Outline of the Topics

1. Trends in the periodic table.
2. Introduction: Heavy Fermions and the Kondo Lattice.
3. Kondo Insulators: the simplest heavy fermions.
4. Large N expansion for the Kondo Lattice
5. **Heavy Fermion Superconductivity**
6. Topological Kondo Insulators
7. Co-existing magnetism and the Kondo Effect.

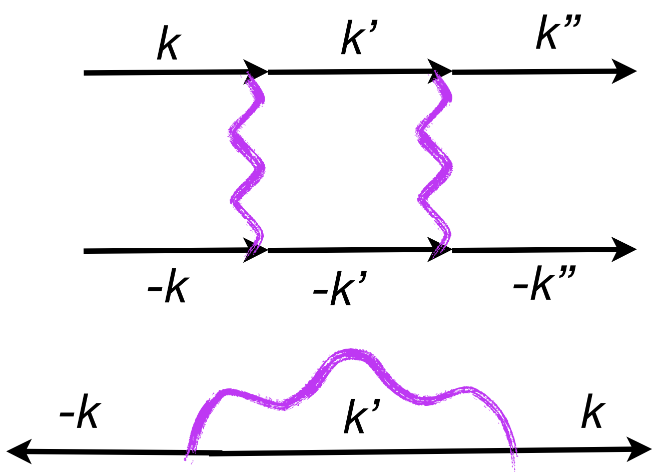
Please ask questions!

Rutgers Center for Materials Theory

Glue vs Fabric.

Glue vs Fabric.

Glue Spin fluctuations = pairing bosons



Eliashberg Approach (cf B. Keimer et al)

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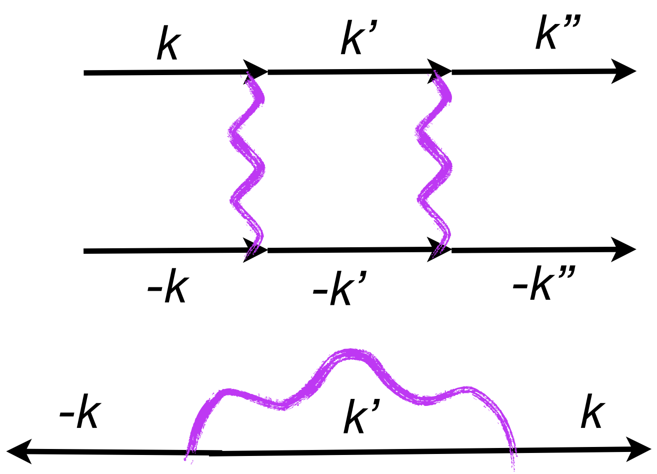

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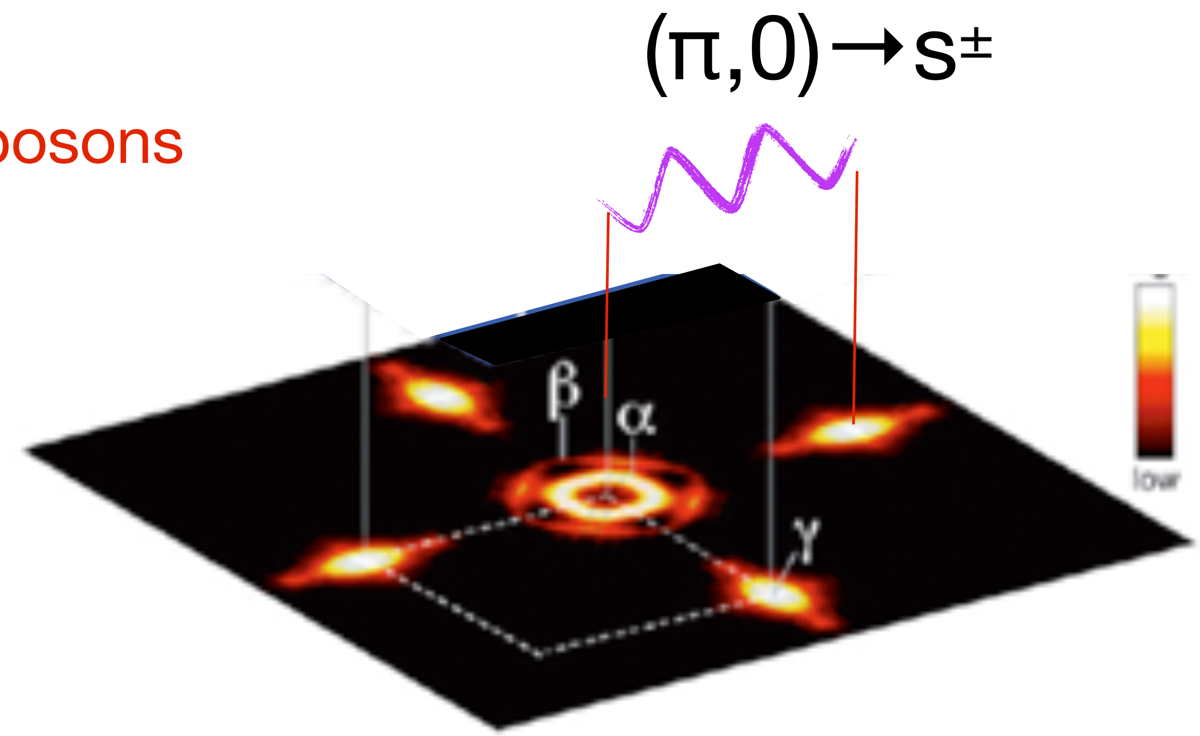

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$(\pi,0) \rightarrow s^{\pm}$

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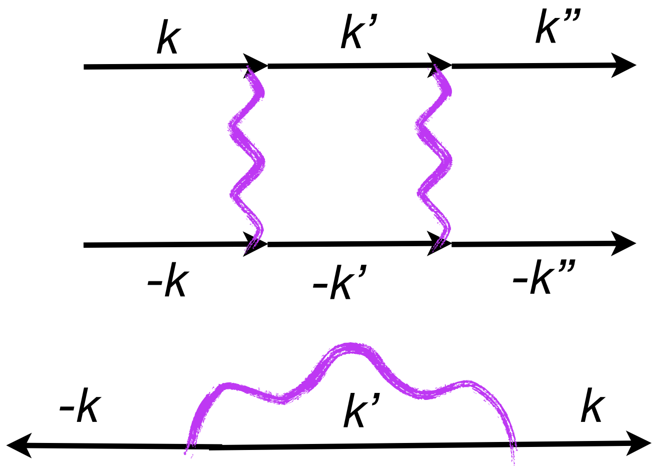

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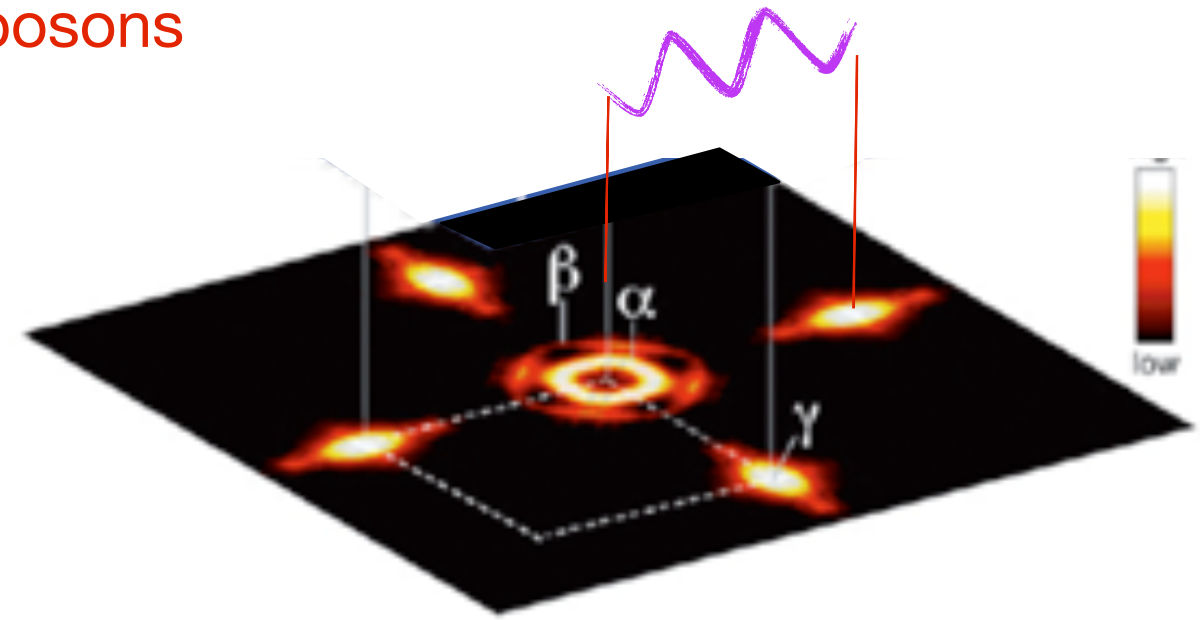

Fabric: spins *make* the pairs

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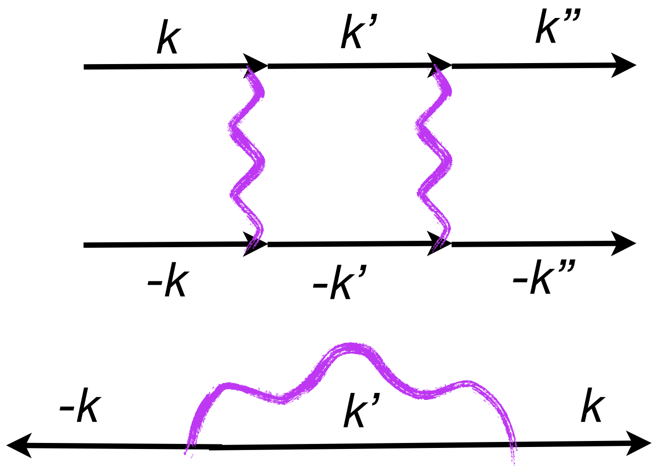

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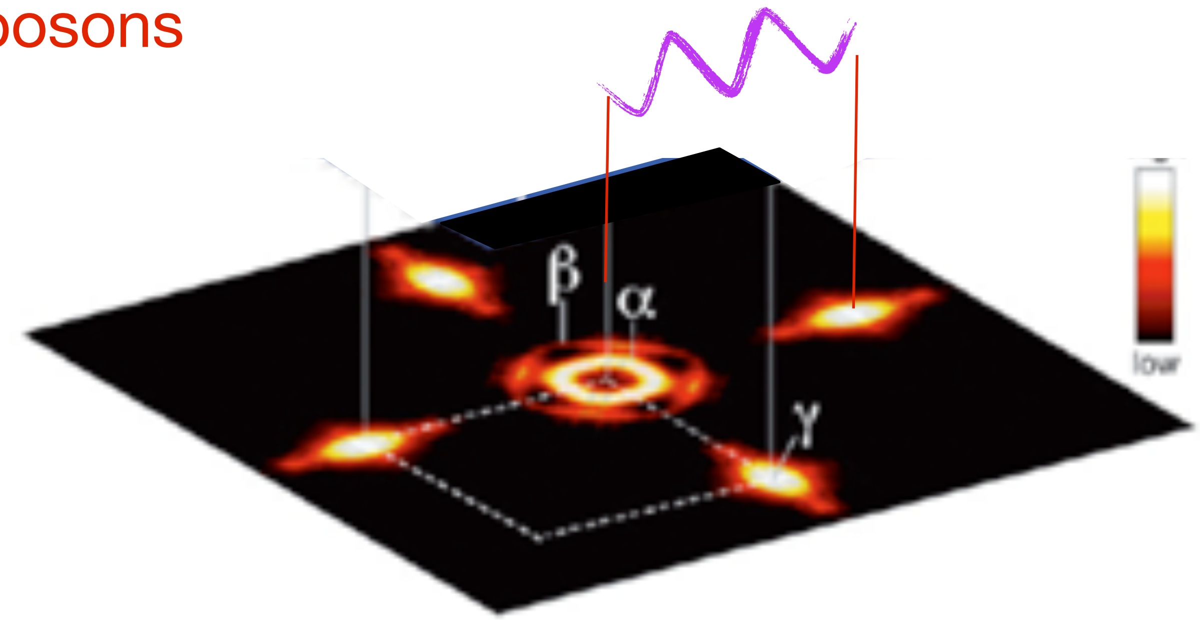

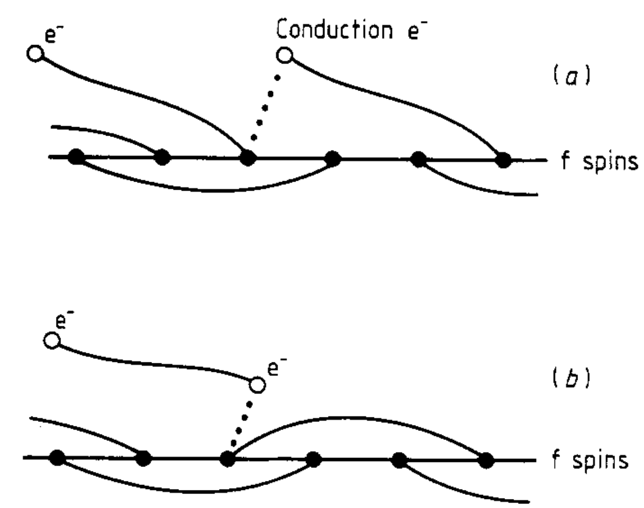

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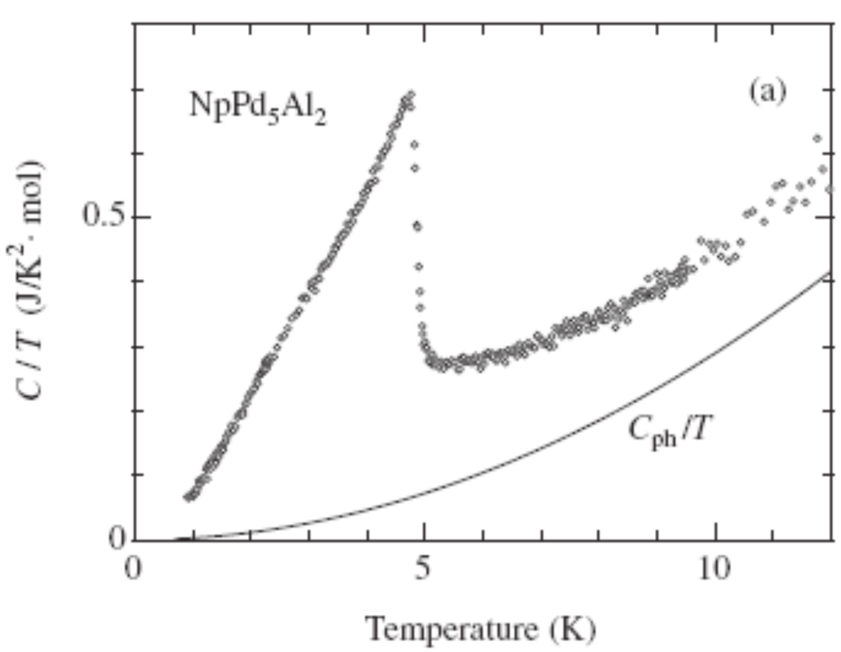

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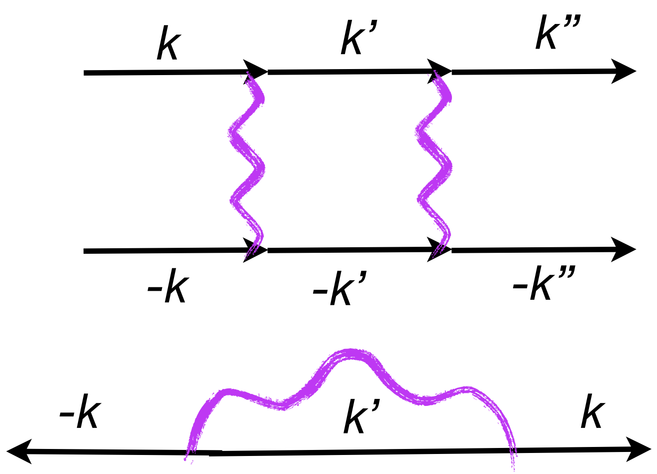

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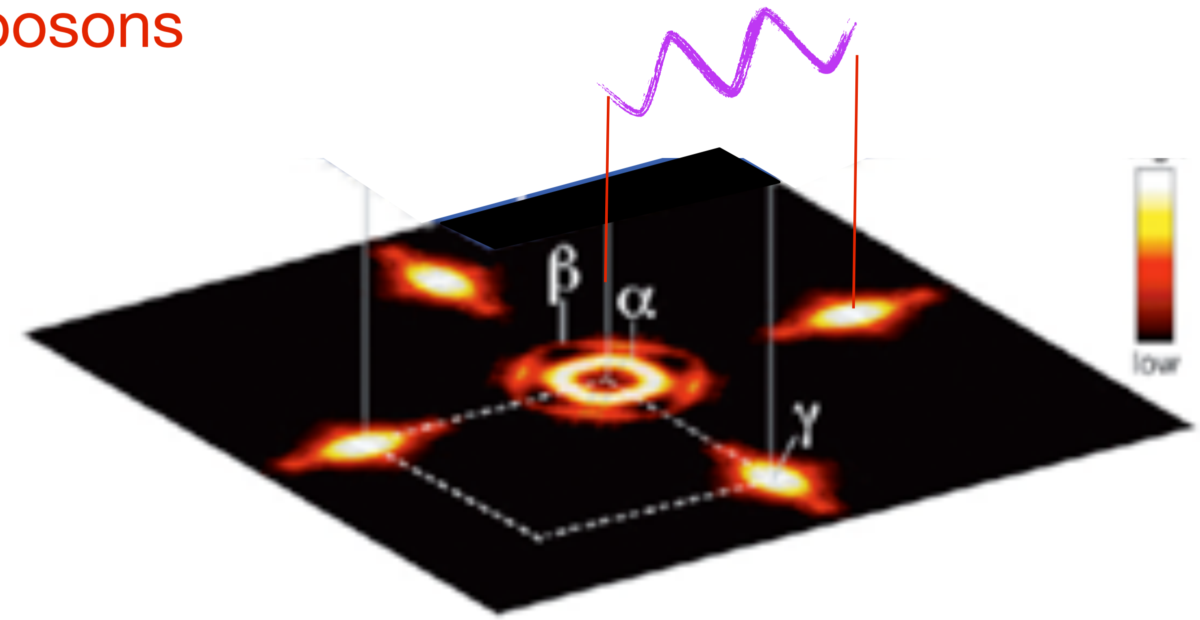

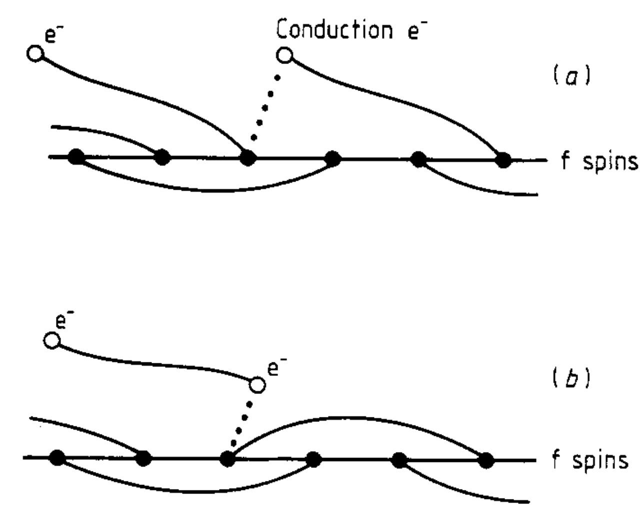

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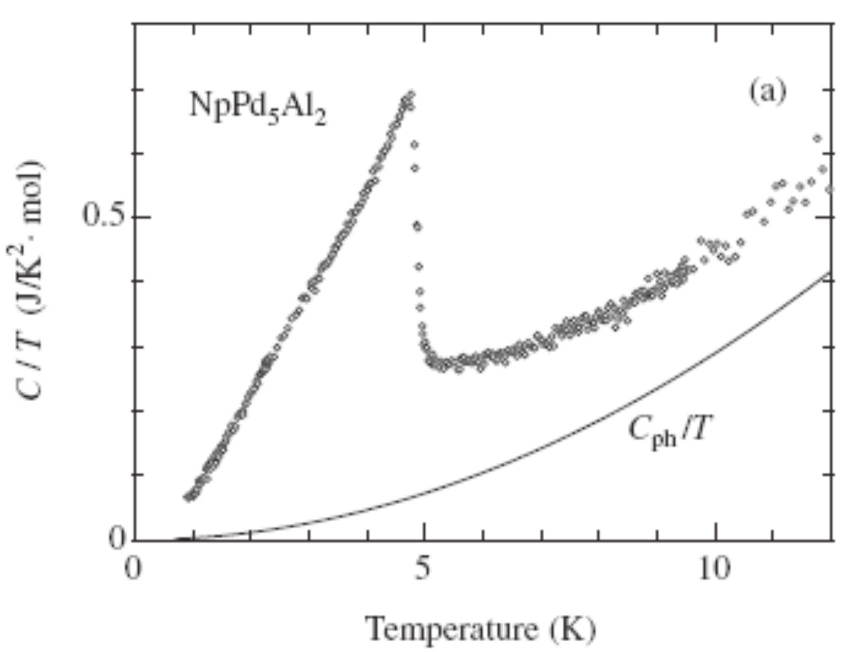

$$R \ln W = \int_0^T dT' \frac{C'}{T'}$$

"Hilbert Space Spectroscopy"

Glue vs Fabric.

Glue

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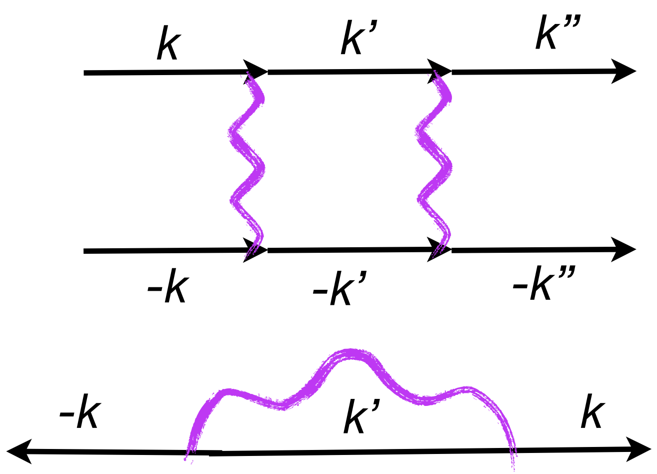

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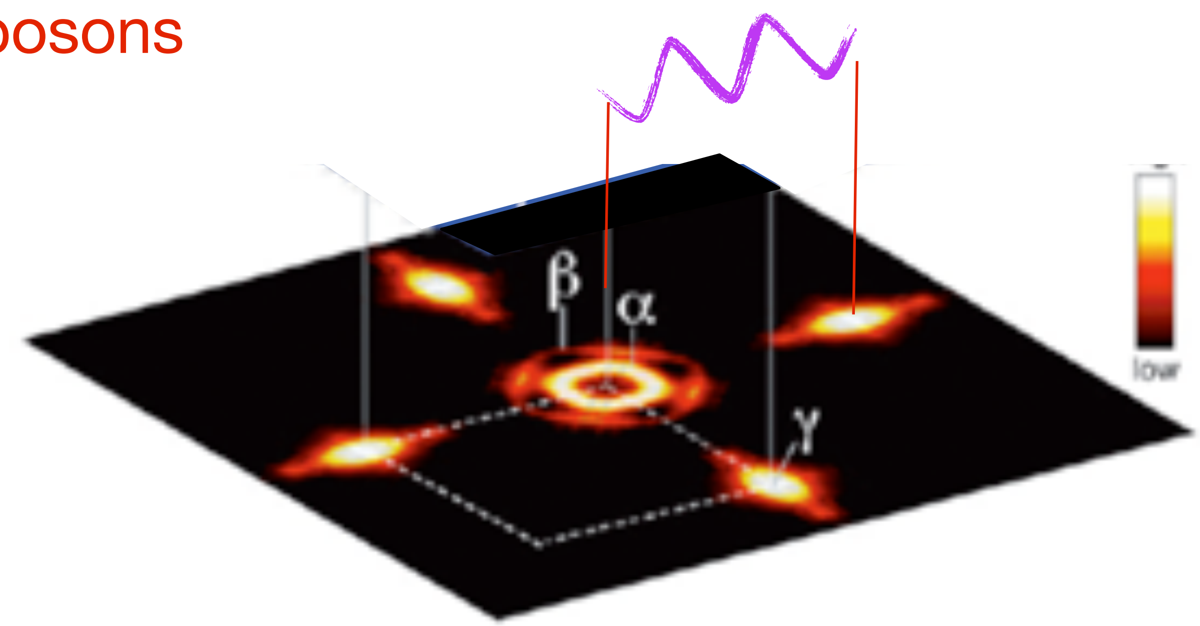

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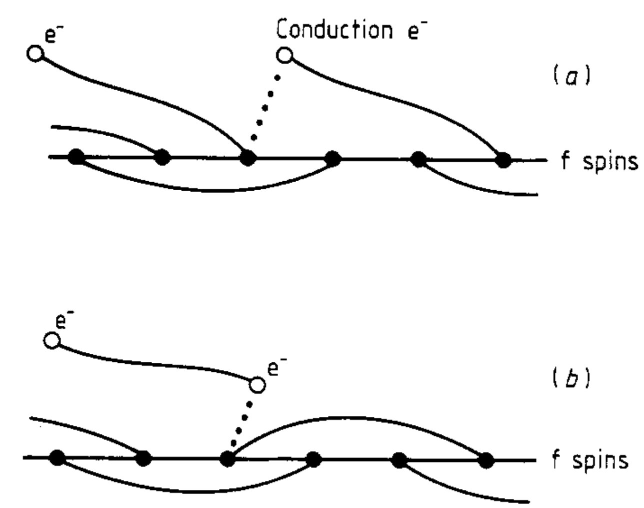

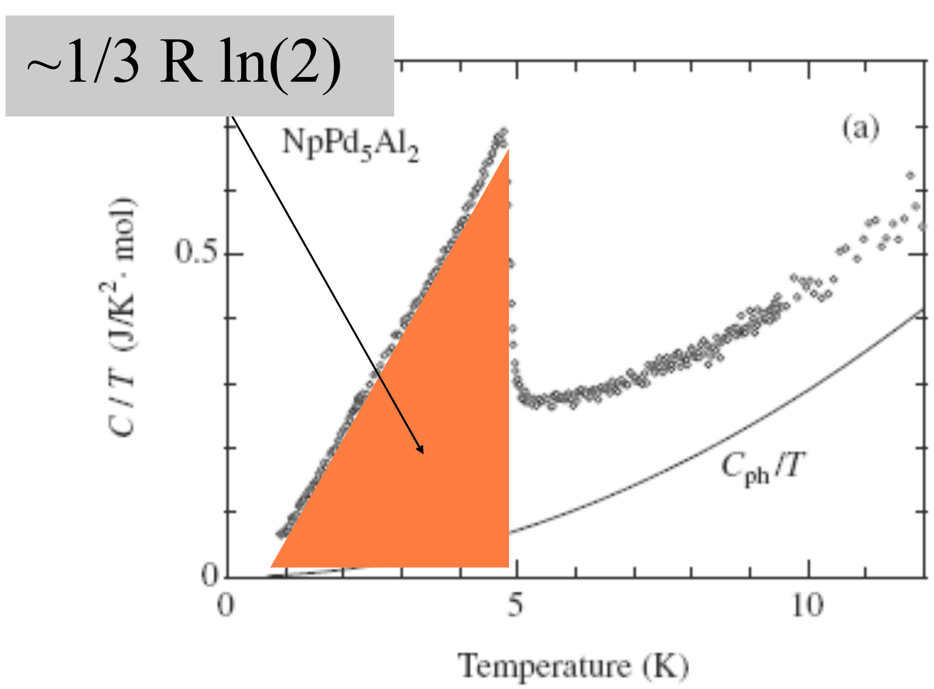

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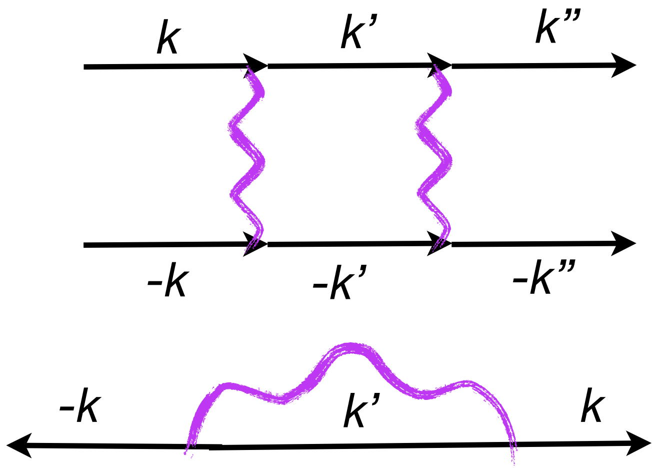

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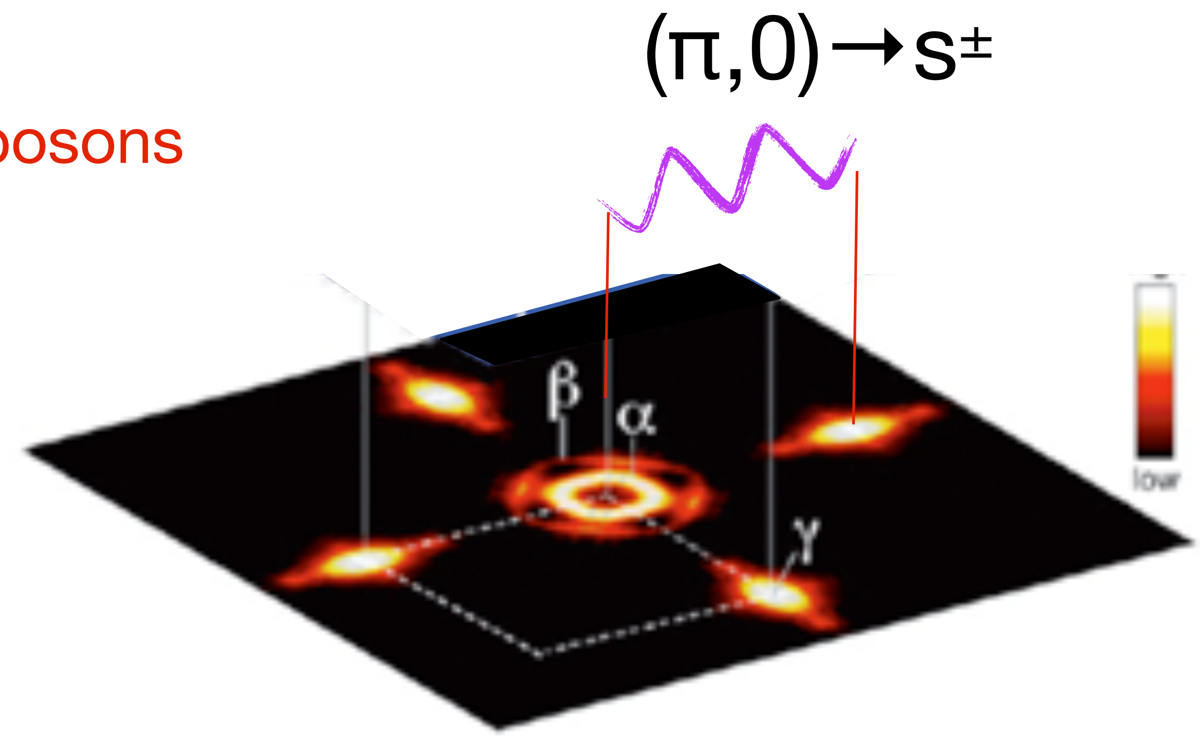

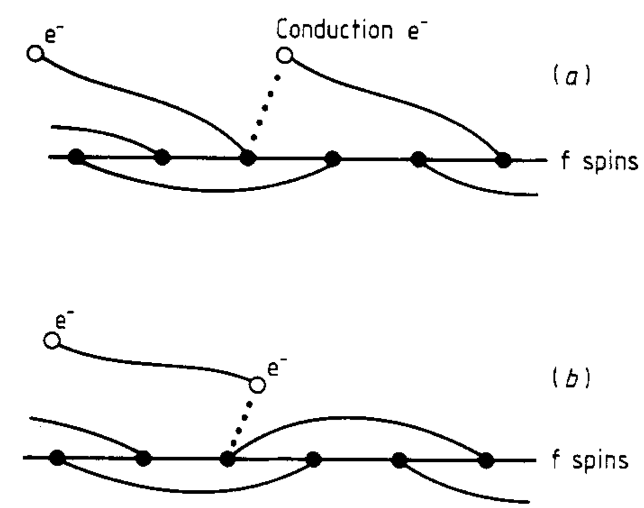

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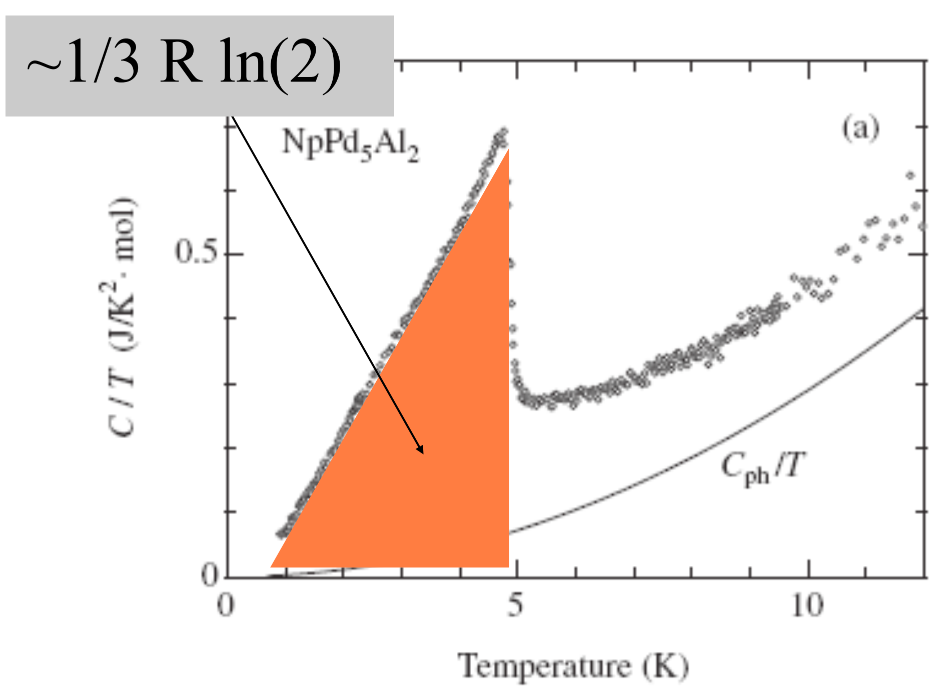

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"Hilbert Space Spectroscopy"

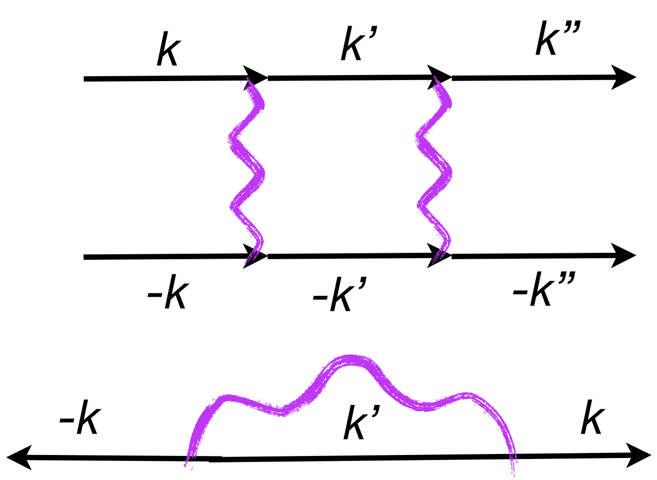
SPIN Hilbert space BUILDS the pairs.

Glue vs Fabric.

(π,0)→s±

Glue

Spin fluctuations = pairing bosons



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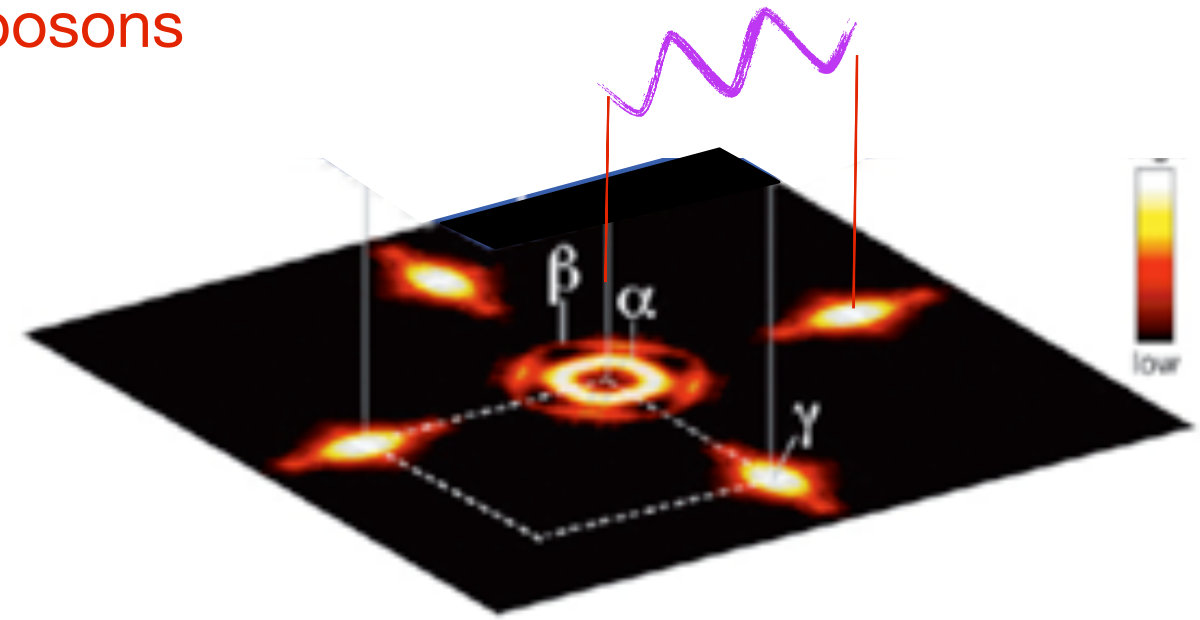

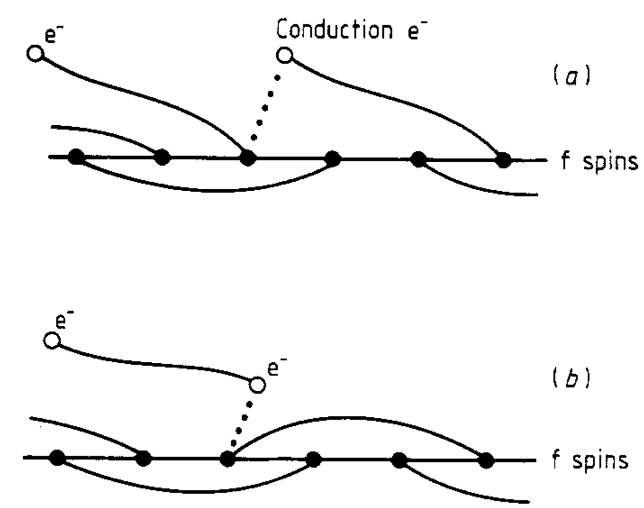

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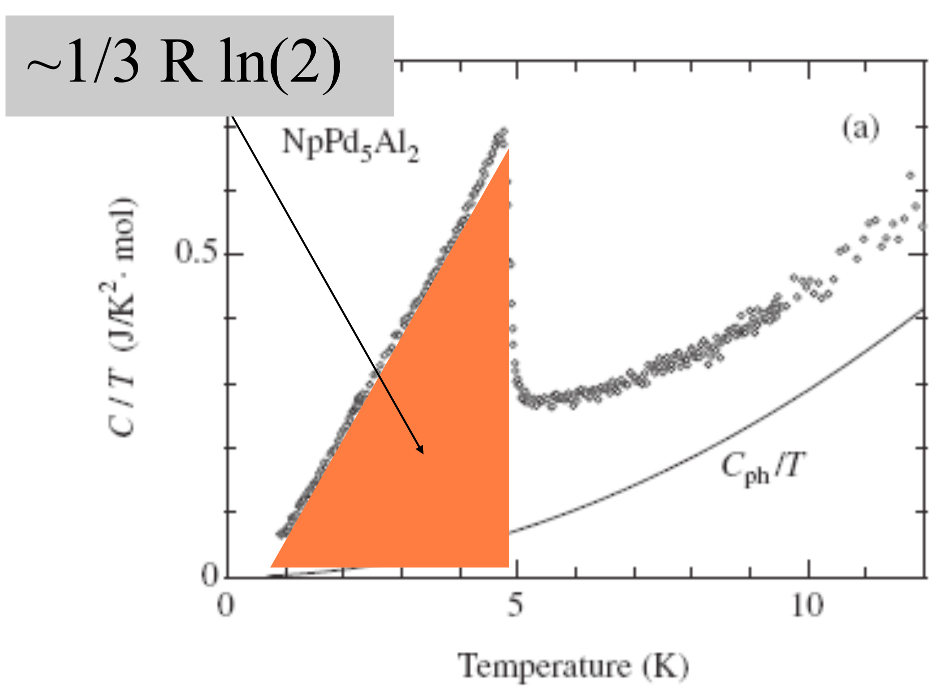

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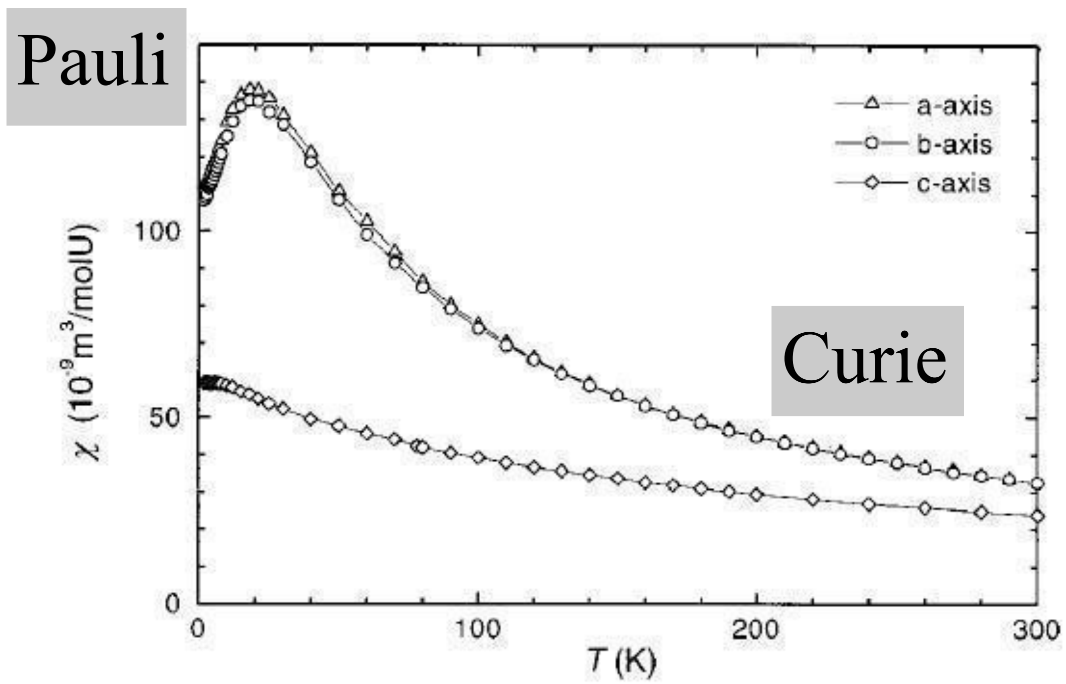
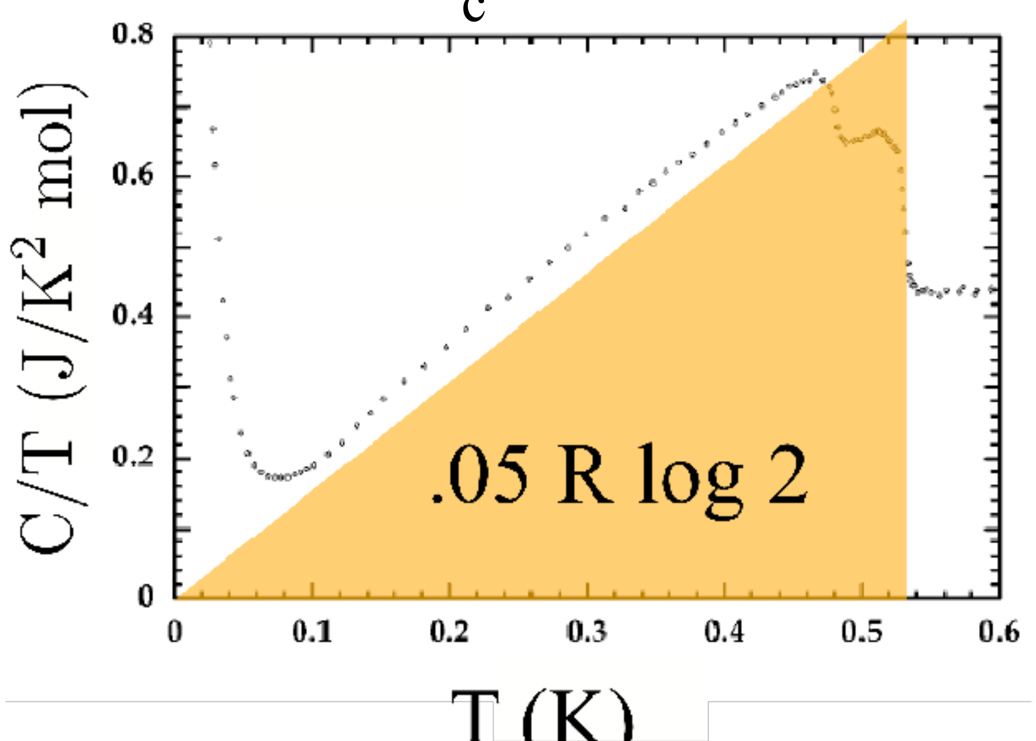
How?

UPt_3



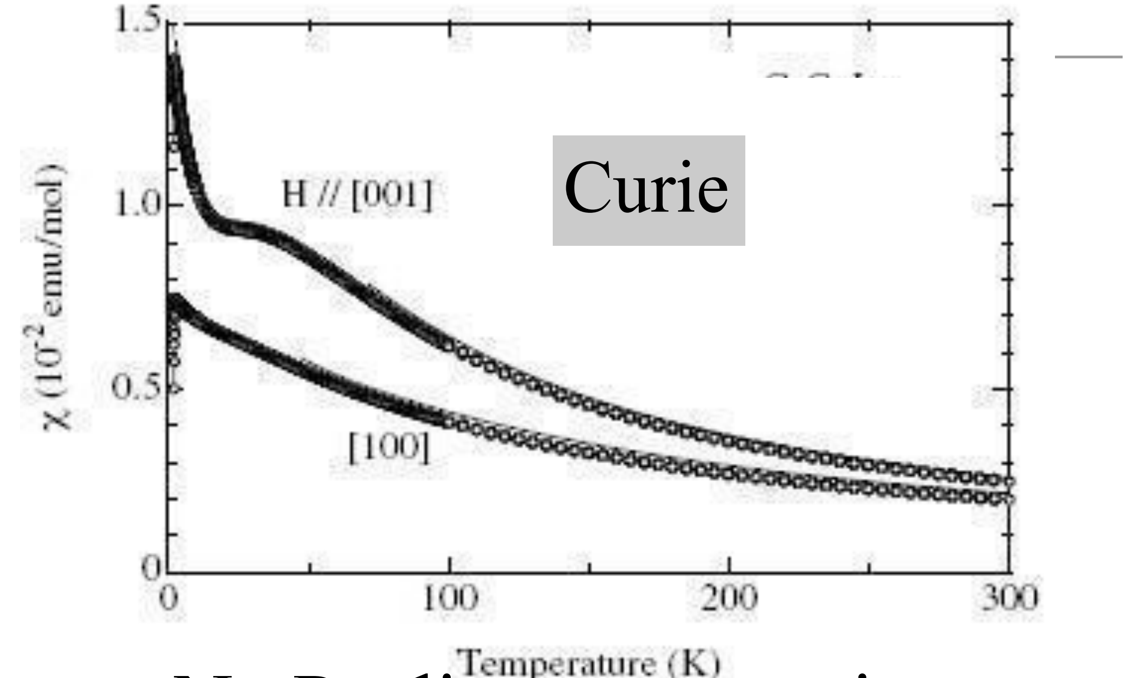
Pauli paramagnetic by 30K

$T_c = 0.5K$



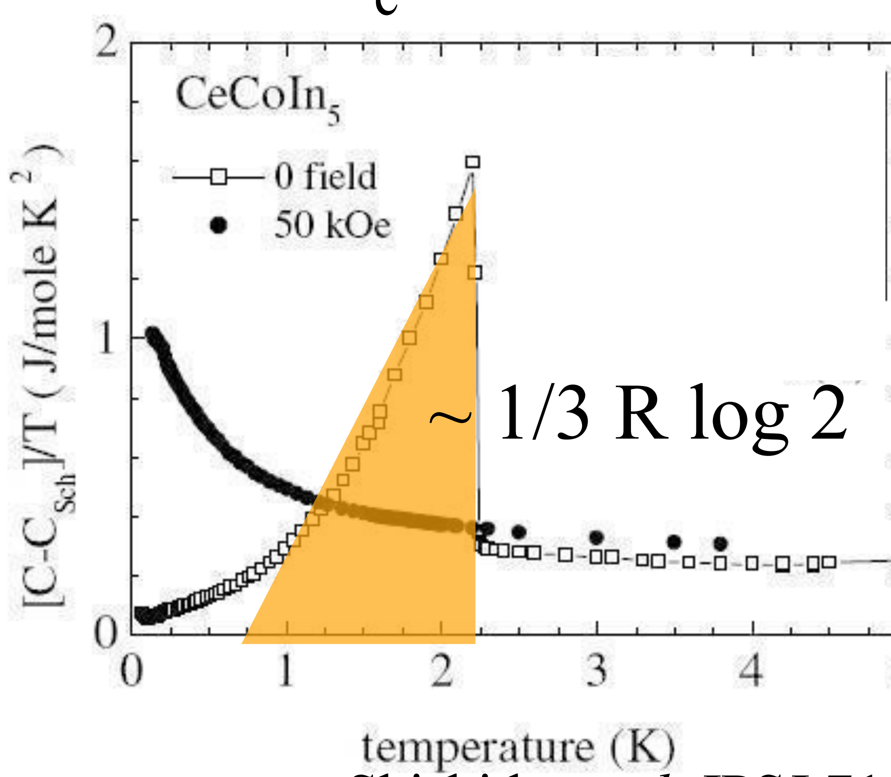
Frings *et al.* J. Magn. Magn. Mater. **31**, 240(1983)
Brison *et al.* J. Low Temp. Phys. **95**, 145*(*1994)

$CeCoIn_5$



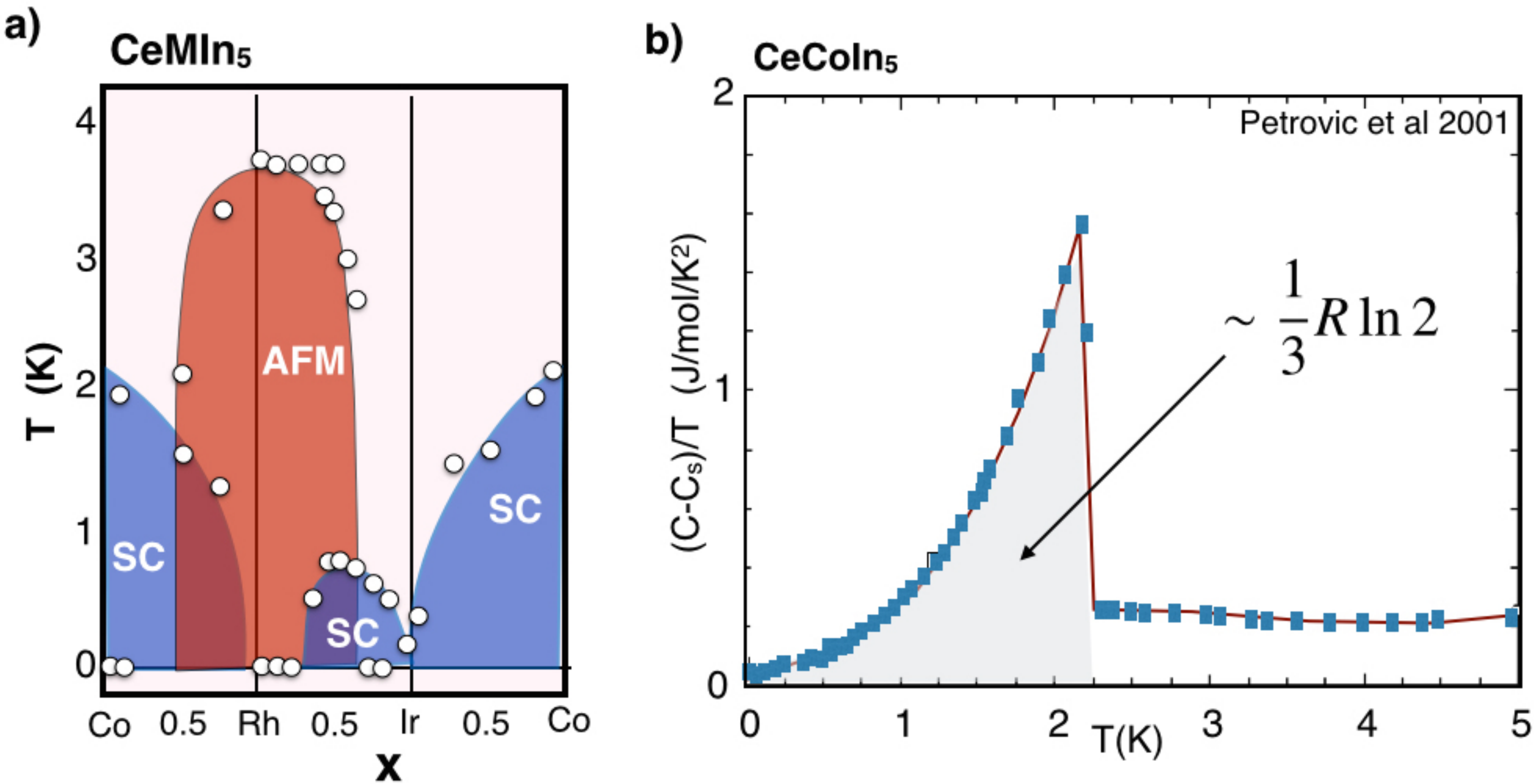
No Pauli paramagnetism

$T_c = 2.3K$

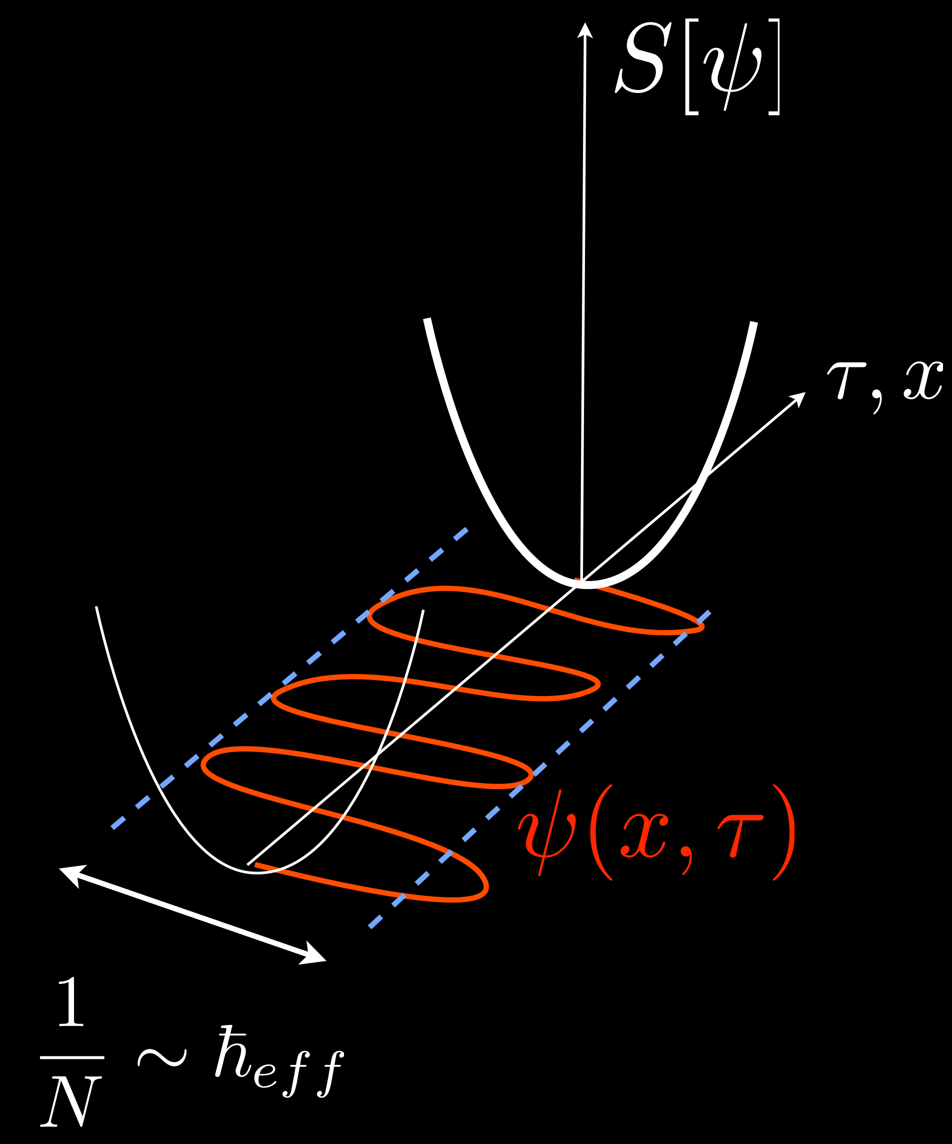


Shishido *et al.* JPSJ **71**, 162 (2002)
Petrovic *et al.* J.Phys Condens. Matter **13** 337 (2001)

115 Materials.

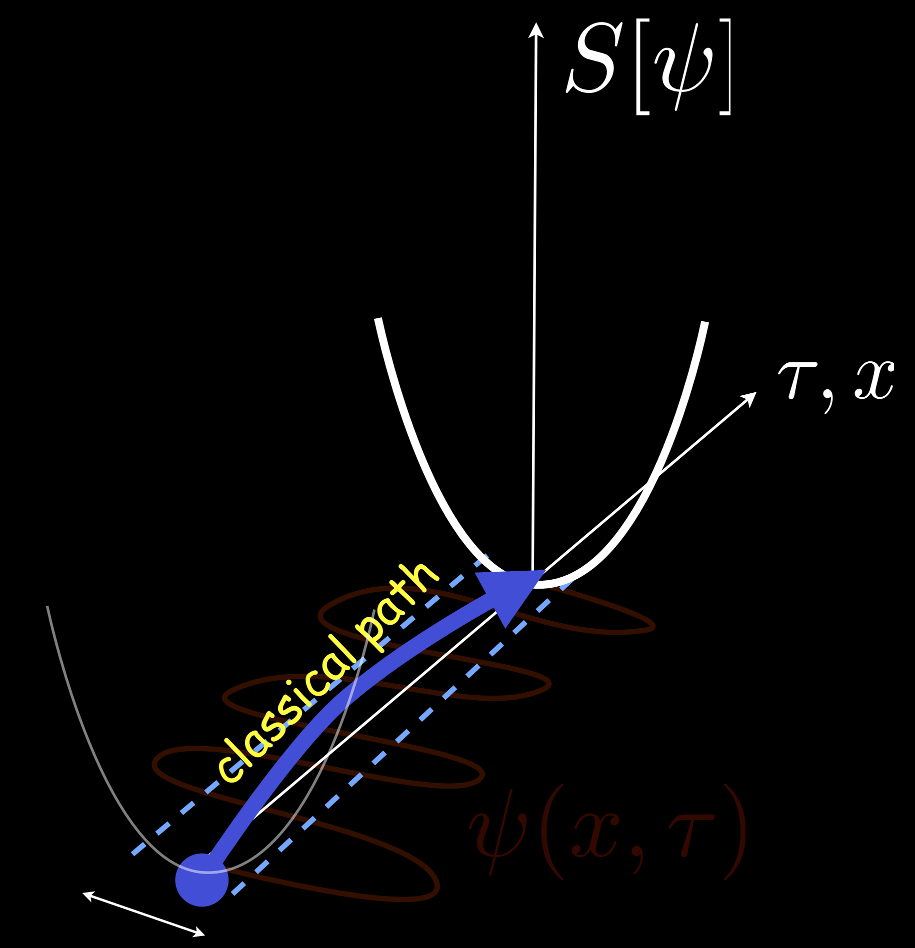


$N \longrightarrow \infty$



$$\sigma \in \left(-\frac{1}{2}, \frac{1}{2}\right) \longrightarrow \left(-\frac{N}{2}, \frac{N}{2}\right)$$

$$N \to \infty$$

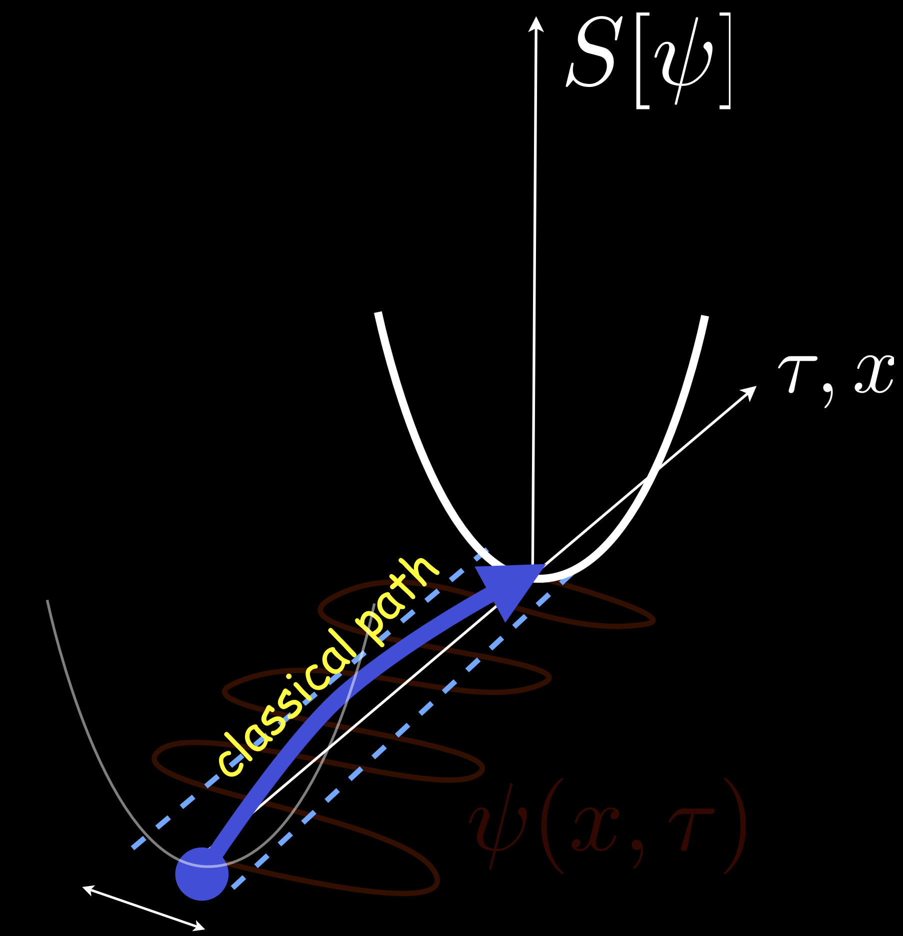


$$\frac{1}{N} \sim \hbar_{eff}$$

$$\sigma \in \left(-\frac{1}{2}, \frac{1}{2}\right) \longrightarrow \left(-\frac{N}{2}, \frac{N}{2}\right)$$

$$H = \sum_{\mathbf{k}\alpha} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha} + \frac{J_K}{N} \sum_{j} c^{\dagger}_{j\alpha} c_{j\beta} S_{\beta\alpha}(j) + \frac{J_H}{2N} \sum_{(i,j)} S_{\alpha\beta}(i) S_{\beta\alpha}(j)$$

$N \to \infty$



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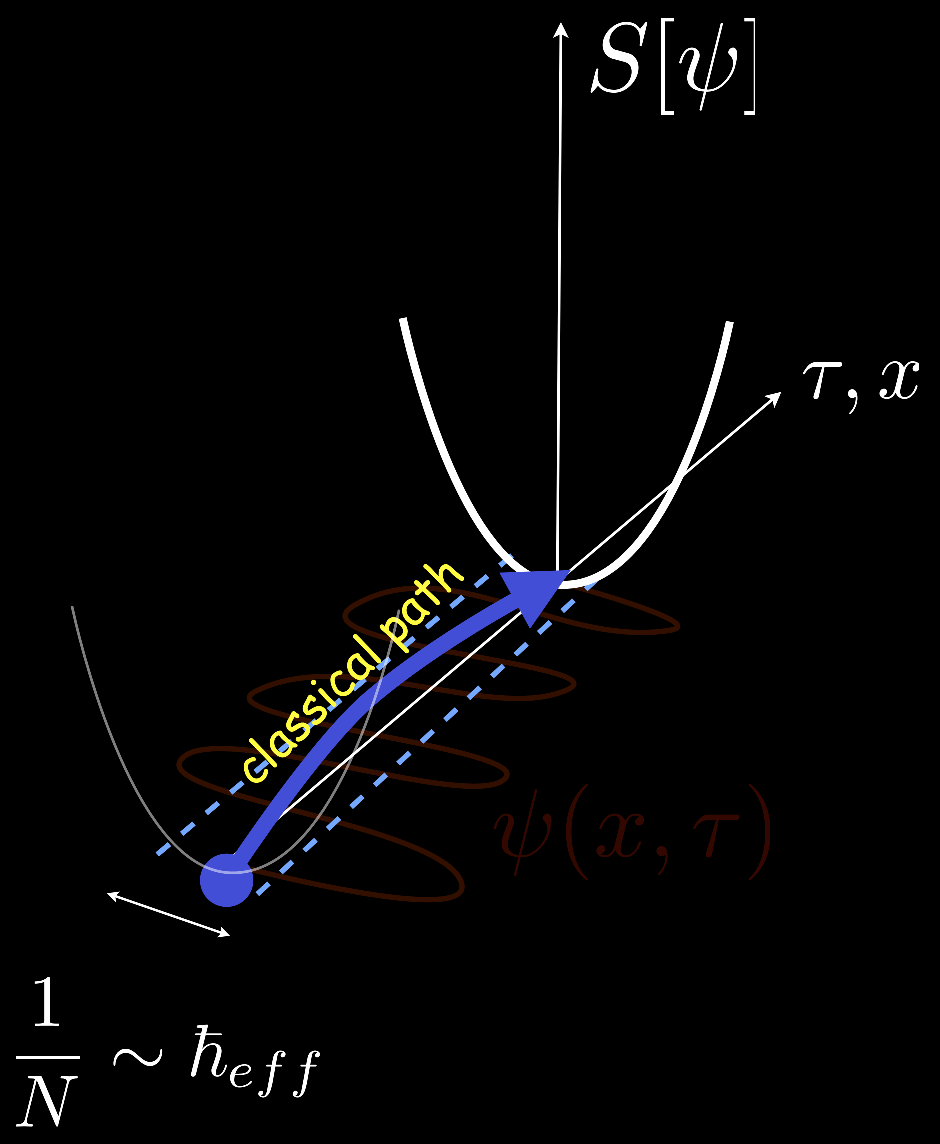

SU(N):

Mesons	*Baryons*
$\bar{q}q$	$q_1 q_2 \dots q_N$

?

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$N \rightarrow \infty$

SU(N):

No Pairs !

Mesons	*Baryons*
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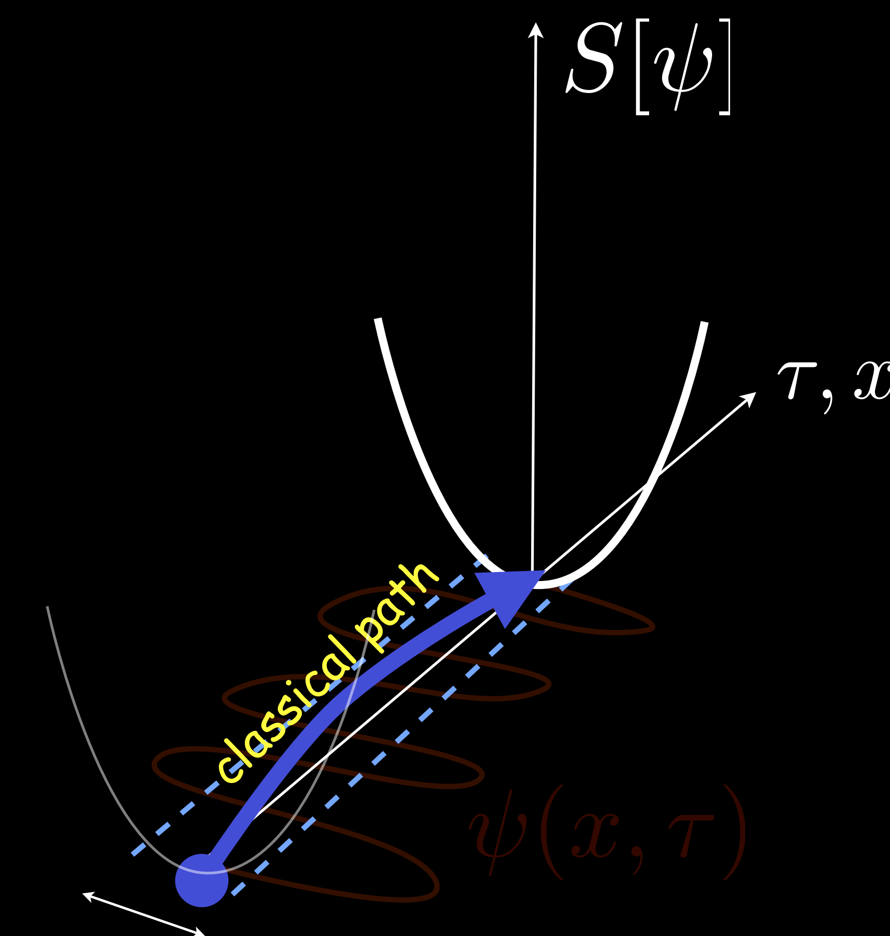

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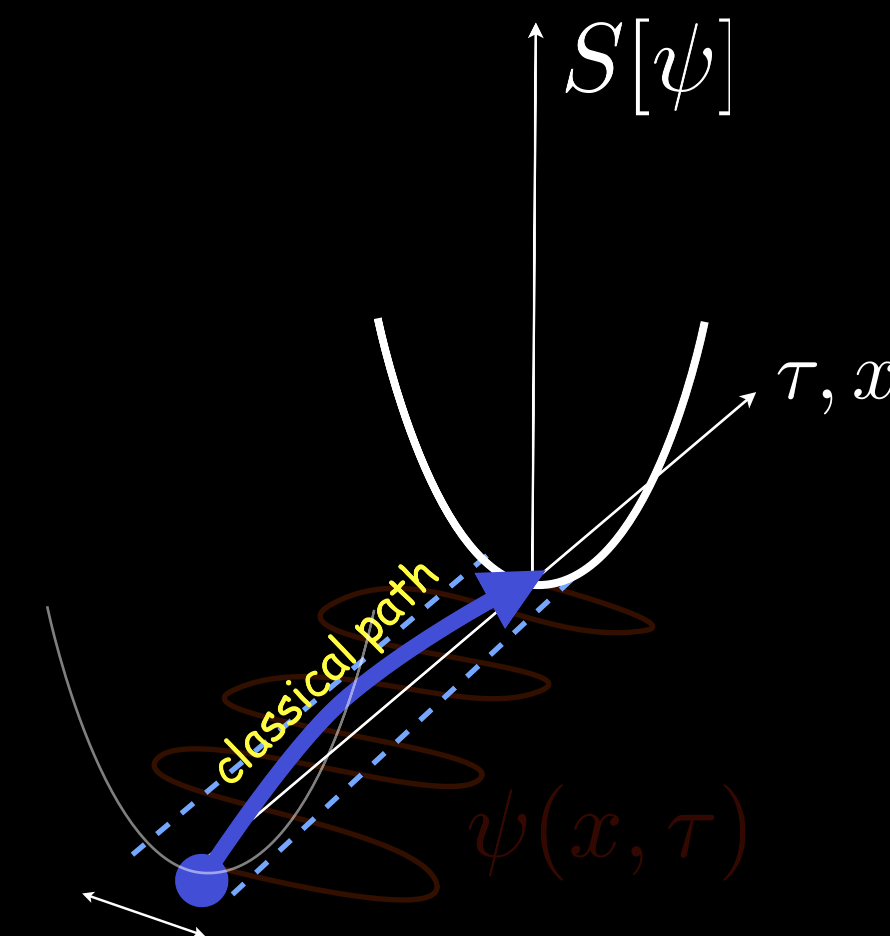

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	Mesons	Baryons
SU(N):	$\bar{q}q$	$q_1 q_2 \cdots q_N$
No Pairs !		
		Cooper pairs
SP(N):	$\bar{q}q$	$q_a q_{-a}$

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$$H = \sum_{\mathbf{k}\alpha} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha} + \frac{J_K}{N} \sum_j c^{\dagger}_{j\alpha} c_{j\beta} S_{\beta\alpha}(j) + \frac{J_H}{2N} \sum_{(i,j)} S_{\alpha\beta}(i) S_{\beta\alpha}(j)$$

$N \to \infty$

"Symplectic Large N"

R. Flint and PC '08

$$S_{\alpha\beta} = f^\dagger_\alpha f_\beta - \mathrm{sgn}(\alpha\beta) f^\dagger_{-\beta} f_{-\alpha}$$

	Mesons	*Baryons*
SU(N):	$\bar{q}q$	$q_1 q_2 \cdots q_N$
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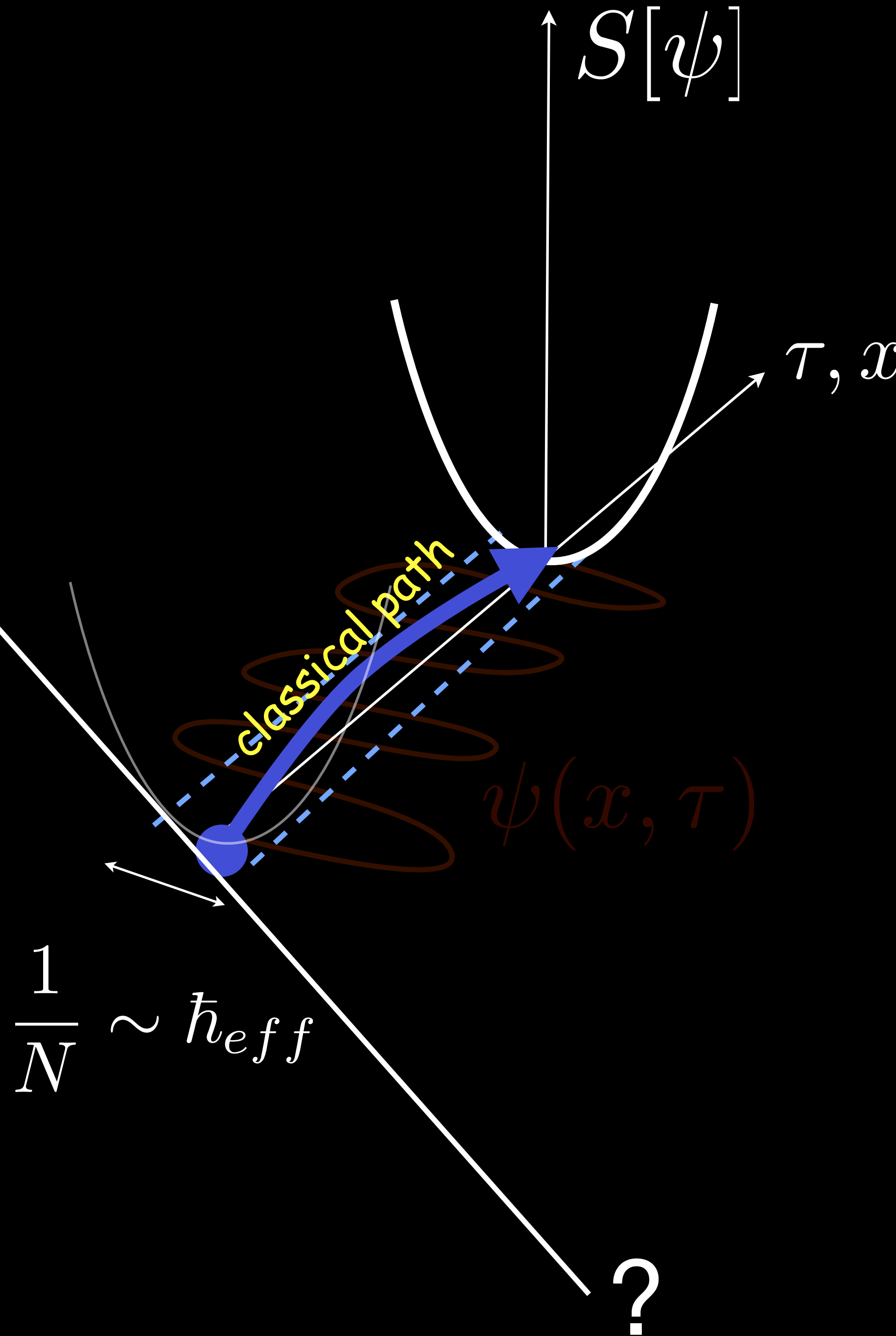

$$H = \sum_{\mathbf{k}\alpha} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha} + \frac{J_K}{N} \sum_j c^\dagger_{j\alpha} c_{j\beta} S_{\beta\alpha}(j) + \frac{J_H}{2N} \sum_{(i,j)} S_{\alpha\beta}(i) S_{\beta\alpha}(j)$$

SP(N) Large N Approach.

$$H = H_c + H_K + H_{RKKY}$$

$$\begin{aligned} H_K &= \frac{J_K}{N}\sum_j c^\dagger{}_{j\alpha} c_{j\beta} S_{\beta\alpha}(j) \to -\frac{J_K}{N}\sum_{i,j}\Big((c^\dagger{}_{j\alpha} f_{j\alpha})(f^\dagger{}_{j\beta} c_{j\beta}) + \tilde{\alpha}\tilde{\beta}(c^\dagger{}_{j\alpha} f^\dagger{}_{j-\alpha})(f_{j-\beta} c_{j\beta})\Big) \\ H_M &= \frac{J_H}{2N}\sum_{(i,j)} S_{\alpha\beta}(j) S_{\beta\alpha}(j) \to -\frac{J_H}{N}\sum_j\Big[(f^\dagger{}_{i\alpha} f_{j\alpha})(f^\dagger{}_{j\beta} f_{i\beta}) + \tilde{\alpha}\tilde{\beta}(f^\dagger{}_{i\alpha} f^\dagger{}_{j-\alpha})(f_{j-\beta} f_{i\beta})\Big] \end{aligned}$$

Uniform solution:

$$H = \sum_{\mathbf{k},\alpha>0} (\tilde{c}^\dagger_{\mathbf{k}\alpha}, \tilde{f}^\dagger_{\mathbf{k}\alpha}) \begin{pmatrix} \epsilon_{\mathbf{k}}\tau_3 & V\tau_3 \\ V\tau_3 & \vec{w}\cdot\vec{\tau} + \Delta_{H\mathbf{k}}\tau_1 \end{pmatrix} \begin{pmatrix} \tilde{c}_{\mathbf{k}\alpha} \\ \tilde{f}_{\mathbf{k}\alpha} \end{pmatrix} + \mathcal{N}_s N \left(\frac{|V|^2}{J_K} + 2\frac{\Delta_H^2}{J_H} \right)$$

SP(N) Large N Approach.

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$$
\begin{aligned}
H_K &= \frac{J_K}{N}\sum_{j} c^{\dagger}{}_{j\alpha} c_{j\beta} S_{\beta\alpha}(j) \rightarrow -\frac{J_K}{N}\sum_{i,j}\Big((c^{\dagger}{}_{j\alpha} f_{j\alpha})(f^{\dagger}{}_{j\beta} c_{j\beta}) + \tilde{\alpha}\tilde{\beta}(c^{\dagger}{}_{j\alpha} f^{\dagger}{}_{j-\alpha})(f_{j-\beta} c_{j\beta})\Big) \\
H_M &= \frac{J_H}{2N}\sum_{(i,j)} S_{\alpha\beta}(j) S_{\beta\alpha}(j) \rightarrow -\frac{J_H}{N}\sum_{j}\Big[(f^{\dagger}{}_{i\alpha} f_{j\alpha})(f^{\dagger}{}_{j\beta} f_{i\beta}) + \tilde{\alpha}\tilde{\beta}(f^{\dagger}{}_{i\alpha} f^{\dagger}{}_{j-\alpha})(f_{j-\beta} f_{i\beta})\Big]
\end{aligned}
$$

$$
\begin{aligned}
H_K &\rightarrow \sum_{j}\Big[c^{\dagger}{}_{j\alpha}\Big(V_j f_{j\alpha} + \tilde{\alpha}\Delta^{K}_{j} f^{\dagger}{}_{j-\alpha}\Big) + \text{H.c}\Big] + N\left(\frac{|V_j|^2 + |\Delta^{K}_{j}|^2}{J_K}\right) \\
H_H &\rightarrow \sum_{(i,j)}\Big[t_{ij} f^{\dagger}{}_{i\alpha} f_{j\alpha} + \Delta_{ij}\tilde{\alpha} f^{\dagger}{}_{i\alpha} f^{\dagger}{}_{j-\alpha} + \text{H.c}\Big] + N\left[\frac{|t_{ij}|^2 + |\Delta_{ij}|^2}{J_H}\right]
\end{aligned}
$$

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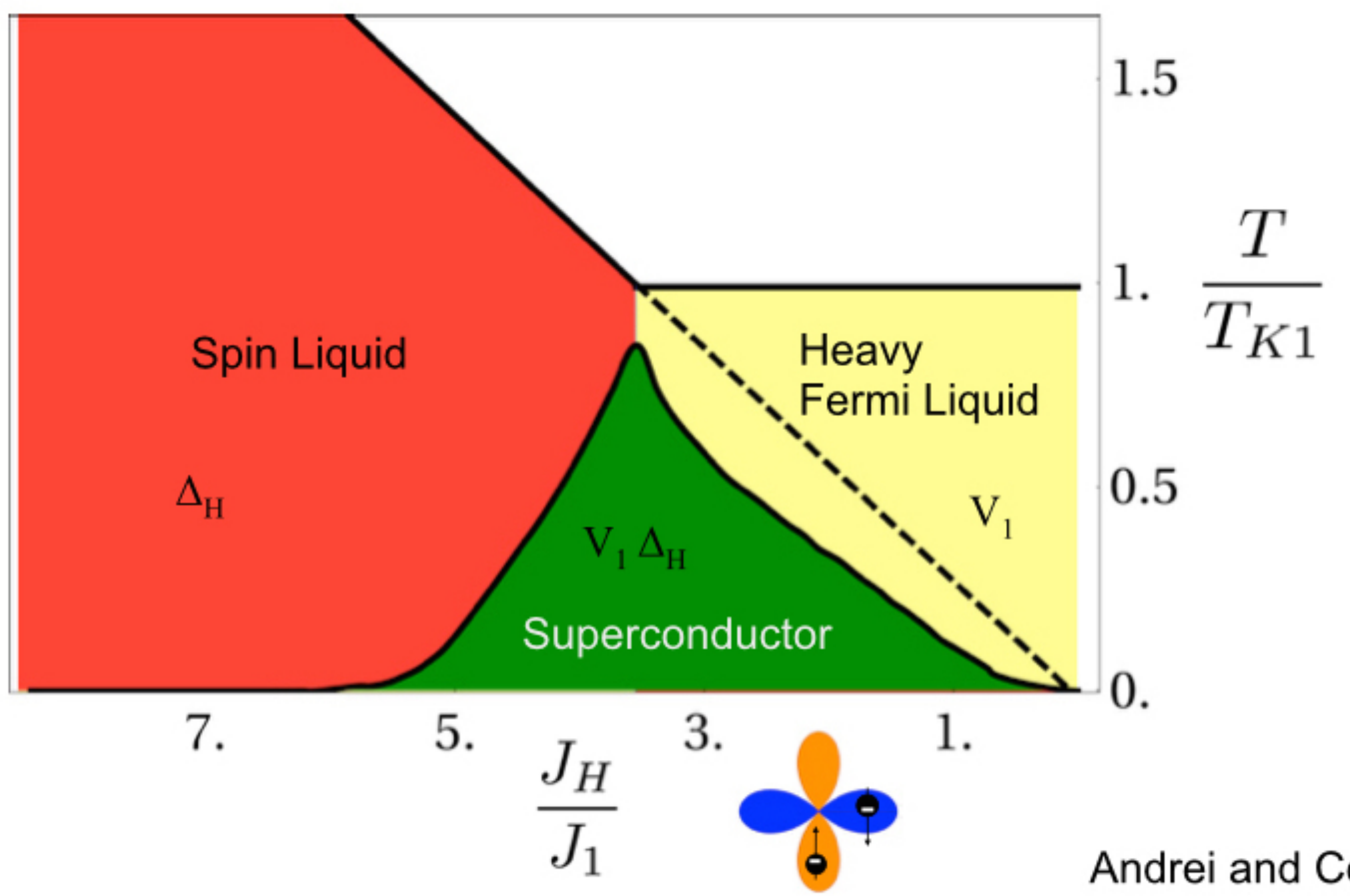

Andrei and Coleman 1989

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