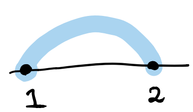
L12 **KITAEV SPIN LIQUIDS**

The idea that quantum zero point fluctuations might destroy antiferromagnetic order was probably first entertained by Landau + Pomeranchuk, who actually speculated that antiferromagnetism would never be stable against these fluctuations.

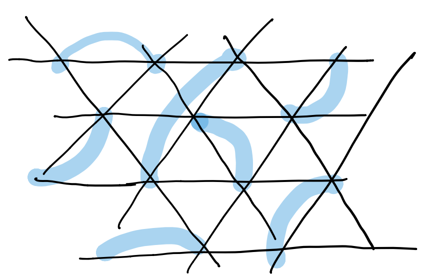
By the early 1950s, it became clear that in higher dimensions, simple antiferromagnets are stable, even for $S=1/2$.

In 1973 Anderson & Fazekas proposed that in a frustrated $S=1/2$ triangular lattice, the ground-state would not be AFM ordered but would instead be a linear combination of resonating valence bonds (RVB).

$$\hat{H} = \sum J_{ij} (\vec{S}_i \cdot \vec{S}_j)$$



$$(1,2) = \frac{1}{\sqrt{2}} \left(|\uparrow_1\rangle |\downarrow_2\rangle - |\downarrow_1\rangle |\uparrow_2\rangle \right)$$

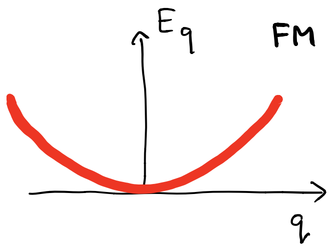


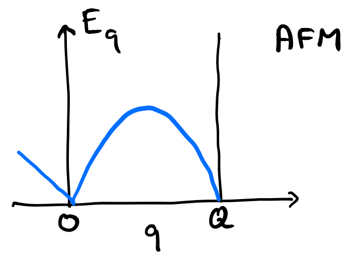
$$|\psi\rangle = \sum_P A_P \prod_i (i, P_i)$$

RVB QSL.

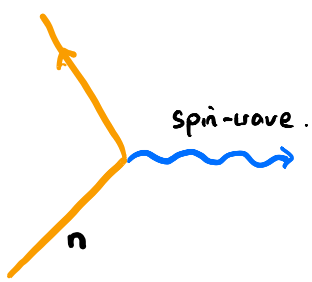
1987 S.C Lasco RVB = Mother state of cuprate S.C

In conventional magnets, the excitations carry $S=1$.





These spin-wave spectra can be seen using neutron scattering.

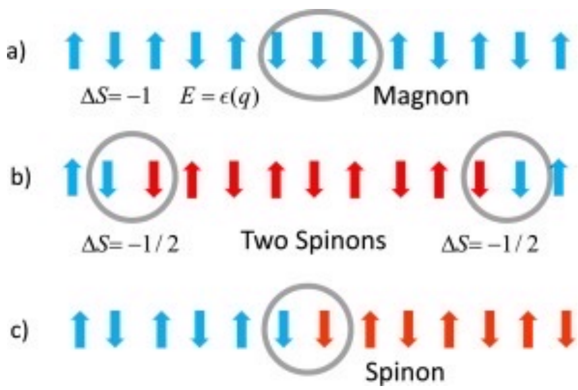


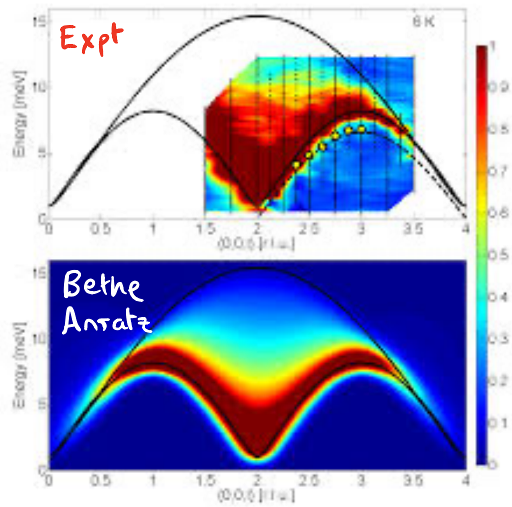
However, in a spin liquid, a spin flip can break-up, or <u>fractionalize</u> into mobile $S=1/2$ quasiparticles, or "spinons".

FRACTIONALIZATION IN 1D HEISENBERG CHAIN

$$\mathcal{H} = J \sum_i \left(\vec{S}_{i+1} \cdot \vec{S}_i \right)$$

Bethe c. 1931





$$\text{“} \vec{S} \longrightarrow f^{\dagger}_{\alpha} \left(\frac{\vec{\sigma}_{\alpha\beta}}{2} \right) f_{\beta} \text{”}$$

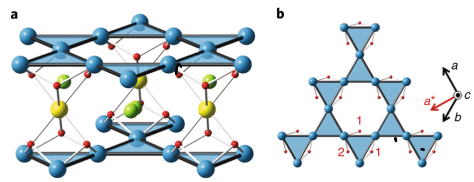
Spin breaks up into Spinons.

$$|\psi\rangle = P_G \prod_{k<k_F} \left(f^{\dagger}_{k\uparrow} f^{\dagger}_{-k\downarrow} \right) |0\rangle$$

$$P_G = \prod_j n_j (2 - n_j)$$

Higher Dimensional Spin Liquid Candidates.

- Organic charge transfer salts e.g κ-$(BEDT\text{-}TTF)_2$.
- "Herbertsmithite"
  $ZnCu_3(OH)_6Cl_2$

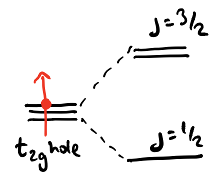


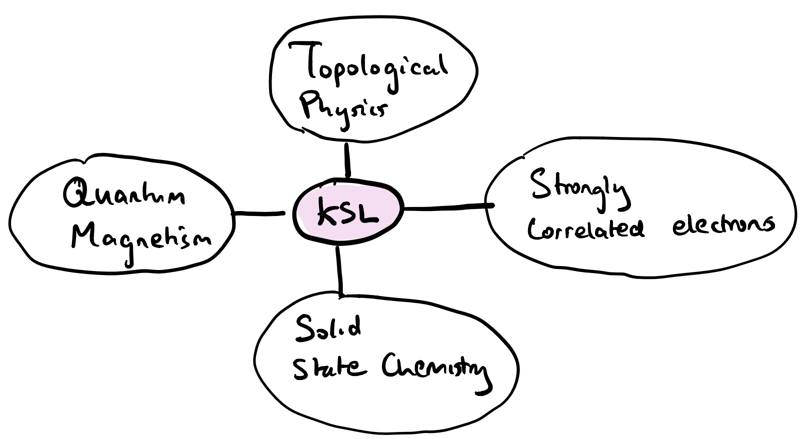
Kagome lattice of Cu $S = 1/2$ moments.

Kitaev Spin Liquids.

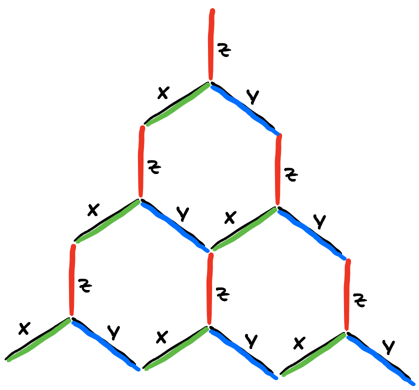
- Kitaev (2006, but worked out around 2002)
  Ann. Phys 321, 2-111 (2006)
- Jackeli + Khaliullin.
  PRL 102, 017205 (2009)

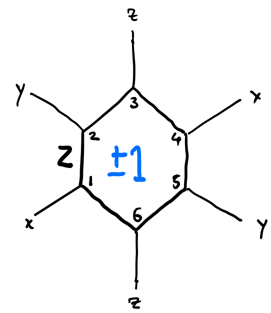
Iridates.





Kitaev Honeycomb Model





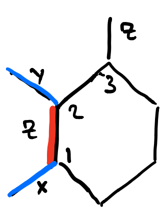
"Vortices"

$$\boxed{W_p = \sigma_1^x \sigma_2^y \sigma_3^z \sigma_4^x \sigma_5^y \sigma_6^z = \pm 1}$$

$$\boxed{H = -J_x \sum_{x\text{-links}} \sigma_j^x \sigma_k^x - J_y \sum_{y\text{-links}} \sigma_j^y \sigma_k^y - J_z \sum_{z\text{-links.}} \sigma_j^z \sigma_k^z}$$

$[W_p, H] = 0$ commute $\therefore$ $W_p = \pm 1$ constants VORTICES

$$\sigma^x \sigma^z = -\sigma^z \sigma^x$$



$$H = -J_z \sigma_1^z \sigma_2^z - J_y \sigma_2^x \sigma_3^x$$

$$W_p = \sigma_1^x \sigma_2^y \sigma_3^z \cdots\cdots$$

$$\sigma_1^x \sigma_2^y H_z = -\sigma_1^x H_z \sigma_2^y = +H_z \sigma_1^x \sigma_2^y$$

$$\sigma_2^y \sigma_3^y H_y = -\sigma_2^y H_y \sigma_3^y = +H_y \sigma_2^y \sigma_3^y$$

$$|\psi\rangle = |\{W_p\}\rangle \qquad \hat{W}_p |\psi\rangle = W_p |\psi\rangle$$

$$W_p = \pm 1$$

$$\left[\begin{aligned}\{d, d^\dagger\} &= 1 \\ d^2 = (d^\dagger)^2 &= 0\end{aligned}\right]$$

Majorana Fermions : (Brief Aside)

Conventional complex (DIRAC) fermions can be divided into a real & imaginary part

$$d^\dagger = \frac{a - ib}{2} \qquad \hat{d} = \frac{a + ib}{2} \qquad \begin{aligned} a = a^\dagger &= d + d^\dagger \\ b = b^\dagger &= \frac{(d - d^\dagger)}{i} \end{aligned}$$

"MAJORANA FERMIONS"

$$\boxed{\begin{aligned} a^2 = b^2 &= \{d, d^\dagger\} = 1 \\ ab &= -ba \end{aligned}}$$

- Many Majoranas : $\{\eta_a, \eta_b\} = 2\delta_{ab}$.

e.g (I) $H = \epsilon\left(d^\dagger d - \frac{1}{2}\right) \qquad E = \pm \epsilon/2.$

$$= \epsilon\left(\frac{(a - ib)(a + ib)}{4} - \frac{1}{2}\right) = \frac{i}{4}(a, b)\underbrace{\begin{pmatrix} 0 & \epsilon \\ -\epsilon & 0 \end{pmatrix}}_{\text{Antisymmetric}}\begin{pmatrix} a \\ b \end{pmatrix}$$

$$= \frac{i\epsilon ab}{2} \qquad \boxed{iab = \pm 1}$$

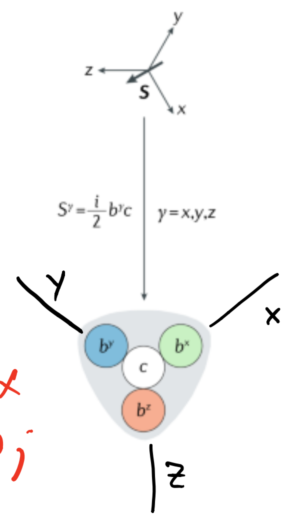
$$iab = (2d^\dagger d - 1)$$

$$d_{\vec{k}} = \sqrt{\frac{1}{2N}} \sum_k d_j e^{-i\vec{k}.\vec{R}_j} \qquad \{d_k, d^\dagger_{k'}\} = \delta_{\vec{k}\vec{k}'}$$

$$d_k^\dagger = d_{-k}$$ Lives in Half Brillouin Zone

$$(\sigma^x)^2 = i^2 (b^x c)(b^x c)$$
$$= -i^2 (b^x)^2 (c^2) = 1$$

Majorana Spin Representation.



$$\tilde{\sigma}^x = i b^x c$$
$$\tilde{\sigma}^y = i b^y c$$
$$\tilde{\sigma}^z = i b^z c$$

$\mathbb{Z}_2$ gauge inv

$$c_j \to (\pm 1)_j c_j \qquad b_j^\alpha \to (\pm 1) b_j$$

Like Pauli operators, they square to unity and they anticommute.

$$(\tilde{\sigma}^x)^2 = 1, \quad \tilde{\sigma}^x \tilde{\sigma}^y = -\tilde{\sigma}^y \tilde{\sigma}^x.$$

But they do not satisfy the correct commutation relations across the entire Fock space: to do this requires a constraint.

$$D|\psi\rangle = |\psi\rangle \qquad D = b^x b^y b^z c = +1$$

$[D, \tilde{\sigma}^a] = 0$, so this commutes with the spin operators.

Check. $\tilde{\sigma}^x \tilde{\sigma}^y \tilde{\sigma}^z = i$,

$$\tilde{\sigma}^x \tilde{\sigma}^y \tilde{\sigma}^z = i^3 b^x c b^y c b^z c = -i^3 b^x b^y b^z c = iD.$$

$\therefore$ correct properties wrt $D = 1$.

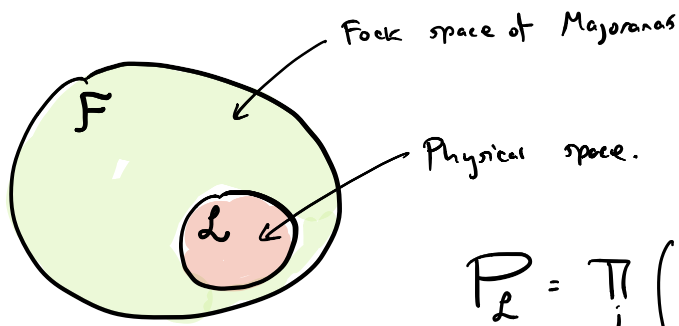
$$\tilde{\sigma}^x \tilde{\sigma}^y = iD\tilde{\sigma}^z \Rightarrow [\tilde{\sigma}^x, \tilde{\sigma}^y] = 2iD\tilde{\sigma}^z$$

$$\tilde{\sigma}_j^\alpha = \sigma_j^\alpha - T_j^\alpha$$

$$D_j = 1 \equiv T_j = 0$$

$$\mathcal{H} = -J_x \sum_x \tilde{\sigma}_l^x \tilde{\sigma}_m^x - J_y \sum_y \tilde{\sigma}_l^y \sigma_m^y - J_z \sum_z \tilde{\sigma}_l^z \tilde{\sigma}_m^z$$

On the lattice we impose $D_j |\psi\rangle = |\psi\rangle$



$$P_{\mathcal{L}} = \prod_j \left(\frac{1 + D_j}{2} \right)$$

$$-J_\alpha \tilde{\sigma}_j^\alpha \tilde{\sigma}_k^\alpha = -J_\alpha (i b_j^\alpha c_j)(i b_k^\alpha c_k)$$

"Z_2 gauge inv"

$$= -J_\alpha c_j c_k (b_j^\alpha b_k^\alpha)$$

$c_j \to -c_j$

$u_{jk} \to -u_{jk}$

$$= J_\alpha c_j c_k \hat{u}_{jk}$$

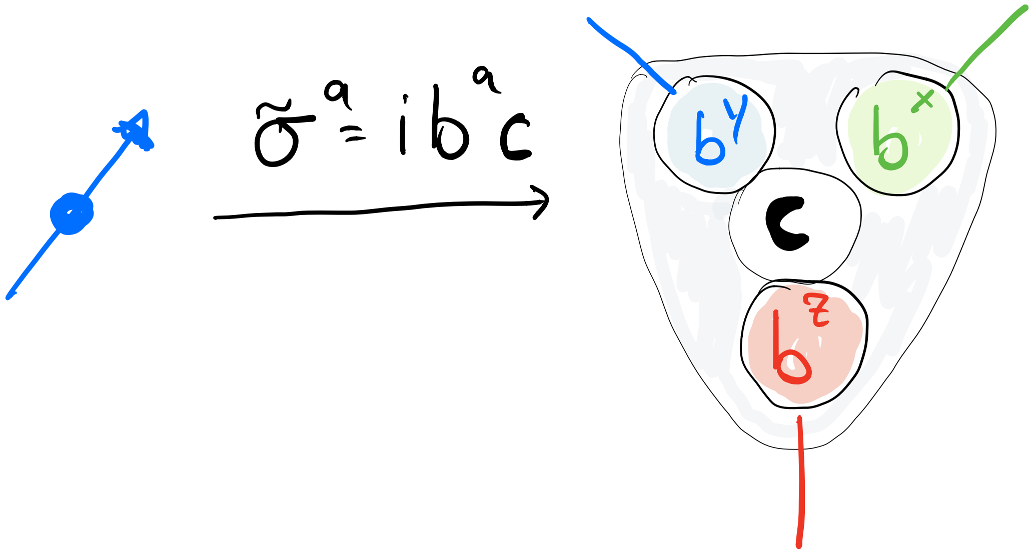
Where $\hat{u}_{jk} = b_j^\alpha b_k^\alpha$. $\alpha \equiv \alpha(j,k)$.

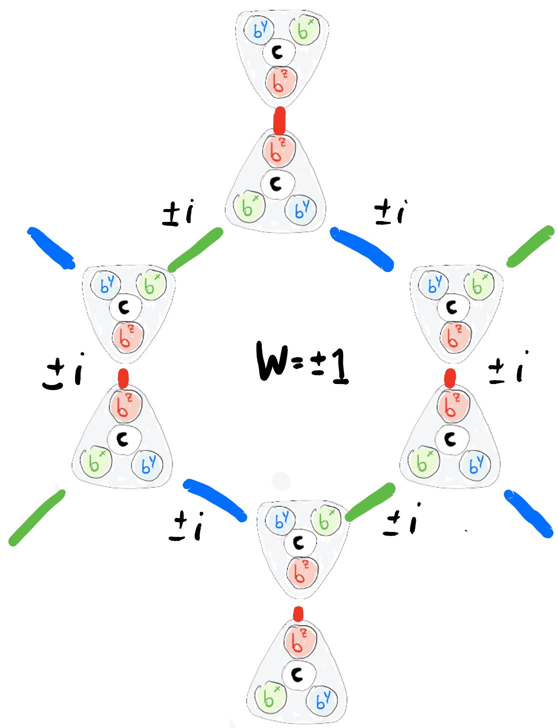
$$[\hat{u}_{jk}, \mathcal{H}] = 0 \implies \hat{u}_{jk} |\psi\rangle = u_{jk} |\psi\rangle$$

$$u_{jk} = \pm i$$

"Z_2 gauge field"

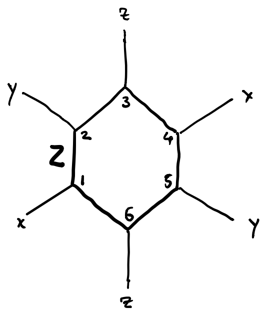
$(\vec{\tilde{\sigma}} = i\vec{b}\,c)$





$\mathbb{Z}_2$ gauge invariance

$$\left.\begin{aligned} c_j &\to Z_j\, c_j \\ u_{jk} &\to Z_j Z_k\, u_{jk} \end{aligned}\right\} \quad \begin{aligned} &\hat{\mathcal{H}} \text{ unchanged} \\ &\hat{W}_p \text{ unchanged} \end{aligned}$$



$$b_j^x c_j b_j^y b_j^z = 1$$

$$b_j^x c_j = b_j^z b_j^y = -b_j^y b_j^z$$

$$\boxed{W_p = \sigma_1^x \sigma_2^y \sigma_3^z \sigma_4^x \sigma_5^y \sigma_6^z = \pm 1}$$

$$= -(b_1^x c_1)(b_2^y c_2)(b_3^z c_3)(b_4^x c_4)(b_5^y c_5)(b_6^z c_6)$$

$$D_j = b_j^x b_j^y b_j^z c_j = 1 \Rightarrow b^x c = -b^y b^z$$

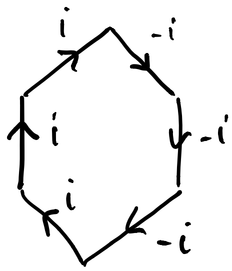
$$\hat{W}_p = -(b_1^y b_1^z)(b_2^z b_2^x)(b_3^x b_3^y)(b_4^y b_4^z)(b_5^z b_5^x)(b_6^x b_6^y)$$

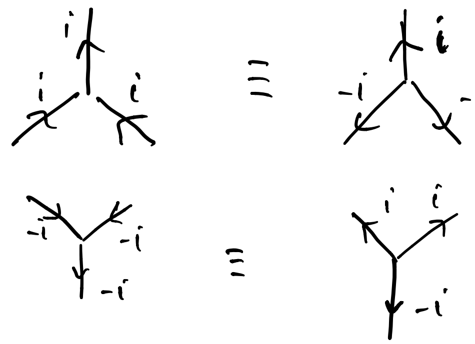
$$= \hat{u}_{12}\hat{u}_{23}\hat{u}_{34}\hat{u}_{45}\hat{u}_{56}\hat{u}_{61}$$

$$= \prod_{\langle j,k\rangle \in \partial p} \hat{u}_{jk} \Rightarrow W_p = \prod_{\langle j,k\rangle \in \partial p} u_{jk}$$

$u_{jk} = -u_{kj}$

Lowest energy state : no vorticity. Choose uniform phase $u_{jk} = i$. in $+x$, $+y$, $+z$ directions

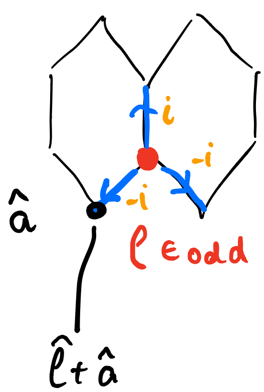




$$\hat{H} = \frac{1}{4} \sum_{\langle j,k \rangle} A_{jk}\, c_j c_k$$

$$A_{jk} = 2J^{\alpha(j,k)}\, u_{jk}$$

$\sqrt{\frac{2}{N}} \sum_k c_k e^{i\vec{k}\cdot\vec{R}_l}$



$$\mathcal{H} = \frac{1}{4} \sum A_{l+\hat{a}\, l}\, \hat{c}_{l+a}\, \hat{c}_{l} + h.c.$$

$\hat{a} = \hat{x}, \hat{y}, \hat{z}$

l = odd subjattice.

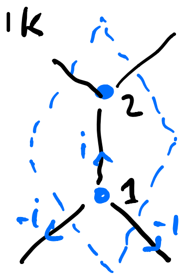
Let
$$c_{\vec k} = \frac{1}{\sqrt{2N}} \sum c_j\, e^{-i\vec k\cdot\vec R_j}$$
$$c^{\dagger}_{\vec k} = \frac{1}{\sqrt{2N}} \sum c_j\, e^{i\vec k\cdot\vec R_j}$$

$$\{c_{k}, c^{\dagger}_{k'}\} = \frac{1}{2N}\sum_{j,l} \overbrace{\{c_j, c_l\}}^{2\delta_{jl}}\, e^{-i\vec k\cdot\vec R_j + i\vec k'\cdot\vec R_l} = \delta_{\vec k,\vec k'}$$

$$c_j = \sqrt{\frac{2}{N}} \sum_{k} c_{\vec k}\, e^{i\vec k\cdot\vec R_j}$$

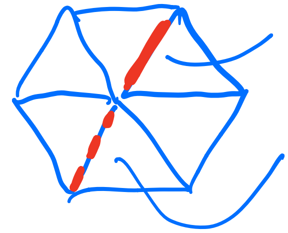
$$\mathcal{H} = \frac{1}{4}\sum_{j\in \text{odd}} \left(\frac{2}{N}\right) A_{l+\hat a, l}\, e^{-i\vec k(\hat R_l + \hat n_{\hat a})}\, e^{i\vec k'\cdot\vec R_l} \left(c^{\dagger}_{2k} c_{1k'}\right) + h.c.$$

$$= \Big[\, \overset{u_{jk}}{\frac{i}{4}} \sum_{k\in BZ} \left(2J_z - 2J_x e^{i\vec k\cdot\vec n_1} - 2J_y e^{i\vec k\cdot\vec n_2}\right) c^{\dagger}_{2k} c_{1k} + h.c.$$

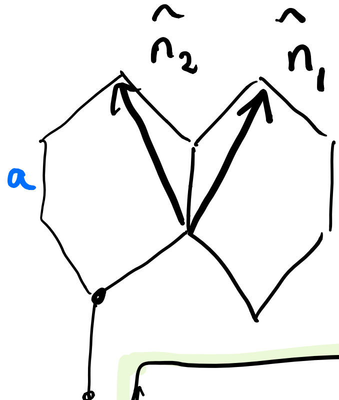


$$= \frac{1}{2}\sum_{k} \left(c^{\dagger}_{2k}, c^{\dagger}_{1k}\right) \begin{pmatrix} 0 & if(k) \\ -if(k) & 0 \end{pmatrix} \begin{pmatrix} c_{2k} \\ c_{1k} \end{pmatrix}$$

$$\overset{'''}{=} \sum_{k\in\frac{1}{2}BZ} \psi^{\dagger}_{k}\, \underline{h}(k)\, \psi_{k}$$



$[u^x_+, u^x_-] \neq 0.$

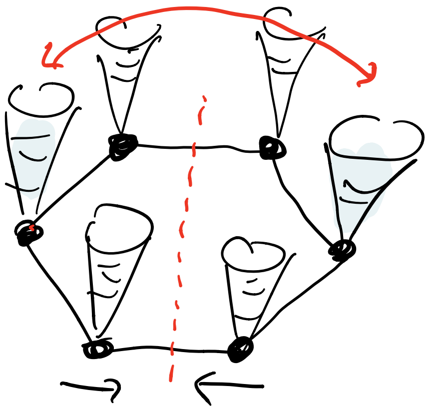


$$\hat{n}_1 = \left(\frac{3}{2}a, \frac{\sqrt{3}}{2}a\right)$$

$$\hat{n}_2 = \left(\frac{3}{2}a, -\frac{\sqrt{3}}{2}a\right)$$

$$\boxed{\begin{aligned} f(k) &= \left(J_z - J_x e^{i\vec{k}\cdot\vec{n}_1} - J_y e^{i\vec{k}\cdot\vec{n}_2}\right) \\ \epsilon(k) &= \pm |f(k)| \end{aligned}}$$

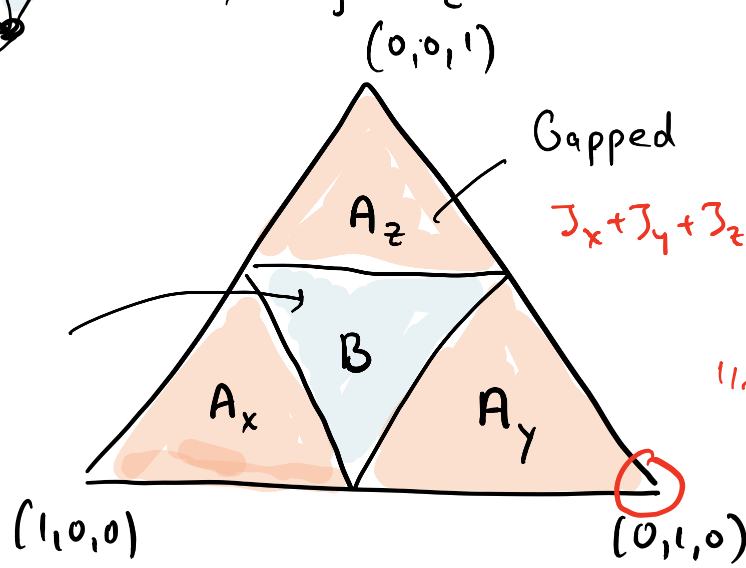
equivalent $c_k \equiv c^\dagger_{-k}$



$J_z > J_x = J_y$

Gapless.

$J_x = J_y = J_z$



$J_x + J_y + J_z = 1$

"Toric code"