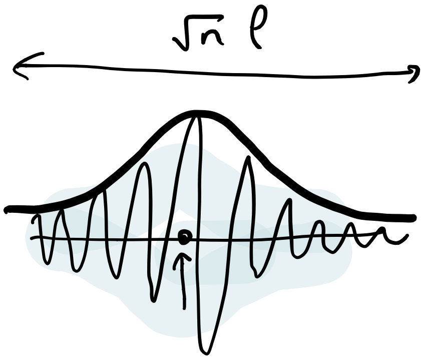
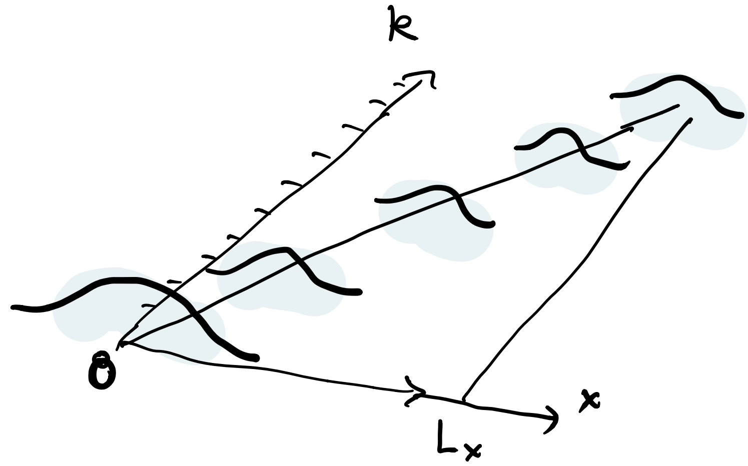




Notice that $X_n = k_n \ell^2 = n\left(\frac{2\pi}{L_y}\right)\ell^2 \ll \sqrt{n}\,\ell.$

In the nth Landau level there are.

$$N = \sum_k 1 = L_y \int_0^{L_x/\ell^2} \frac{dk}{2\pi} = \frac{L_x L_y}{2\pi \ell^2}$$

$$= \frac{A\,eB}{2\pi\hbar} = \frac{AB}{(h/e)}$$

$$N = \left(\frac{AB}{\Phi_0}\right) = \frac{\Phi}{\Phi_0} = N_\Phi$$

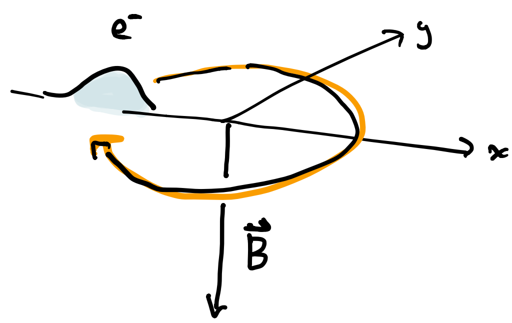
For ν levels $N = \nu N_\Phi$.

Dynamics.

$$\boxed{\Psi(r,t) = \frac{L_y}{2\pi} \sum_n \int \frac{dk}{2\pi} \underbrace{a_n(k)\, \psi_{nk}(\vec{r})}_{\text{wavepacket.}}\, e^{-i(n+\frac{1}{2})\omega_c t}}$$

$$\psi\left(\vec{r}, t+\frac{2\pi}{\omega_c}\right) = -\psi(\vec{r},t)$$

semiclassical motion.



cyclotron motion of wavepacket

Current flowing in single Landau level state ψ_{kn} ?

Warm up problem for the case with an electric field.

Heuristically $\quad I_y = -e\, v_y = -\frac{e}{\hbar}\frac{\partial E}{\partial k} = 0$

Microscopically:

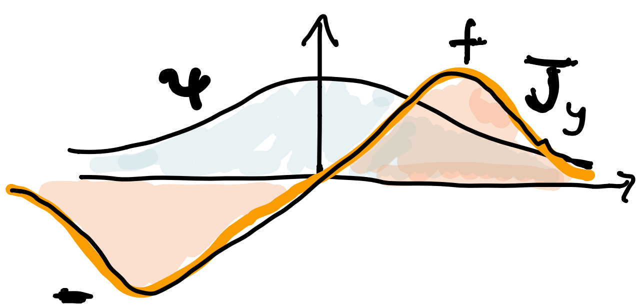
$$\psi_k = \frac{1}{\sqrt{\pi^{1/2}\ell L_y}} \, e^{iky} \, e^{-\frac{1}{2\ell^2}(x-k\ell^2)}$$

Lowest Landau Level.

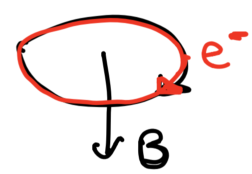
$$\langle J_y(x,y)\rangle = -\frac{e}{m}\, \psi^*_{nk}(x,y)\left[\overbrace{-i\hbar\vec{\nabla}_y}^{\hbar k} + \overbrace{e\vec{A}_y}^{-eBx.}\right]\psi_{nk}(x,y)$$

$$\langle I_y\rangle = \int_0^{L_x}\langle J_y(x,y)\rangle\, dx$$

$$= -\frac{e}{m}\,\frac{1}{\sqrt{\pi}\,\ell L_y}\int dx\; e^{-\frac{1}{\ell^2}(x-k\ell^2)^2}\underbrace{(\hbar k - eBx)}_{-eB(x-k\ell^2)}$$



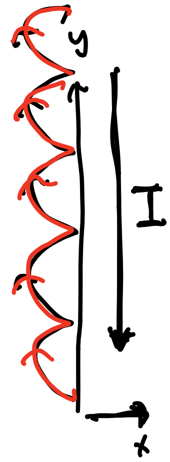
c.f



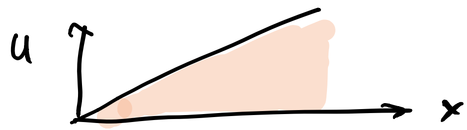
$$\langle I_y\rangle = e\omega_c\,\frac{1}{\sqrt{\pi}\,\ell}\,\frac{1}{L_y}\int dx\; e^{-\frac{1}{\ell^2}(x-k\ell^2)}(x-k\ell^2) = 0.$$

As expected

Current in a Field



$$\vec{E} = [E_x, 0, 0]$$



$$V = -Ex$$

$$U = (-e)V = eEx$$

$$U(x) = eEx$$

$$H = \frac{p_x^2}{2m} + \frac{1}{2} m\omega_c^2 (x - k\ell^2)^2 + eEx.$$

$$= \frac{p_x^2}{2m} + \frac{1}{2} m\omega_c^2 \left(x^2 - 2k\ell^2 x + (k\ell^2)^2\right) + eEx$$

$$= \frac{p_x^2}{2m} + \frac{1}{2} m\omega_c^2 \left(x^2 - 2\left(k\ell^2 - \frac{eE}{m\omega_c^2}\right)x + \left(k\ell^2 - \frac{eE}{m\omega_c^2}\right)^2\right)$$

$$+ \frac{1}{2} m\omega_c^2 \left((k\ell^2)^2 - \left(k\ell^2 - \frac{eE}{m\omega^2}\right)\right)$$

$$= \frac{p_x^2}{2m} + \frac{1}{2} m\omega_c^2 (x - X_k')^2 + \underbrace{\frac{1}{2} m\omega_c^2 \left(X_k^2 - X_k'^2\right)}$$

$$\boxed{\begin{aligned} X_k' &= k\ell^2 - \frac{eE}{m\omega_c^2} = X_k - \frac{eE}{m\omega_c^2} \\ &= \frac{1}{\omega_c}\left(\frac{\hbar k}{m} - \bar{v}\right) \\ \bar{v} &= \left(\frac{E}{B}\right) \end{aligned}}$$

$$\frac{1}{2} m\omega_c^2 \left(2X_k' + \frac{eE}{m\omega_c^2}\right)$$

$$= eEX_k' + \frac{1}{2} m \left(\frac{E}{B}\right)^2$$

$$H = \frac{p_x^2}{2m} + \frac{1}{2} m\omega_c^2 (x - X_k')^2 + eEX_k' + \frac{1}{2} m v^2$$

$$\epsilon_{kn} = \hbar\omega_c\left(n+\tfrac{1}{2}\right) + eEX_k' + \tfrac{1}{2}mv^2$$

$$v_y = \frac{1}{\hbar}\frac{\partial \epsilon_{kn}}{\partial k} = \frac{1}{\hbar}\frac{\partial}{\partial k}\left(eEk\ell^2\right) = \frac{eE\ell^2}{\hbar} = \left(\frac{E}{B}\right) = \tilde{v}$$

Physically

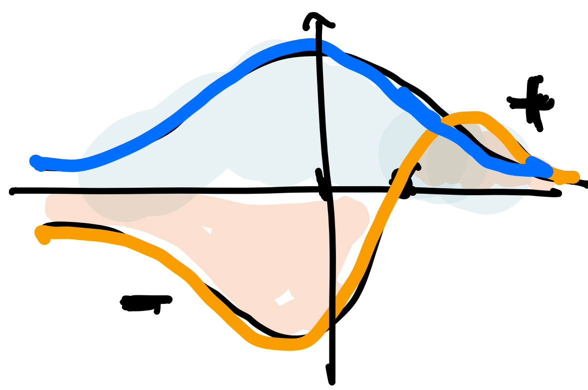
$$I_y = -nev_yL_x = -\left(\frac{e}{L_xL_y}\right)L_x\bar{V} = \frac{-e}{L_y}\bar{V}$$

Microscopically:

$$I_y = -\frac{e}{m}\frac{1}{\sqrt{\pi}\,eL_y\ell}\int dx\; e^{-\frac{1}{\ell^2}(x-X_k')^2}\Big[\overbrace{\hbar k - eB(x)}^{-eB\left(x-\left(X_k'+\frac{eE}{m\omega_c^2}\right)\right)}\Big]$$

$$= -\frac{e}{m}\underbrace{\frac{1}{\sqrt{\pi}\,eL_y\ell}\int dx\; e^{-\frac{1}{\ell^2}(x-X_k')}}_{\frac{1}{L_y}}\Big[\cancel{\hbar k} - eB(x-\cancel{X_k'}) - \cancel{eB}\left(k\ell^2-\frac{eE}{m\omega_c^2}\right)\Big]$$

$$I_y = -\frac{e\omega_c}{L_y}\left(\frac{Em}{B\frac{eB}{m}}\right) = -\frac{e}{L_y}\bar{V}$$



Total current in a Landau level is then.

$$I_y = \frac{-e}{L_y} \overbrace{\int dk \left(\frac{L_y}{2\pi}\right)}^{\sum_k} \overline{\frac{1}{\hbar}\frac{\partial \epsilon_{kn}}{\partial k}}$$

$$= \frac{-e}{h} \int dk \frac{d\epsilon_{kn}}{\partial k} = -\frac{e}{h}\left[\mu_R - \mu_L\right]$$

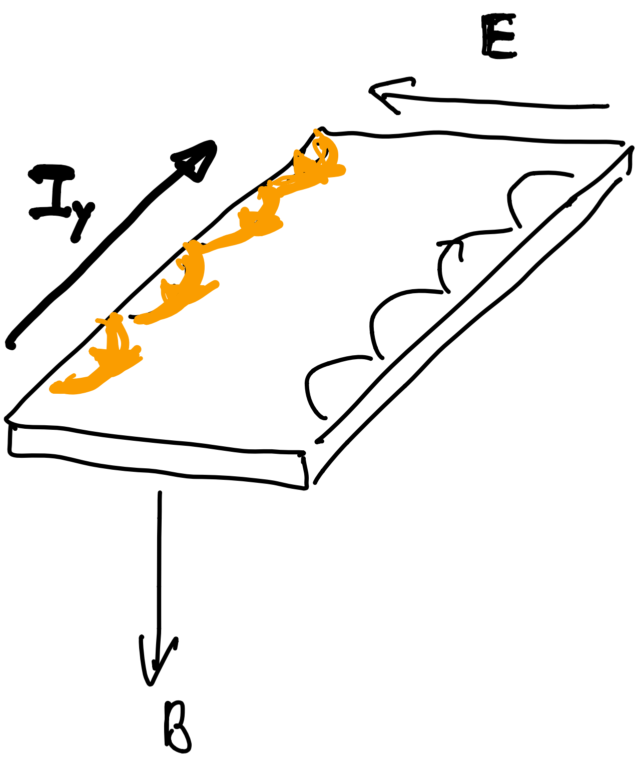
$$-eV_H = -e\left(V_R - V_L\right) = \left(\mu_R - \mu_L\right)$$

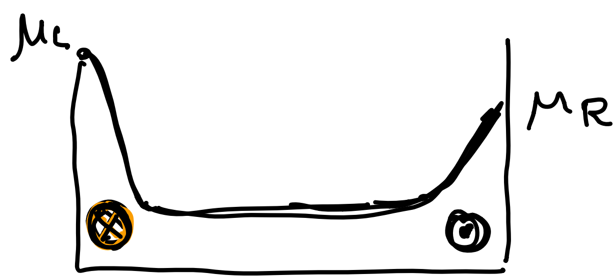
$$I_y = \frac{e^2}{h} \nu V_H$$

$$\sigma_{xy} = \frac{e^2 \nu}{h}.$$

$$\sigma_{xx} = 0$$

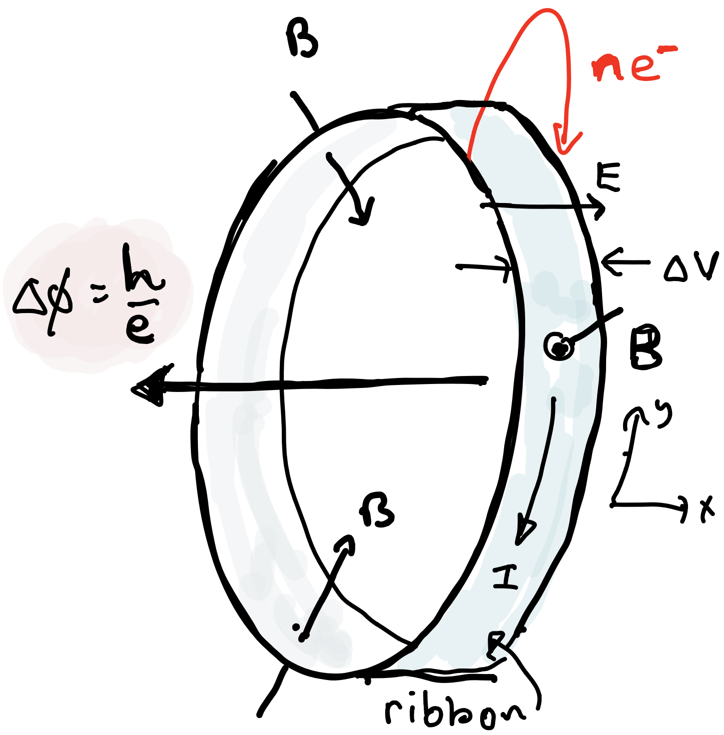
$$\rho_{xy} = \frac{h}{e^2 \nu}.$$





Edge-states run along equipotentials.

LAUGHLIN'S FLUX ARGUMENT



$$I = \frac{\partial U}{\partial \phi} = \frac{\partial U}{L\,\partial A} = \frac{\Delta U}{\Delta \phi}$$

$$\Delta\phi = \frac{h}{e} \qquad \psi \to \psi\, e^{\frac{ieAy}{\hbar}}$$

$$A = n\left(\frac{h}{e}\right)\frac{1}{L}$$

NO CHANGE TO ENERGIES OF BULK

$$\psi_{nk} = e^{iky}\,\phi_n(x - x_0)$$

$$x_0 = \frac{1}{\omega_c}\left(\frac{\hbar k}{m} - \frac{E}{B} + \frac{eA}{m}\right)$$

NO CHANGE TO BULK ENERGIES

$$x_0 \longrightarrow x_0 + \frac{A}{B}$$

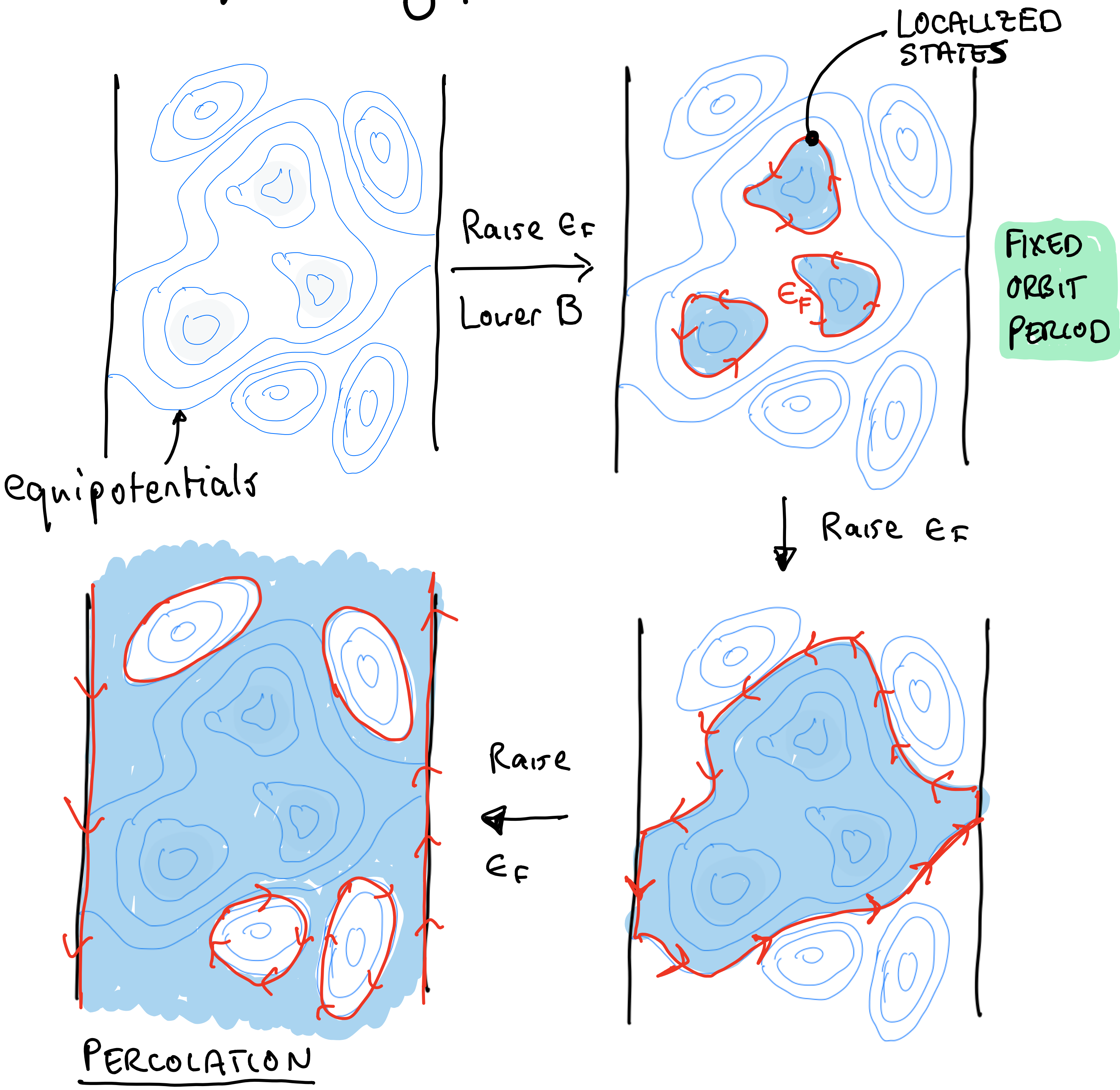
TRANSFERING CHARGE FROM LEFT TO RIGHT

$$\left.\begin{aligned} \Delta U &= ne\,\Delta V \\ \Delta\phi &= h/e \end{aligned}\right\} \qquad I_n = \frac{ne}{(h/e)}\Delta V = \frac{ne^2}{h}\Delta V$$

$\Rightarrow$ INTEGER σ_{xy}.

Percolation Picture

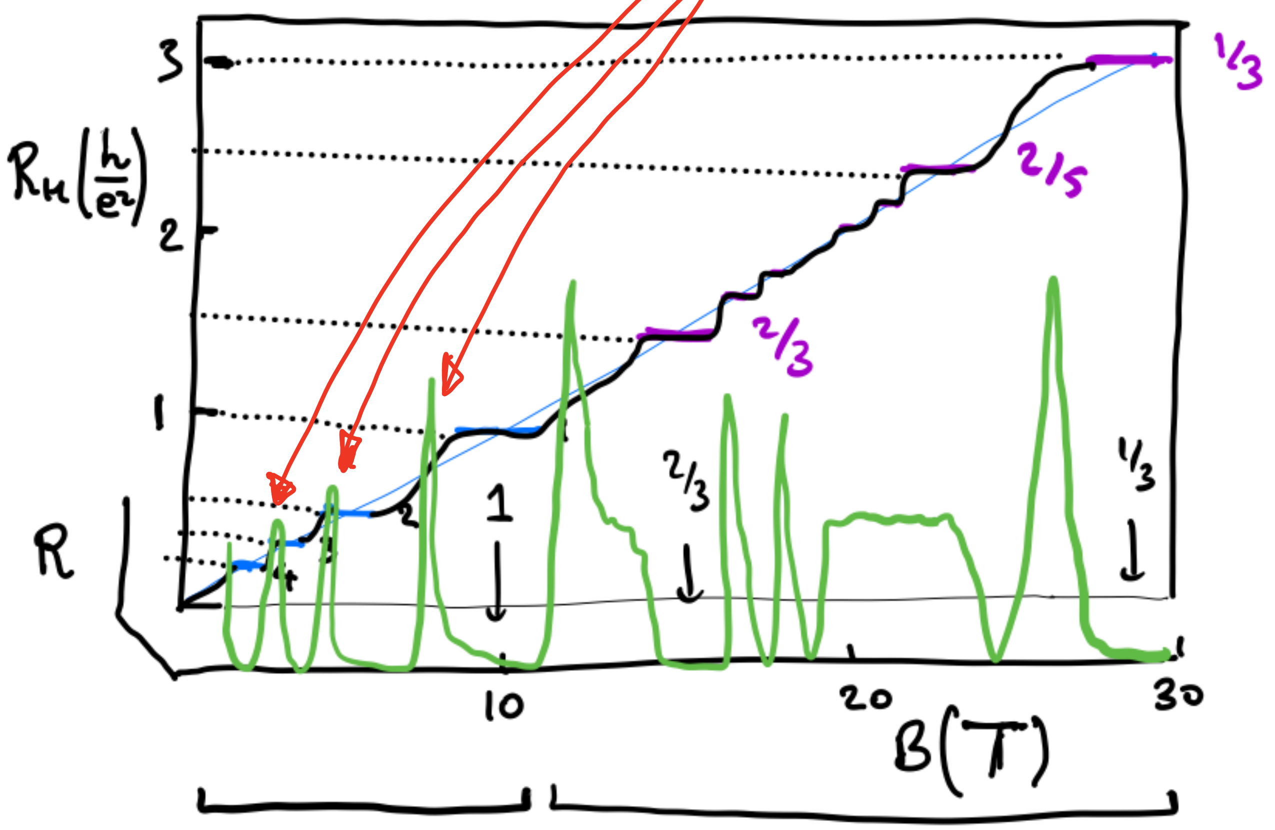
- Disorder is essential for the IQHE
- Require a gap in the excitation spectrum



Percolation will occur @

$$\mu_n^* = \left(n + \frac{1}{2}\right)\hbar\omega_c$$

When the shoreline percolates, the orbital period diverges + the excitation gap vanishes, so that dissipation takes place at the transition between plateaux.



5. Fractional Quantum Hall Effect

- Early expts showed that for $\nu < 1$, σ_{xx} & σ_{xy} vanished, as expected from the perspective of the IQHE. However, with the invention of modulation doping in GaAs quantum wells, much higher quality samples were possible, with much less disorder.

- A Wigner crystal was expected for low enough e^- concentrations in pure samples. Pinned by disorder it would be insulating.

- 1982 Tsui, Störmer & Gossard discovered a QH plateau at $\nu = 1/3$, with $\sigma_{xx} \to 0$ & $\sigma_{xy} = \frac{1}{3} e^2/h$.

- Tsui joked that it might be quarks! The effect did not involve quarks, but incredibly, the electrons in the $\nu = 1/3$ FQH state have indeed condensed into a state with fractional charge excitations $q^* = \frac{1}{3} e$.

- Many more fractions observed

- Because $\sigma_{xx} \to 0$, this is a dissipationless state, with a gap, presumeably driven by the $e^- - e^-$ Coulomb interaction.

- The new ideas developed to explore & understand these new phases of matter & their topological properties have had profound implications for physics, both in the lab, and the cosmos!

5.1 PRELIMINARIES: Mechanical Momentum + Guiding Centers

Recall
$$H = \frac{(\vec{p} + e\vec{A})^2}{2m} = \frac{\vec{\Pi}^2}{2m}$$

where
$$\vec{\Pi} = (\Pi_x, \Pi_y) = \vec{p} + e\vec{A}(\vec{r})$$

MECHANICAL MOMENTUM

This is a gauge-invariant quantity, but unlike the canonical momentum, Π_x & Π_y do not commute

$$[\Pi_x, \Pi_y] = [p_x, eA_y] + [eA_x, p_y]$$
$$= e\left([-i\hbar\partial_x, A_y] - [-i\hbar\partial_y, A_x]\right)$$

$$\boxed{[\Pi_x, \Pi_y] = -i\hbar e \underbrace{(\partial_x A_y - \partial_y A_x)}_{B_z = -B \text{ (for convenience)}} = i\hbar e B = \frac{i\hbar^2}{\ell^2} \quad \left(\frac{\hbar}{eB} = \ell^2\right)}$$