STRANGE METALS

The term "Strange Metal" is used to describe the unusual metallic behavior, epitomized by the normal state of cuprate high temperature superconductors, in which the resistivity is linear in temperature over a wide range of temperature,

$$\rho = \rho_0 + AT$$

where the residual resistivity ρ_0, is a small fraction of the linear rise, and in which the "scattering time", determined from a fit to the Drude formula

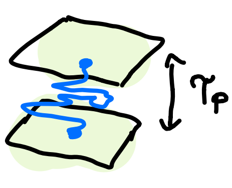
$$\rho = \frac{m}{ne^2\tau} \qquad (1)$$

appears to be of order the "Planck time"

$$Z_Q = \sum_{\text{HISTORIES}} e^{-S/\hbar}$$

$$\Delta t \sim \frac{\hbar}{\Delta E} \sim \frac{\hbar}{k_B T}$$

$$\tau_p \sim \frac{\hbar}{k_B T}, \qquad (2)$$



that is

$$\rho \approx \frac{m}{ne^2}\left(\frac{k_B T}{\hbar}\right)$$

What is so strange about this? Afterall, isn't the resistivity of copper linear in temperature? Well — if we look at conventional metals, they do show linear resistivity over limited regions of temperature, infact the scattering rate of electrons induced by scattering off phonons, is linear in conventional metals

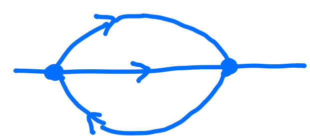
$b = \frac{\ell}{a} \sim \frac{\ell}{\lambda}$

$$\frac{\hbar}{\tau_{tr}} = 2\pi \lambda k_B T$$

where λ is the electron-phonon coupling constant, however such behavior usually extends over a very limited intermediate scale, and does not extend to either low temperatures (where $\tau_{tr}^{-1} \propto T^5$) or high temperatures, where the scattering tends to saturate. Moreover, in conventional "Fermi liquids", the low temperature resistivity is governed by electron-electron scattering for which

$$\Gamma \sim \frac{\omega^2 + (\pi k_B T)^2}{T_F}$$

$$\frac{1}{\tau_{tr}} \propto \left(T^2 / T_F \right)$$

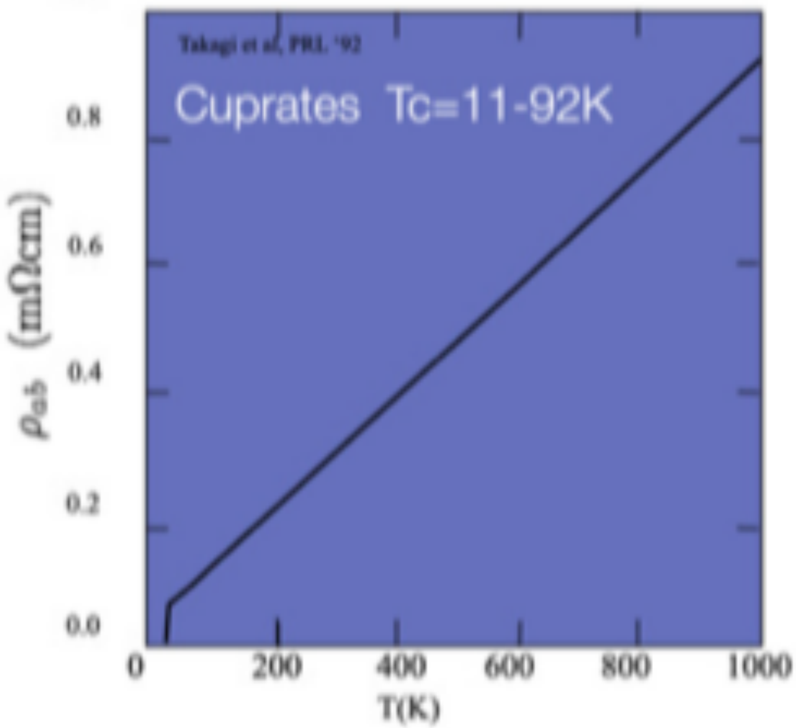


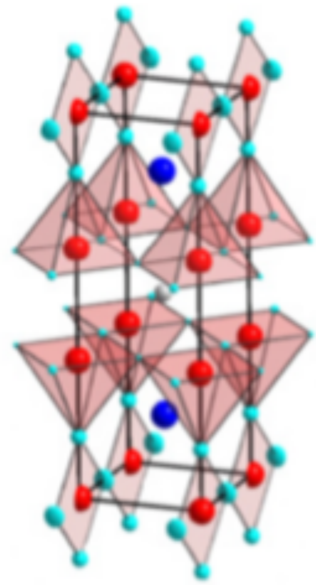
$$\boxed{\omega \to 0 \qquad \frac{\Gamma(\omega)}{\omega} \to 0}$$

where T_F is the Fermi temperature. None of these features are seen in strange metals: indeed at optimal doping, the linear resistance of cuprate superconductors extends from their superconducting transition temperature, T_c, up to 1000K

Moreover, strange metallic behavior is also seen in certain heavy electron materials, such as $CeCoIn_5$, in iron based high temperature superconductors, and most recently, in twisted-bilayer graphene. These are metals with widely differing electronic structure, yet their commonalities suggest a uniform underlying explanation.

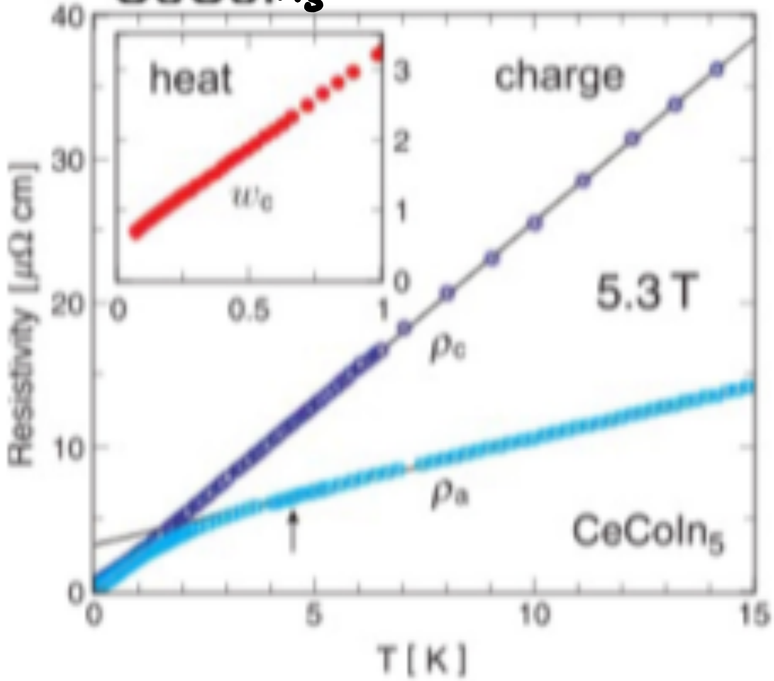
$LaSr_2Cu_1O_4$

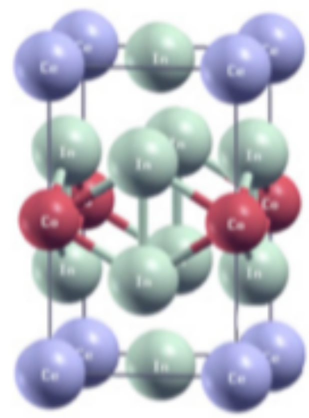




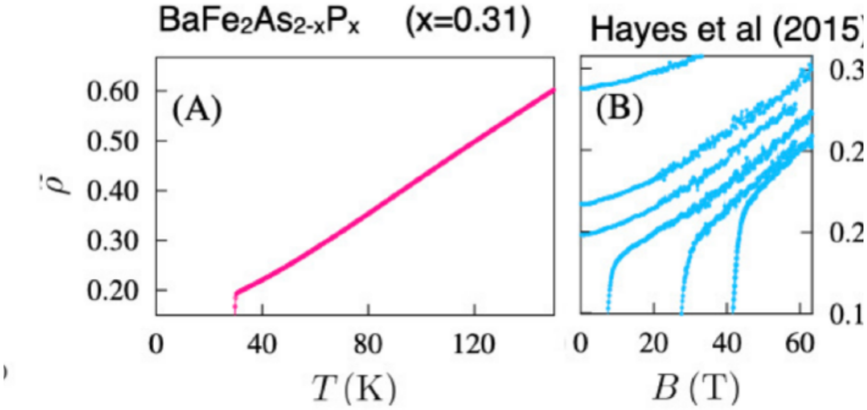
$YBa_2Cu_3O_7$

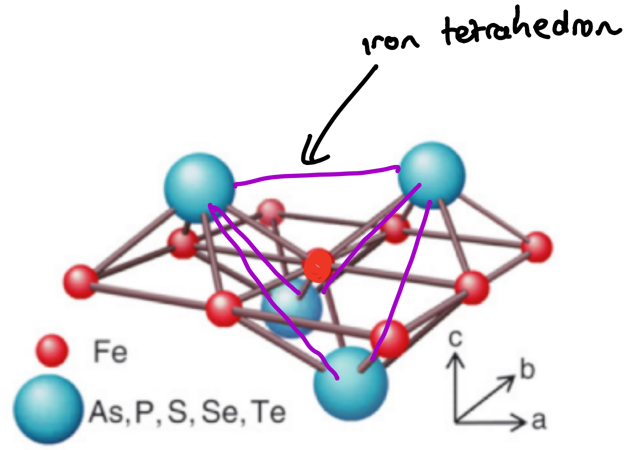
$CeCoIn_5$

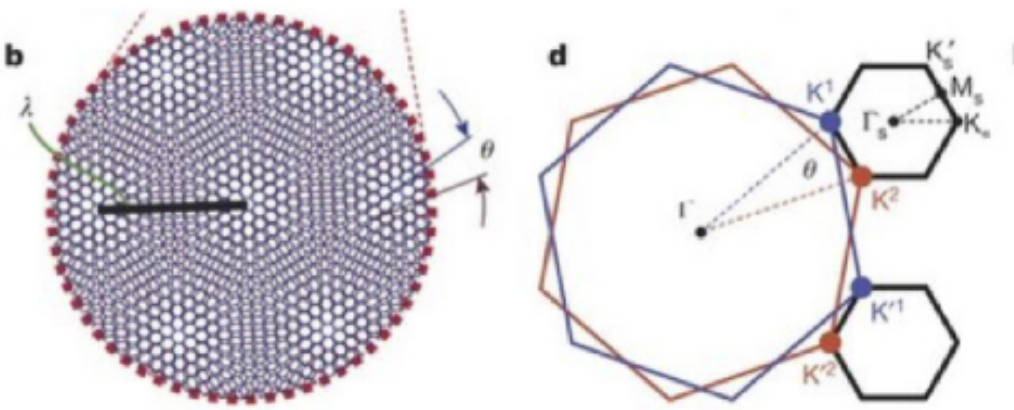


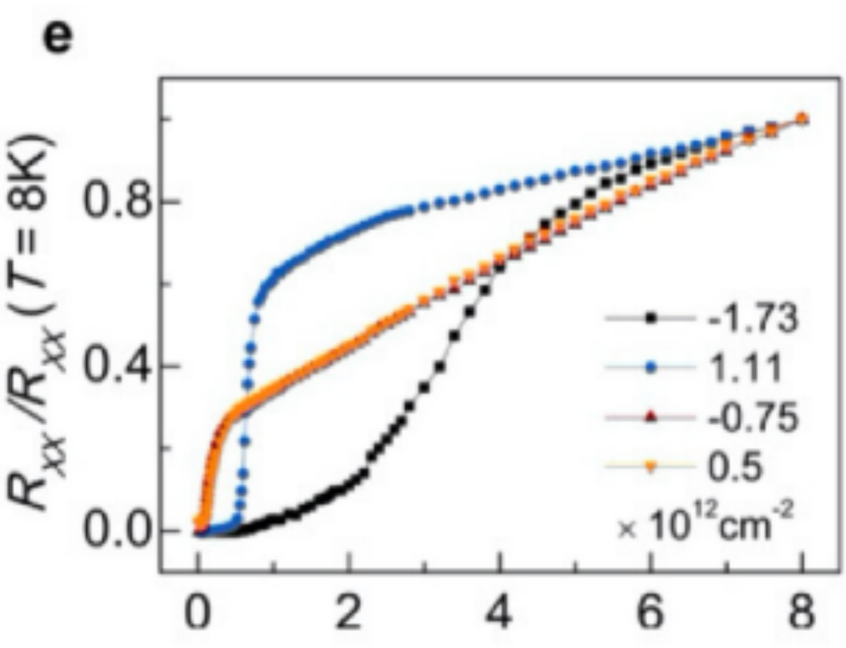


$CeCoIn_5$









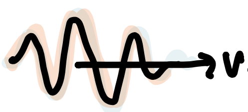
Candidate Strange Metal.

In short, in 2020, some 30 odd years after they were first discovered, Strange metals remain a major unsolved problem in condensed matter physics, and they make us believe that there is a new kind of metal, a counterpart to the well-established Landau Fermi liquid, that we have yet to understand.

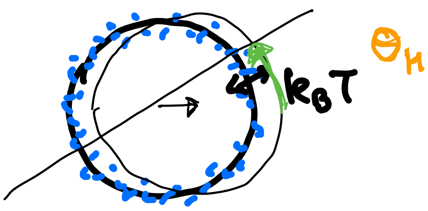
In the next three lectures we will explore the phenomenology and the current state of understanding of strange metals. The outline of the lectures is

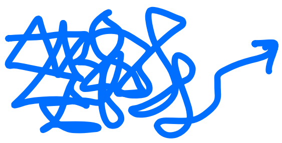
1. Conventional & Strange Metals Contrasted.
2. The Marginal Fermi Liquid Phenomenology.
3. The Ioffe-Larkin Model.
4. A survey of current theories and experiments.

Conventional Metals



$$\omega_c = \frac{eB}{m}$$

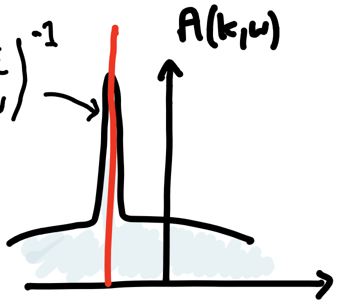




Diffusion

$$V_D \sim V_F \tau$$

$$Z \sim \left(1 - \frac{\partial \varepsilon}{\partial \omega}\right)^{-1}$$



"Quasiparticle"

$$\tau^{-1} \sim \frac{\omega^2 + (2\pi T)^2}{\epsilon_F}$$

LANDAU FERMI LIQUID

$$\delta\rho = \rho(T,B) - \rho(T,0)$$
$$\rho = \rho(T,0)$$

$\boxed{\text{DRUDE}}$

$$\left.\begin{aligned}
&\sigma_{xx} \sim \frac{ne^2 \tau_{tr}}{m} \\
&\sigma_{xy} \sim \sigma_{xx} \overbrace{(\omega_c \tau)}^{\Theta_H} \\
&\Delta\sigma_{xx} \sim -\sigma_{xx} \overbrace{(\omega_c \tau)^2}^{\Theta_H^2}
\end{aligned}\right\}$$

ONE RELAXATION TIME.

$$\rho_{xy} = R_H B = \frac{\sigma_{xy}}{\sigma_{xx}^2} \sim \frac{\sigma_{xx}(\omega_c \tau)}{\sigma_{xx}^2}$$

$$= \frac{B}{ne}$$ HALL CONST.

CANCELATION BECAUSE $\tau_H = \tau$

$$\left(\frac{\delta\rho}{\rho}\right) \propto (\omega_c \tau)^2 \propto \left(\frac{B}{\rho}\right)^2$$

"Kohlers Rule"

ONE RELAXATION TIME.

STRANGE METALS

Violate many of these properties

- $\tau^{-1} \approx \left(\frac{k_B T}{\hbar}\right)\eta$

$\eta \approx 1$

Quasiparticles not well defined.

"MARGINAL FERMI LIQUID"

"NON FERMI LIQUID"

"PLANKIAN DISSIPATION"

- VIOLATE KOHLER'S RULE

$$\tau_H^{-1} \approx \frac{T^2}{W} + b_i$$

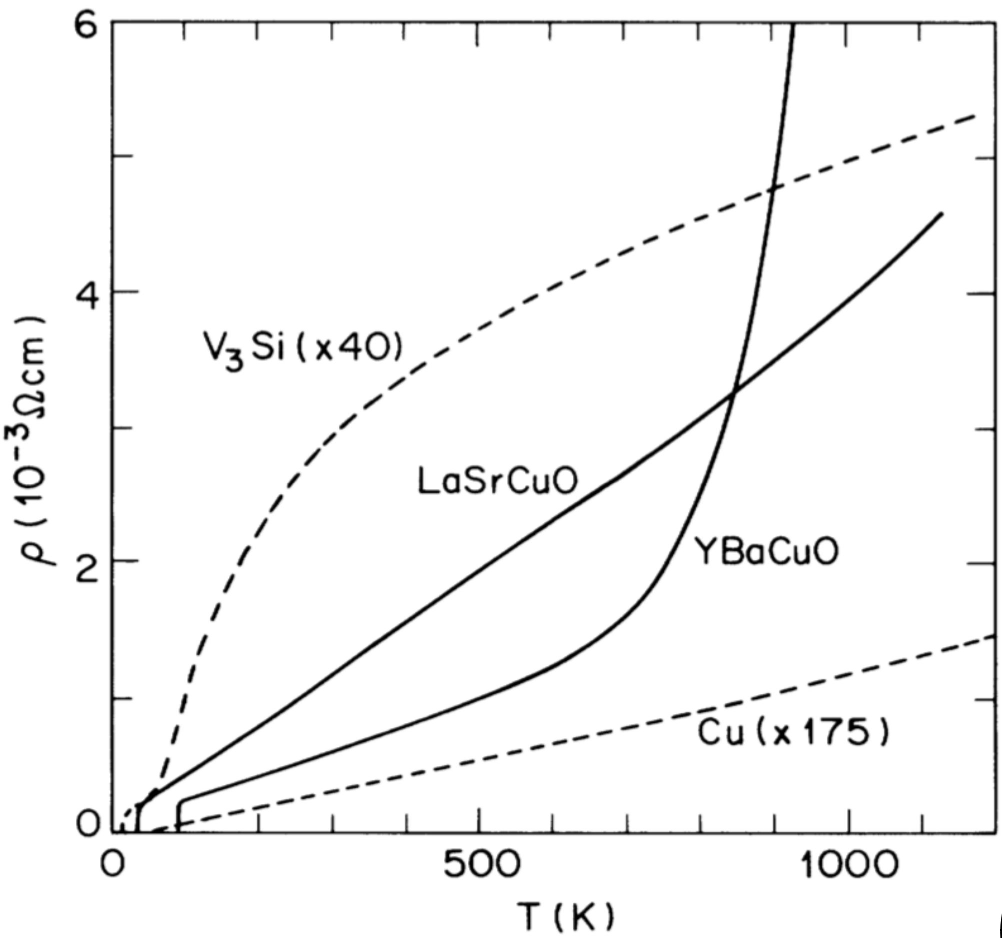
Impurities

TWO RELAXATION TIMES (AT EACH POINT ON THE FS ?)

ORIGIN OF A MODIFIED KOHLER'S RULE

Resistivity of $La_{1.825}Sr_{0.175}CuO_4$ and $YBa_2Cu_3O_7$ to 1100 K: Absence of Saturation and Its Implications

M. Gurvitch and A. T. Fiory
AT&T Bell Laboratories, Murray Hill, New Jersey 07974
(Received 29 July 1987)



$$\left(\sigma = \frac{\omega_p^2}{4\pi}\tau\right)$$

$$\sigma = \frac{ne^2}{m}\tau = (\epsilon_0 \omega_p^2)\tau$$

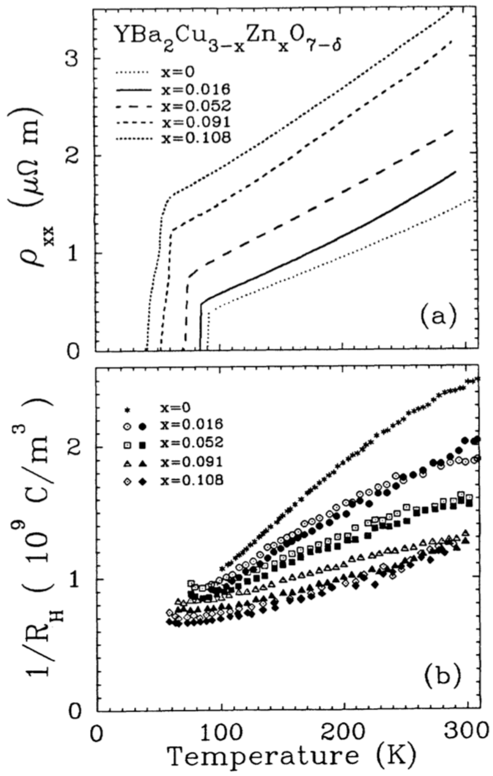
$$\tau^{-1} = 2\pi\lambda\left(\frac{k_B T}{\hbar}\right) = \eta\left(\frac{k_B T}{\hbar}\right)$$

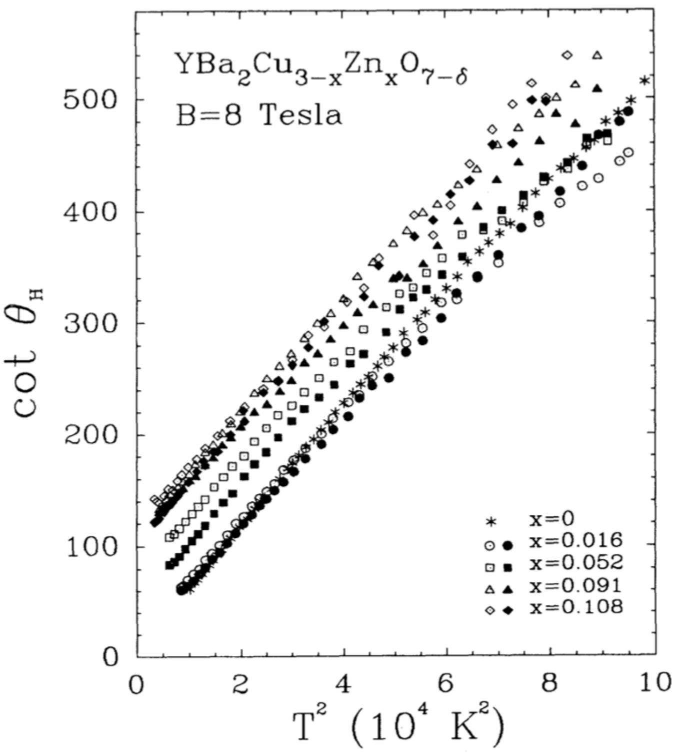
	λ	$\eta = 2\pi\lambda$	$b = \ell_{MFP}/a$ (300K)
LaSCO	0.1	0.6	7.6
YBCO	0.3	1.8	5.5
V_3Si			4

- No saturation despite closeness to Ioffe Regel.
- Rules out an e^--phonon mechanism for S.C.

Effect of Zn Impurities on the Normal-State Hall Angle in Single-Crystal $YBa_2Cu_{3-x}Zn_xO_{7-\delta}$

T. R. Chien, Z. Z. Wang,[a] and N. P. Ong
Joseph Henry Laboratories of Physics, Princeton University, Princeton, New Jersey 08544
(Received 17 July 1991)





- Hall constant is strongly T- dependent, and behaves strangely under Zn doping.

- $\cot\theta_H = \frac{\sigma_{xx}}{\sigma_{xy}} = \frac{1}{\omega_c \tau_H} \sim T^2 + c_i$

- $\tau_H^{-1} \sim \frac{(k_B T)^2}{W} + b_i$

  $W \sim 800K$

Do not obey Köhlers Rule

• $\sigma_{xy} = \frac{ne^2}{m}\,\overset{\frac{1}{T}}{\tau_{tr}}\,\overset{\frac{1}{T^2+c}}{\tau_H}$

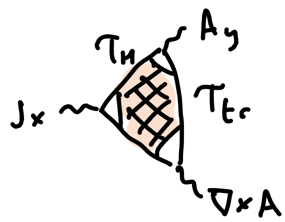
Multiplicative combination of relaxation times

$\sigma_{xx} \sim$ 

$J = \sigma_{xx}\dot{A}_x$

$J_y \sim \sigma_{xy} E_y \sim A^2$

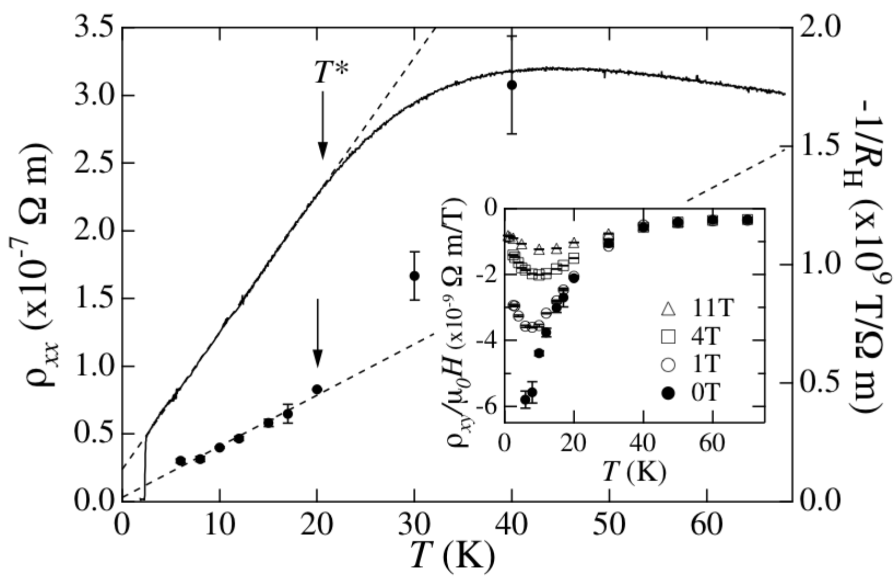
$\propto \vec{\nabla}\times\vec{A}$

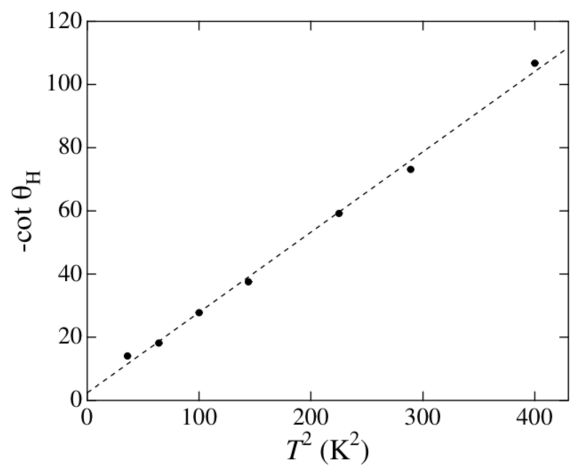


Similar Behavior in CeCoIn5

Normal-state Hall Angle and Magnetoresistance in quasi-2D Heavy Fermion $CeCoIn_5$ near a Quantum Critical Point

Y. Nakajima[1], K. Izawa[1], Y. Matsuda[1], S. Uji[2], T. Terashima[2], H. Shishido[3], R. Settai[3], and Y. Onuki[3], and H. Kontani[4]





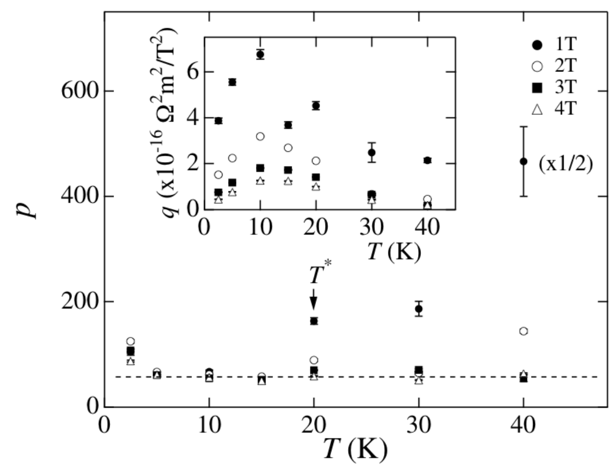
If $\Delta\sigma_{xx} \sim \sigma_{xx}(\omega_c\tau_H)^2$

$\left(\frac{\sigma_{xy}}{\sigma_{xx}}\right) \sim (\omega_c\tau_H)$

$\tan\theta_H$

$\frac{\Delta\rho_{xx}}{\rho_{xx}} \sim (\sigma_{xy}\rho_{xx})^2 \Rightarrow p = \frac{\Delta\rho_{xx}}{\sigma_{xy}^2\rho_{xx}^3} = $ cons?

MODIFIED KOHLER RULE.



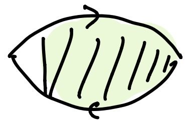
Marginal Fermi Liquid Theory

VOLUME 63, NUMBER 18 PHYSICAL REVIEW LETTERS 30 OCTOBER 1989

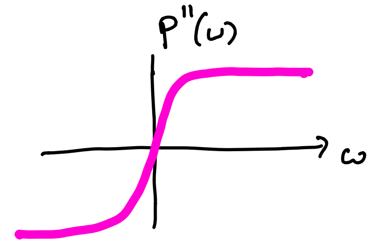
Phenomenology of the Normal State of Cu-O High-Temperature Superconductors

C. M. Varma, P. B. Littlewood, and S. Schmitt-Rink
AT&T Bell Laboratories, Murray Hill, New Jersey 07974

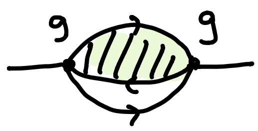
E. Abrahams and A. E. Ruckenstein
Serin Physics Laboratory, Rutgers University, Piscataway, New Jersey 08855
(Received 7 August 1989)

$P(\vec{q},\omega) =$  Charge or spin polarizeability.

$$\mathrm{Im}\, P(\vec{q},\omega) \sim -N(0) \times \begin{cases} (\omega/T) & |\omega| \ll T \\ \mathrm{sgn}(\omega) & |\omega| \gg T \end{cases}$$



$$P''(\omega) \sim \tanh\left(\frac{\omega}{T}\right)$$

$\Sigma =$ 

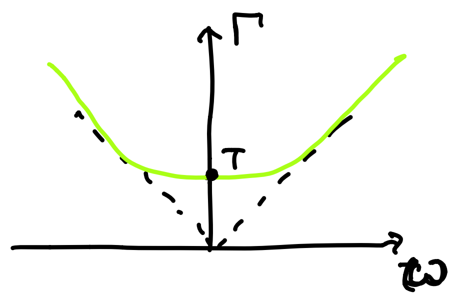
$$P(\omega) \sim \int_{-\infty}^{\infty} \frac{d\nu}{\pi} \frac{P''(\nu)}{\omega - \nu}$$

$$\sim \ln \omega$$

$$\Rightarrow \Sigma(\vec{k},\omega) \sim \overbrace{(g\, N(0))^2}^{\lambda} \left[\omega \ln \frac{x}{\omega_c} - \frac{i\pi}{2} x \right]$$

$x = \max(|\omega|, T)$

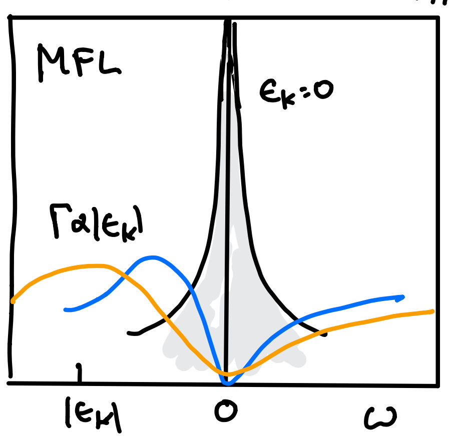
$$\Rightarrow \Sigma''(k,\omega) = \Gamma = \frac{\pi}{2} \lambda \begin{cases} |\omega| & \omega > T \\ T & \omega < T \end{cases}$$

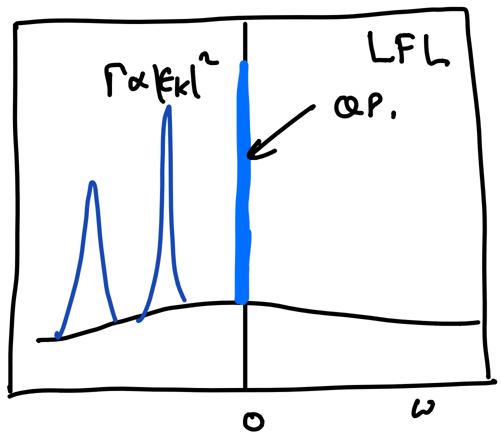


Quasiparticles are no longer well defined. $\Gamma / |\epsilon_k| = \text{constant}.$

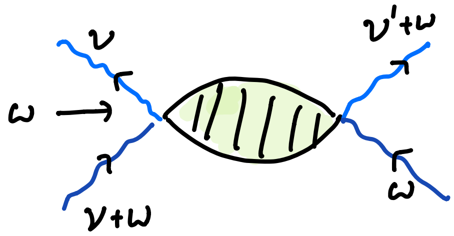
$$G(k,\omega) = \frac{1}{\omega - \epsilon_k - \Sigma(\omega)} = \frac{1}{\omega - \epsilon_k - \lambda\left(\omega \ln \frac{\omega}{\omega_c} + \frac{i\pi}{2}\omega\right)}$$

$$A(\omega) = G(\omega - i\delta)''/\pi$$





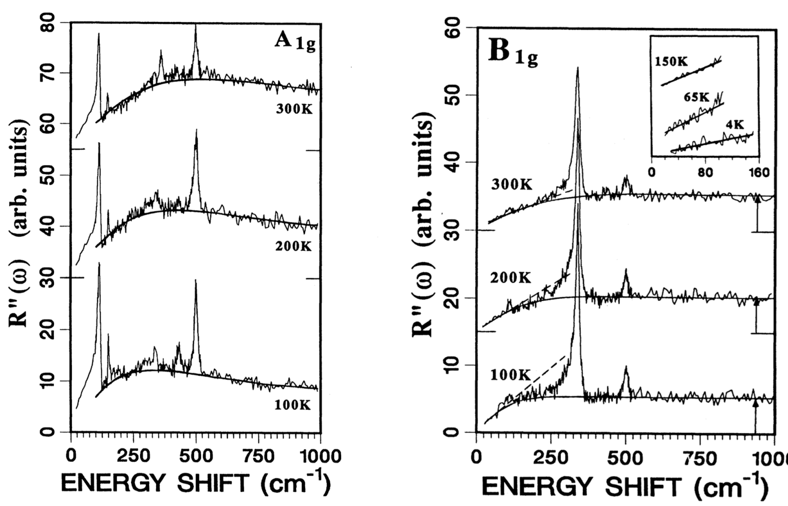
Raman Scattering



$$I(\omega) \propto -[1 + n(\omega)] \, \mathrm{Im}\, \epsilon^{-1}(0,\omega)$$

$$\sim [1 + n(\omega)] \, \mathrm{Im}\, P(\omega)$$

$$\sim \begin{cases} \frac{T}{\omega} \, \mathrm{Im}\, P(\omega) & \omega < T \\ \mathrm{Im}\, P(\omega) & \omega > T. \end{cases}$$



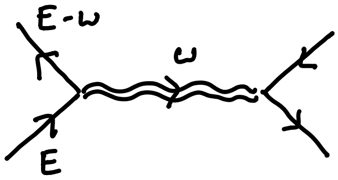
UNICATIONS
AL REVIEW B VOLUME 43, NUMBER 4 1 FEBRU

Raman investigation of the $YBa_2Cu_3O_7$ imaginary response function

F. Slakey, M. V. Klein, J. P. Rice, and D. M. Ginsberg
Department of Physics, Materials Research Laboratory, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, Illinois 61801

(Received 9 August 1990; revised manuscript received 12 October 1990)

Electron Energy Loss Spectroscopy



Anomalous density fluctuations in a strange metal

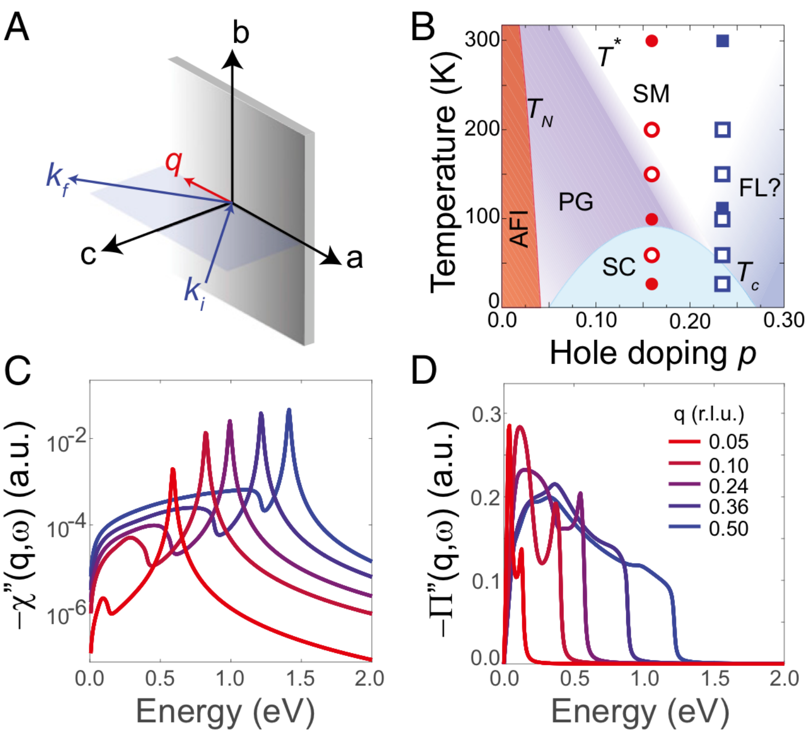
M. Mitrano[a,b,1], A. A. Husain[a,b], S. Vig[a,b], A. Kogar[a,b,2], M. S. Rak[a,b], S. I. Rubeck[a,b], J. Schmalian[c], B. Uchoa[d], J. Schneeloch[e], R. Zhong[e], G. D. Gu[e], and P. Abbamonte[a,b,1]

5392–5396 | PNAS | May 22, 2018 | vol. 115 | no. 21

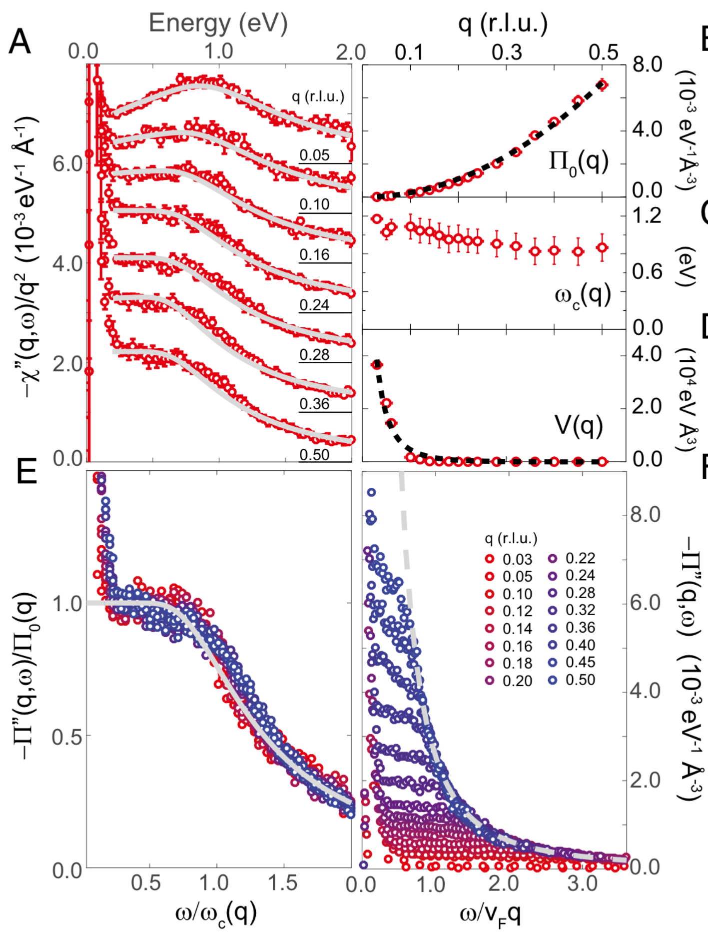
$$\chi(q,\omega) = \frac{P(q,\omega)}{1 - \epsilon_\infty^{-1} V(q) P(q,\omega)}$$

At large q $\chi \sim P$.

$V(q) \sim \dfrac{e^{-qz}}{q}$ is fit from data.



$(\Pi \equiv P)$



$$\Pi''(q,\omega) = -\Pi_0(q)\tanh\left[\frac{\omega_c^2(q)}{\omega^2}\right].$$

FIT FORM.

Essentially local (q independent)

MFL particle-hole

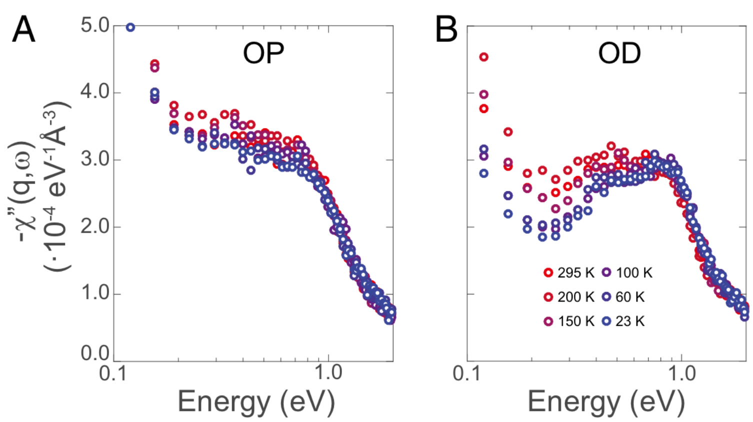
continuum.

"Local criticality"

$$\xi_\tau \sim \frac{\hbar}{k_B T}$$

$$\xi_\ell^z = \xi_\tau$$

z = dynamical critical exp.



An energy scale is seen to develop in the overdoped samples.

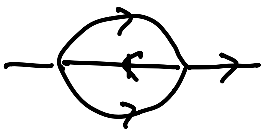
$$\xi_\ell \sim \ln \xi_\tau \sim \xi_\tau^0$$

$$\Rightarrow z = \infty \,!$$

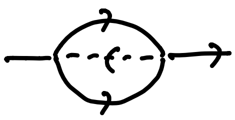
"Local quantum criticality"

WHAT IS THE MICROSCOPIC ORIGIN OF $P(\omega) \sim$ constant?

Helpful to look in the time domain

FL  $\propto \frac{1}{\tau^3} \Longleftrightarrow \Sigma''(\omega) \sim \int \Sigma(\tau) e^{i\omega\tau} d\tau \sim \omega^2$ ✓

$G(\tau) \sim 1/\tau$

MFL  $\propto \frac{1}{\tau^2} \Longleftrightarrow \Sigma''(\omega) \sim \int \frac{1}{\tau^2} e^{i\omega\tau} d\tau \sim \omega$

$\longrightarrow \quad \frac{1}{\tau}$

$- - - - \quad \mathrm{Sgn}(\tau)$

This is one way to achieve a MFL

- seen in the two channel kondo model
- but what would stabilize a local fermion in the lattice?
- "local quantum criticality".

See Coleman, Ioffe + Tsvelik (1994).