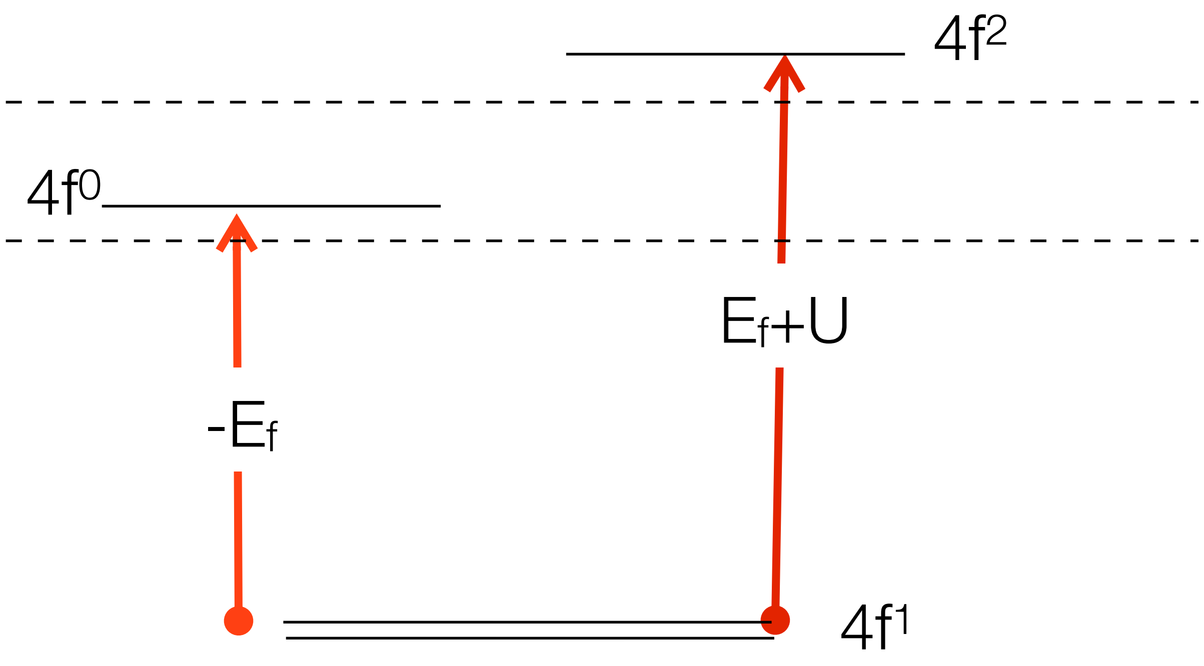
Kondo effect

1. Schrieffer Wolff Transformation



Virtual Valence fluctuations in the singlet channel, induced by hybridization

$$e^-_\uparrow + f^1_\downarrow \leftrightarrow f^2 \leftrightarrow e^-_\downarrow + f^1_\uparrow \qquad \Delta E_I \sim U + E_f$$
$$h^+_\uparrow + f^1_\downarrow \leftrightarrow f^0 \leftrightarrow h^+_\downarrow + f^1_\uparrow \qquad \Delta E_{II} \sim -E_f$$

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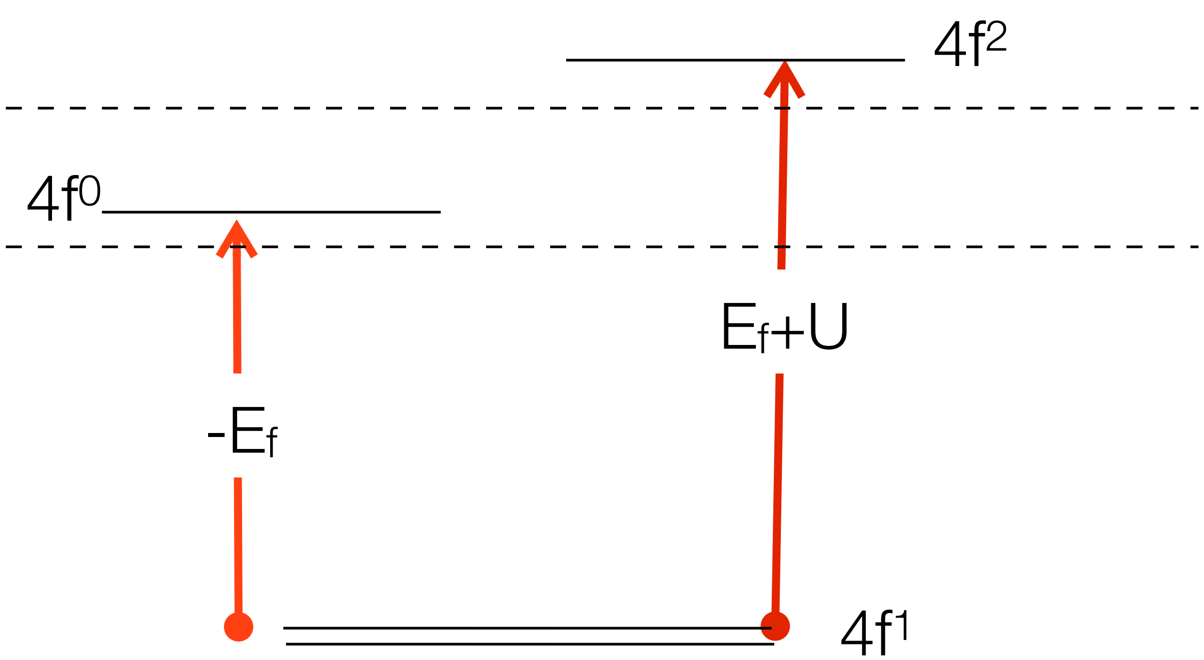

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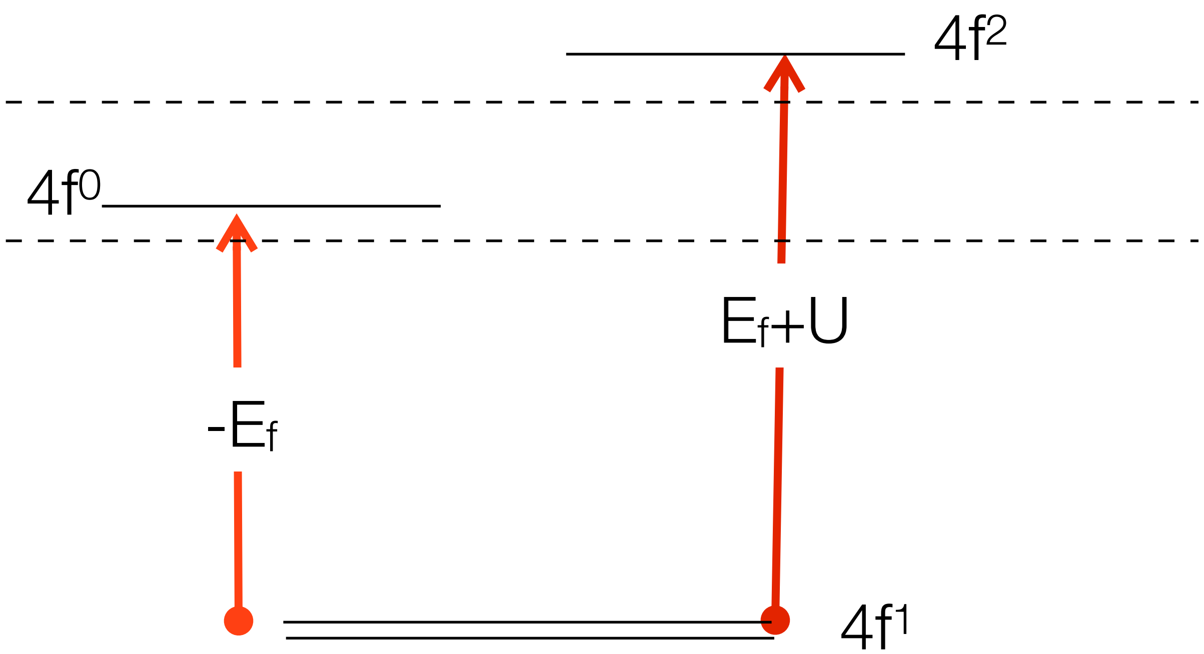

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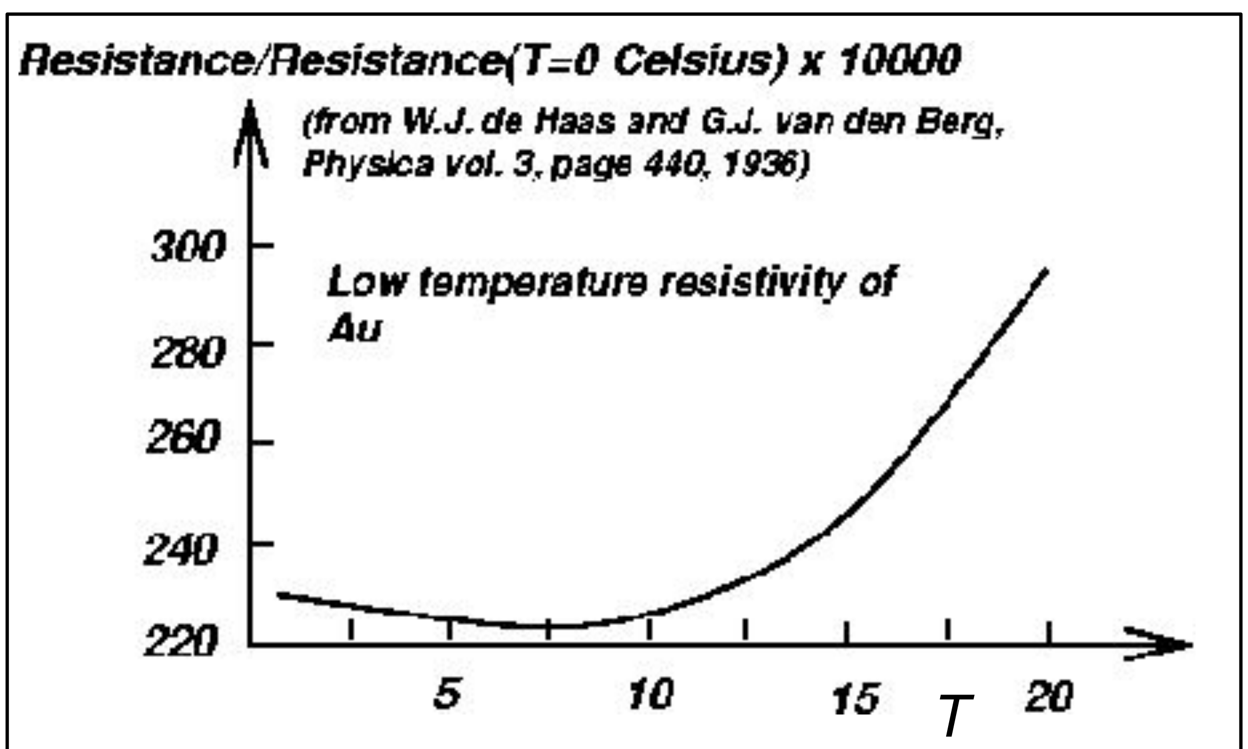
$$H_K = -2JP_{S=0} = -2J \left[\frac{1}{4} - \frac{1}{2} \vec{\sigma}_c(0) \cdot \vec{S}_f \right] \longrightarrow J\vec{\sigma}_c(0) \cdot \vec{S}_f$$

Antiferromagnetic interaction

Kondo effect



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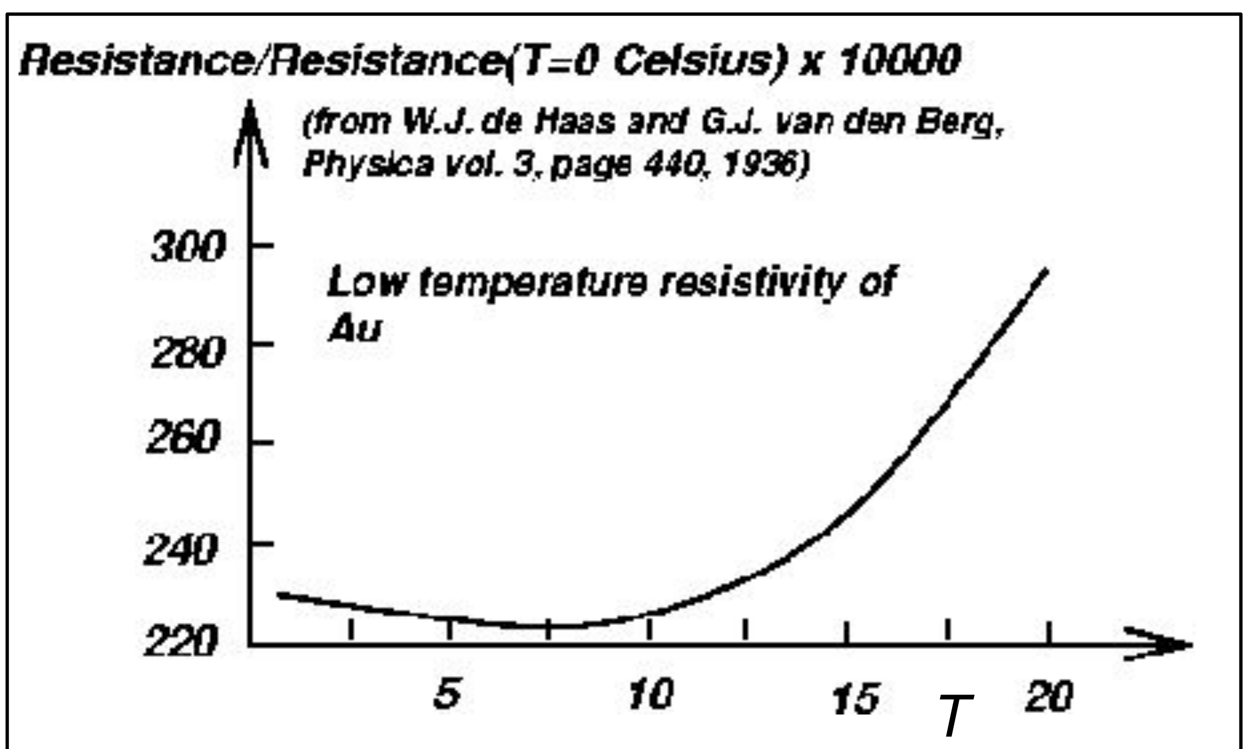
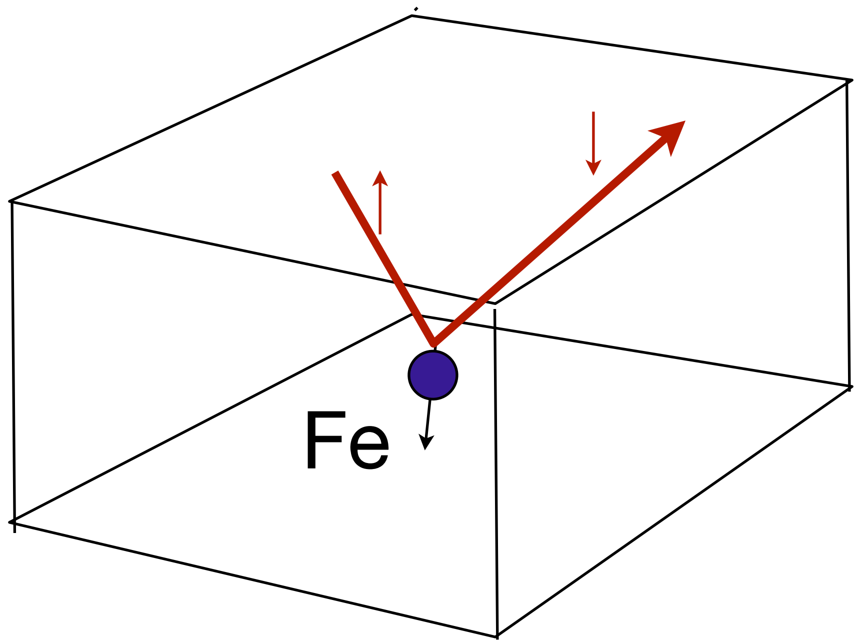


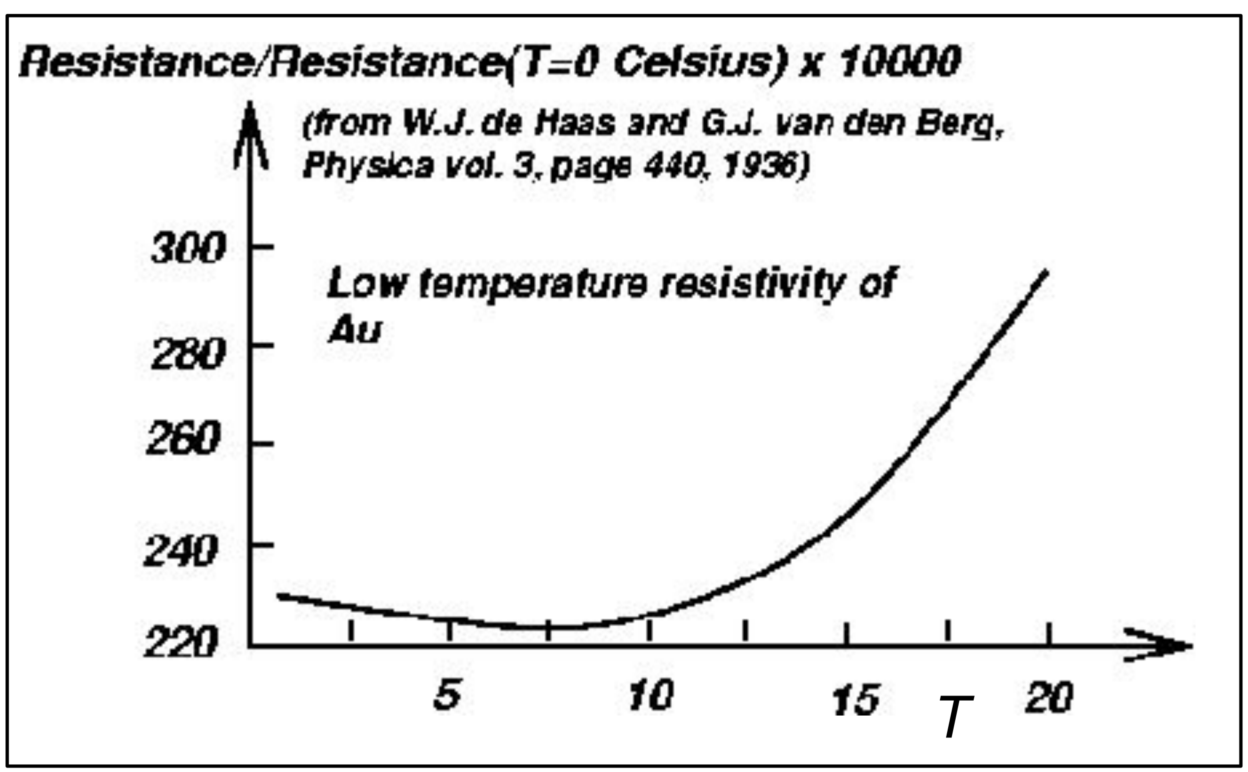
“A 75 year odyssey”

Kondo effect



$$H = \sum \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + J(\psi^\dagger \vec{\sigma}\psi) \cdot \vec{S}$$

Kondo (1962)



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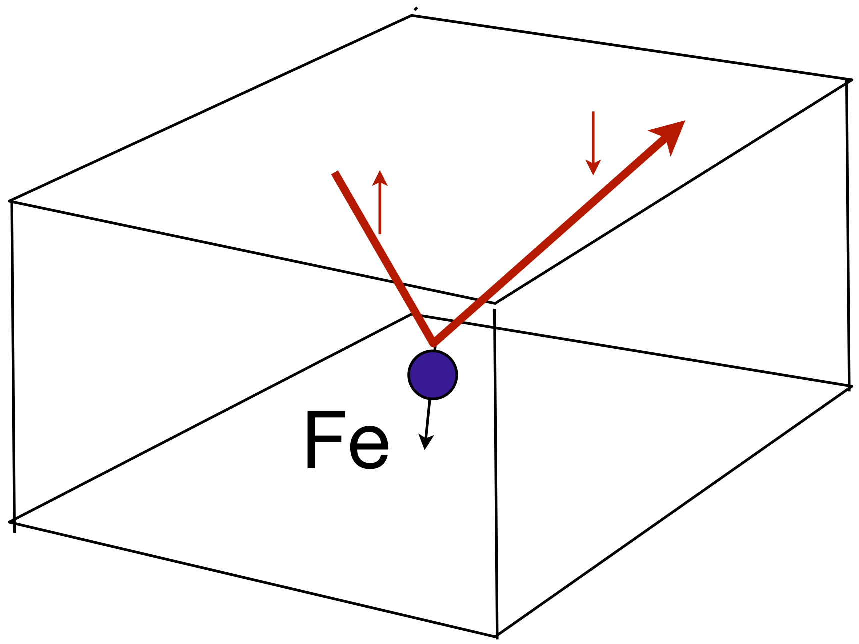
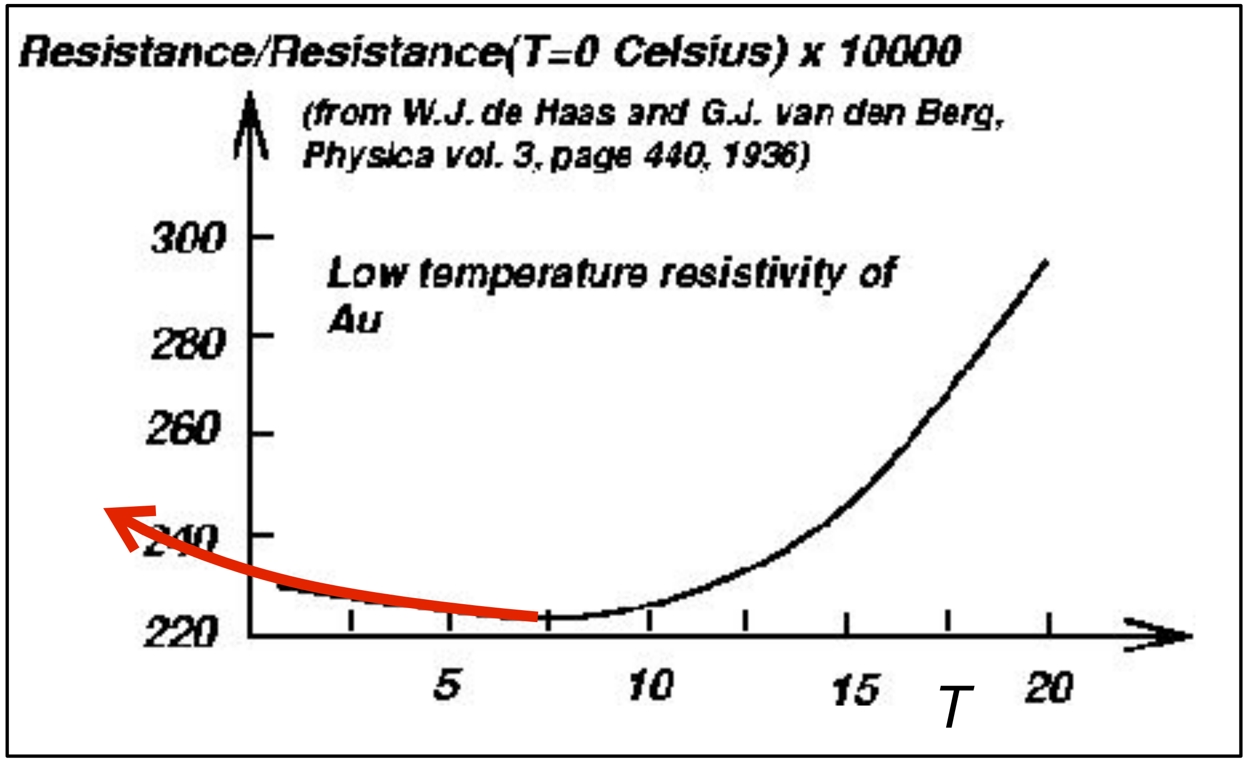


$$H = \sum \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + J(\psi^\dagger \vec{\sigma} \psi) \cdot \vec{S}$$

Kondo (1962)

$$J\rho \to J\rho + 2(J\rho)^2 \ln(D/T) + \ldots$$



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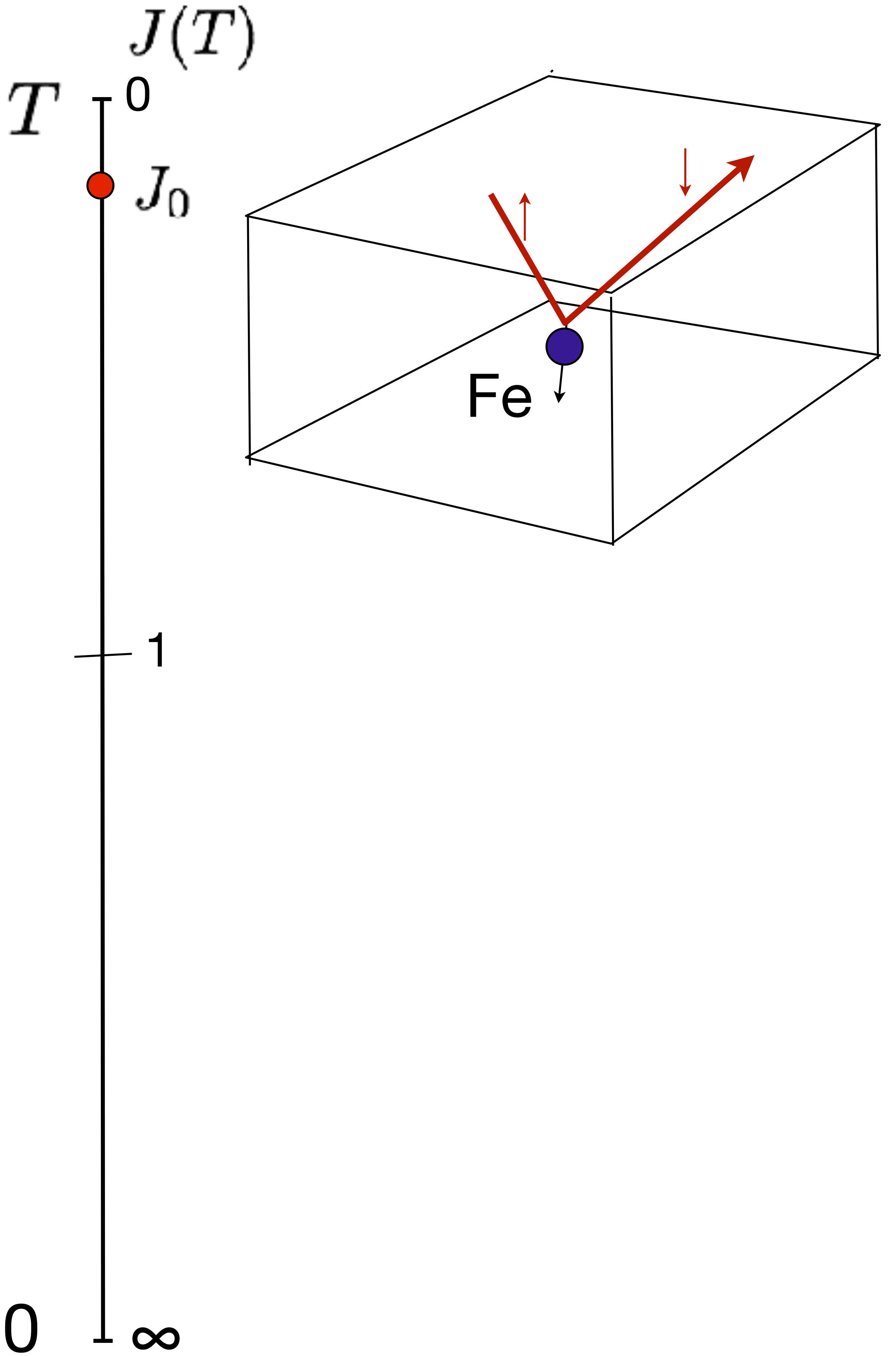

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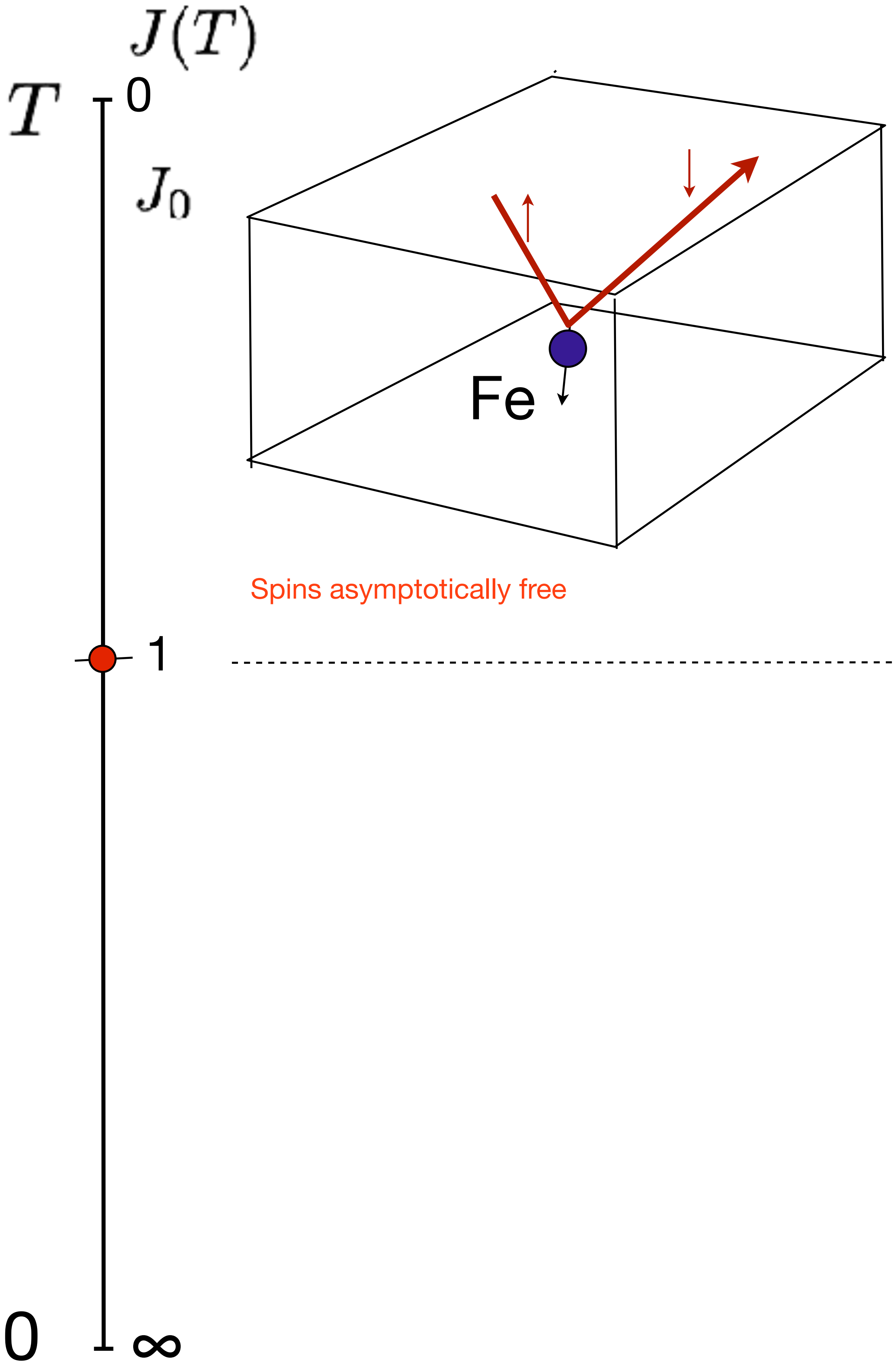

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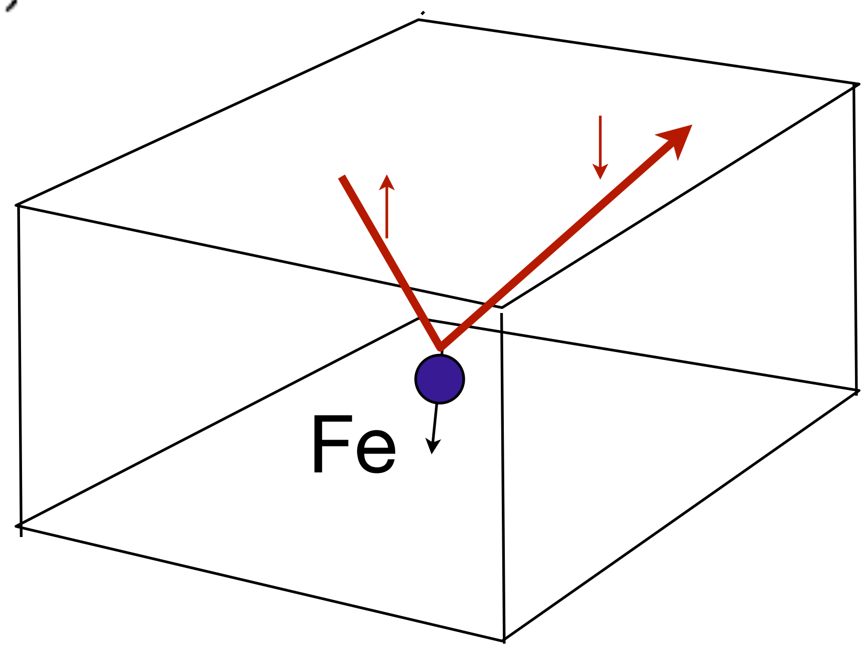
"Kondo Temperature"

Kondo effect

$J(T)$

T 0

J_0



Spins asymptotically free

1

Formation of Kondo Singlet

0 ∞

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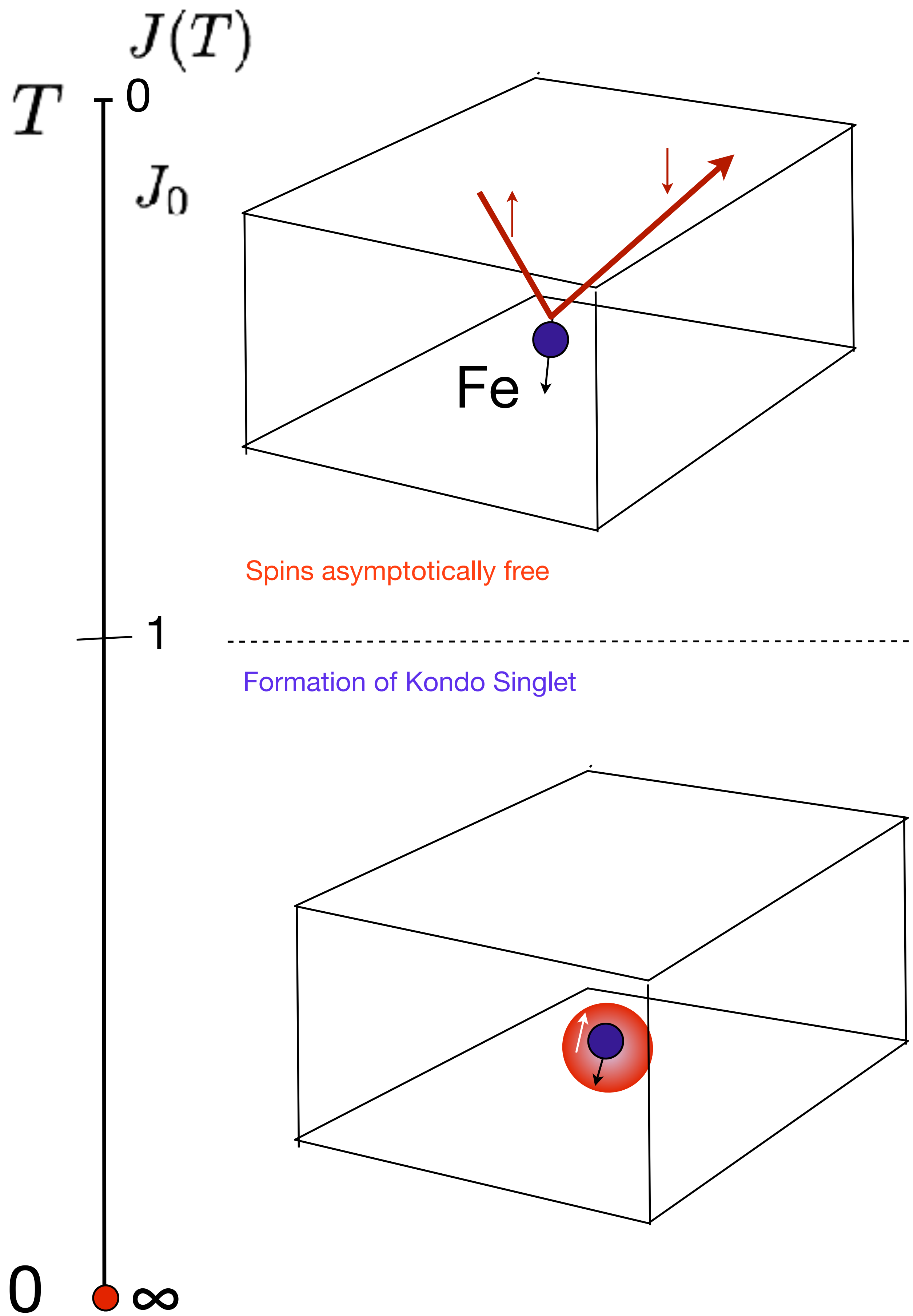

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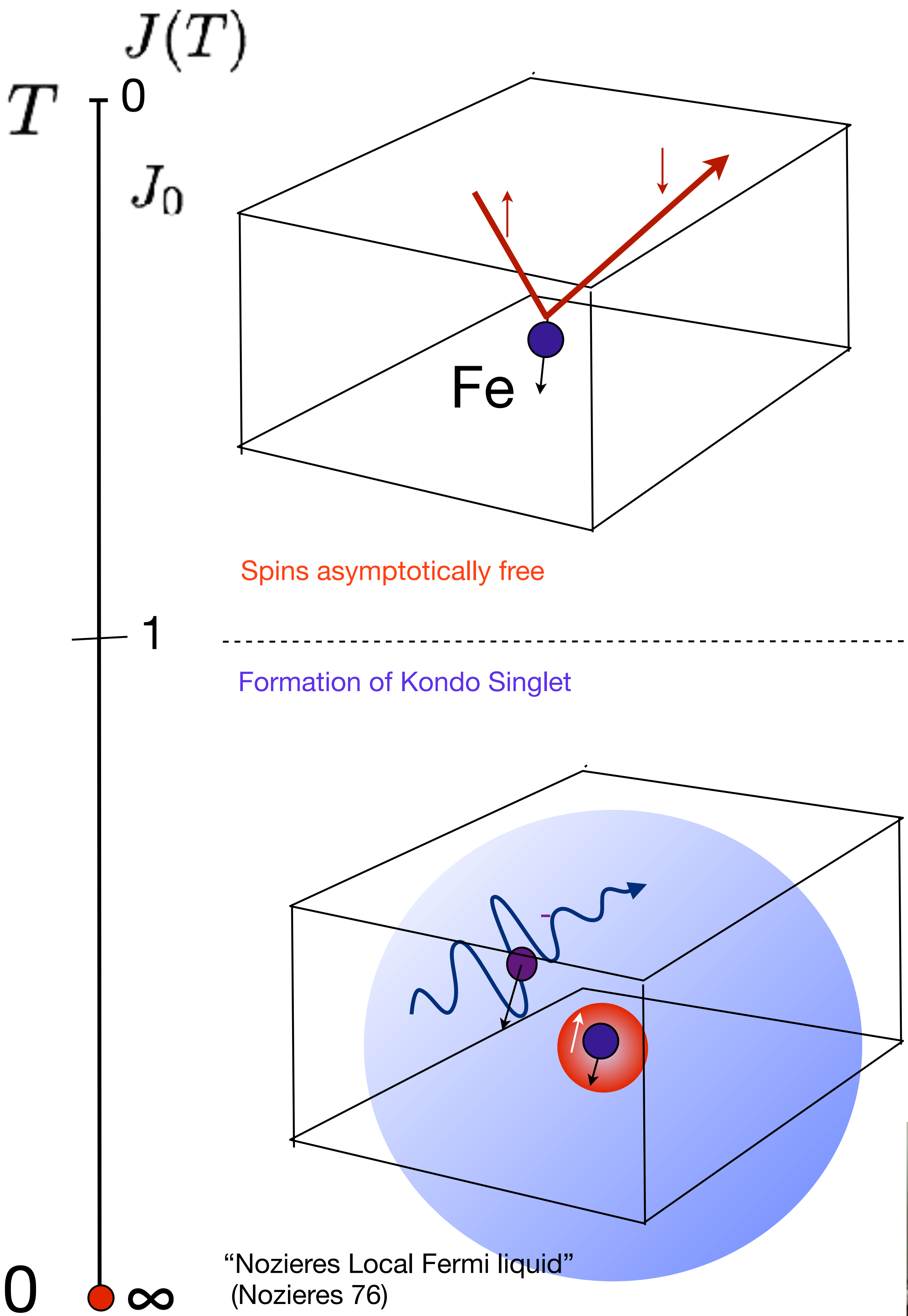

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"Kondo Temperature"

Kondo effect

T $J(T)$

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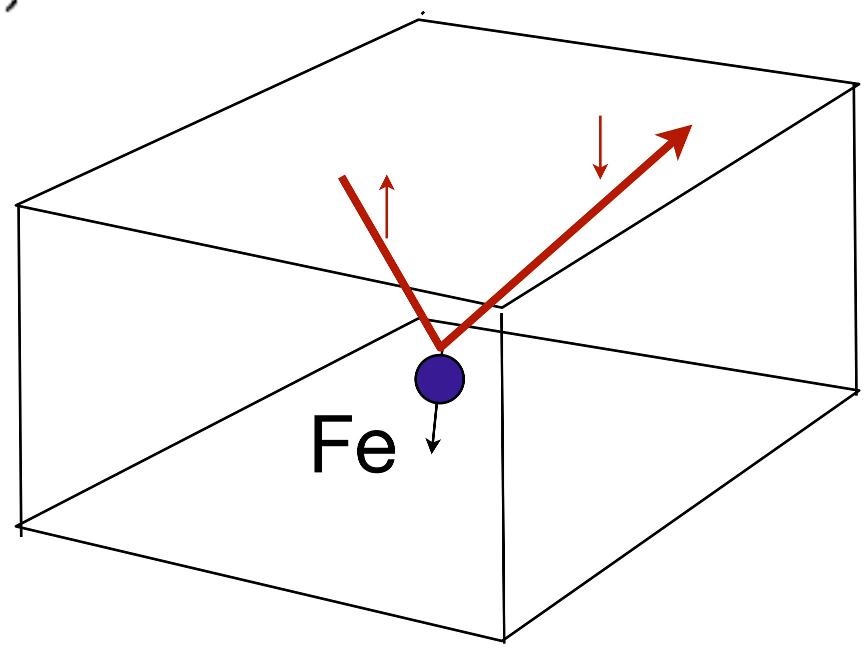

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Kondo (1962)

1

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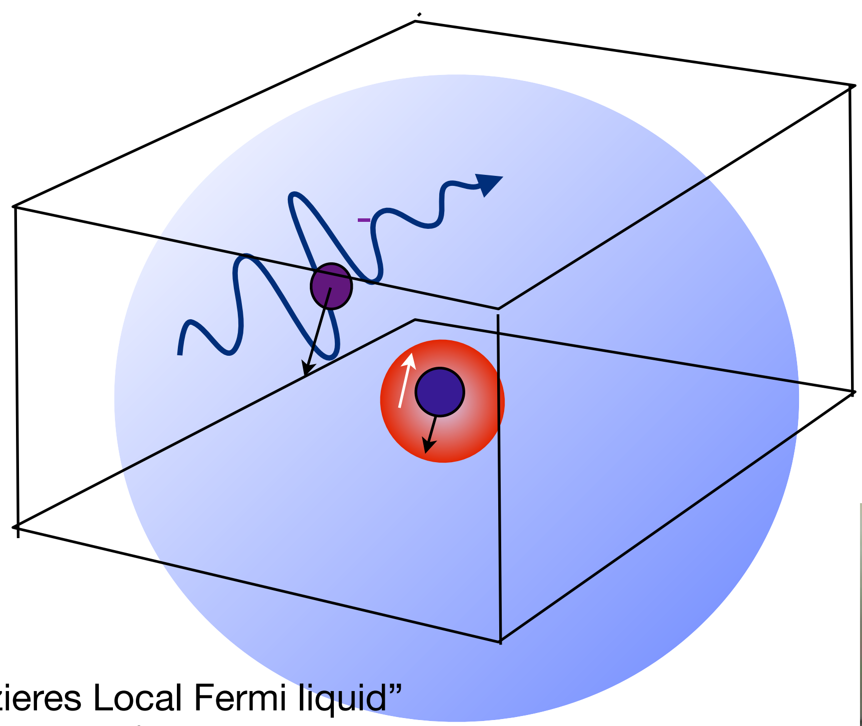
"Kondo Temperature"

Formation of Kondo Singlet

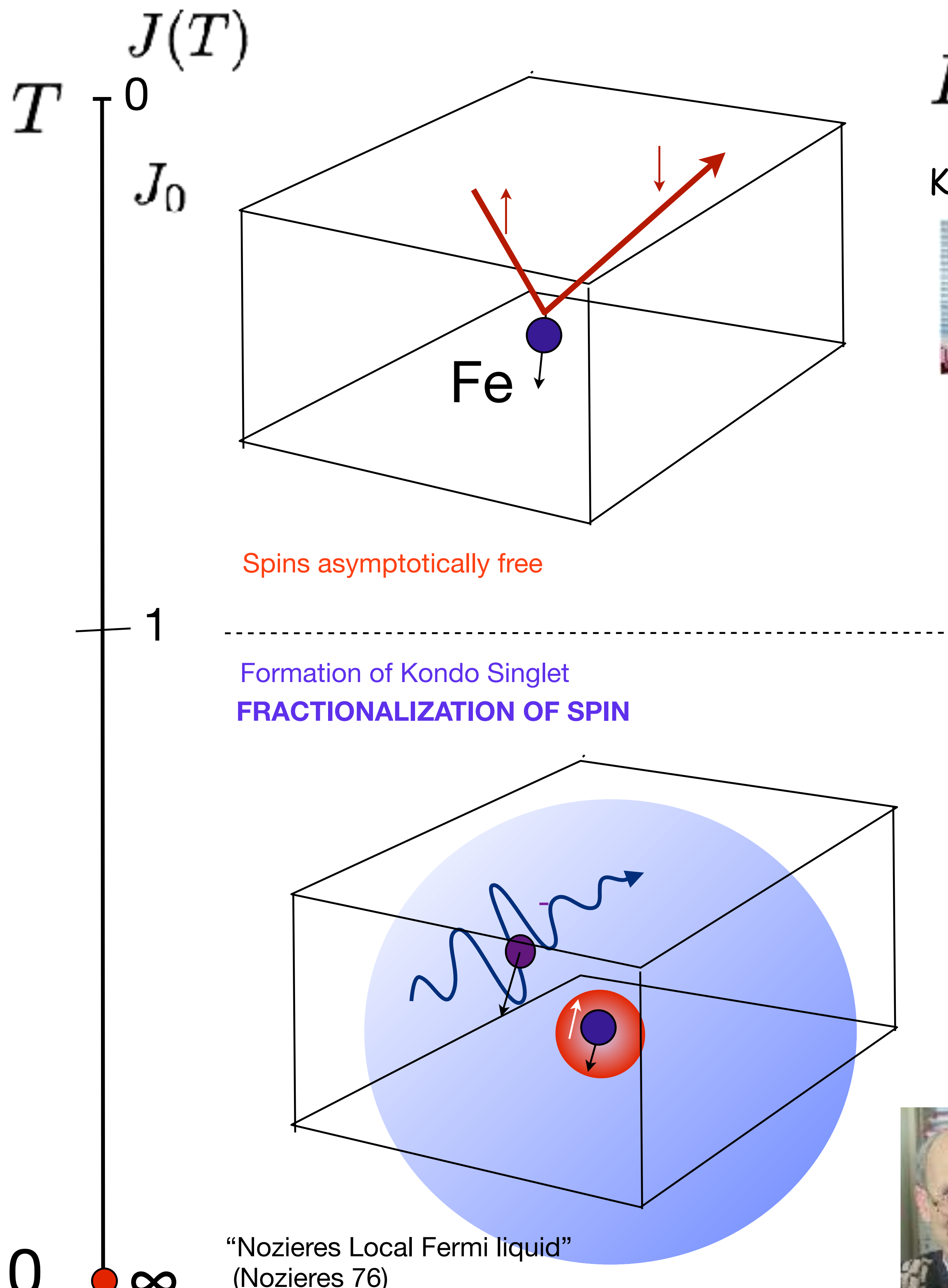
FRACTIONALIZATION OF SPIN



0 ∞

"Nozieres Local Fermi liquid"
(Nozieres 76)

Kondo effect



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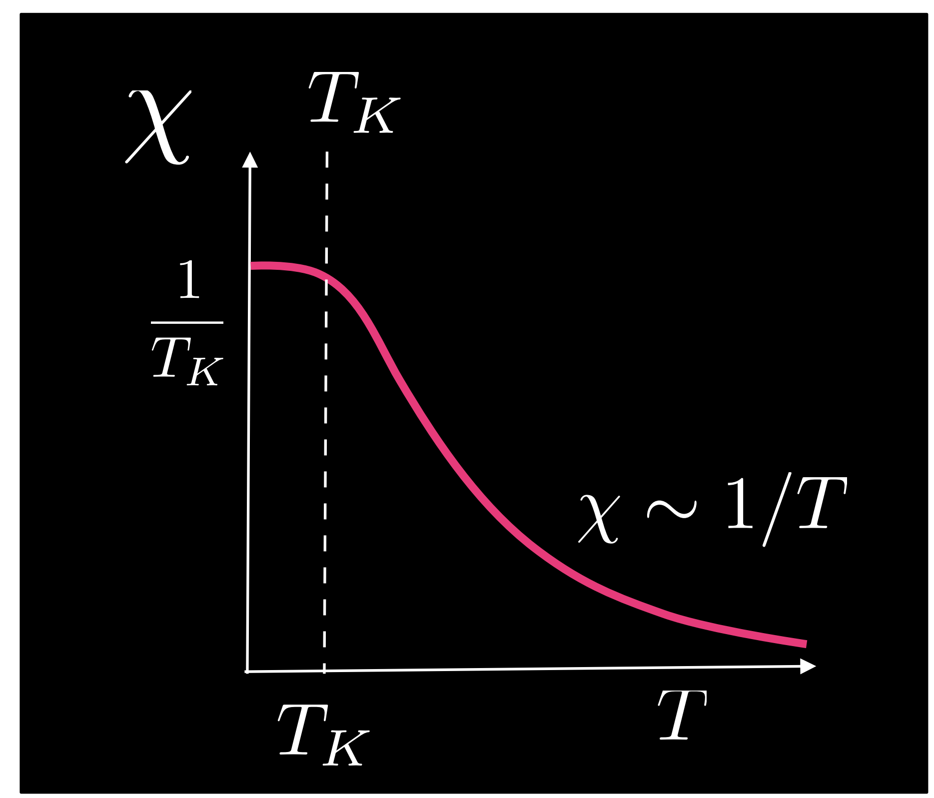
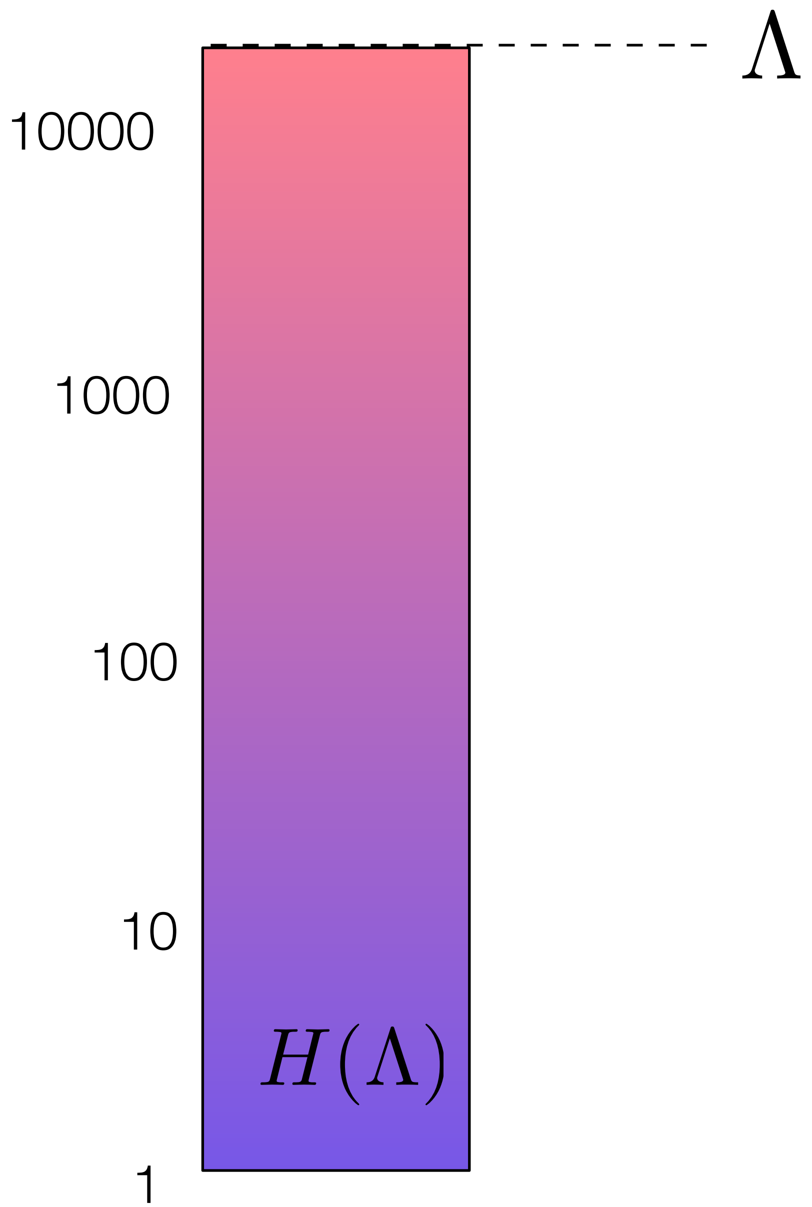


“Poor Man’s” Scaling

Anderson and Yuval 1969,
Anderson 1973



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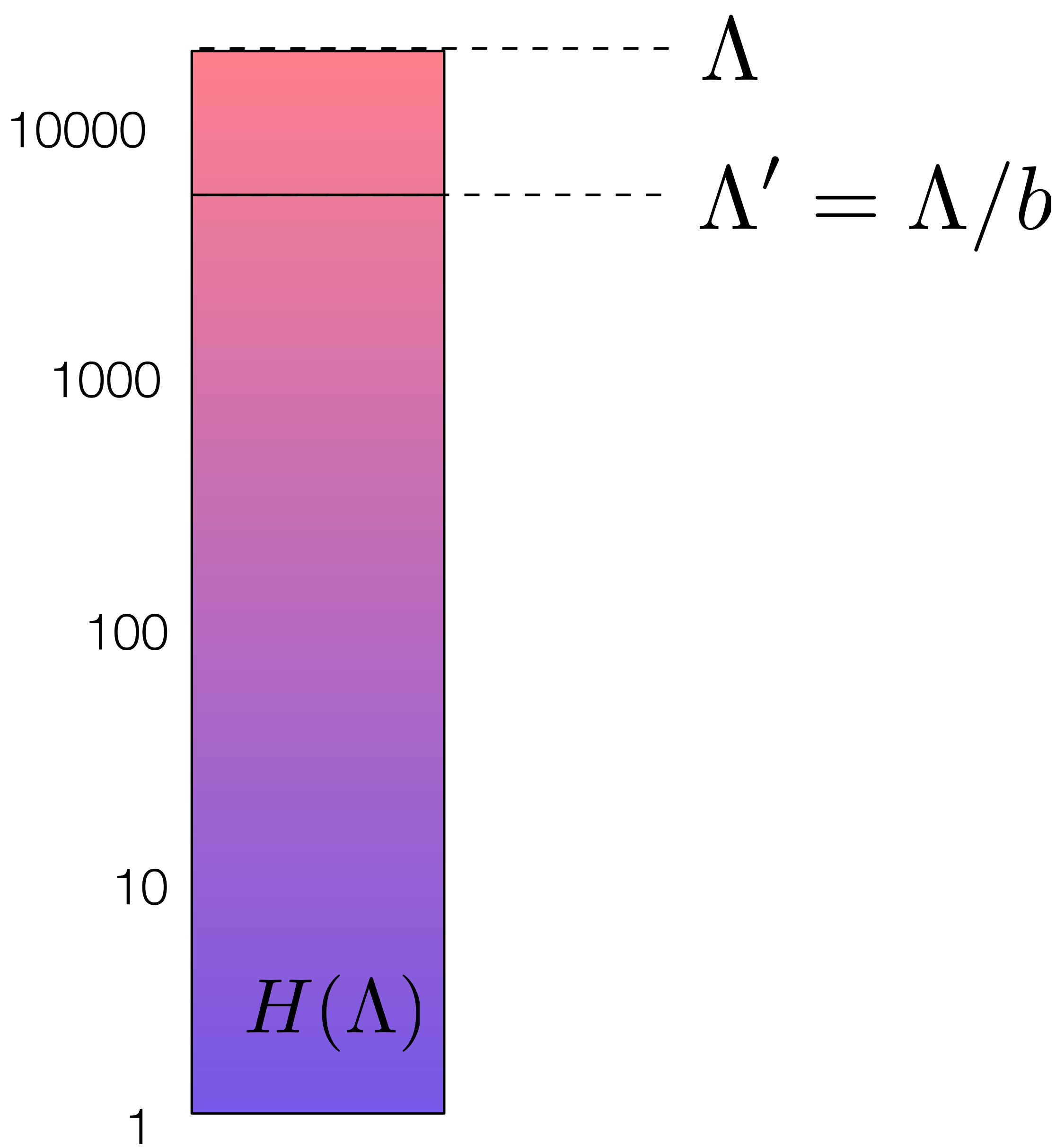

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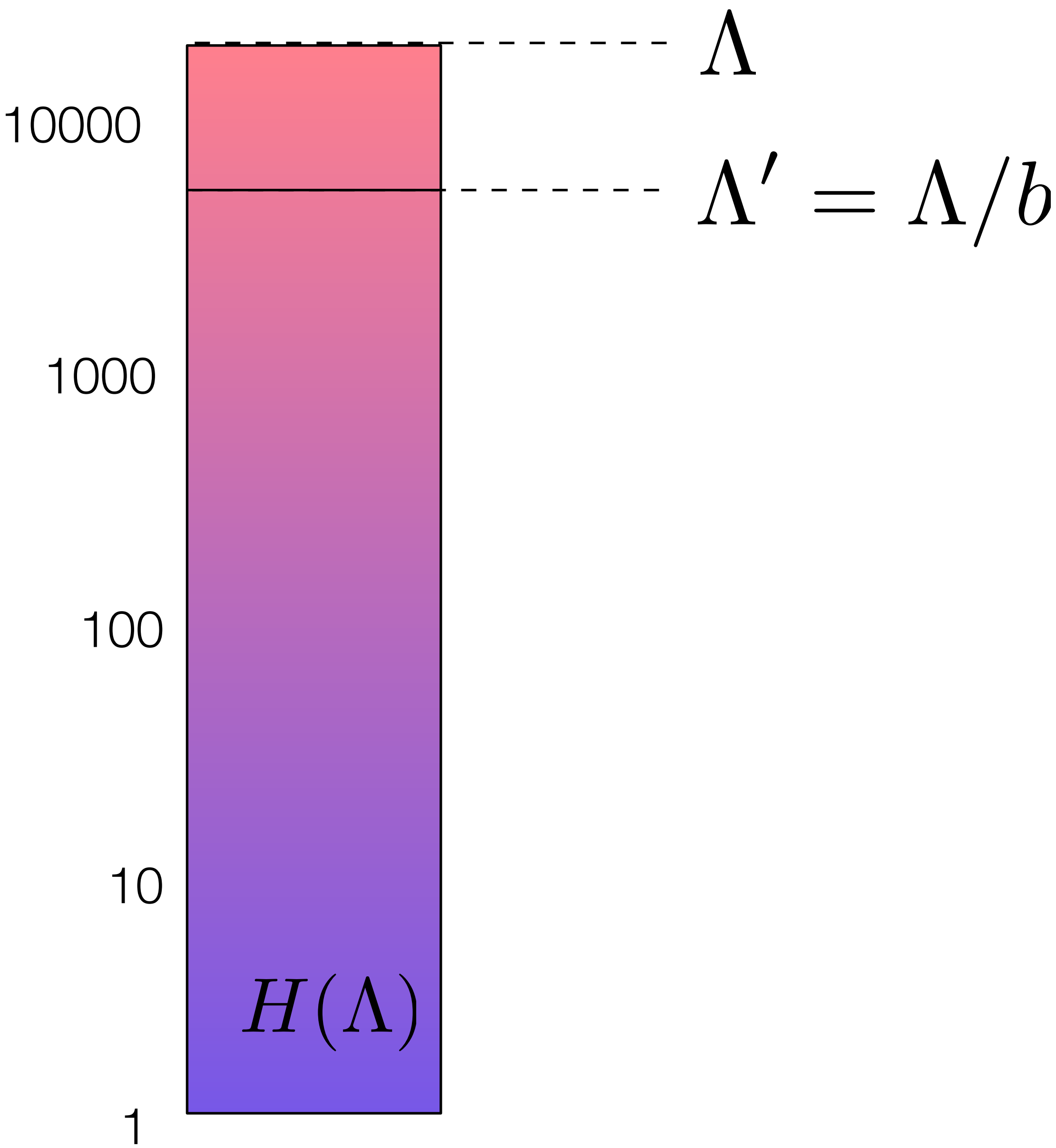

$$H(\Lambda) = \left[\begin{array}{c|c} H_L & V^\dagger \\ \hline V & H_H \end{array} \right],$$

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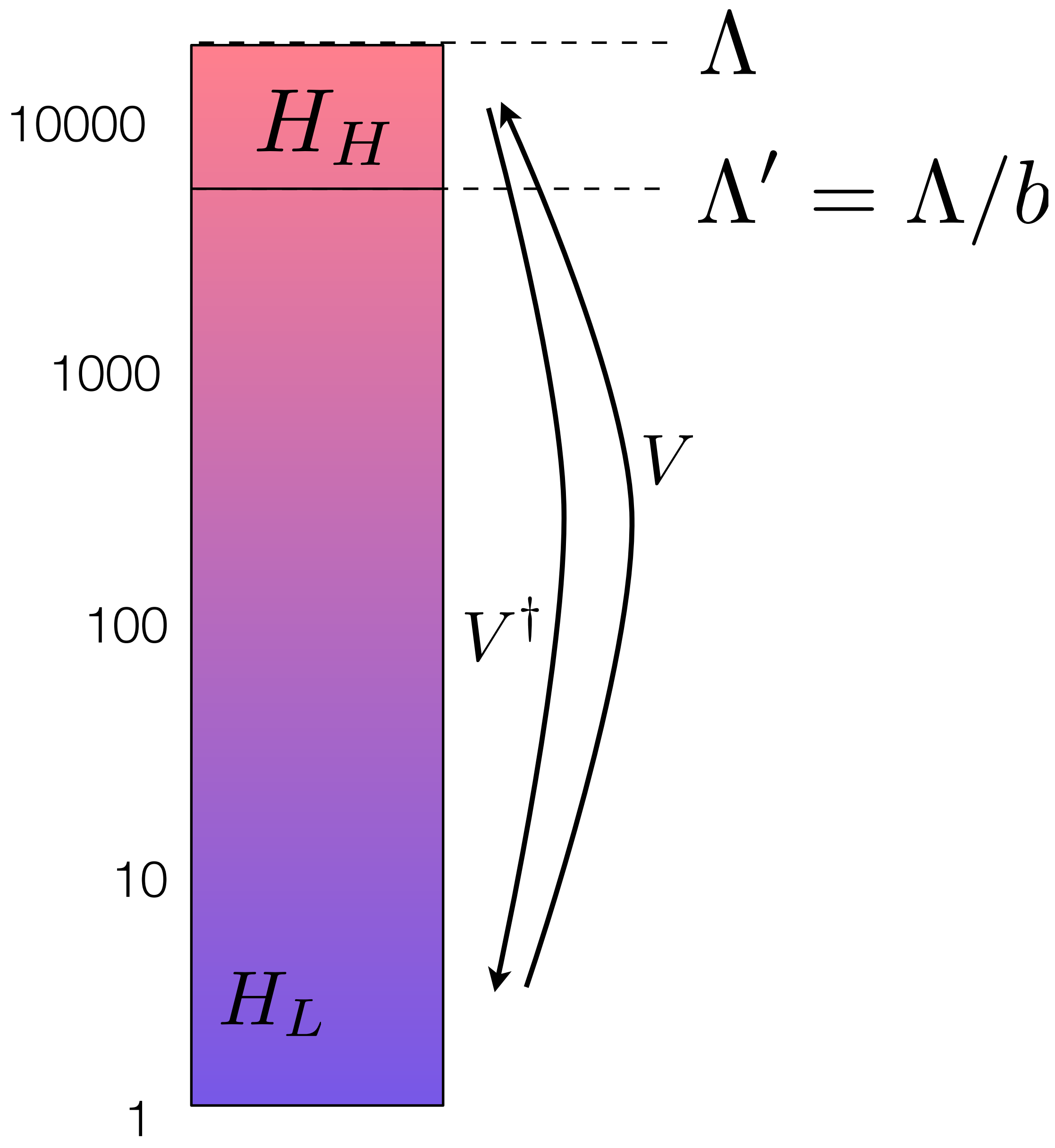

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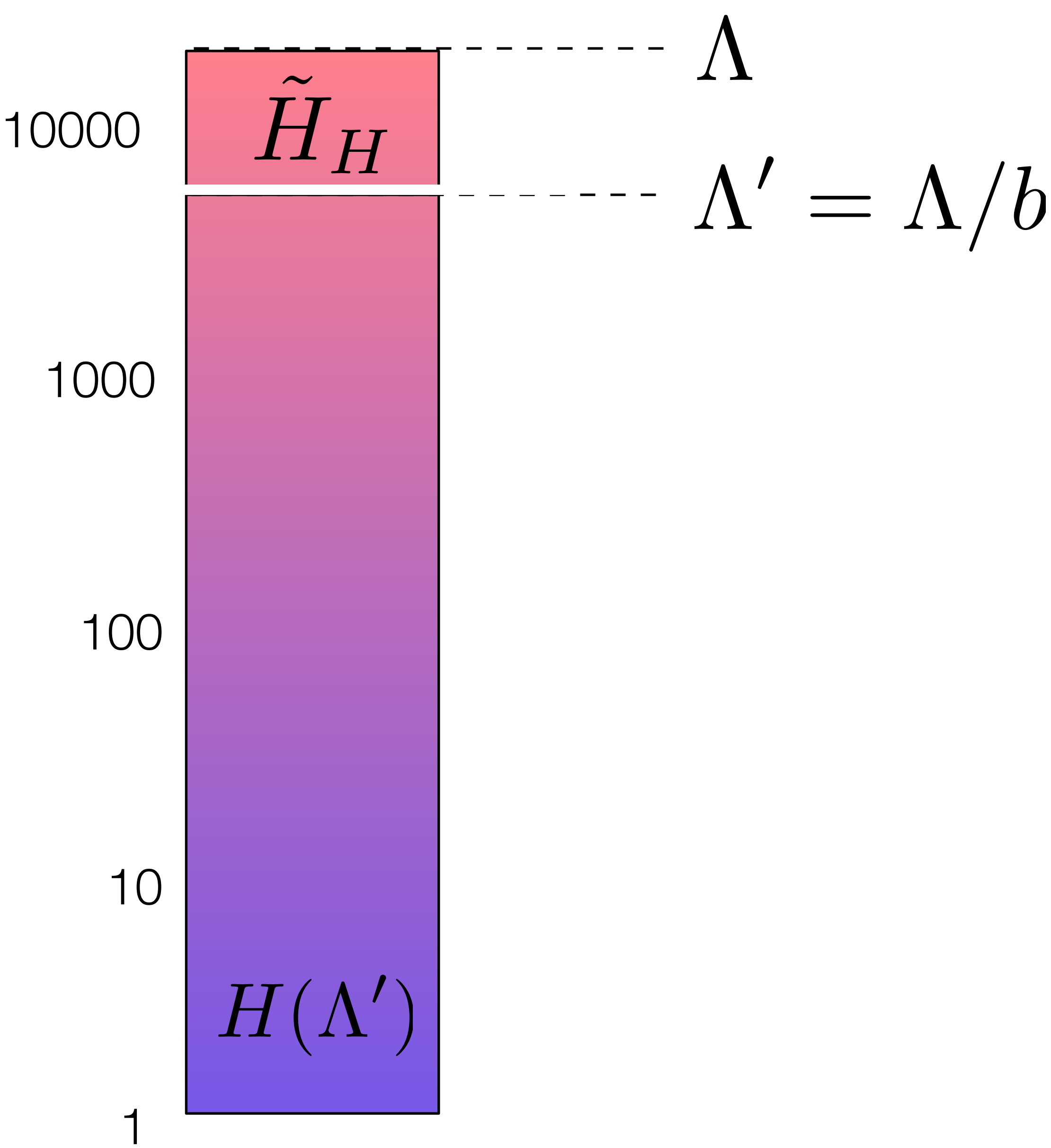

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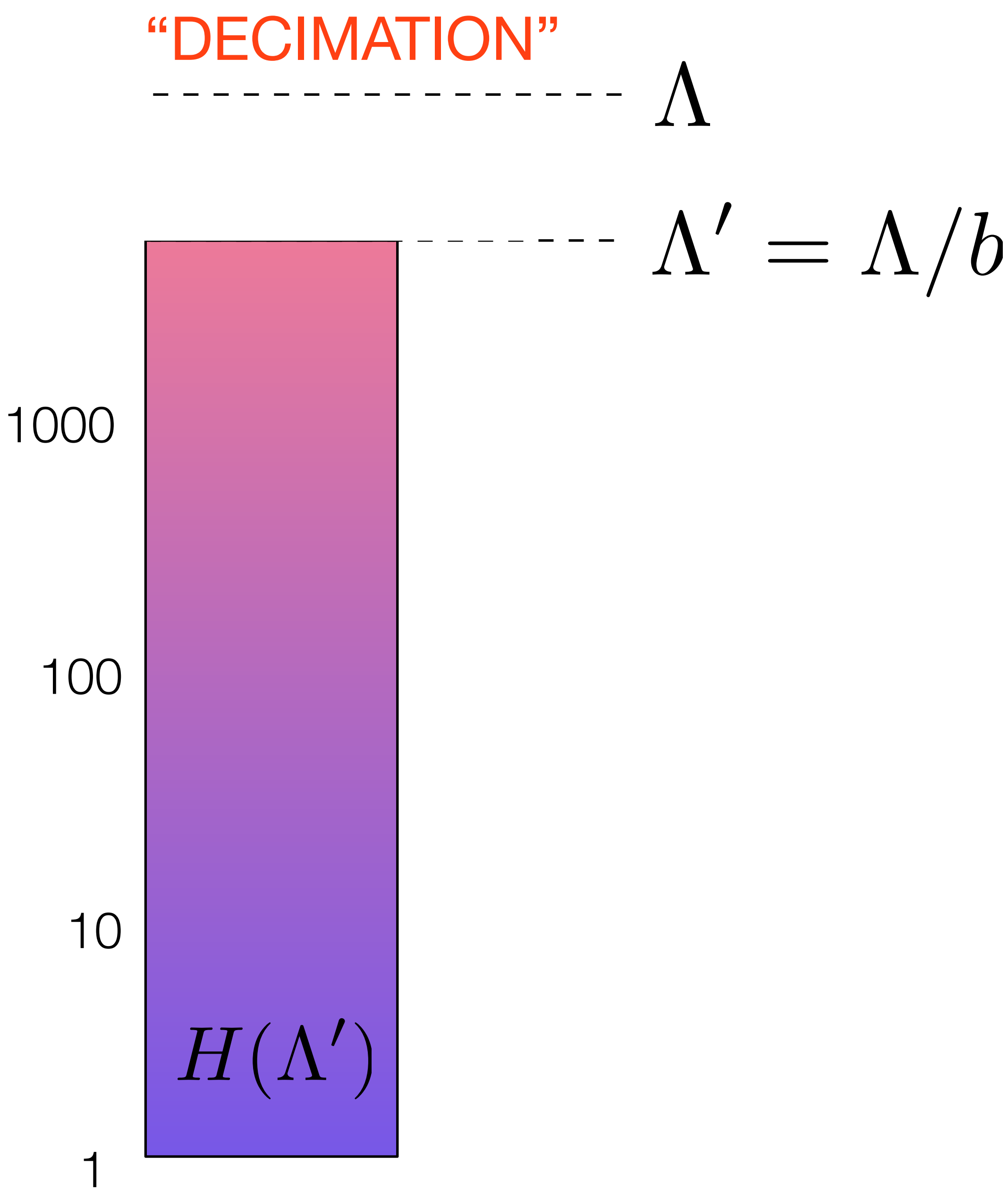

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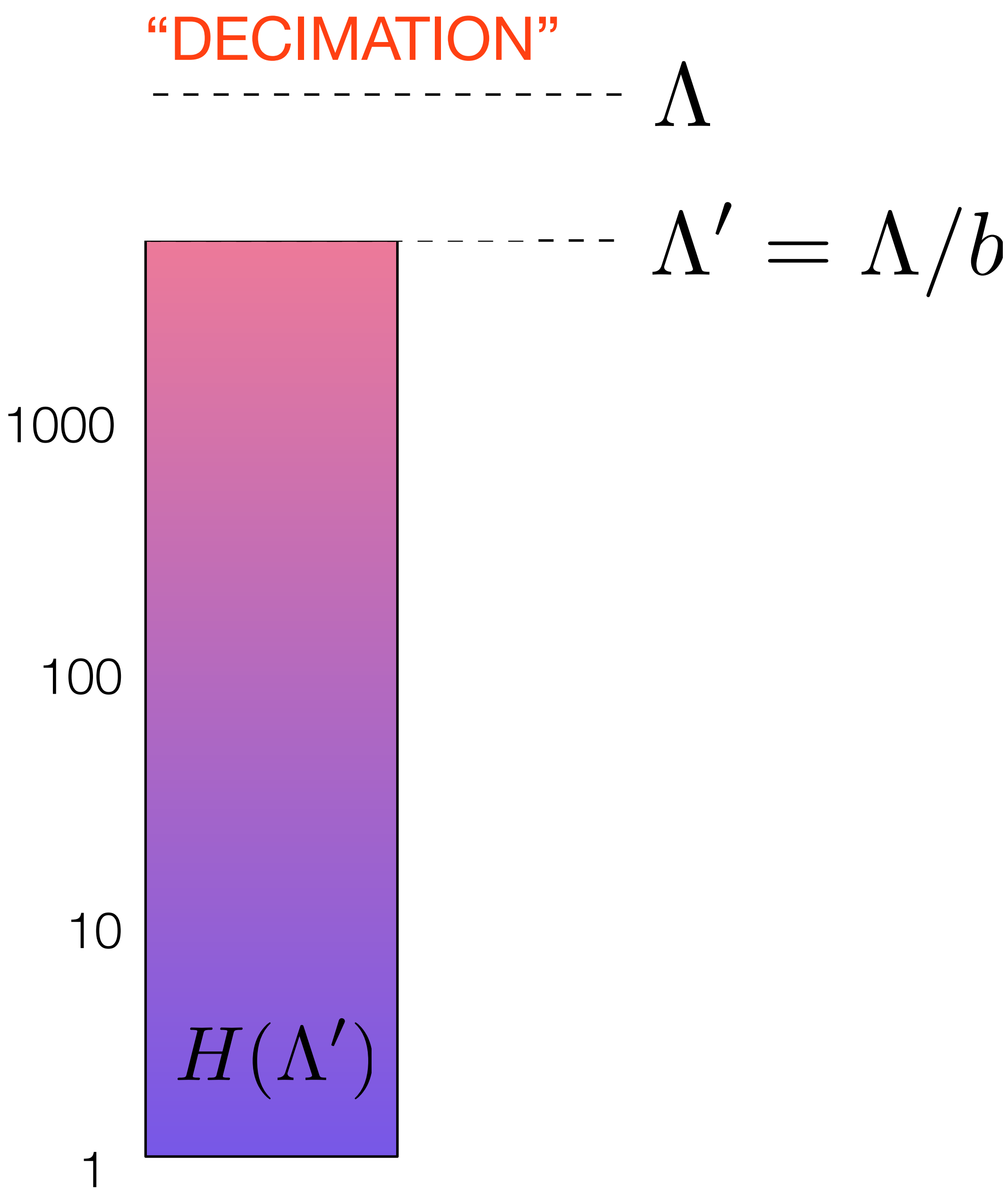

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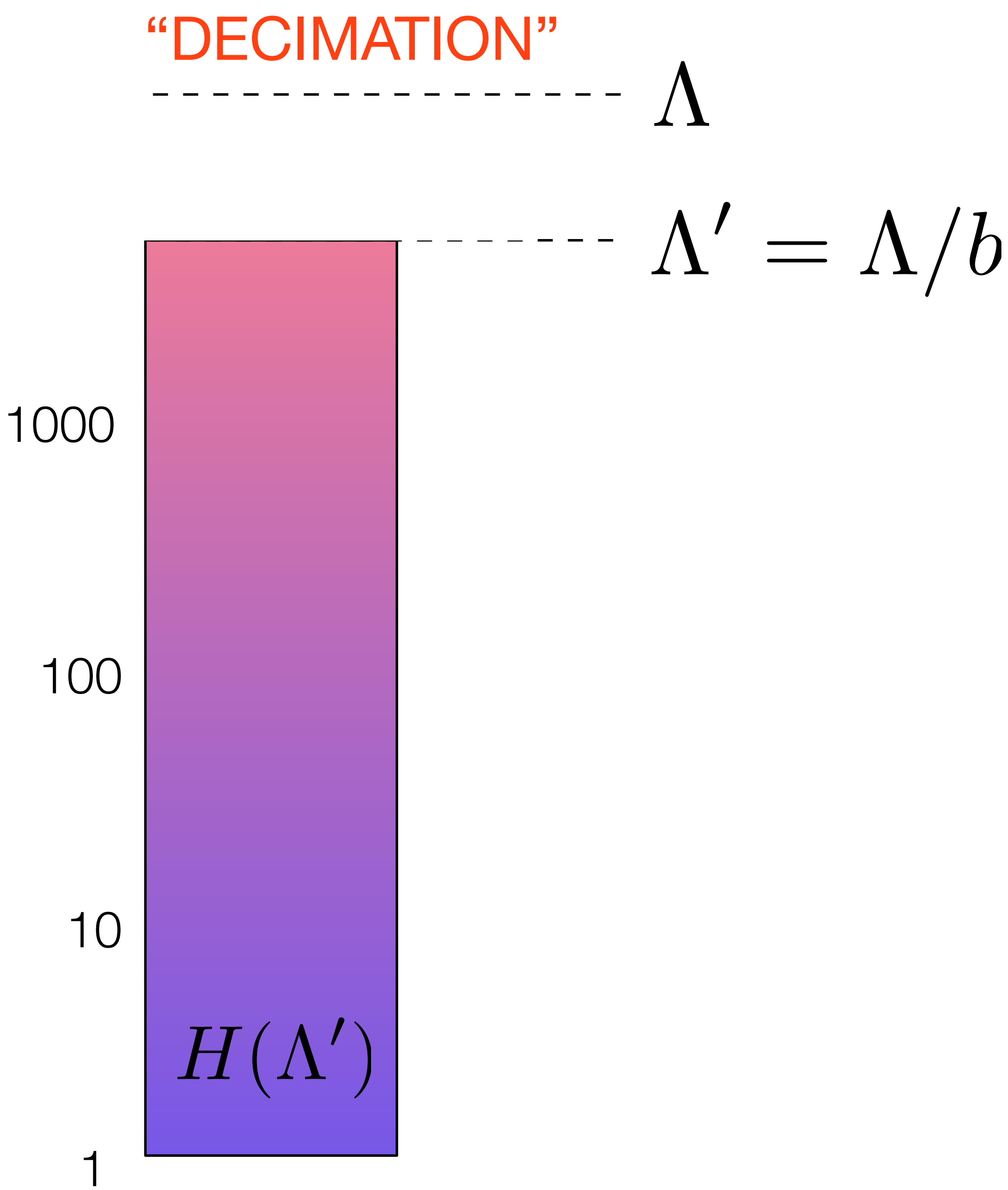

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$$\delta H = P_L V^\dagger \left(\frac{1}{E - \tilde{H}_H} \right) V P_L \Big|_{E \sim E_L}$$

Second-order ptbn theory

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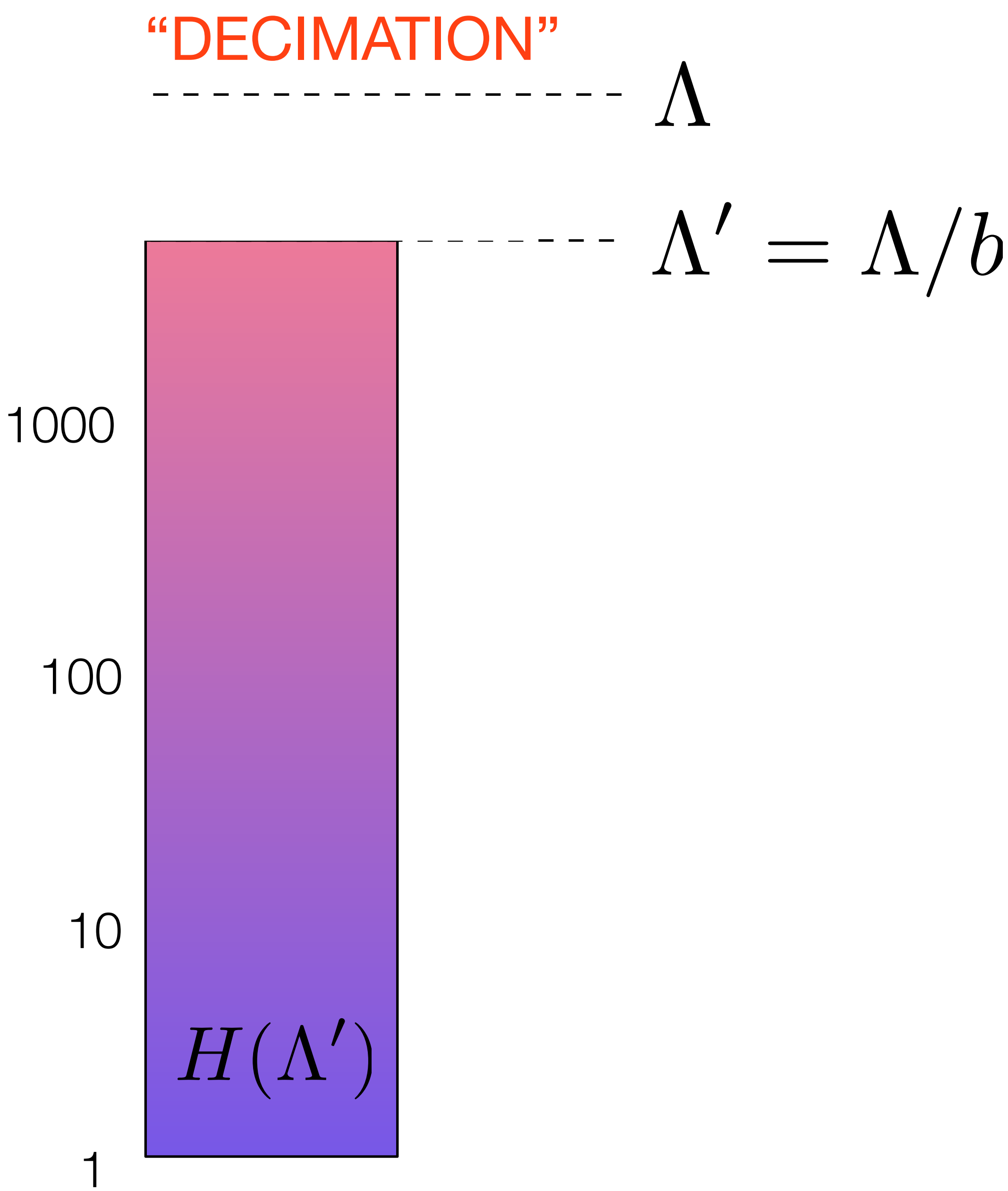

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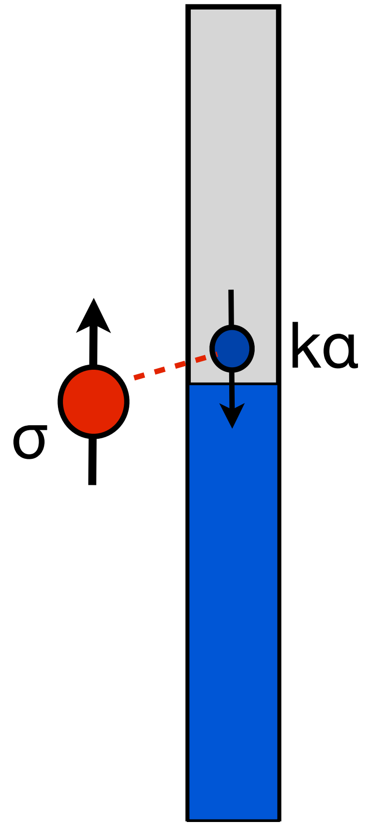

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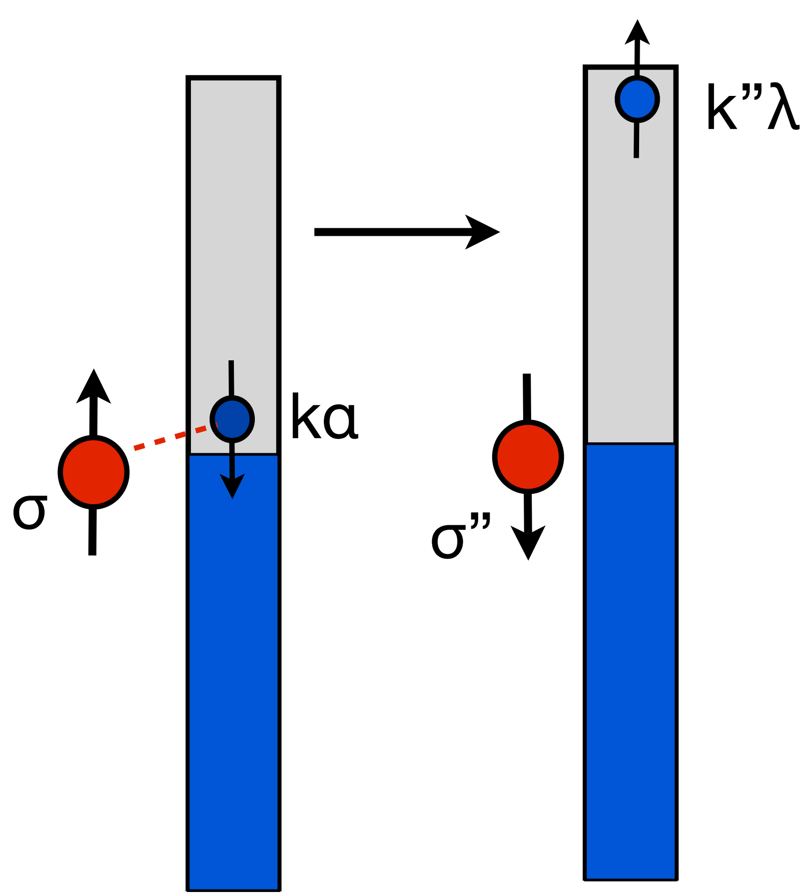

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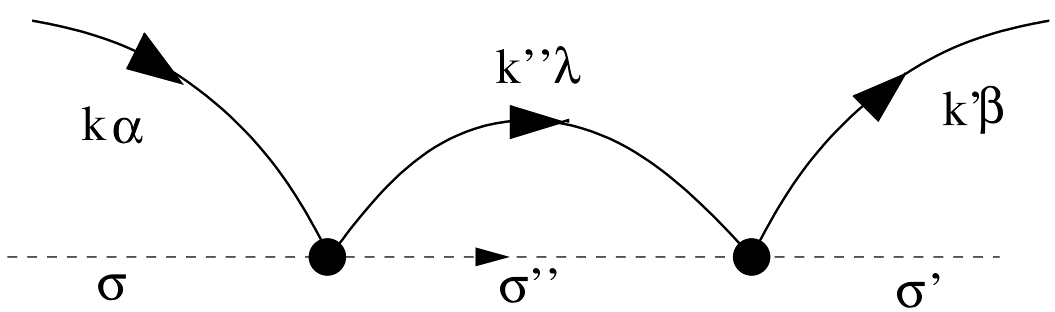

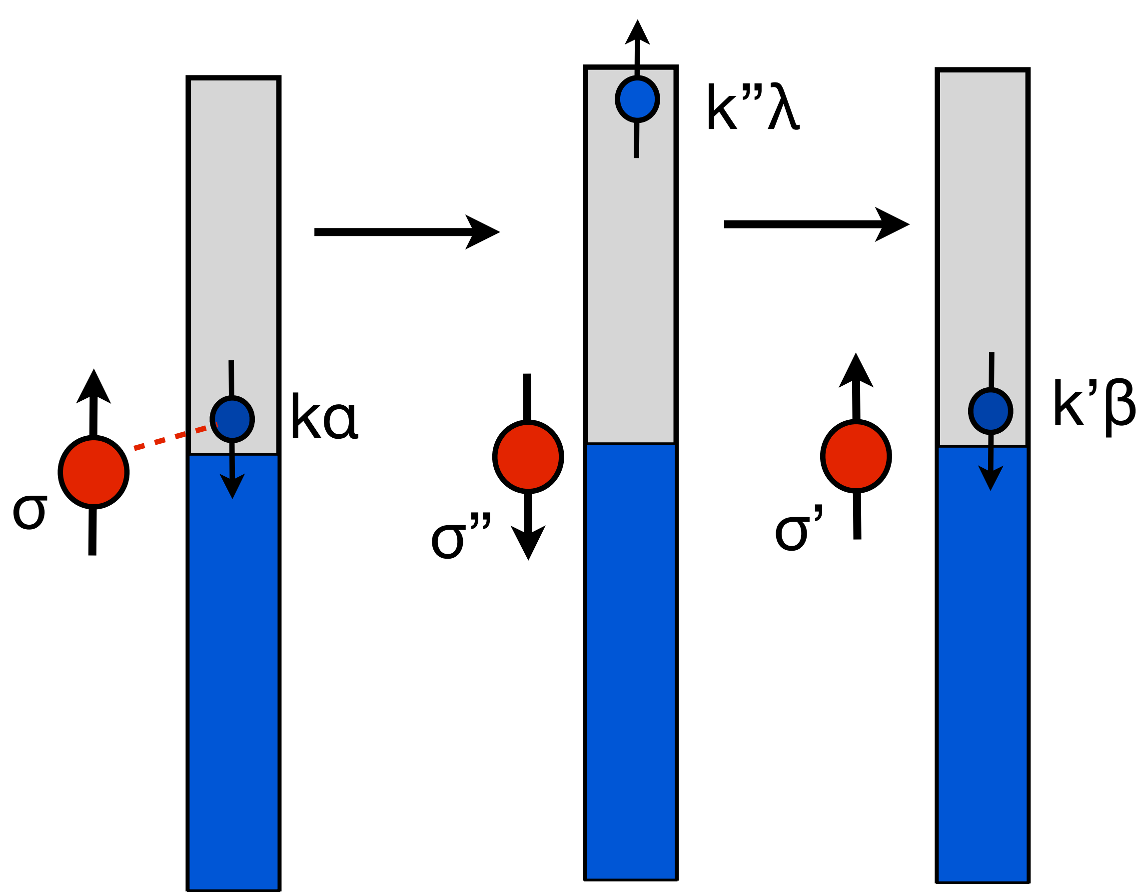

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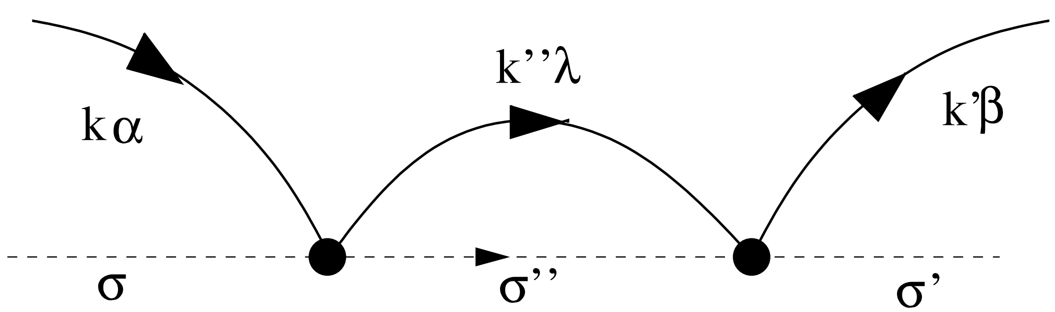

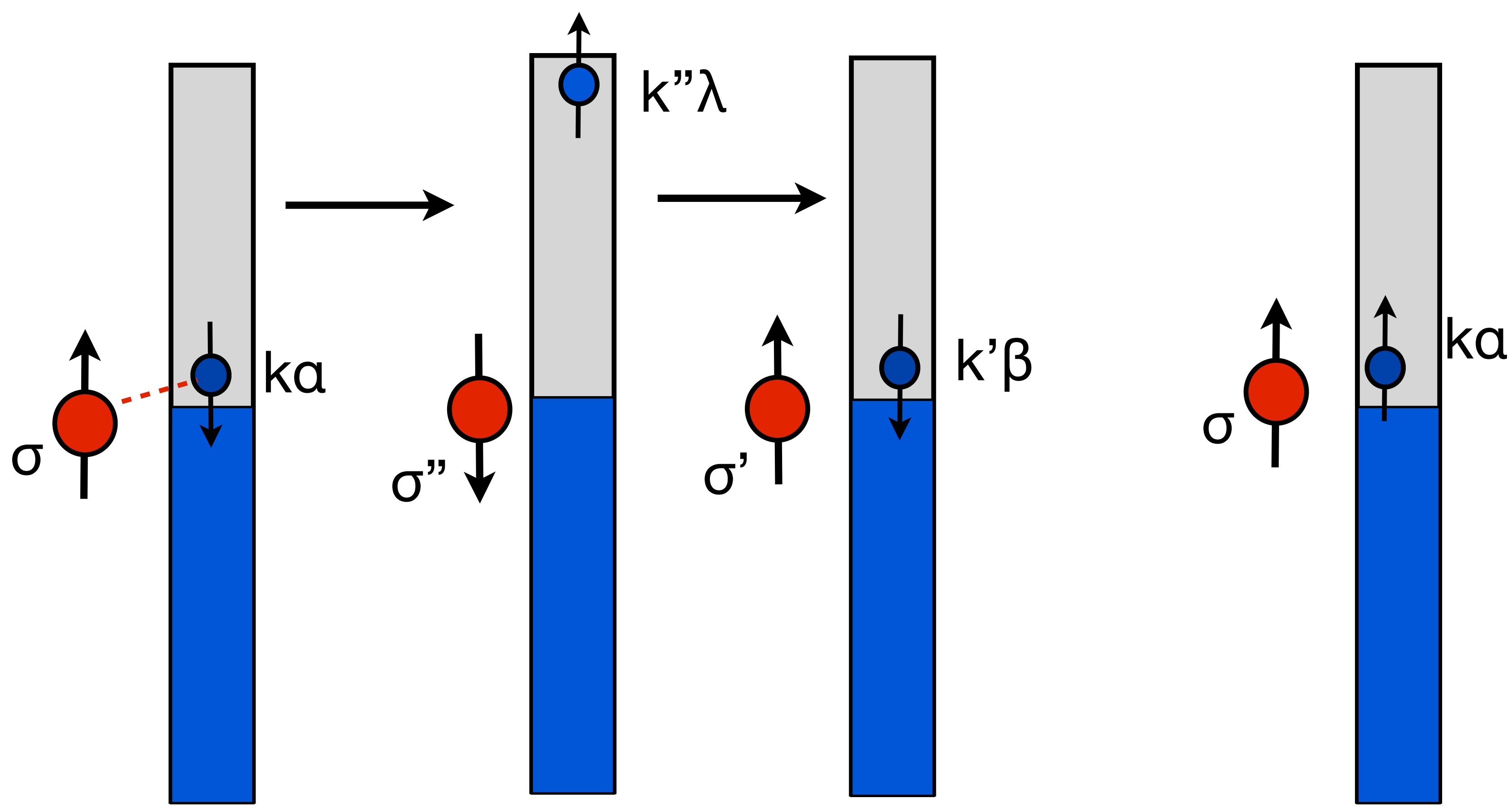

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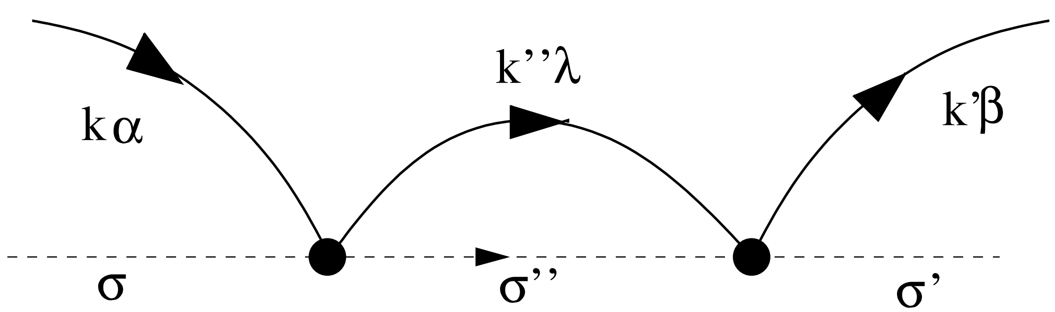

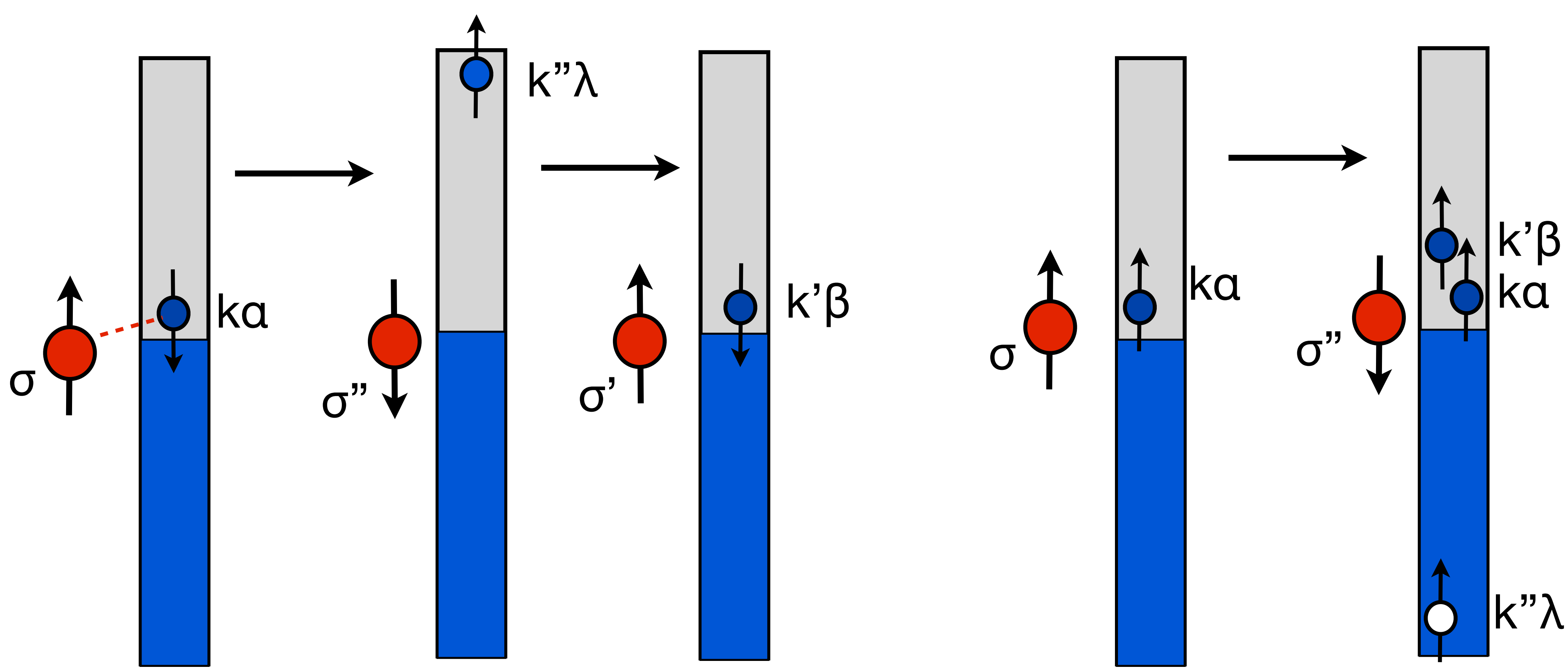

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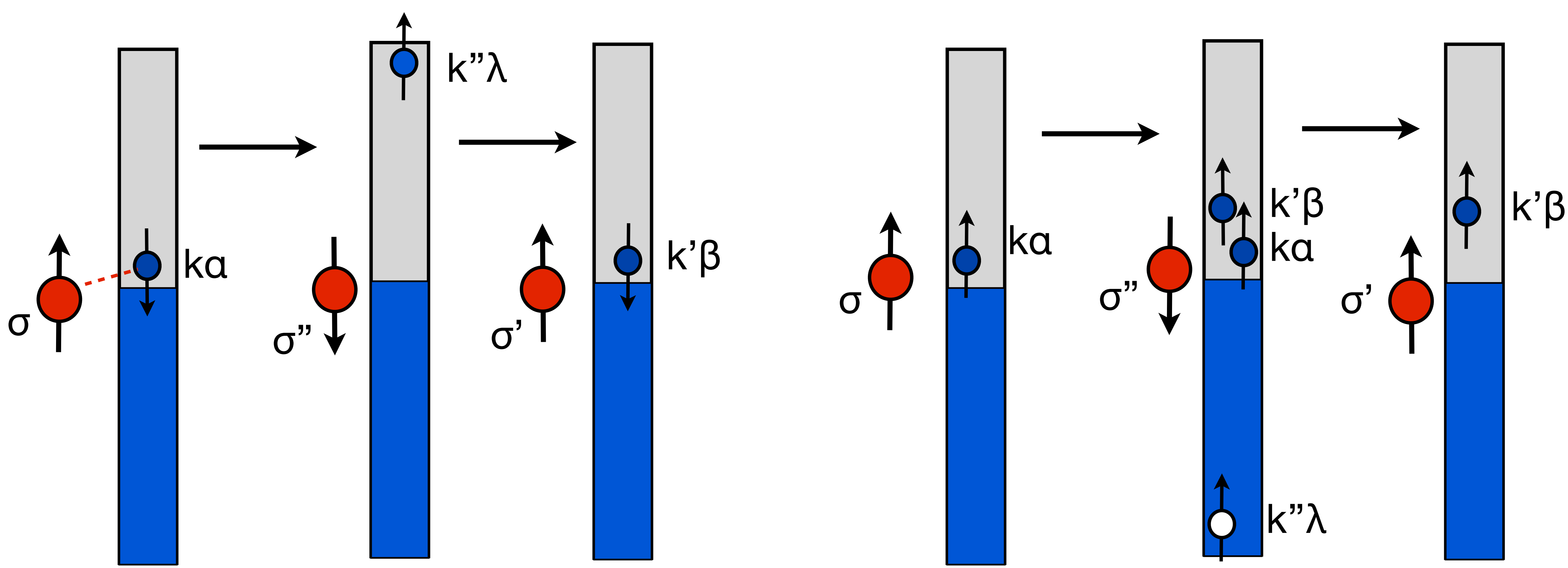
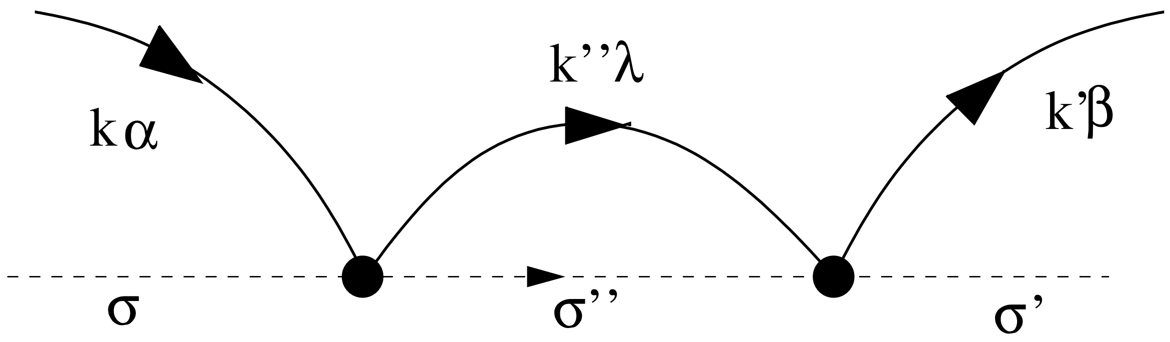
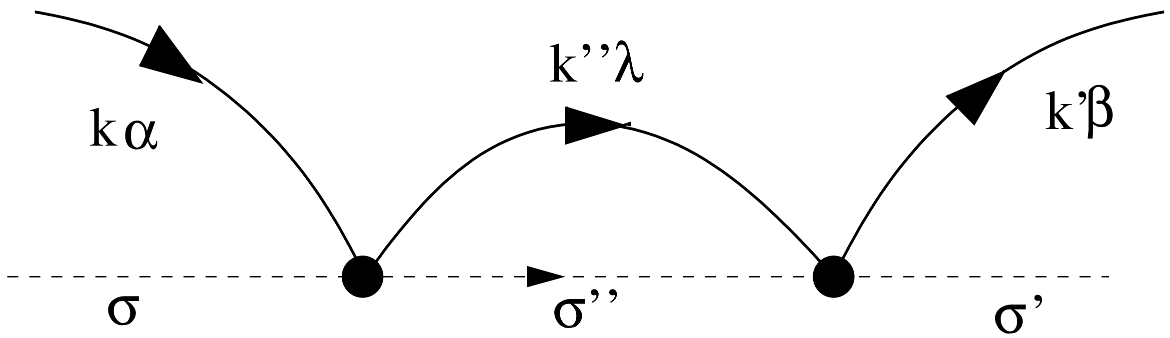
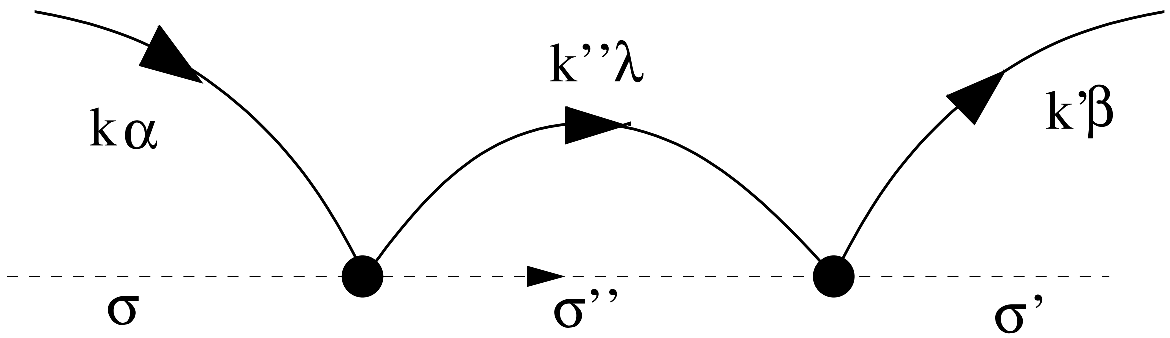




k''λ
kα
k'β
σ
σ''
σ'



$$T^{(I)}(E)_{k'\beta\sigma';k\alpha\sigma} \quad = \quad \sum_{\epsilon_{k''}\in[D-\delta D,D]} \left[\frac{1}{E-\epsilon_{k''}}\right] J^2(\sigma^a\sigma^b)_{\beta\alpha}(S^aS^b)_{\sigma'\sigma}$$



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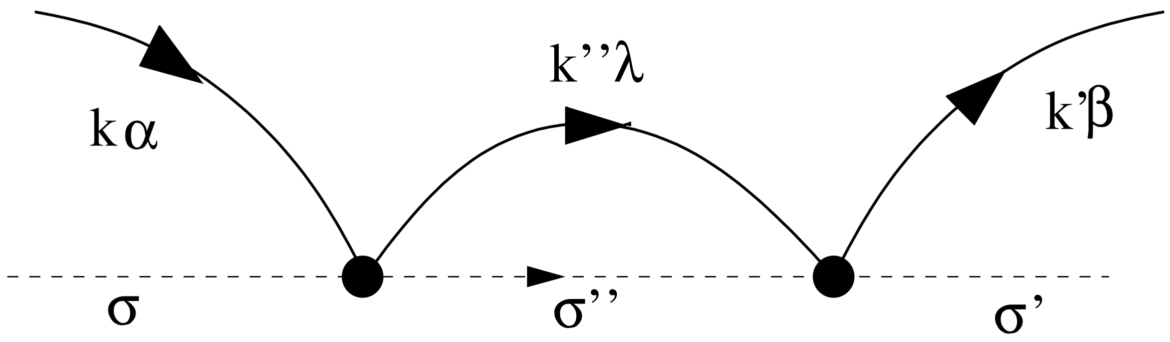
\end{aligned}
$$



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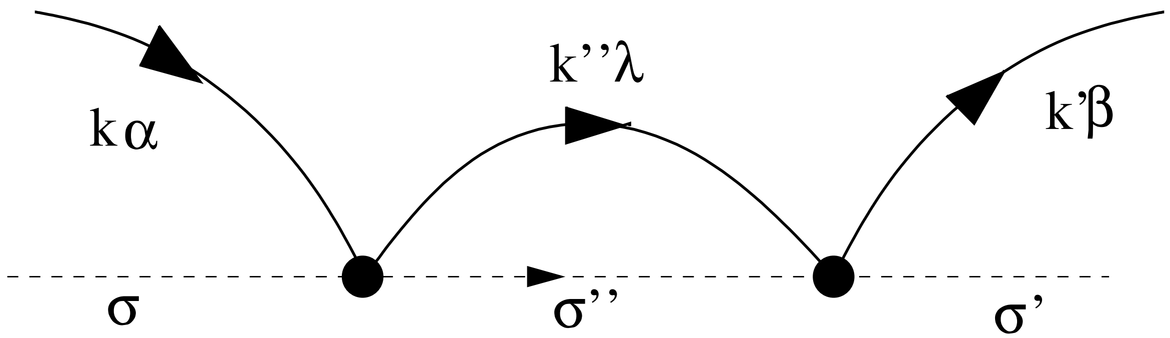

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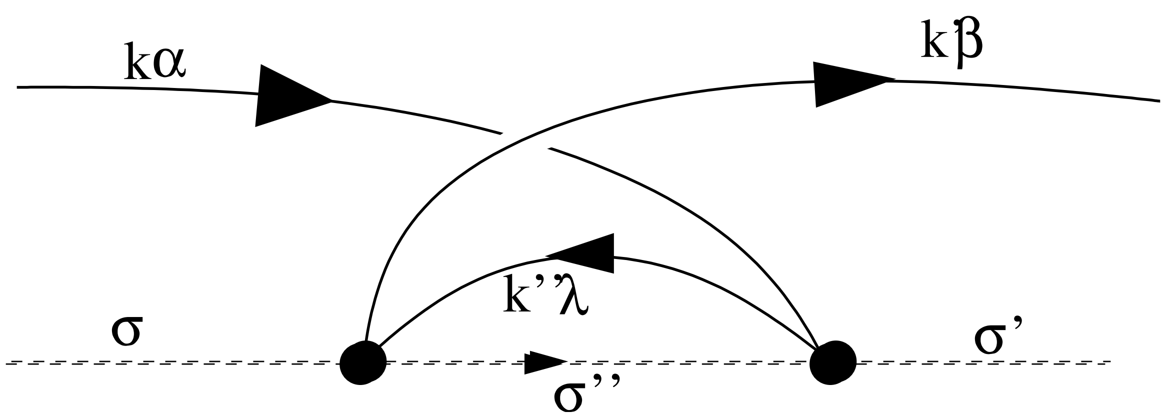

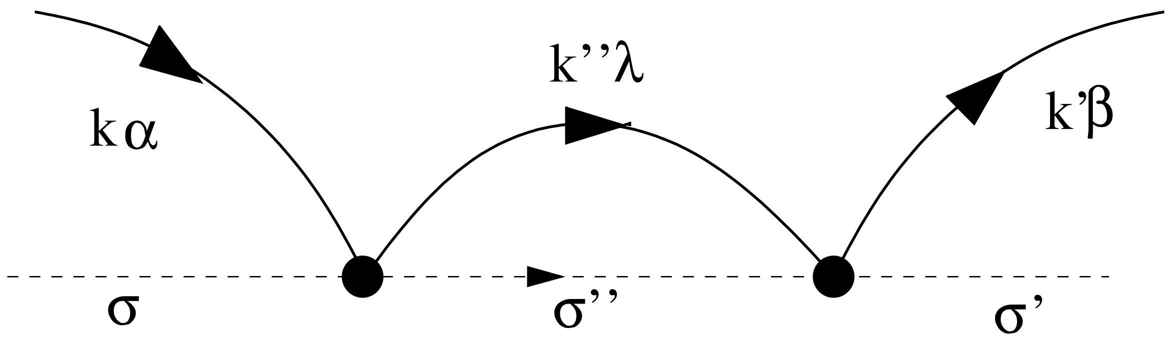

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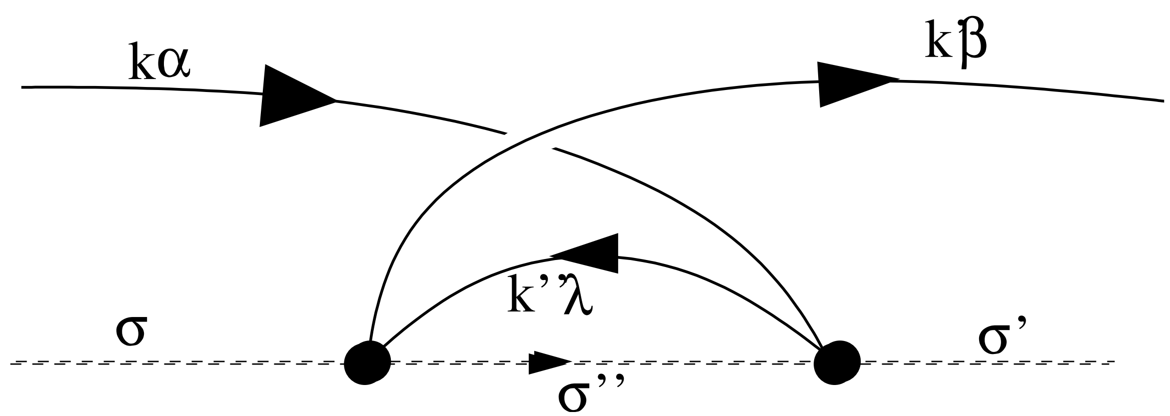
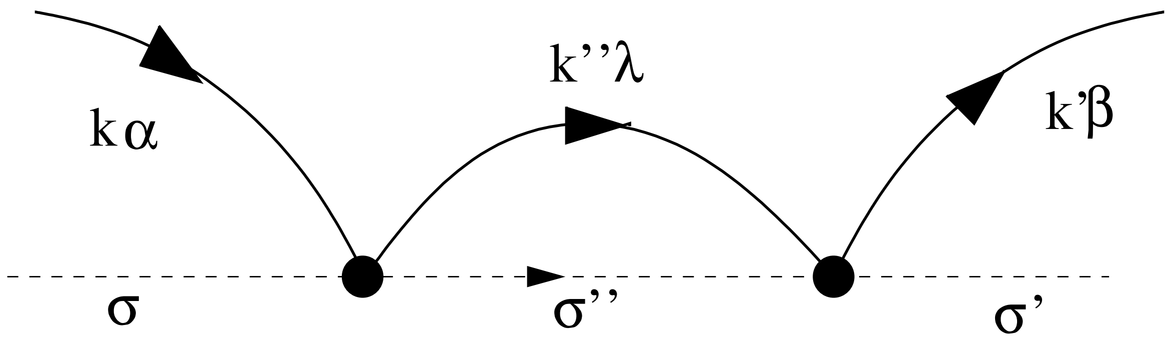


$$T^{(II)}(E)_{k'\beta\sigma';k\alpha\sigma} = -\sum_{\epsilon_{k''}\in[-D,-D+\delta D]}\left[\frac{1}{E-(\epsilon_k+\epsilon_{k'}-\epsilon_{k''})}\right]J^2(\sigma^b\sigma^a)_{\beta\alpha}(S^aS^b)_{\sigma'\sigma}$$



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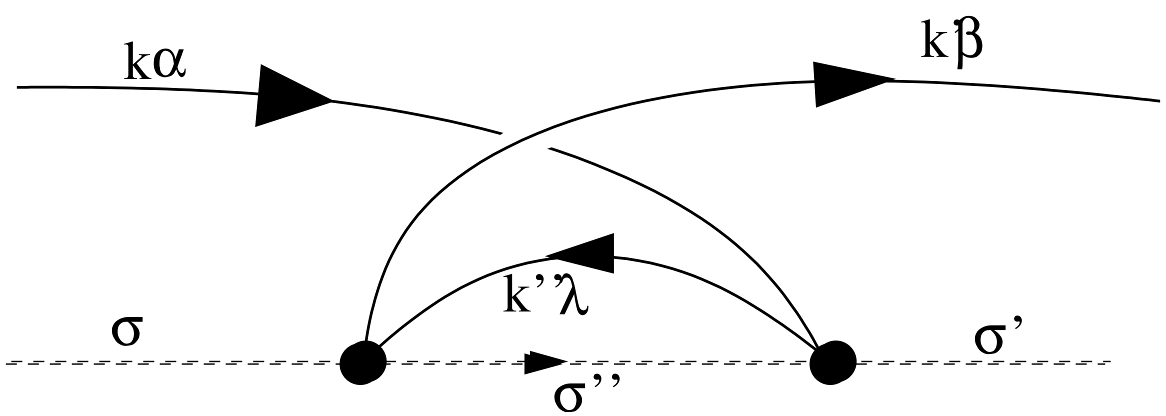

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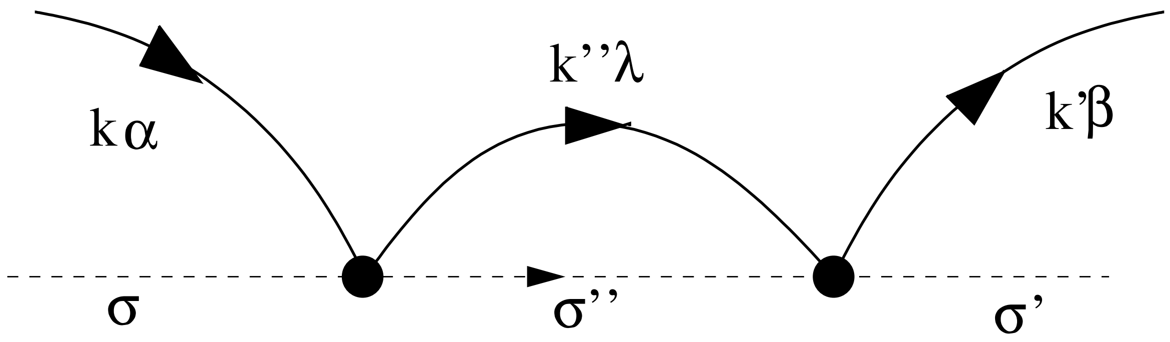

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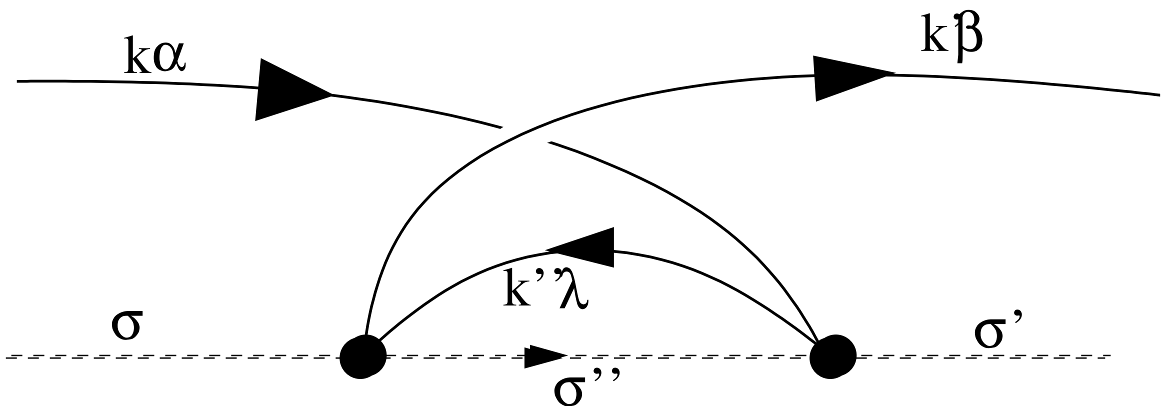

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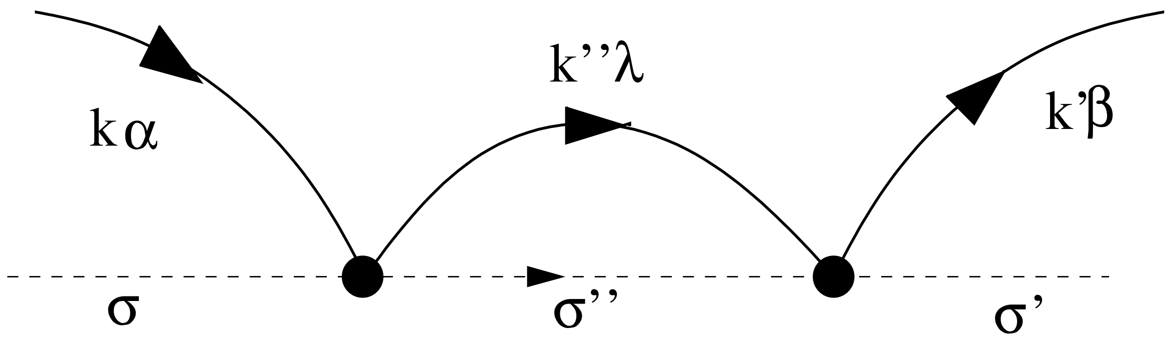

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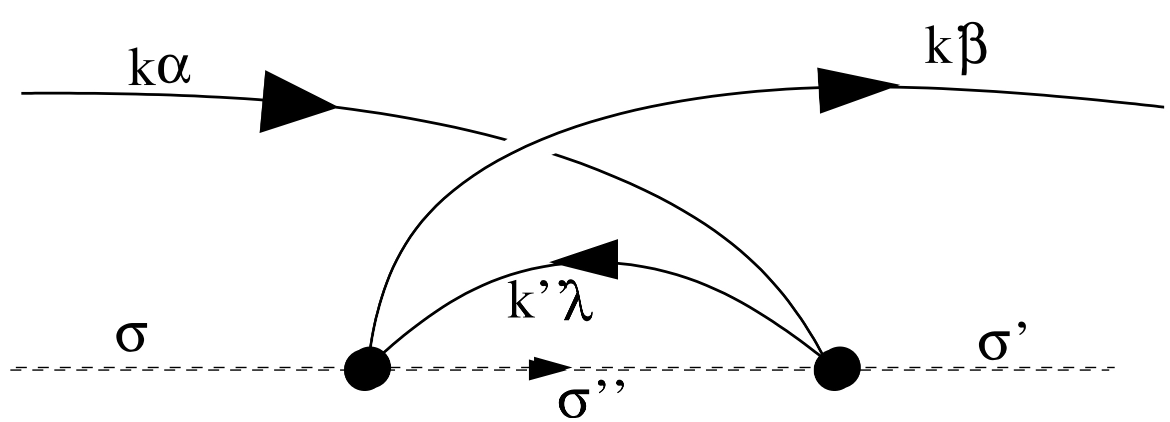

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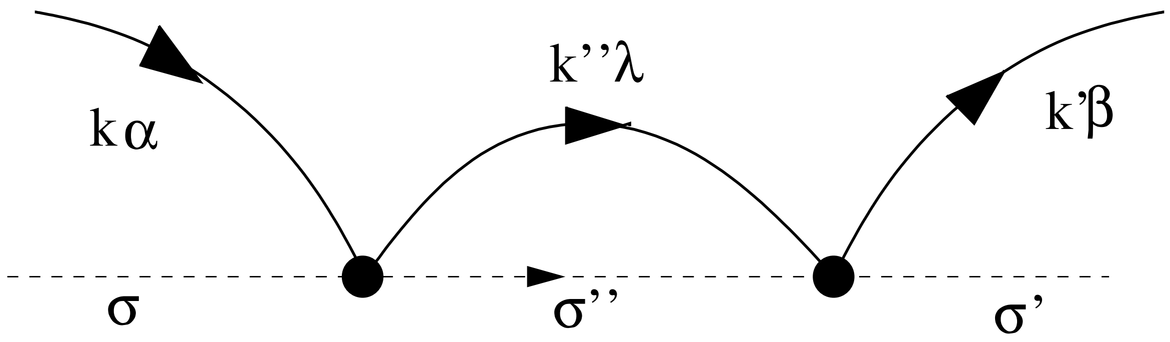

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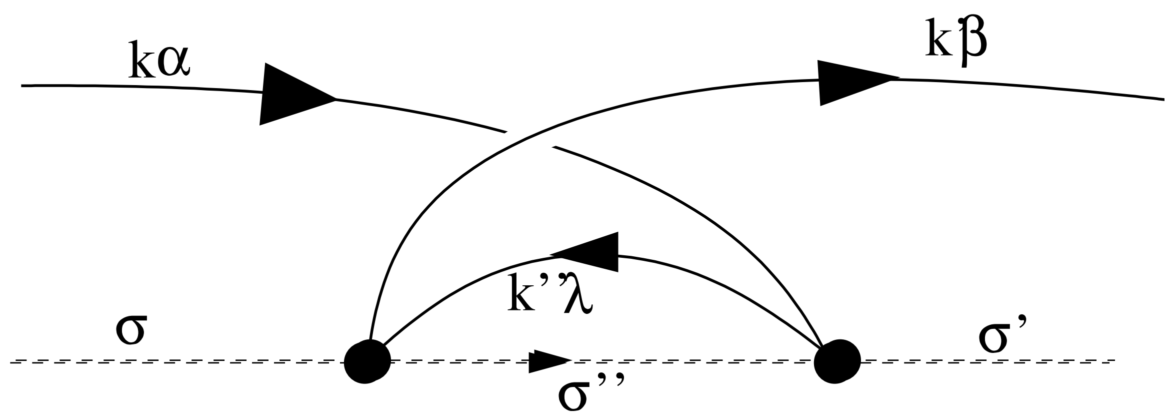

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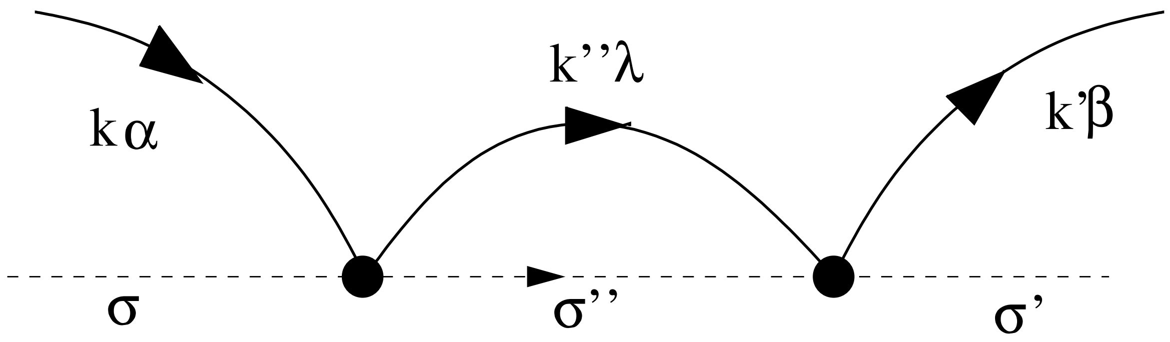
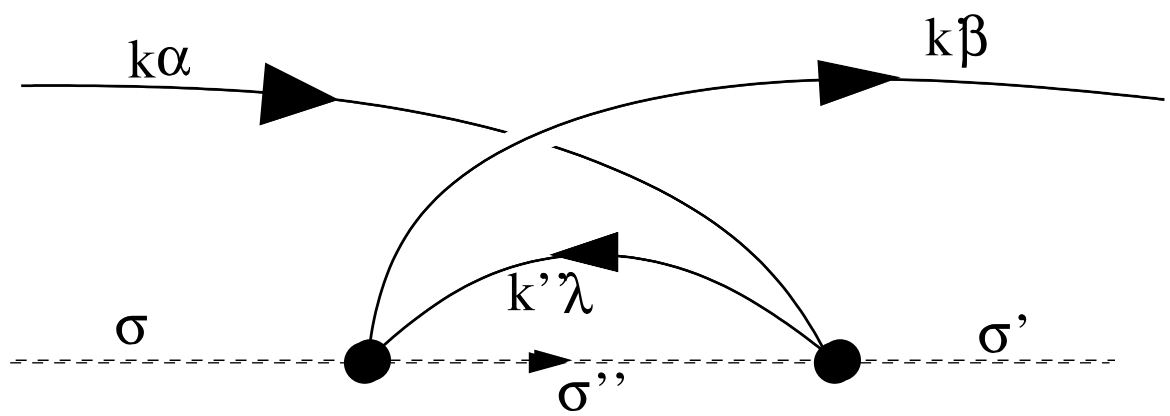


$$\hat{T}^{I} \approx -\frac{J^2 \rho \delta D}{D} (\sigma^b \sigma^a)_{\beta\alpha} (S^a S^b)_{\sigma' \sigma}$$



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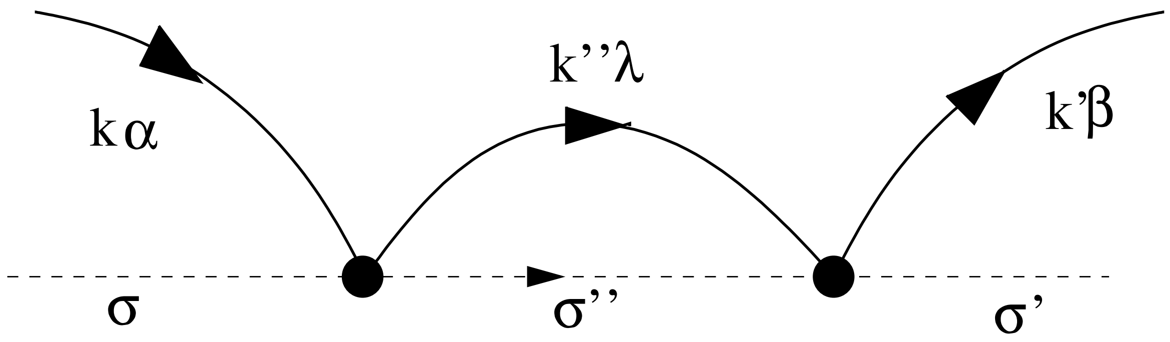

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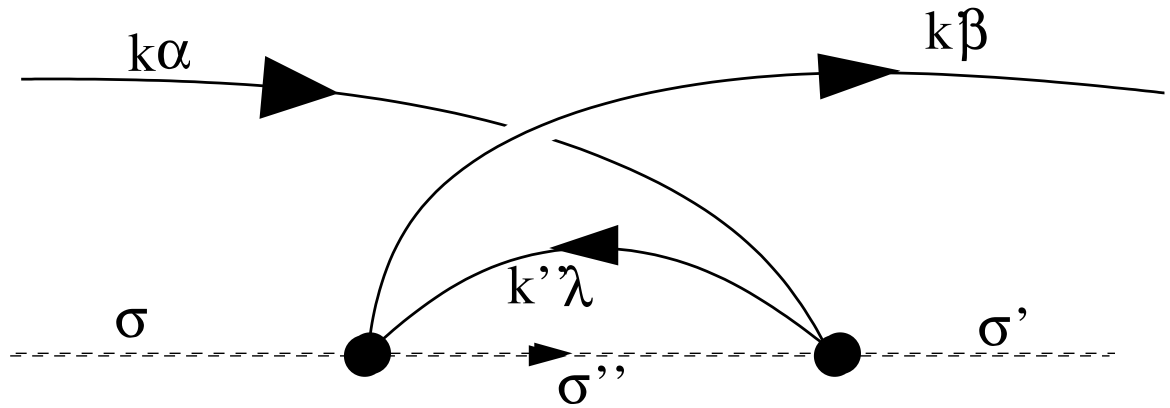

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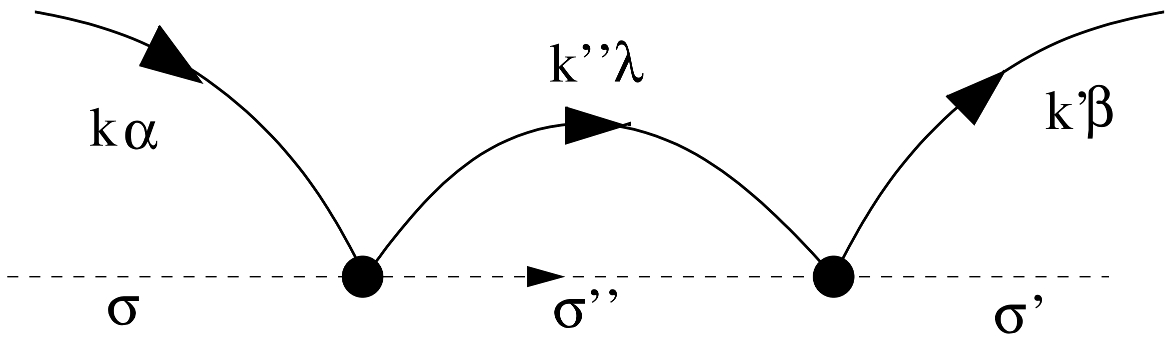
$$\delta H^{int}_{k'\beta\sigma';k\alpha\sigma} \quad = \quad \hat{T}^{I} + T^{II} = -\frac{J^2 \rho \delta D}{D} [\sigma^a, \sigma^b]_{\beta\alpha} (S^a S^b)_{\sigma' \sigma}$$

.



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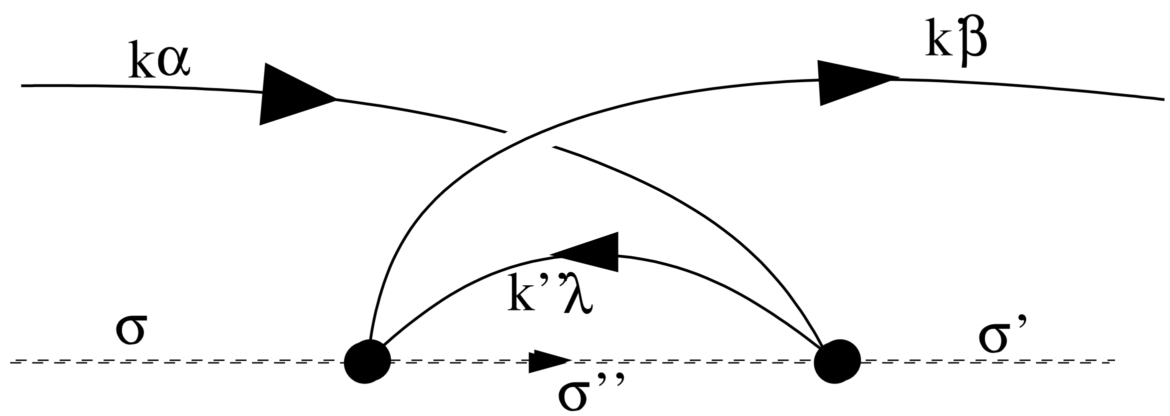

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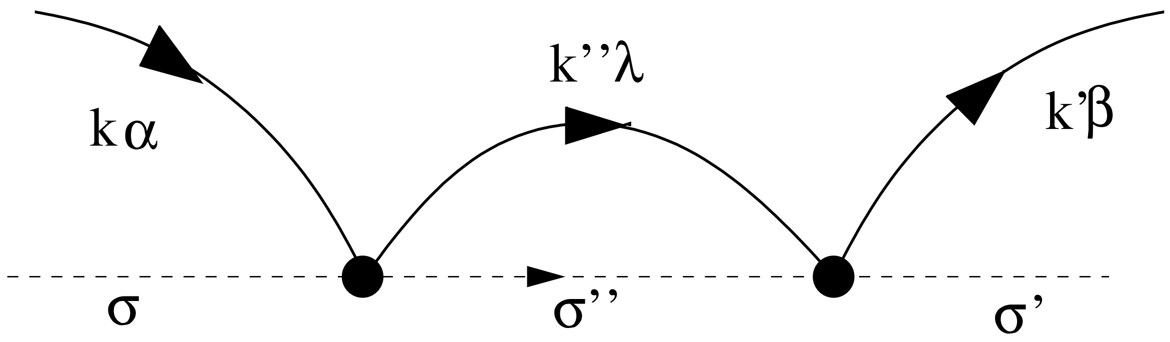

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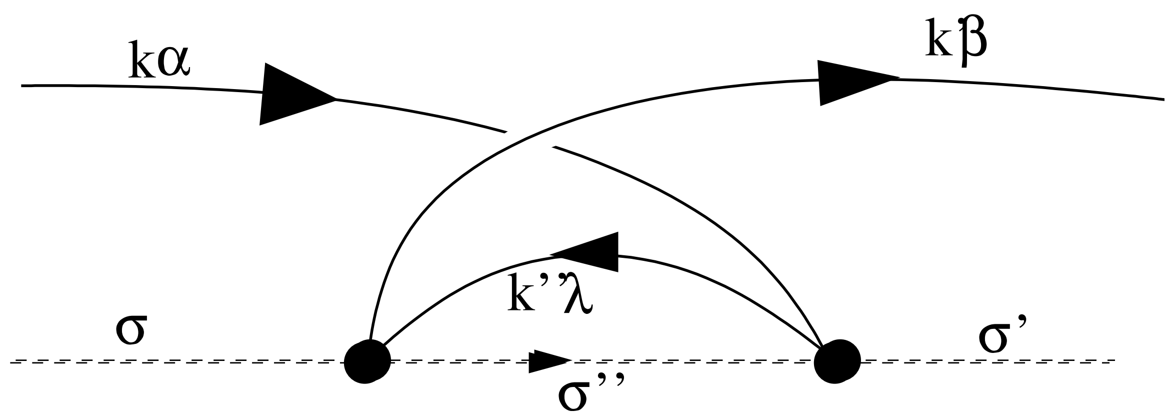

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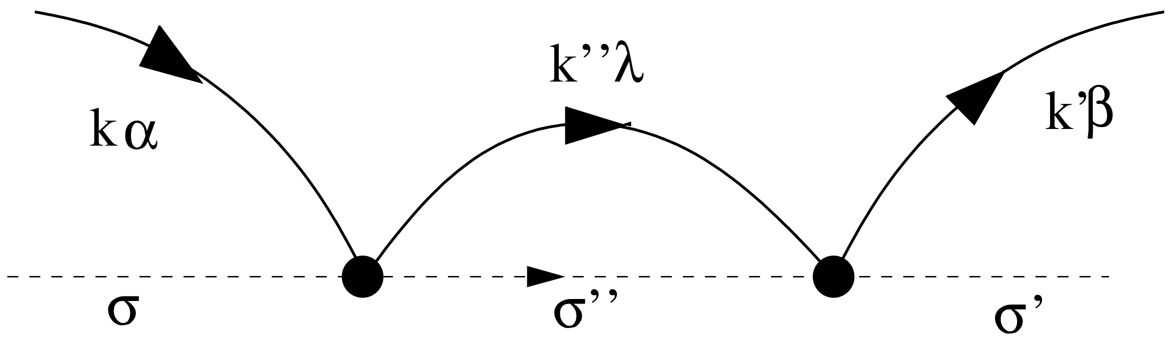

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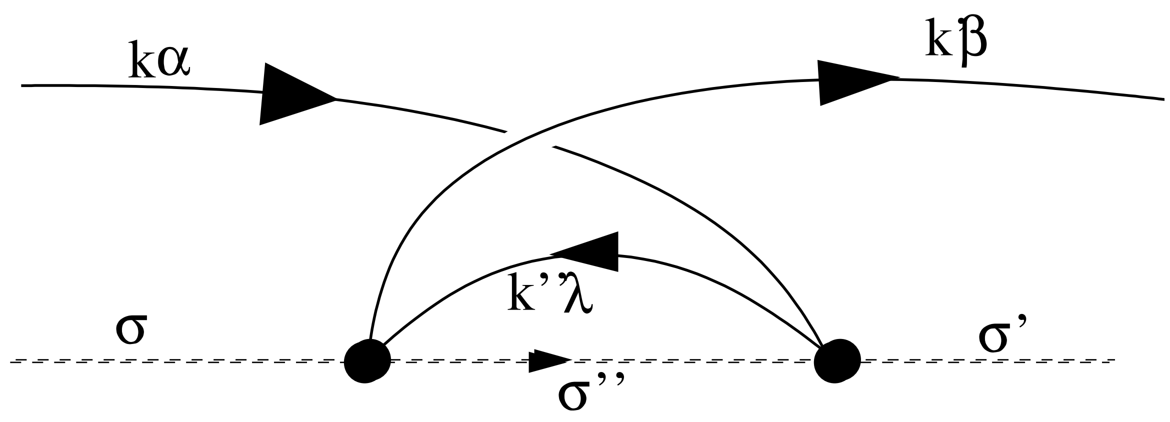

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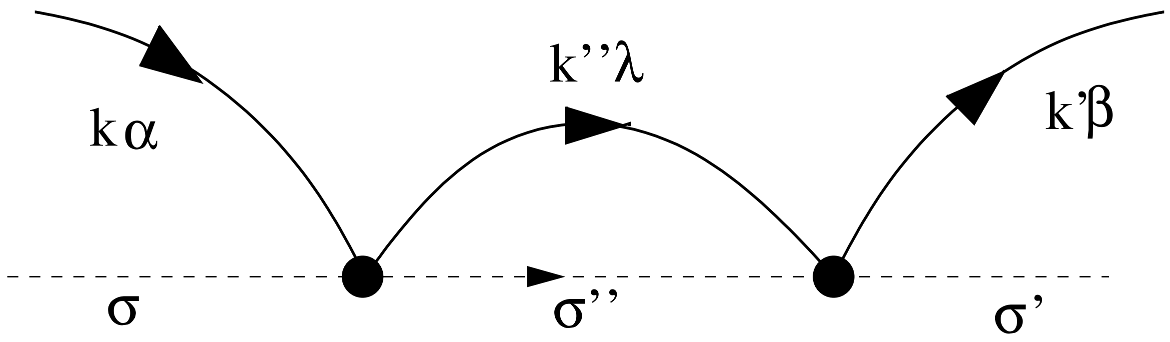

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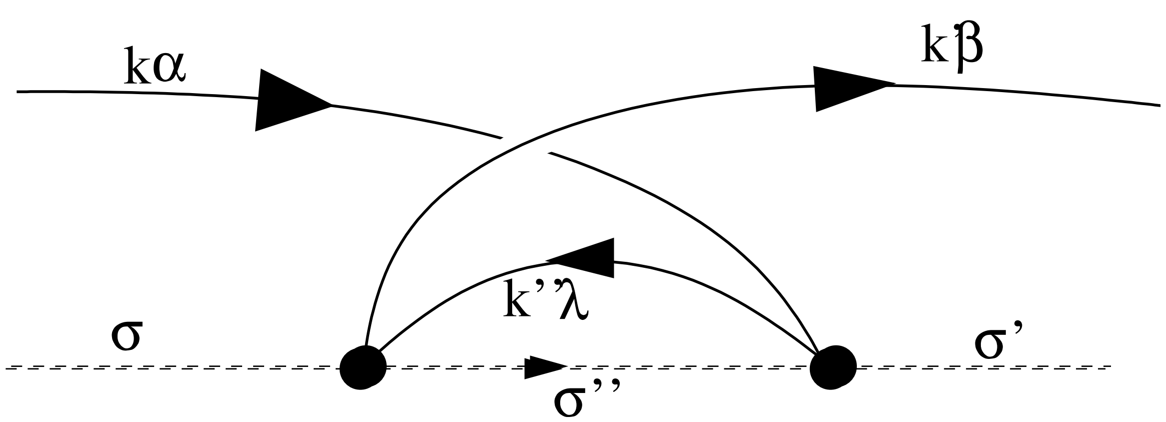

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$$J(D') = J(D) + 2J^2 \rho \frac{\delta D}{D}$$

$$\boxed{\frac{\partial g}{\partial \ln D} = \beta(g) = -2g^2 + O(g^3).}$$

$$(g = \rho J)$$

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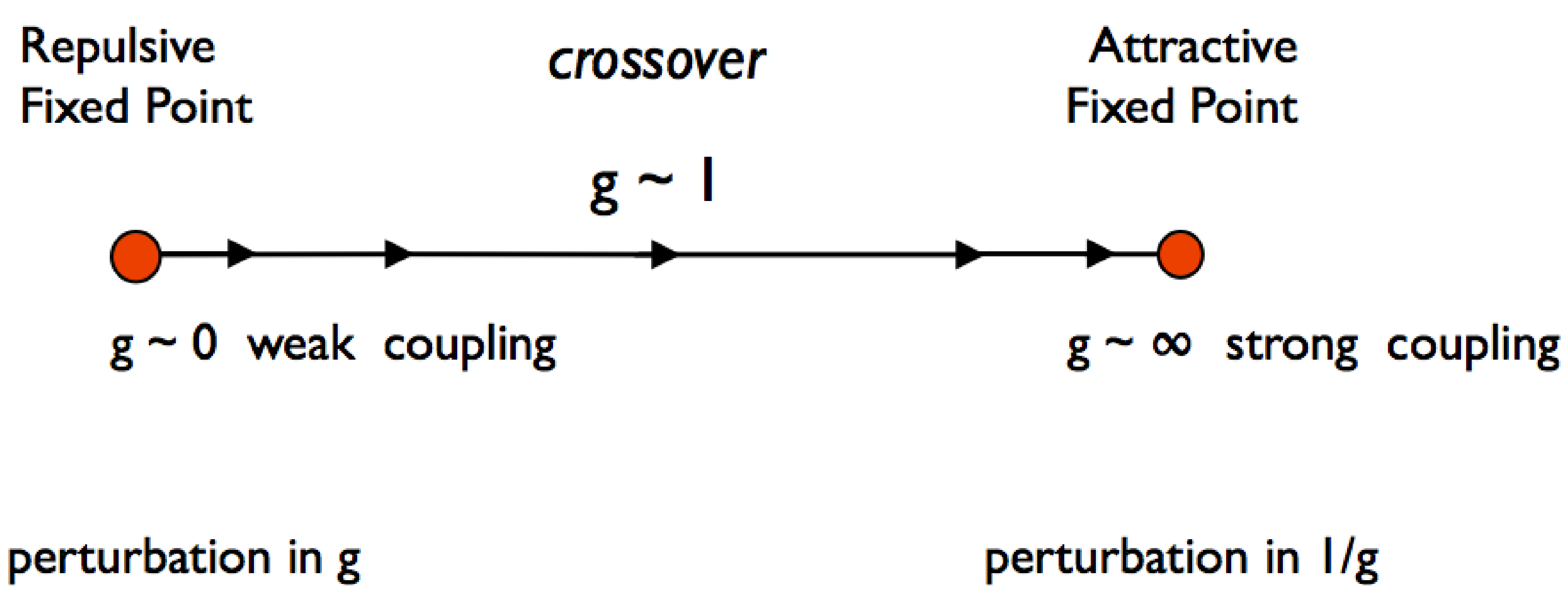
$(g = \rho J)$

$$g(D') = \frac{g_o}{1 - 2g_o \ln(D/D')}$$

$$T_K = D \exp\left[-\frac{1}{2g_o}\right]$$

$$2g(D') = \frac{1}{\ln(D'/T_K)}$$



Outline of the Topics

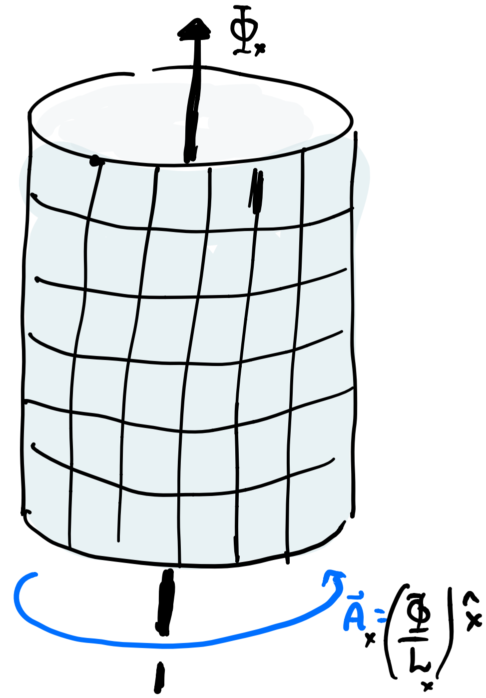
1. Trends in the periodic table.
2. Introduction: Heavy Fermions and the Kondo Lattice.
3. From Anderson to Kondo
4. **Kondo Insulators: the simplest heavy fermions.**
5. Oshikawa's Theorem.
6. Large N expansion for the Kondo Lattice
7. Heavy Fermion Superconductivity
8. Topological Kondo Insulators
9. Co-existing magnetism and the Kondo Effect.

Rutgers
Center for Materials Theory

OSHIKAWA

Flux argument

$$H[\Phi] = -t_{jk} e^{-i\int_k^j \vec{A}\cdot d\vec{x}} c_j^+ c_j + H_{int}$$



Consider insertion of flux Φ.

$$A_x = \frac{\Phi_x}{L_x}$$

$$\mathcal{E} = -\frac{\partial \phi}{\partial t}, \quad \vec{E} = -\frac{\partial \vec{A}}{\partial t}.$$

Electrons accelerated ∴ change in momentum.

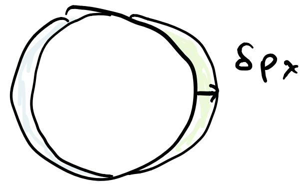
If $\Phi = \left(\frac{h}{e}\right) = 2\pi\left(\frac{\hbar}{e}\right) =$ "2π", then the Hamiltonian $H[\Phi_0]$ is related.

Momentum conservation means every electron picks up a momentum

$$P_x = e\int \frac{\partial A_x}{\partial t} dt = \frac{e\,\Phi_0}{L_x} = \frac{h}{L_y} = \frac{2\pi\hbar}{L_x} \equiv \frac{2\pi}{L_x}$$

$$\Delta P_x = \left(\frac{2\pi}{L_x}\right) N = 2\pi L_y \nu$$

Actually, this is the Mechanical momentum: the canonical momentum is unchanged.



Fermi-Liquid (2D)

$$\delta\vec{p} = \frac{2\pi}{L_x}\hat{x} = \delta p_x \hat{x}$$

$$\Delta\vec{P} = \sum \delta n_p \vec{p}$$

$$\Delta P_x = \sum \delta n_p p_x = \frac{L_x L_y}{(2\pi)^2}\int \left(\delta\vec{p}\cdot d\vec{S}\right) p_x$$

$$= \frac{L_x L_y}{(2\pi)^2}\int \left(\delta p_x\, p_x\, \hat{x}\right)\cdot d\vec{S}$$

$$= \frac{L_x L_y}{(2\pi)^2}\int_{FS} dV\, \overbrace{\nabla_{\vec{p}}^{x}\left(\delta p_x\, p_x\right)}^{\delta p_x} = \frac{L_x L_y}{(2\pi)^2}\delta p_x \int dV$$

$$\boxed{\Delta P_x = \frac{2\pi}{L_x} L_x L_y \frac{V_{FS}}{(2\pi)^2}}$$

$$2\pi\nu L_y = \frac{2\pi}{L_x}\frac{V_{FS}}{(2\pi)^2}L_x L_y + 2\pi m_x$$

$$2\pi\nu L_x = \frac{2\pi}{L_y}\frac{V_{FS}}{(2\pi)^2}L_x L_y + 2\pi m_y$$

$$\nu = \frac{N}{L_x L_y} \qquad N = \nu L_x L_y.$$

$$2\pi \frac{N}{L_x} = \frac{2\pi}{L_x} \frac{V_{FS}}{(2\pi)^2} L_x L_y + 2\pi m_x$$

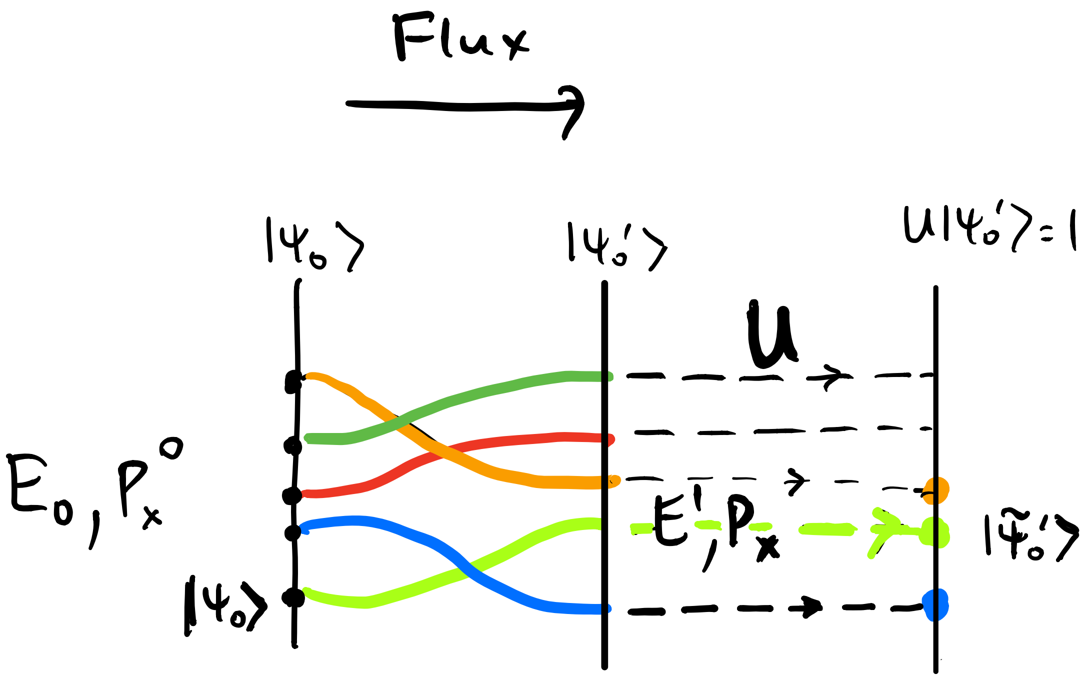
$$N - \frac{V_{FS}}{(2\pi)^2} L_x L_y = m_x L_x$$

$$N - \frac{V_{FS}}{(2\pi)^2} L_x L_y = L_y m_y.$$

$$m_y L_x = m_y L_y = p L_x L_y$$

$$\Rightarrow \quad N - \frac{V_{FS}}{(2\pi)^2} L_x L_y = p L_x L_y.$$

$$\Rightarrow \quad \boxed{\nu = \frac{V_{FS}}{(2\pi)^2} + p.}$$



$$H[0] \xrightarrow{\text{Flux}} H[\Phi_0] \xleftarrow{U^\dagger H_0 U} H$$

$$H[\Phi_0] = U^\dagger H[0]\, U$$

$$U^\dagger c_j U = e^{\frac{2\pi i}{L} x_j} c_j$$

$$\Rightarrow \quad U^\dagger c_i^\dagger c_j U = c_i^\dagger e^{i\frac{2\pi}{L}(x_i - x_j)} c_j$$

$$U = \exp\left[\frac{2\pi i}{L} \sum n_j x_j\right]$$

Unitary Transform that inserts Flux

$$H[\Phi_0]|\Psi_0'\rangle = E|\Psi_0'\rangle$$

$$U^{\dagger}H[0]U|\Psi_0'\rangle = E|\Psi_0'\rangle$$

$$\Rightarrow \quad H[0]U|\Psi_0'\rangle = E\,\overbrace{U|\Psi_0'\rangle}^{|\tilde{\Psi}_0'\rangle}$$

eigenstate of $H[\Phi]$ — eigenstate of $H[0]$

$$\boxed{U|\Psi_0'\rangle = |\tilde{\Psi}_0'\rangle}$$

$$T_x|\Psi_0\rangle = e^{-iP_x^0}|\Psi_0\rangle \qquad |\Psi_0\rangle \longrightarrow \boxed{|\Psi_0'\rangle}$$

$$T_x|\Psi_0'\rangle = e^{-iP_x^0}|\Psi_0'\rangle$$

CANONICAL MOMENTUM UNCHANGED (TRANSLATIONALLY INVARIANT)

To determine Mechanical Momentum need to carry out Unitary Tfm back to original Hamiltonian.

$$T_x|\tilde{\Psi}_0'\rangle = e^{-iP_x}|\tilde{\Psi}_0'\rangle \Rightarrow U^{\dagger}T_xU|\Psi_0'\rangle = e^{-iP_x}|\Psi_0'\rangle$$

$$U^{\dagger}\hat{T}_xU = \exp\left[-\frac{2\pi i}{L_x}\sum n_x\right]\hat{T}_x$$

$$\Rightarrow U^{\dagger}T_xU|\Psi_0'\rangle = \exp\left[-\frac{2\pi i}{L_x}\sum n_x\right]T_x|\Psi_0'\rangle$$

$$\exp[-iP_x]|\tilde{\Psi}_0'\rangle = \exp\left[-i\left(P_x^0 + 2\pi\frac{N}{L_x}\right)\right]|\tilde{\Psi}_0'\rangle$$

$$U^{+} T_{x} U = U^{+} \exp\left[\frac{2\pi i}{L} \sum x (\hat{n}_{x+a})\right] T_{x}$$

$$= U^{+} \exp\left[\frac{2\pi i}{L_x}\left(\sum (x+a)\, \hat{n}_{x+a} - a \sum \hat{n}_{x+a} + L_x \hat{n}_{x_1}\right)\right] T_{x}$$

$$= \exp\left[-\frac{2\pi i}{L_x} \sum_j \hat{n}_j\right] T_x$$

$$P_x = P_x^0 + \frac{2\pi}{L_x} N + 2\pi m_x$$

$$\overbrace{\frac{2\pi}{L_x} \cdot \frac{V_{FS}}{(2\pi)^D} L_x L_y \ldots L_D}^{P_x - P_x^0} = -2\pi m_x + \frac{2\pi}{L_x} N$$

$$\Rightarrow \quad N - \frac{V_{FS}}{(2\pi)^D} V = m_x . L_x$$

$$N - \frac{V_{FS}}{(2\pi)^D} V = m_y L_y$$

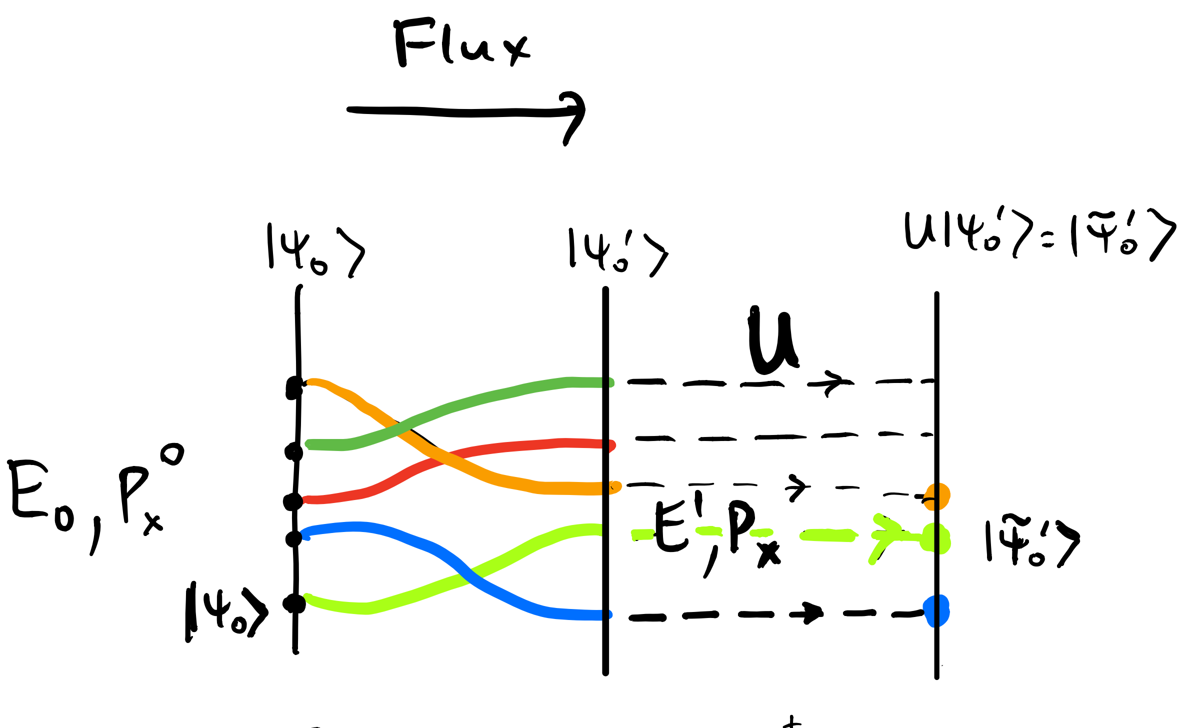
$$= p \, L_x L_y L_z \ldots L_D$$

$$\Rightarrow \quad \boxed{\frac{N}{V} = \frac{V_{FS}}{(2\pi)^D} + p.}$$

LUTTINGER SUM RULE.

Kondo Lattice

$$H = \sum t_{ij}\, c^\dagger_{i\sigma} c_{j\sigma} + J \sum c^\dagger_j \vec{\sigma} c_j \cdot \vec{S}_j + U \sum c^\dagger_{j\uparrow} c_{j\uparrow}\, c^\dagger_{j\downarrow} c_{j\downarrow}$$



$$H[0] \xrightarrow{\text{FLUX}} H[\Phi_0] \xleftarrow{U^\dagger H_0 U} H$$

Flux Insertion Single Spin Component

$$H \to \sum t_{ij} c^{\dagger}_{i\sigma} e^{-i \int_i^j \vec{A}_\sigma \cdot dx} c_{j\sigma} + J \sum \vec{\sigma}_j \cdot \vec{S}_j + U \sum n_{c\uparrow}(j) n_{c\downarrow}(j)$$

$$A_\sigma = \begin{cases} A_\uparrow & \sigma = \uparrow \\ 0 & \text{otherwise} \end{cases} \qquad A_\uparrow = \frac{2\pi}{L_x}.$$

$$H[\Phi] = U^{\dagger} H[0] U$$

We require a U so that:

$$U^{\dagger} c_{j\sigma} U = e^{\frac{2\pi i}{L_x} x_j} c_{j\sigma}.$$

$$U^{\dagger} (\sigma_j \cdot S_j) U = (\sigma_j \cdot S_j).$$

$$U_\uparrow = \exp\left[\frac{2\pi i}{L} \sum x \left(n_{j\uparrow} + S_{zj}\right)\right]$$

$$\sigma_j \cdot S_j = \sigma_j^z S_j^z + \sigma_j^+ S_j^- + \sigma_j^- S_j^+$$

$$U^\dagger \sigma_j^z U = \sigma_j^z$$

$$U^\dagger \sigma_j^+ U = \exp\left[-\frac{2\pi i}{L_x} x_j\right] \sigma_j^+$$

$$U^\dagger S_j^- U = \exp\left[+\frac{2\pi i}{L_x} x_j\right] S_j^-$$

$$U^\dagger H[0] U = H[\Phi_0]$$

$$U^\dagger T_x U = ?$$

$$T_x U T_x^{-1} = \exp\left[\frac{2\pi i}{L} \sum x \left(n_{j+a\uparrow} + S^z_{j+a}\right)\right]$$

$$= \exp\left[\frac{2\pi i}{L_x} \sum_j (x+a)\left(n_{j+a\uparrow} + S^z_{j+a}\right) - a\left(n_{j+a\uparrow} + S^z_{j+a}\right)\right]$$

$$
\begin{aligned}
(L+a) S^z_{L+a} &= (L+a) S^z_1 \\
&= a S^z_1 + L S^z_1 \\
&= x_1 S^z_1 + L S^z_1
\end{aligned}
$$

$$
L\, n_{L+a} = x_1 n_1 + L x_1
$$

$$
\begin{aligned}
T_x U = \exp\Big[&\frac{2\pi i}{L_x} \sum_j x \left(n_{j\uparrow} + S^z_j\right) + \left(n_{j\uparrow} + S^z_j\right) \delta_{x_j,1} \\
&+ \frac{2\pi i}{L} \sum a \left(n_{j\uparrow} + S^z_j\right) \Big] T_x
\end{aligned}
$$

$$
U^\dagger T_x U = \exp\left[-\frac{2\pi i}{L_x}\left(N_\uparrow + M^z\right) + \frac{2\pi i}{L_x} \sum_{\vec{y}} S^z_{1\vec{y}} \right] T_x
$$

$$
\begin{aligned}
U^\dagger T_x U |\psi_0'\rangle &= e^{-iP_x} |\psi_0'\rangle \\
&= e^{-iP_x^0} \exp\left[\frac{-2\pi i}{L_x}\left(N_\uparrow + M^z\right) + 2\pi i \sum S^z_{1y} \right] |\psi_0'\rangle
\end{aligned}
$$

$$\frac{2\pi}{L_x} \frac{V_{FS\uparrow}}{(2\pi)^D} V = \frac{2\pi}{L_x}\left(N_\uparrow + M^z\right) + \frac{2\pi S}{L_x} V \quad + \overset{2\pi}{P_\uparrow}$$

$$\frac{2\pi}{L_x} \frac{V_{FS\downarrow}}{(2\pi)^D} V = \frac{2\pi}{L_x}\left(N_\downarrow - M^z\right) + \frac{2\pi S}{L_x} V \quad + \overset{2\pi}{P_\downarrow}$$

$$2\frac{V_{FS}}{(2\pi)^D} V = \left(N_e + 2S\right) V \quad + \quad m_x L_x$$

$$m_y L_y$$

$$n_z L_z$$

$$= r V$$

$$\Rightarrow \boxed{\frac{2 v_{FS}}{(2\pi)^D} = (\nu + 2S) + r}$$

$$\nu = \frac{N_e}{V}$$