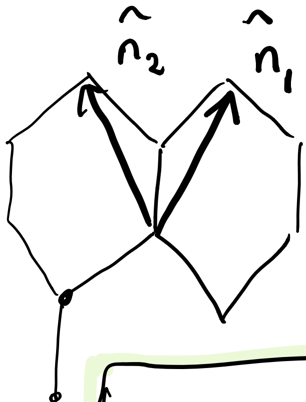
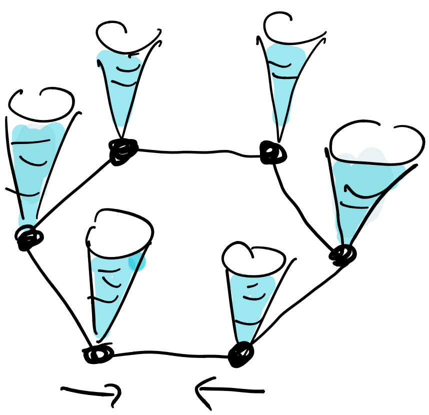
Continuing discussion of the Kitaev Honeycomb Model. Oct 23rd.



$$\hat{n}_1 = \left(\frac{\sqrt{3}}{2}a, \frac{3}{2}a\right)$$

$$\hat{n}_2 = \left(-\frac{\sqrt{3}}{2}a, \frac{3}{2}a\right)$$

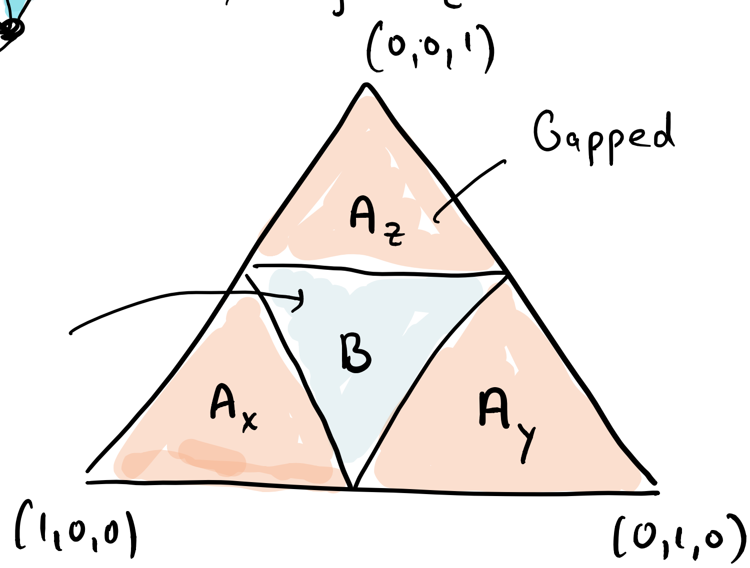
$$\boxed{\begin{aligned} f(k) &= \left(J_z + J_x e^{i\vec{k}\cdot\vec{n}_1} + J_y e^{i\vec{k}\cdot\vec{n}_2}\right) \\ \epsilon(k) &= \pm\,|f(k)| \end{aligned}}$$



$J_z > J_x = J_y$.

Gapless.

$J_x = J_y = J_z$



FERMION QPS

$$\begin{pmatrix} 0 & i f_{\vec{k}} \\ -i f^*_{\vec{k}} & 0 \end{pmatrix} \begin{pmatrix} 1 \\ \mp i e^{-i\theta_k} \end{pmatrix} = \begin{pmatrix} 0 & i|f|e^{i\theta} \\ -i|f|e^{-i\theta} & 0 \end{pmatrix} \begin{pmatrix} 1 \\ \mp i e^{-i\theta} \end{pmatrix}$$

$$f_k = |f_k| e^{i\theta_{\vec{k}}} \qquad = \pm |f_k| \begin{pmatrix} 1 \\ -i e^{-i\theta_k} \end{pmatrix}$$

$$|\psi_\pm\rangle = |i\rangle\langle i|\psi\rangle \Rightarrow a^\dagger_\pm = c^\dagger_i \langle i|\psi_\pm\rangle$$

$$a^\dagger_\pm(k) = \frac{1}{\sqrt{2}}\left(c^\dagger_{1k} \mp i e^{\mp i\theta_k} c^\dagger_{2k}\right)$$

$$\hat{\mathcal{H}} = \frac{1}{2}\sum |f| \left[a^\dagger_+(\vec{k}) a_+(\vec{k}) - a^\dagger_-(\vec{k}) a_-(\vec{k})\right]$$

$$f_k = f^*_{-k} \qquad a^\dagger_-(-k) = \frac{1}{\sqrt{2}}\left(c^\dagger_{1-k} + i e^{+i\theta_k} c^\dagger_{2-k}\right)$$

$$\theta_k = -\theta_{-k} \qquad = a^\dagger_+(k)$$

$$\mathcal{H} = \sum_{k \in BZ} |f_k| \left[a_+^\dagger(k) a_+(k) - \frac{1}{2} \right]$$

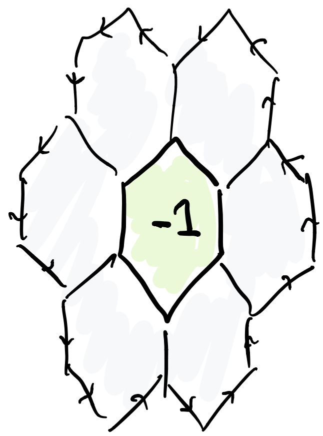
- Only positive energy excitations : MAJORANA's

- G.S.E $\quad E_g = -\frac{1}{2} \sum_k |f_k|.$

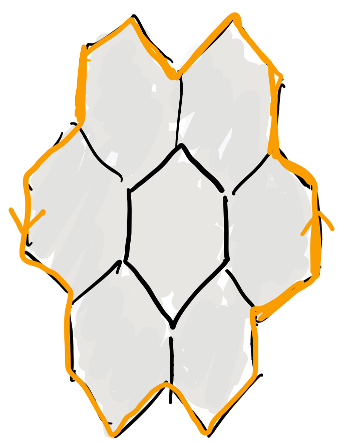
Types of excitation

- Fermions

- Vortices.

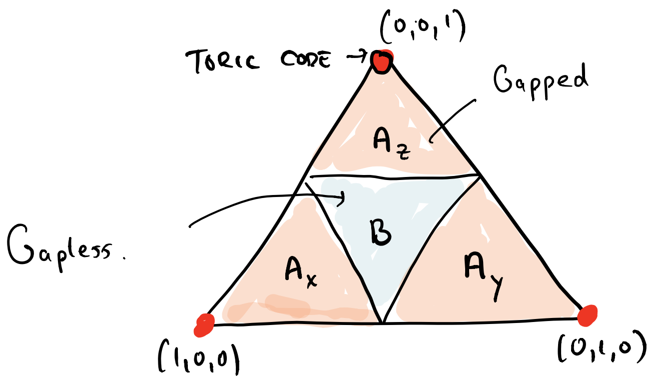


- Edge excitations (broken T.R.)



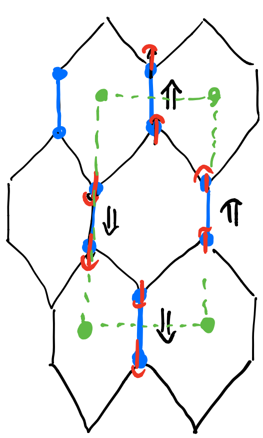
Neutral, chiral edge states

Properties of gapped phases



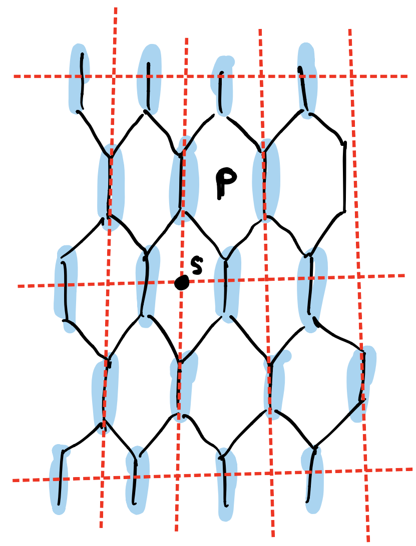
- Around a vortex $\prod u_{ij} = -1$

- Additional insight is gained from the extreme limit $J_x = J_y = 0$, which maps onto the "Toric code"



- $|\Uparrow\rangle = |\uparrow\uparrow\rangle$
  $|\Downarrow\rangle = |\downarrow\downarrow\rangle$ } effective spin.

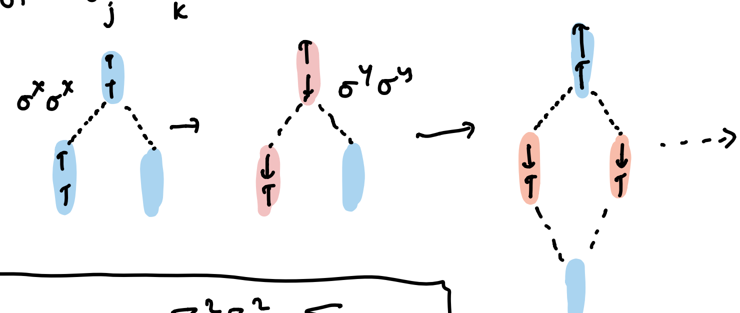
- Lie on bonds of a new lattice.



- Doing perturbation theory on the weak links, acting sequentially around a hexagon $\quad H_{eff}^{(4)} = V G_0 V G_0 V G_0 V$

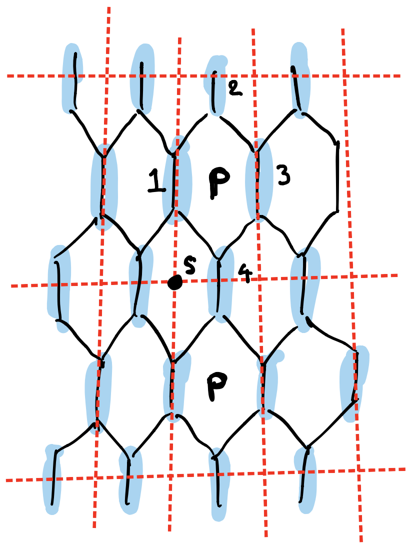
$V = \sigma_j^x \sigma_{jk}^x \quad$ or $\quad \sigma_j^y \sigma_k^y$

$\Delta E = 4J_z$



Gives

$$\boxed{\begin{aligned} H_{eff}^{(4)} &= \text{const} - \frac{J_x^2 J_y^2}{16 J_z^3} \sum_p Q_p \\ Q_p &= \sigma_{p_1}^y \sigma_{p_2}^x \sigma_{p_3}^y \sigma_{p_4}^x \end{aligned}}$$



"Plaquet terms" B_p

"Star terms" A_s

Rotate about z $\begin{cases} \sigma_y \to \sigma_x & \text{on odd S.L} \\ \sigma_x \to -\sigma_y & \text{on odd SL} \end{cases}$

Rotate about x on all sites $\sigma_y \to \sigma_z$.

$$H_T = -J_e \sum_s A_s - J_m \sum_p B_p$$

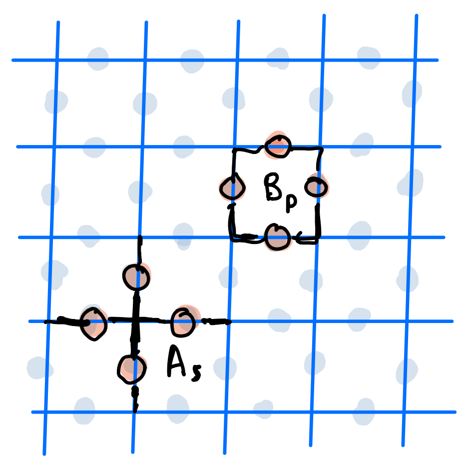
$$A_s = \prod_{\text{star}} \sigma_j^x \qquad B_p = \prod_{\text{plaquet}} \sigma_j^z$$

star

plaquet

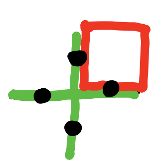
(FLIP SPINS)

TORIC CODE



- A's & B's all commute + each other

(SHARE ZERO OR TWO EDGES



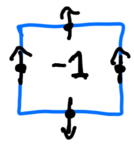
- Vortices correspond to loops with an odd # of spin flips

$$\boxed{W_p = \prod_{j \in \partial p} s_j}$$

$W_p = -1$ VORTEX.

$$\hat{\sigma}_j^z |\psi\rangle = s_j |\psi\rangle$$

$s_j = \pm 1$



Minimize energy $\Rightarrow$ $B_p |\psi\rangle = |\psi\rangle$

Requires states with <u>NO</u> <u>VORTICES</u>

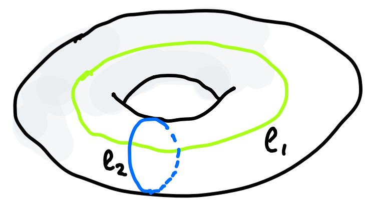
$$\Longrightarrow \quad |\psi\rangle = \sum_{\{s : W_p(s) = 1 \ \forall p\}} c_s |s\rangle$$

The condition $A_s|\psi\rangle = |\psi\rangle$ implies the state is unaffected by flipping spins on one star.

$\Rightarrow$ c_s equal for all configurations with no vortices.

TOPOLOGICAL G.S

$$W_\ell(s) = \prod_{j \in \ell} s_j \,, \quad \ell = \ell_1, \ell_2$$



LARGE CYCLES.

Absence of vortices means the $W_\ell(s)$ are the same for all paths ℓ_1 & all paths ℓ_2.

$$W_\ell(s) = \pm 1$$

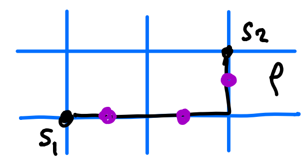
"Cohomology class of vortex free states"

A_s star operator does not affect $W_\ell(s)$.

$$|\psi\rangle = \sum c_{W_{\ell_1} W_{\ell_2}} |s\rangle$$

$$\boxed{D = (4)^g}$$

EXCITATIONS



"Electric Charges"

$$\hat{W}_\ell^{(E)} = \prod_{j \in \ell} \sigma_j^z$$

$$[W_\ell^{(E)}, B_p]$$

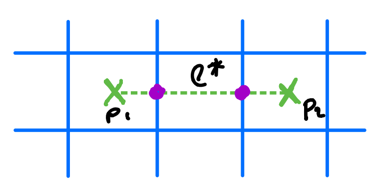
$$W_\ell^E A_{s_1} = - A_{s_1} W_\ell^E$$

$$|\psi^E_{s_1, s_2}\rangle = \hat{W}_\ell^E |\psi_0\rangle$$

costs energy $\Delta E = 2J_e$.

CHARGES ARE "DECONFINED"

"Magnetic Charges"



$$W_{\ell^*}^M = \prod_{j \in \ell^*} \sigma_\ell^z$$

$$\{B_{p_1}, W_{\ell^*}\} = 0$$

$$|\Psi^{M}_{p_1 p_2}\rangle = W^{M}_{\ell^*} |\Psi_0\rangle \qquad \Delta E = 2J_m$$

In the Honeycomb model, these excitations are degenerate because $J_m = J_e = \dfrac{J_x^2 J_y^2}{16 J_z^3}$.