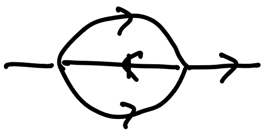
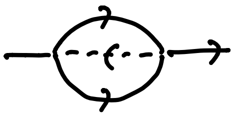
WHAT IS THE MICROSCOPIC ORIGIN OF $P(\omega) \sim$ constant?

Helpful to look in the time domain

FL  $\propto \frac{1}{\tau^3} \Longleftrightarrow \Sigma''(\omega) \sim \int \Sigma(\tau) e^{i\omega\tau} d\tau \sim \omega^2$ ✓

$G(\tau) \sim 1/\tau$

MFL  $\propto \frac{1}{\tau^2} \Longleftrightarrow \Sigma''(\omega) \sim \int \frac{1}{\tau^2} e^{i\omega\tau} d\tau \sim \omega$

$\rightarrow \quad \frac{1}{\tau}$

$- - - - \quad \mathrm{sgn}(\tau)$

This is one way to achieve a MFL

- seen in the two channel kondo model
- but what would stabilize a local fermion in the lattice?
- "local quantum criticality".

See Coleman, Ioffe + Tsvelik (1994).

Fractionalization in the Hubbard Model

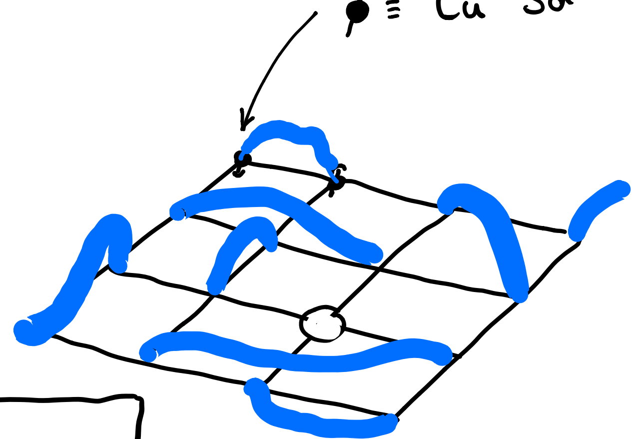
P.W. Anderson, Science 235, 1196 (1987)

Hi T_c = doped Mott Insulator

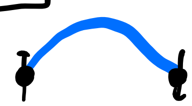
P.W. Anderson + Z. Zou PRL 60, 132 (88).

J. Marston + I. Affleck, PRB, 39, 11538 (89)

$\equiv Cu^{2+}\ 3d^9$

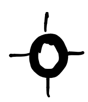


$$\boxed{\mathcal{H} = -\sum t\left(c^\dagger_{i\sigma} c_{j\sigma} + c^\dagger_{j\sigma} c_{i\sigma}\right) + J \sum_{(i,j)} \vec{S}_i \cdot \vec{S}_j}$$



$\equiv \dfrac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$

"Resonating valence bond"

 hole (from doping)

$n_i \leq 1$ Non holonomic constraint

$$\boxed{\begin{aligned} |i; \uparrow : \sigma\rangle &= f^\dagger_{i\sigma} |0\rangle \\ |i: \text{hole}\rangle &= b^\dagger_i |0\rangle \end{aligned}}$$

"slave boson"

Coleman, PRB 29, 3035 (1984).
Barnes, J Phys F, 6, 1375 (1976).

$n_c(i) \leq 1 \longrightarrow \boxed{b^\dagger_i b_i + f^\dagger_{i\sigma} f_{i\sigma} = 1}$

Holonomic constraint

Loss of Hilbert Space
$\rightleftharpoons$
Gauge Fields

PHYSICAL REVIEW B VOLUME 39, NUMBER 13 1 MAY 1989

Gapless fermions and gauge fields in dielectrics

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(Received 20 June 1988; revised manuscript received 28 November 1988)

To study the nonmagnetic dielectric state and Mott transitions we consider an example of a two-dimensional modified Hubbard model with a large number of colors. Low-energy excitations in this phase are fermionic excitations and Bose excitations described by gauge fields of the U(1) group. The transition into the metal state has little effect on the fermionic spectrum, but it results in the local U(1) symmetry being broken and fermions becoming able to transfer charge excitations. Apart from the half-filling, scalar Bose excitations also appear. Due to the presence of additional gauge fields the physical conductivity is determined by the lowest conductivity of the Fermi or Bose subsystems.

PHYSICAL REVIEW B VOLUME 42, NUMBER 16 1 DECEMBER 1990

Transport phenomena near the Mott transition

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(Received 4 May 1990; revised manuscript received 7 August 1990)

We consider the transport properties of a strongly (antiferromagnetically) correlated electron system in the temperature regime where the Fermi-liquid coherence ceases to exist. We find that the resistivity is linear in temperature, the thermal conductivity is almost temperature independent obeying approximately the Wiedemann-Franz law, while the Hall coefficient acquires a temperature dependence. The sign of the thermopower and Hall coefficient are hole-like. We calculate the residual resistivity caused by a random potential using the slave-boson technique. The disorder changes the slope of the temperature-dependent resistivity, but the Fermi surface remains relatively sharp.

- Spin Liquid "Dielectric Phases"
- Fluctuations of the Phase of the OP
- Metal Dielectric Transformation (Mott transition)
- Ioffe - Larkin Formula
- Linear resistance? Ioffe - Kotliar.

Dielectric Phases

$$\boxed{\begin{aligned} H = & -\sum_{i,j} J_{ij} \left(c^\dagger_{i\alpha} c_{j\alpha}\right)\left(c^\dagger_{j\beta} c_{i\beta}\right) - \tilde{\mu} \sum_i c^\dagger_{i\alpha} c_{j\alpha} \\ & + i \sum \varphi_i \left(n_i - n_0\right) \end{aligned}}$$

$$\left.\begin{aligned} \mathcal{L} &= \sum c^\dagger_{i\sigma} \partial_\tau c_{i\sigma} + H \\ Z &= \int \mathcal{D}[c]\, e^{-S} \end{aligned}\right\} \qquad S = \int_0^\beta d\tau\, \mathcal{L}$$

$$-J_{ij} \left(c^\dagger_{i\alpha} c_{j\alpha}\right)\left(c^\dagger_{j\beta} c_{i\beta}\right) \longrightarrow -\left(\bar{\Delta}_{ij} c^\dagger_{i\alpha} c_{j\alpha} + h.c\right) + \frac{|\Delta_{ij}|^2}{J_{ij}}$$

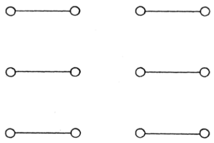
Mean field theory $\qquad \langle c^\dagger_{i\alpha} c_{j\alpha} \rangle = J_{ij} \Delta_{ij}$

$$\begin{aligned} \mathcal{L}_F = & \sum \bar{c}_{j\alpha} \partial_\tau c_{j\alpha} - \sum_{(ij)} \left(\bar{\Delta}_{ij} c^\dagger_i c_j + h.c\right) \\ & + \sum \left(i \varphi_j - \tilde{\mu}\right)\left(n_j - n_0\right) \end{aligned}$$

$$\mathcal{G} = \frac{1}{i\omega_n + \tilde{\mu} + \hat{\Delta}}$$

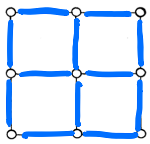
$$\left(T \sum_n \frac{1}{(i\omega_n + \tilde{\mu} + \hat{\Delta})}\right)_{ij} = J_{ij} \Delta_{ij}$$

$$\Delta_{ij} = |\Delta_{ij}| e^{i\phi_{ij}}$$



(a)

Dimer



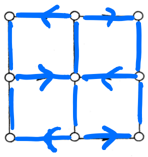
(b)

Spin Liquid

$\Delta_{ij} = \Delta_0$

$\epsilon_k = 2\Delta(\cos k_x + \cos k_y) - \tilde{\mu}$

$\Rightarrow \boxed{C_v \sim \gamma T.}$



(c)

Flux Phase

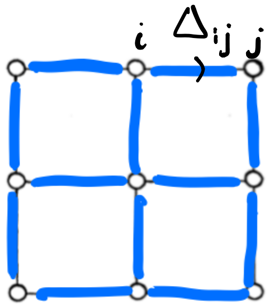
$\prod e^{i\phi_{ij}} = -1$

$\Delta_k = 2\Delta(\cos k_x + i\cos k_y)$

$\epsilon_k = -\tilde{\mu} \pm |\Delta_k|$

(Dirac Metal?)

Gauge Field + Fluctuations of the Order Parameter.



(b)

$$\Delta_{ij} = \Delta_{ij}^{(0)} \exp[i\phi_{ij}]$$

$$\phi_{ij} = \varphi_i - \varphi_j + a_{ij}$$

$$a_{ij} = \int_j^i \vec{A}\cdot d\vec{x}$$

$$e^{-S_{eff}[a]} = \int \mathcal{D}[c]\, e^{-S_F[\varphi, a]}$$

does not depend on static part of φ_i

METAL- INSULATOR TRANSITION

$$\boxed{H = \sum_{i,j} \left(-J_{ij} (c_i^\dagger c_j)(c_j^\dagger c_i) + t_{ij} c_i^\dagger c_j \right) - \mu \sum n_j + \frac{1}{2} U \sum n_j^2}$$

$$U n_j^2 \longrightarrow i \varphi_j n_j + \frac{1}{2U} \varphi_j^2$$

$$-J_{ij} (c_i^\dagger c_j)(c_j^\dagger c_i) \longrightarrow -\bar{\Delta}_{ij} c_j^\dagger c_i + h.c. + \frac{|\Delta_{ij}|^2}{J_{ij}}$$

$$\Delta_{ij} c_i^\dagger c_j = \Delta_{ij}^{(0)} e^{i(\psi_i - \psi_j - a_{ij})} c_i^\dagger c_j$$

$$\downarrow$$

$$c_j \rightarrow c_j e^{i\psi_j} \qquad \Delta_{ij}^{(0)} e^{-i a_{ij}} c_i^\dagger c_j$$

$$c_j^\dagger \partial_\tau c_j \rightarrow c_j^\dagger (\partial_\tau + i\dot{\psi}_j) c_j$$

$$\mathcal{L} = c_j^\dagger \left(\partial_\tau + i\dot{\psi}_j + i\varphi_j\right) c_j$$

$$+ \sum_{(ij)} \frac{|\Delta_{ij}|^2}{J_{ij}} + \left[\left(t_{ij} e^{i(-\psi_i + \psi_j)} + \Delta_{ij} e^{-ia_{ij}}\right) c_i^\dagger c_j + h.c\right]$$

$$+ \sum_j \frac{\varphi_j^2}{2u}$$

$$\varphi_j \to \varphi_j - \dot{\psi}_j$$

$$\mathcal{L} = c_j^\dagger \partial_\tau c_j$$

$$+ \sum_{(ij)} \frac{|\Delta_{ij}|^2}{J_{ij}} + \left[\left(t_{ij} e^{i(-\psi_i + \psi_j)} + \Delta_{ij} e^{-ia_{ij}}\right) c_{i\alpha}^\dagger c_{j\alpha} + h.c\right]$$

$$+ \sum_j \left[\left(i\varphi_j - \mu\right) c_{j\alpha}^\dagger c_{j\alpha} + \frac{1}{2u}\left(\varphi_j - \dot{\psi}_j\right)^2\right]$$

Steady state

$$i \langle c^\dagger_{j\alpha} c_{j\alpha} \rangle = \frac{1}{u} \langle \dot{\psi}_j - \varphi_0 \rangle$$

$$L_r = \frac{1}{2u} \left(\dot{\psi} - i\varphi_0 \right)^2$$

$$\frac{\partial \psi}{\partial \tau} = \frac{\partial \psi}{i \partial t}$$

Real time.

$L_r = -L_E$.

$$P = \frac{\partial L}{\partial \dot{\psi}} = \frac{1}{u} \left(\dot{\psi} - i\varphi_0 \right)$$

$$\Rightarrow \dot{\psi} = (uP + i\varphi_0)$$

$$H = P\dot{\psi} - L. = \frac{1}{u}\left(\dot{\psi} - i\varphi_0\right)\dot{\psi} - \frac{1}{2u}\left(\dot{\psi} - i\varphi_0\right)^2$$

$$= \frac{1}{u} uP\left(uP + i\varphi_0\right) - \frac{u}{2} P^2$$

$$= \frac{uP^2}{2} + iP\varphi_0$$

$$= \frac{u}{2}\left(P + i\varphi_0/u\right)^2 + \frac{\varphi_0^2}{2u}.$$

$$\psi(\varphi) = \psi(\varphi + 2\pi) \Rightarrow \psi(\varphi) = e^{im\varphi}$$

$$P = -i\frac{\partial}{\partial\varphi}$$

$\lambda = \varphi_0$ s.p

$$\frac{u}{2}\left(-i\frac{\partial}{\partial\varphi} + \frac{\lambda}{u}\right)^2 \psi = E\psi.$$

$$E = \frac{u}{2}\left(m + \frac{\lambda}{u}\right)^2$$

m = integer closest to $-\lambda/u$. $= n_0$

$\frac{\dot{\varphi}}{u} \equiv m \equiv$ # of conduction e^-

Integer charge.

Electromagnetic Properties