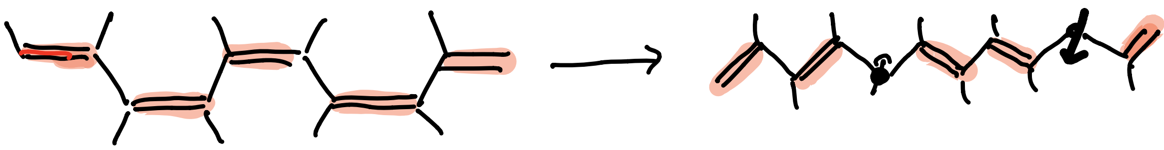
METHODS TO CREATE SOLITONS

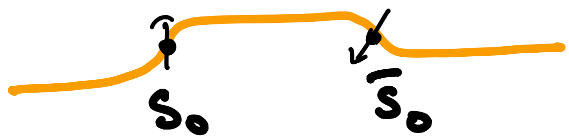
1. Neutral solitons

Warming of cis $-(C_2H_2)_x$ causes it to isomerize into trans-$(CH)_{2x}$, filled with neutral defects



trans

cis.



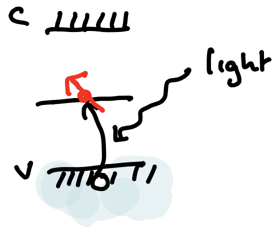
PARAMAGNETIC

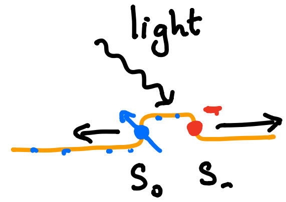
- Exhibit $\chi \sim n_{solitons}/T \Rightarrow$ paramagnetic
- No IR activity $\Rightarrow$ neutral
- Electron nuclear double resonance (ENDOR!) $\xi \sim 7a$
(Interplay of NMR & ESR)

2. Charged Solitons

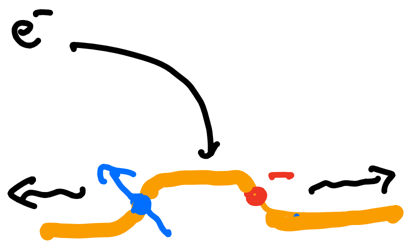
- Photo-excitation

(MID GAP ABSORPTION)



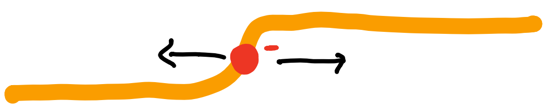


- Charge transfer doping $(CH)_x^+ \, Br^-$

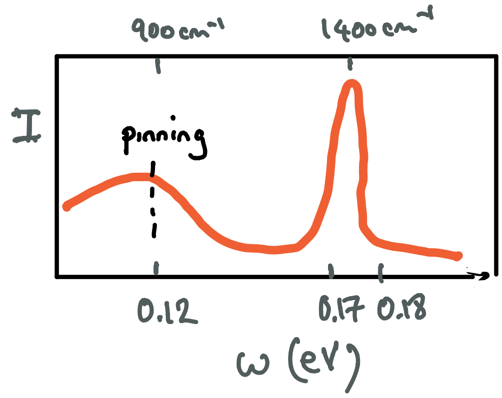


Generates infra-red modes

infra-red absorption



vibrational modes



- ESR confirms that

$N_s / N_{ch} << 1$

N_s: # spins

N_{ch}: # charges

LOCAL NEUTRALITY

The neutral soliton does not affect the **local** charge density. Let $\phi_\lambda(n)$ be the wavefunction of the λth state, then the local density of states is

$$\rho_{nn}(E) = \sum_\lambda |\phi_\lambda(n)|^2 \delta(E-E_\lambda)$$

Now

$$\int \rho_{nn}(E)\,dE = 1$$

at each site. In the presence of a soliton,

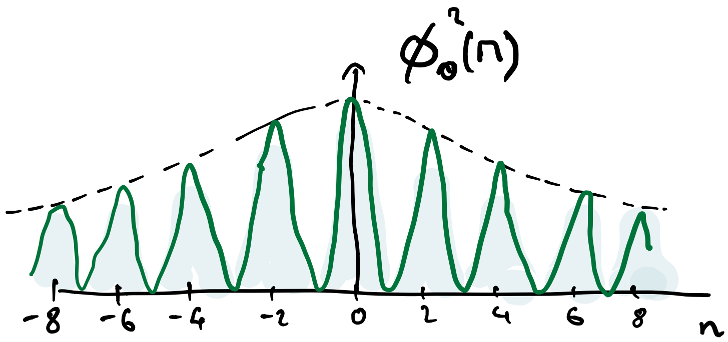
$$\rho^s_{nn}(E) = \phi_0^2(n)\,\delta(E) + \underbrace{\tilde{\rho}^s_{nn}(E)}_{\text{valence \& conduction band}}$$

$$\int_{-\infty}^{\infty} \rho^s_{nn}(E)\,dE = \int_{-\infty}^{-\Delta} \rho^s_{nn}(E)\,dE + \int_{\Delta}^{\infty} \rho^s_{nn}(E)\,dE + \phi_0^2(n) = 1$$

$$\Rightarrow \quad 2\int_{-\infty}^{-\Delta} dE\, \rho^{(s)}_{nn}(E) + \phi_0^2(n) = 1$$

$$2 \int_{-\infty}^{-\Delta} dE \, \Delta\rho_{nn}(E) + \phi_0^2(n) = 0$$

$$\Rightarrow \underbrace{2 \int_{-\infty}^{-\Delta} dE \, \Delta\rho_{nn}(E)}_{\text{Compensating reduction in charge density of valence band.}} \qquad \underbrace{\phi_0^2(n)}_{\text{increase in charge density due to occupancy of zero mode}}$$



For the $Q_- = -e$ charged soliton, the zero mode is empty, so now

$$q_-(n) = 2 \int dE \, \Delta\rho_{nn}(E) = -\phi_0^2(n)$$

For the $Q_+ = e$ soliton

$$q_+(n) = 2 \int dE \, \Delta\rho_{nn}(E) + 2\phi_0^2(n) = +\phi_0^2(n)$$

Calculation of $\phi_0(n) \equiv \phi_n$

We will show $$\boxed{\phi_0(n) \sim \frac{1}{\sqrt{\ell}} \cos\frac{\pi n}{2} \frac{1}{\cosh\left(\frac{n}{\ell}\right)}}$$

Since $E_0 = 0$, we have $H_{nn'} \phi_{n'} = E_0 \phi_n = 0$

$$\Rightarrow \quad t_{n+1,n+2}\,\phi_{n+2} + t_{n+1,n}\,\phi_n = E\phi_{n+1} = 0$$

$$\Rightarrow \quad \phi_{n+2} = \left(-\frac{t_{n+1,n}}{t_{n+2,n+1}}\right)$$

$$\phi_n = \left(-\frac{t_{n-1,n-2}}{t_{n,n-1}}\right)\left(-\frac{t_{n-3,n-4}}{t_{n-2,n-3}}\right)\cdots\left(\frac{-t_{1,0}}{t_{2,1}}\right)\phi_0$$

$$\phi_n = (-1)^{n/2} \exp\left[\sum_{\substack{r=2\\ \text{even}}}^{n} \ln\left(\frac{t_{r-1,r-2}}{t_{r,r-1}}\right)\right]\phi_0$$

Let $$(u_{n+1} - u_n) \sim (-1)^n u \tanh\frac{n}{\ell}, \text{ then}$$

$$\ln\left[\frac{t_{r-1,r-2}}{t_{r,r-1}}\right] = \ln\frac{t - 2\alpha u \tanh r/\ell}{t + 2\alpha u \tanh r/\ell} \approx -\frac{4\alpha u}{t}\tanh\left(\frac{r}{\ell}\right)$$

$$\therefore \quad \phi_n = \cos\left(\frac{\pi n}{2}\right) \exp\left[-\frac{4\alpha u}{t}\sum_{r=2}^{r=n} \tanh\frac{x_r}{\ell}\right]$$

$$= \cos\left(\frac{\pi n}{2}\right) \exp\left[-\frac{2\alpha u}{t}\int_0^x dx \tanh\frac{x}{\ell}\right]$$

$$= \cos\frac{\pi n}{2} \exp\left[-\frac{2\alpha u \ell}{t}\ln\left[\cosh\frac{x}{\ell}\right]\right]$$

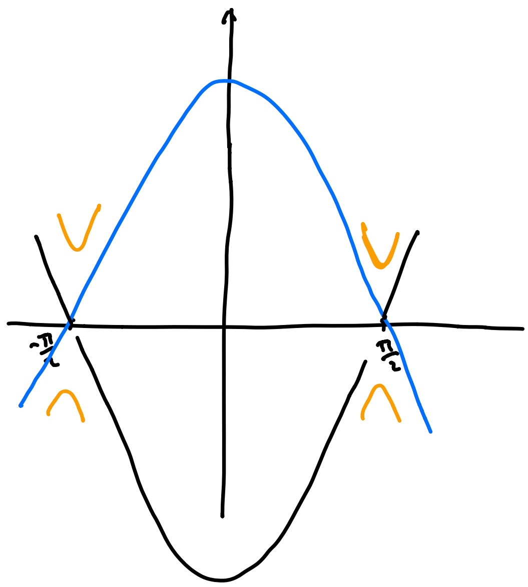
But $\xi = \frac{v_F}{\Delta} = \frac{2t}{4\alpha u} = \frac{t}{2\alpha u}$ is the o.p coherence length, and so $\frac{2\alpha u \ell}{t} = \left(\ell/\xi\right)$

$$\phi_n = \cos\frac{\pi n}{2} \frac{1}{\left(\cosh\frac{x}{\ell}\right)^{\ell/\xi}} \sim \cos\frac{\pi n}{2}\frac{1}{\cosh\left(\frac{x}{\ell}\right)}$$

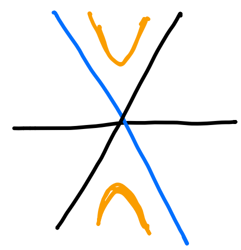
if $\ell \sim \xi$.

Normalization requires $\phi_n \sim \frac{\cos \pi n/2}{\sqrt{\ell}} \frac{1}{\cosh\left(\frac{x}{\ell}\right)}$

Continuum Model for Soliton



Lattice



Continuum

$$H = \sum_k \psi_k^+ \left(\epsilon_k \tau_3 + \Delta_k \tau_2\right) \psi_k$$

$$\mathcal{H} = \int dx\, \tilde{\psi}^+(x) \left[v_F \left(-i\partial_x\right) \tau_3 + \Delta(x)\, \tau_2 \right] \psi(x)$$

RELATIVISTIC THEORY.

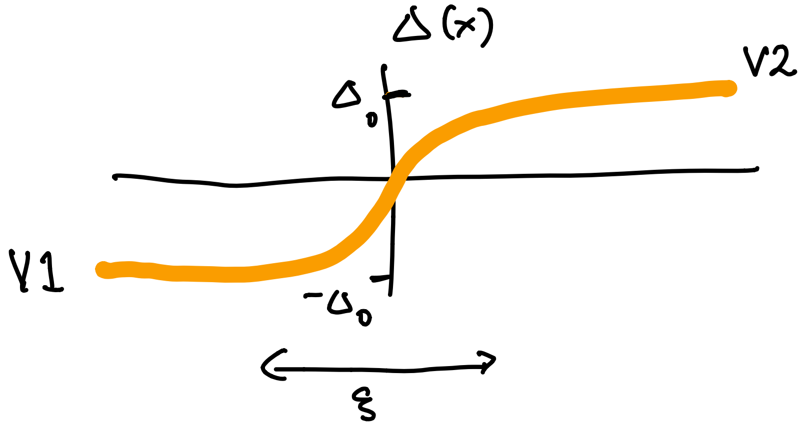
$$H = \int dx \left[\frac{\Pi^2}{2} + \frac{1}{2}\left(\frac{d\Phi}{dx}\right)^2 + V[\Phi] + \psi^+\left(\alpha\left(-i\partial_x\right) + \beta g \Phi\right)\psi \right]$$

JACKIW + SCHRIEFFER '81

$$v_F\left(-i\partial_x \tau_3 + \Delta(x)\, \tau_2\right)\phi = 0 \qquad \text{zero mode.}$$

$$\partial_x \phi(x) = -\frac{\Delta(x)}{v_F} \tau_1 \phi(x) \qquad \left(\tau_3 \tau_2 = -i\tau_1\right)$$

$$\Rightarrow \quad \phi(x) = \exp\left[-\int_0^x \frac{\Delta(x')}{v_F}\, dx' \right] \phi(0)$$



$$\boxed{\Delta(x) = \Delta_0 \tanh\left(\frac{x}{\ell}\right) \quad \text{Topological defect.}}$$

$$\phi(x) = \exp\left[-\frac{\Delta_0}{v_F} \int^x \tanh\frac{x'}{\xi}\, dx'\, \tau_1 \right] \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

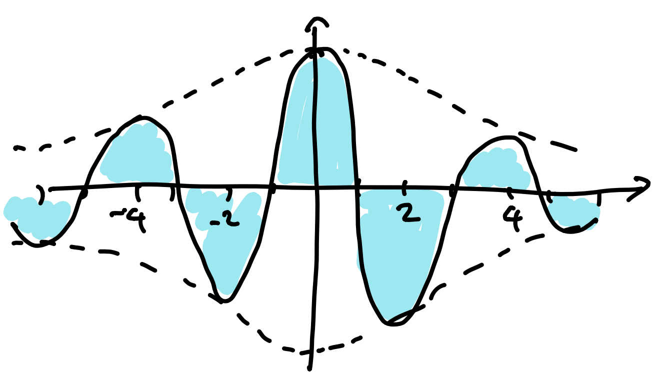
$$= \phi_0 \exp\left[-\frac{\Delta_0 \xi}{v_F} \ln\left(\cosh x/\xi\right) \right] \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

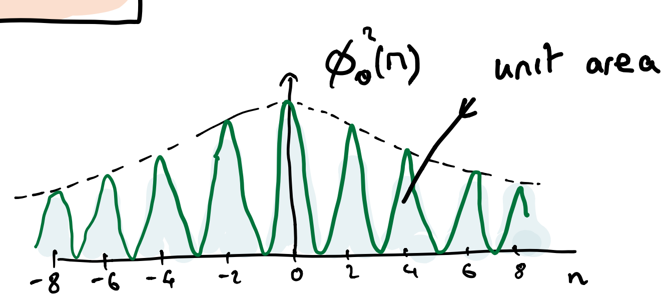
$$= \phi_0 \left(\frac{1}{\cosh x/\xi} \right)^{\alpha} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\xi \sim v_F/\Delta_0 \Rightarrow \alpha = \frac{\Delta_0 \xi}{v_F} \sim 1.$$

$$\psi_j = \frac{1}{\sqrt{2}} \left(\phi_j^1 e^{\frac{i\pi x_j}{2}} + \phi_j^2 e^{-\frac{i\pi x_j}{2}} \right)$$

$$\psi_j = \phi_0 \left(\operatorname{sech} x_j/\xi \right)^{\alpha} \cos\left(\frac{\pi}{2} x_j \right)$$





$$\phi_0 \sim \frac{1}{\sqrt{\ell}}$$

DETAILS

$$\psi_j = \frac{1}{\sqrt{N}} \sum_{k \in [0,\pi]} \left(c_k + (-1)^j c_{k+\pi} \right) e^{ikR_j}$$

$$= \frac{1}{\sqrt{N}} \sum_k \left(c_{k-\frac{\pi}{2}} e^{i(k-\frac{\pi}{2})R_j} + c_{k+\frac{\pi}{2}} e^{i(k+\frac{\pi}{2})R_j} \right)$$

$$\left(k+\tfrac{3\pi}{2} \equiv k-\tfrac{\pi}{2} \right)$$

$$= \frac{1}{\sqrt{N}} \sum \left(\tilde{\psi}_k^2 e^{i(k-\frac{\pi}{2})R_j} + \tilde{\psi}_k^1 e^{i(k+\frac{\pi}{2})R_j} \right)$$

$$= \frac{1}{\sqrt{2}} \left(\tilde{\psi}^2(x_j) e^{-i\frac{\pi}{2}R_j} + \tilde{\psi}^1(x_j) e^{+i\frac{\pi}{2}R_j} \right)$$

$$\tilde{\psi}^{1,2}(x) = \sqrt{\frac{2}{N}} \sum \psi_k^{(1,2)} e^{ikx}$$

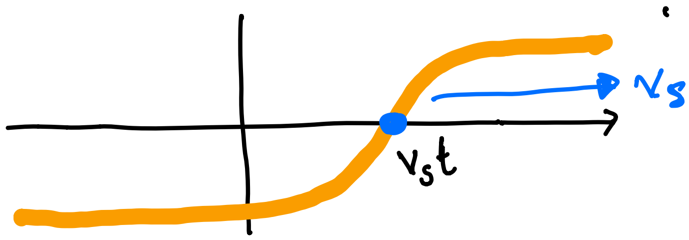
$$\epsilon_{k-\frac{\pi}{2}} \left(c^+_{k+\frac{\pi}{2}} c_{k+\frac{\pi}{2}} - c^+_{k-\frac{\pi}{2}} c_{k-\frac{\pi}{2}} \right) \equiv v_F k \left(\psi_k^{1+} \psi_k^1 - \psi_k^{+2} \psi_k^2 \right)$$

$$+ i \Delta_{k-\frac{\pi}{2}} \left(c^+_{k+\frac{\pi}{2}} c_{k-\frac{\pi}{2}} - c^+_{k-\frac{\pi}{2}} c_{k+\frac{\pi}{2}} \right) = -i\Delta \left(\psi_k^{1+} \psi_k^2 - \psi_k^{+2} \psi_k^1 \right)$$

$$\epsilon_{k-\frac{\pi}{2}} \tau_3 - \Delta_{k-\frac{\pi}{2}} \tau_2 \longrightarrow v_F k \tau_3 + \Delta \tau_2$$

$$\longrightarrow \left(v_F -i\partial_x \tau_3 + \Delta(x) \tau_2 \right)$$

Mass of Soliton



$$\psi_n(t) = u_0 \tanh\left[(na - v_s t)/\xi\right]$$

$$\frac{1}{2} M v_s^2 = \frac{1}{2} M \sum_n \left(\dot{\psi}_n(t)\right)^2$$

$$= \frac{1}{2} M \sum_n \left(\frac{u_0 v_s}{\xi} \operatorname{sech}^2\left[(na - v_s t)/\xi\right]\right)^2$$

$$= \frac{1}{2} \frac{M u_0^2 v_s^2}{\xi^2} \int_{-\infty}^{\infty} \frac{dx}{a} \operatorname{sech}^4\left(\frac{x}{\xi}\right)$$

$$= \frac{1}{2}\left(\frac{M u_0^2}{a\,\xi} \underbrace{\int_{-\infty}^{\infty} du \operatorname{sech}^4 u}_{4/3}\right) v_s^2$$

$$= \frac{1}{2} M_s v_s^2$$

$$M_s = \frac{4}{3(\xi/a)} M \left(\frac{u_0}{a}\right)^2 \underset{\text{SSH}}{\sim} 6\, m_e$$

High Mobility

QUANTUM

$u_0 \sim 0.04\,\text{Å}$

$a \sim 1.22\,\text{Å}$

$\xi/a \sim 7$

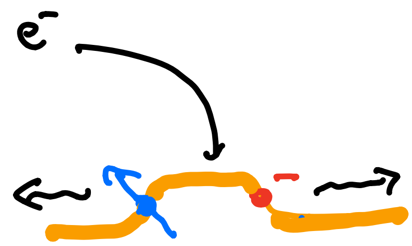
$M \sim 13 \times 2000\ m_e$

I find:

$$\frac{4}{3(7)} (13 \times 2000) \times \left(\frac{0.04}{1.22}\right)^2 = 5.3$$

TAKE HOME MESSAGES FROM THE SSH MODEL

- First demonstration of fractionalization with experimental support.
- Below the gap electrons fractionalize into spinons + holons.



- A link between topological defects & fractionalization
- In their 1981 paper, Schrieffer + Jackiw write:–

The ideas on charge fractionalization can be carried further. It' has been suggested that arbitrary charge fractions can be obtained in fermion-soliton systems, provided charge conjugation is abandoned and the vacuum structure is sufficiently complex. An example with 1/3rd units of charge, which is not obscured by the two spin states has been discussed by SSH' in the condensed matter context of TTF-TCNQ; while related ideas for particle physics have been investigated by Goldstone and Wilczek.