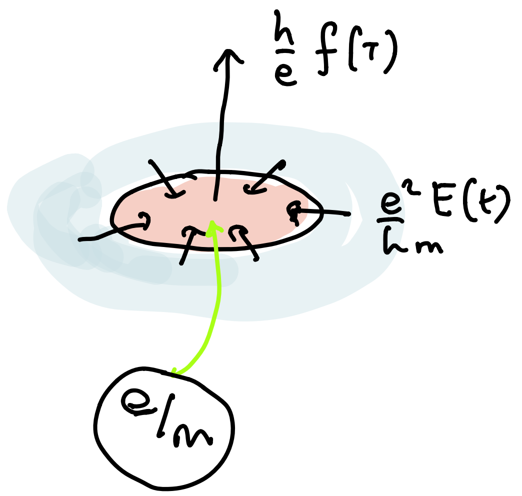
The azimuthal electric field will creat a radial current

$J(t)$, and this will create a build up of charge $(+)$

$$Q = \sigma_{xy} \int_0^T dt \underbrace{\oint \vec{E}.d\ell}_{\text{anticlockwise}} = \sigma_{xy} \int_0^T \frac{\partial \Phi}{\partial t} dt = \frac{e^2}{h} \frac{1}{m} \left(\frac{h}{e} \right)$$

$$= \frac{e}{m} \equiv e^*$$

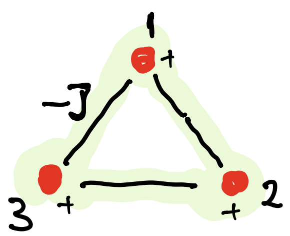
$$\left(\vec{J} \sim (\hat{b} \times \hat{E}) \sigma_{xy} E = -\sigma_{xy} |E| \hat{r} \right)$$



Anyon $\equiv$ injecting a flux into the Laughlin liquid.

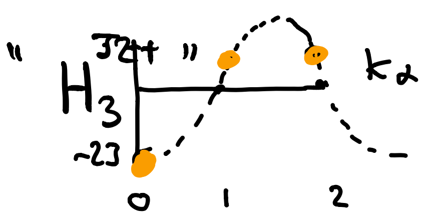
DISCUSSION ON FRACTIONAL CHARGE

- "Imposter fractional charge"



$$\psi_k = \frac{1}{\sqrt{3}} \sum_{j=1}^{3} e^{ikj} |j\rangle$$

$$\epsilon_{k_\alpha} = -E_{1s} - 2J \cos k_\alpha$$



$$k_\alpha = \frac{2\pi}{3} \alpha \quad (\alpha = 0, 1, 2.)$$

$P_n = |n\rangle\langle n|$ ——— projection onto site n $(n=1,2,3)$

$$\langle \psi_{k\alpha}|P_n|\psi_{k\alpha}\rangle = \frac{1}{3}$$

$Q_n = -eP_n$ $\quad\therefore\quad$ $\langle Q_n\rangle = -\frac{e}{3}$

However

$$\langle P_n^2\rangle - \langle P_n\rangle^2 = \frac{1}{3} - \frac{1}{9} = \frac{2}{9}$$

$$\therefore \quad \sqrt{\langle Q_n^2\rangle - \langle Q_n\rangle^2} = \frac{\sqrt{2}}{3}e$$

FLUCTUATING CHARGE.

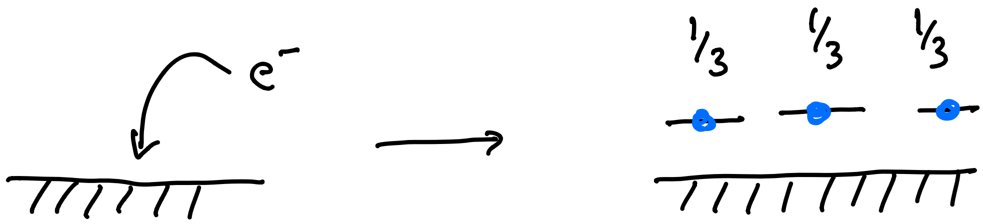
$1/3$ time we measure $Q=-e$, $\frac{2}{3}$ time $Q=0$.

Characteristic time for the fluctuations $\tau \sim \frac{\hbar}{\Delta E} \sim \frac{\hbar}{3}$

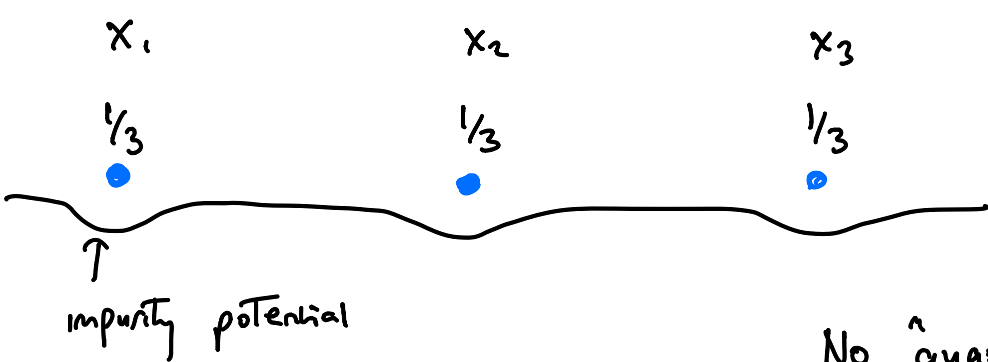
As we separate the sites, the states become degenerate

& the fluctuations easy to observe.

LAUGHLIN $\nu = 1/3$ STATE.



The qp can be separated without destroying the gap.



$|x_1, x_2, x_3\rangle$

No "quasidegeneracy".

The state is uniquely specified by the position of the Anyons.

"EMERGENT PARTICLES" (elementary).

The charge operator, projected into the low energy manifold is a <u>sharp</u> operator.

In the three site example, if one introduces the projected charge operator

$$P_n^{(\Omega)} = P^{\Omega} P_n P^{\Omega}$$

where

$$P^{\Omega} = \Theta\left(\Omega - (\hat{H} - E_0)\right) \qquad \text{Finite if } \Delta E < \Omega .$$

then

$$P_\alpha^{(\Omega)} = \sum_{\alpha=1,2,3} |\psi_{k\alpha}\rangle \, \Theta\left(\Omega - (\epsilon_{k\alpha} - \epsilon_{k0})\right) \langle \psi_{k\alpha}|$$

If $\Omega < \Delta = 3J$, then

$$P_\alpha^{(\Omega)} = |\psi_{k0}\rangle\langle\psi_{k0}|$$

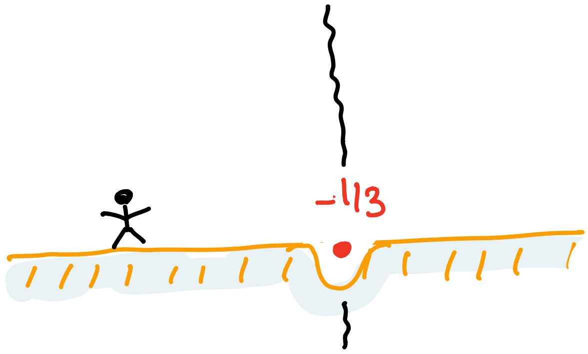
Now $\qquad P_n^{(\Omega)} = |\psi_{k0}\rangle \frac{1}{3} \langle \psi_{k0}|$

and

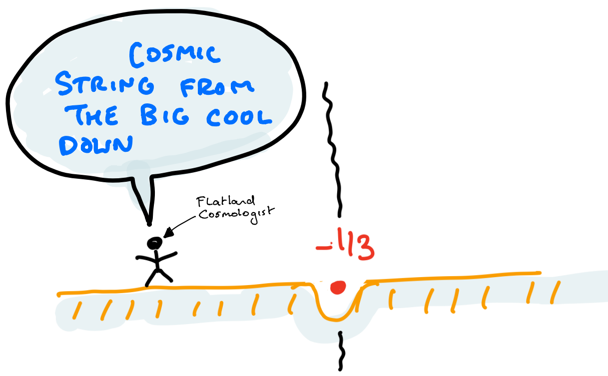
$$\langle \psi_{k0}|\left(P_n^{(\Omega)}\right)^2|\psi_{k0}\rangle - \left|\langle \psi_{k0}|P_n^{(\Omega)}|\psi_{k0}\rangle\right|^2 = 0$$

$$1/9 \qquad - \qquad 1/3 \cdot 1/3 .$$

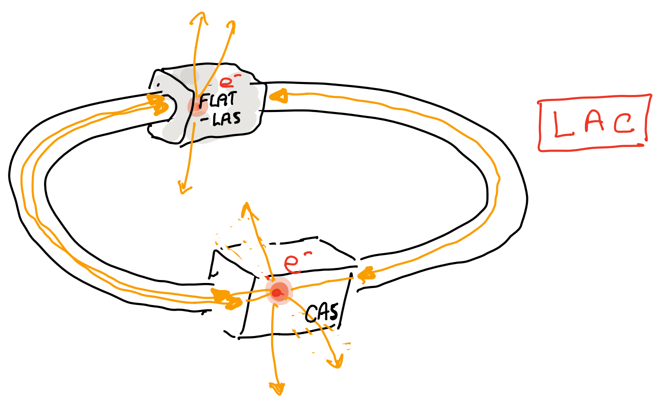
FLATLAND

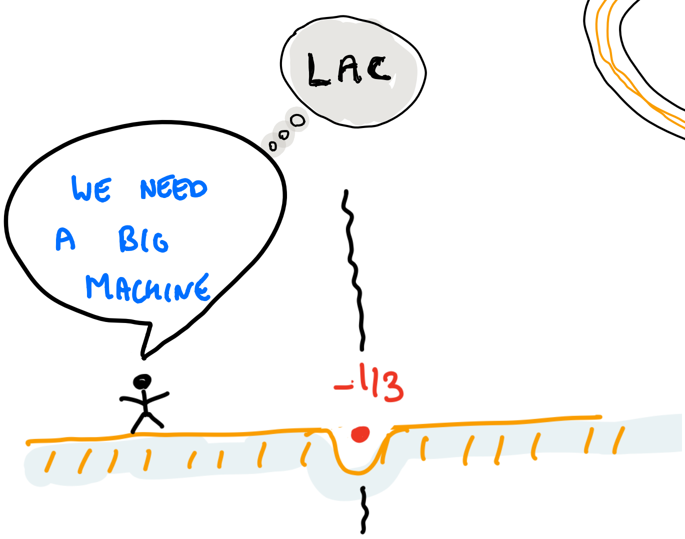


Flatland cosmologists speculate:

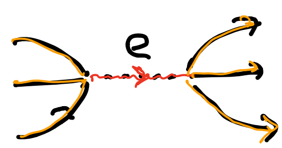


Flatland experimentalists call for the creation of a NATIONAL ACCELERATOR FACILITY





$E_{CM} = 10K$



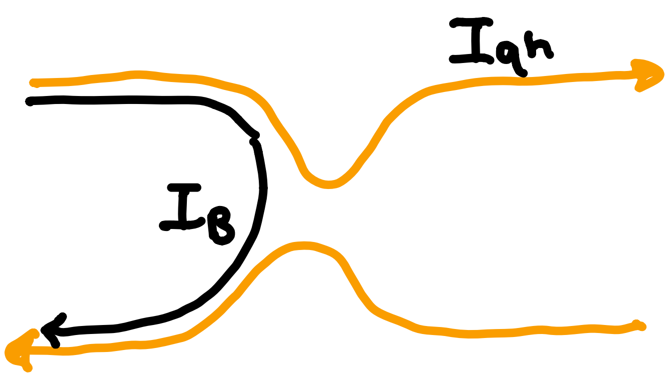
Virtual electrons

• Charge excitation gap $\underbrace{\Delta = \Delta_+ + \Delta_-}_{\text{only produced in pairs.}}$

• Conductivity $\rho_{xx} \sim \gamma \frac{h}{e^2} e^{-\beta\Delta_c/2}$

• $\Delta_+ \neq \Delta_-$

• Noise



$I = I_{qh} - I_B$

• $S_I(\omega) = e^* \langle I_B \rangle$