- W 10.20 F 1.40 Zoom
- Office hour Monday 5-6 (?)

Concepts
- FRACTIONALIZATION
- STRANGE METALS

1980 — beginings of ideas of "EMERGENCE"

fractionalization almost unknown

SSH model. — mobile solitons

"Fractionalization" *

"Topological order"

"Quantum entanglement"

"SPT ≡ Symmetry protected Topological Phases"

This course will present a brief introduction to the concept of fractionalization and the unsolved problem of strange metals. Fractionalization is the phenomenon whereby a system develops excitations that carry quantum numbers that are fractions of the microscopic degrees of freedom, such as 1/3rd charges in the fractional quantum Hall effect, or neutral spin 1/2 solitons in polyacetylene. Strange metals is the term we give to the unusual metallic state found at optimal doping in high temperature superconductors and related materials, characterized by a robust linear resistivity and electrical transport properties that can not be understood in terms of Landau Fermi liquid theory. Since the strange metal problem is unsolved, I can't say whether it has any link to fractionalization, but many people suspect it does, and for this reason I thought it would be fun to treat the two subjects in a single course.

I started graduate school in 1980, not long after physicists had renamed the older field of solid state as condensed matter physics. Though the early condensed matter physicists were optimistic about the future, like every era in physics, few could have imagined the discoveries and the huge transformations in ideas that lay ahead. Solid state physics, the field that lasted from about 1925-1980, took an essentially reductionist view of electronic matter - in which the excitations were built up out of electrons, and their bosonic condensates, plus the collective excitations of the electrons, the order parameter and ions. The idea of fractionalization was more-or-less unknown.

The discoveries that have taken place in the intervening decades have fundamentally changed our view of physics. They have opened our eyes to the huge possibilities of emergence, the idea that collective behavior of matter leads to properties that require new conceptual frameworks for their understanding. Out of these discoveries has arisen a new lexicon - words like

"Fractionalization",
"Topological order",
"Quantum Entanglement",
"SPT = Symmetry protected Topological phases"

Some of these ideas are linked, and the community is still arguing about the true meaning of words like "topological order". In this course we'll focus on the first concept, though we will allude to the others, and we'll look at some simple examples of fractionalization and the experiments that have inspired them. We'll see that topology and fractionalization are often, but not exclusively linked.

Useful texts:

Soliton Excitations in Polyacetylene, W. P. Su, J. Robert Schrieffer and Alan J. Heeger PRB 22, 2099 (1980)

Solitons with Fermion Number 1/2 in Condensed Matter and Relativistic Field Theories, Roman Jackiw and J. Robert Schrieffer, Nucl Phys. B 190 253-265 (1981)

Lectures on the Quantum Hall Effect, David Tong

Topological Superconductors and Category Theory, Titus Neupert and Andrei Bernevig

Topological Phases and Quantum Computation, Alexei Kitaev and Chris Laumann arXiv: 0904.2771

Heavy Fermions and the Kondo Lattice, a 21st C. perspective, Piers Coleman, arXiv: 1509.05769

Universal T-linear resistivity and Planckian dissipation in overdoped cuprates, A. Legions eat al, Nature Physics, 15, 142-147 (2019).

Fractionalization: *the concept that excitations of a many-body system can carry fractional quantum numbers.*

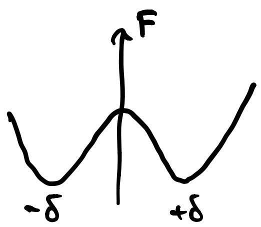
Examples

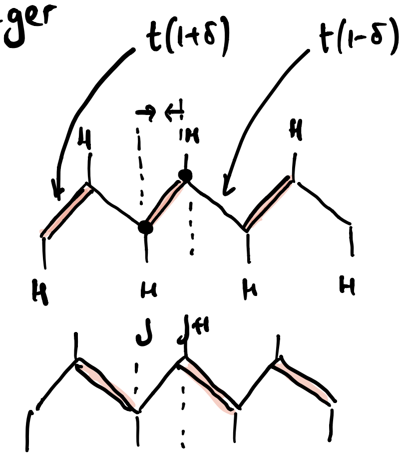
Solitons in Polyacetylene $(CH)_n$

Su, Schrieffer + Heeger 1980

"Peierls distortion"

$2k_F$



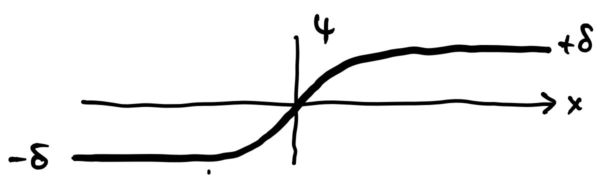


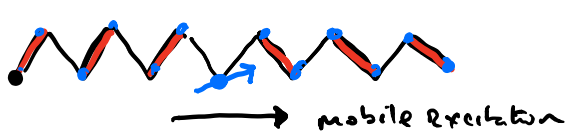
$$t_{j+1,j} = t - \underbrace{\alpha(u_{j+1}-u_j)}_{= +\psi(-1)^j}$$

$\psi = -\delta$

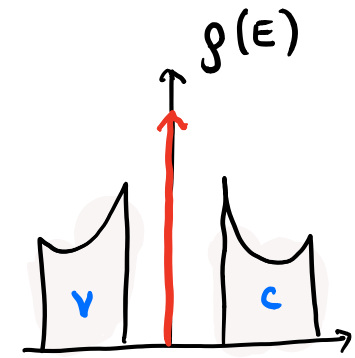
$\psi = +\delta$

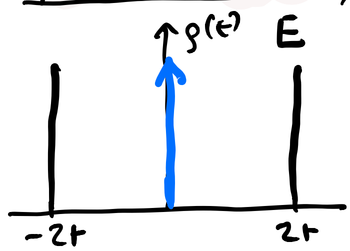
BROKEN Z_2 SYMMETRY





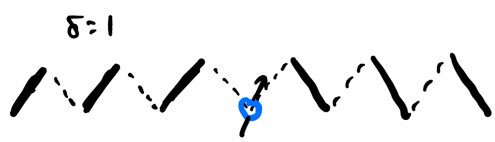
$q = 0 \quad s = 1/2$

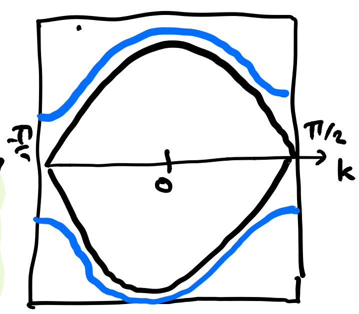




$$\rho(E) = \sum_\lambda \delta(E - E_\lambda)$$

$\delta = 1$ extreme limit (adiabatically connected to $\delta < 1$)



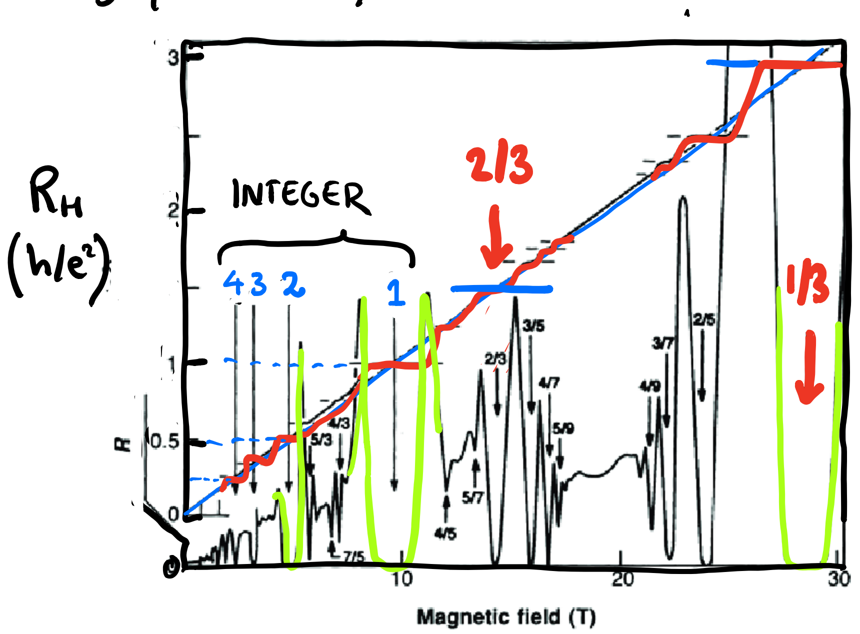


$H = (-i\underline{\alpha}\,\partial_x + \Delta(x)\underline{\beta})$

Topological defects of Dirac Eqn

"ANYONS"

$$R_H \hat{=} \rho_{xy} = V_x / I_y.$$



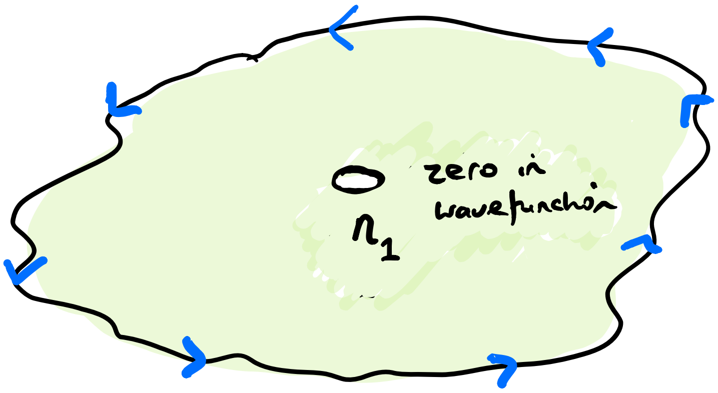
$$R_H = \rho_{xy} = \frac{h}{e^2 \nu}$$

$\nu = n$ INTEGER	$\nu = p/m$ FRACTIONAL

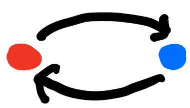
m = integer.

Stormer, Tsui, Gossard (1982)

edge currents



"Anyon"



$$e^{i\pi/m}$$

FRACTIONAL STATISTICS.
(Arovas, Schrieffer, Wilczek)

$$\Psi_0 = \prod_{i,j} (z_i - z_j)^m \, e^{-\sum \frac{|z_j|^2}{4\ell_0^2}}$$

LAUGHLIN 1982

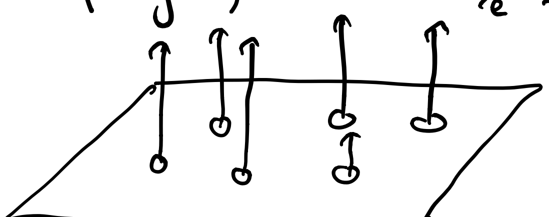
$$q = +\frac{|e|}{m}$$

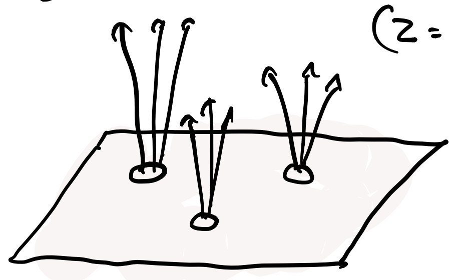
FRACTIONAL CHARGE
(Laughlin)

$$\Psi = \prod_{j=1}^{N} (z_j - \eta_1)\, \Psi_0$$

$h/e = \Phi_0$

$(z = x + iy)$





$$\nu = \left(\frac{\Phi}{N\Phi_0}\right)$$

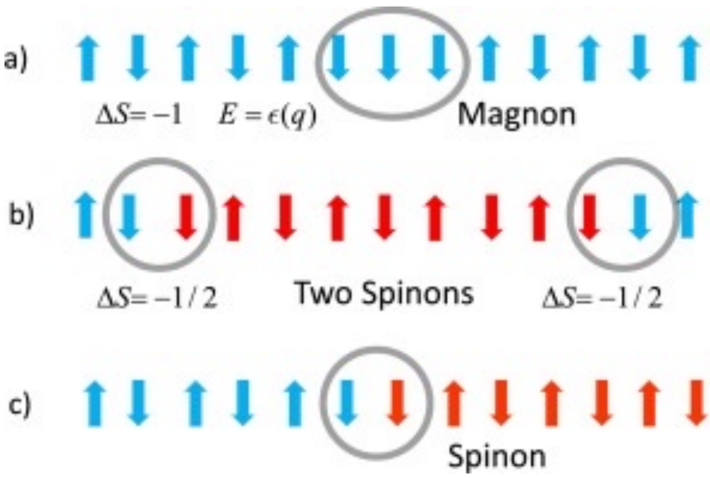
$\nu = 1$

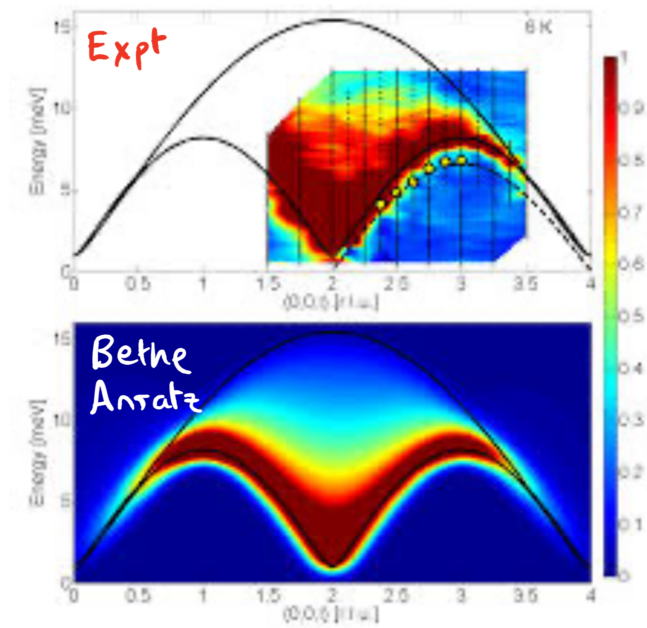
"composite fermions"
(Read + Jain).

FRACTIONALIZATION IN HEISENBERG CHAIN

$$\mathcal{H} = J \sum_i (\vec{S}_{i+1} \cdot \vec{S}_i)$$

AFM $\underline{\underline{J > 0}}$





$$\vec{S} \longrightarrow f^{+}_{\alpha} \left(\frac{\vec{\sigma}_{\alpha\beta}}{2} \right) f_{\beta}$$

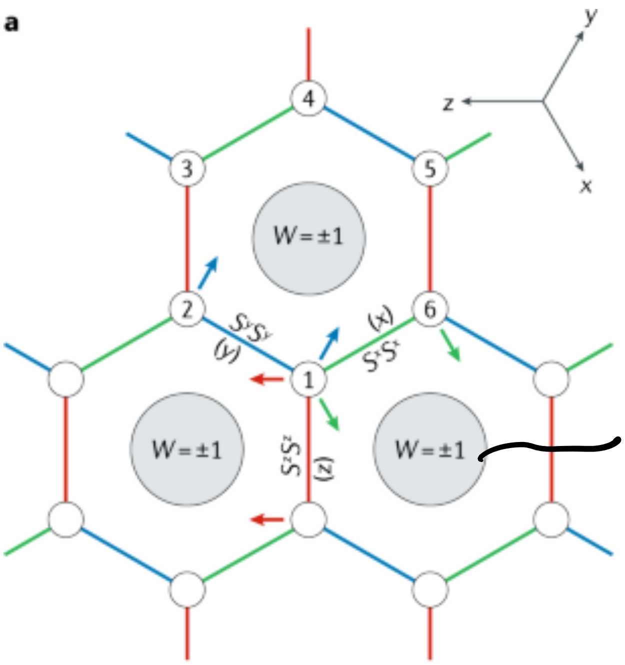
Spin breaks up into Spinons.

Kitaev Spin Liquid.

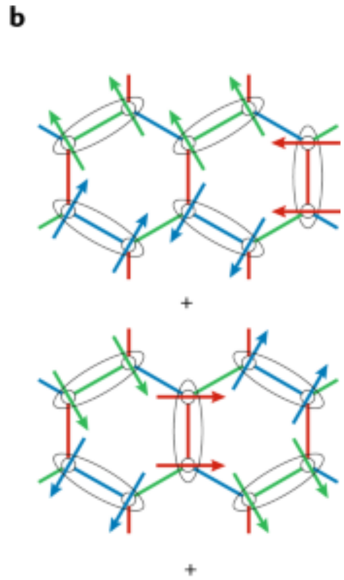
Kitaev, 2006

$$H = -J\left(\sum_{(i,j)\in x} S_i^x S_j^x + \sum_{(i,j)\in y} S_i^y S_j^y + \sum_{(i,j)\in z} S_i^z S_j^z\right)$$

x bonds, y bonds, z bonds

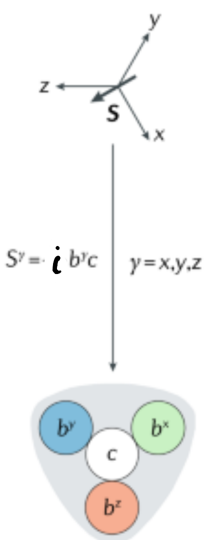


"viton"



FRACTIONALIZATION

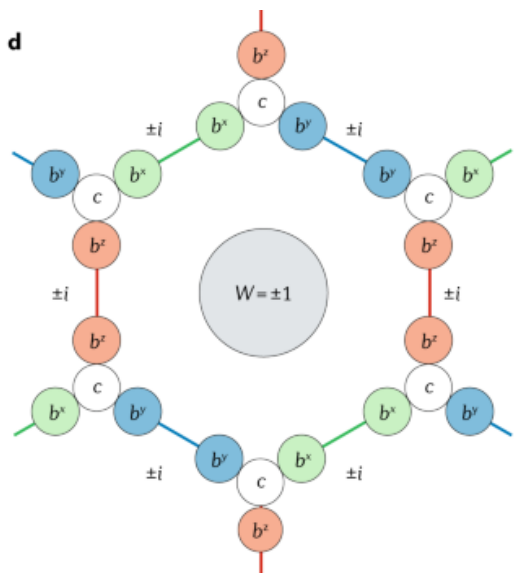
$$\vec{S} = i\,\vec{b}\,c$$



$b^y = (b^y)^\dagger$

$c = c^\dagger$

(c.f. $c = \frac{a+a^\dagger}{\sqrt{2}}$)



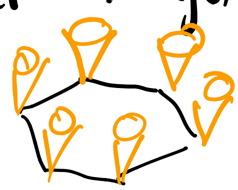
"MAJORANA FERMIONS"

$1 = \{c, c^\dagger\} = \{c, c\} = 2c^2$

$c = \frac{a+id}{\sqrt{2}} \Leftrightarrow a = \frac{c+c^\dagger}{\sqrt{2}} = a^\dagger$

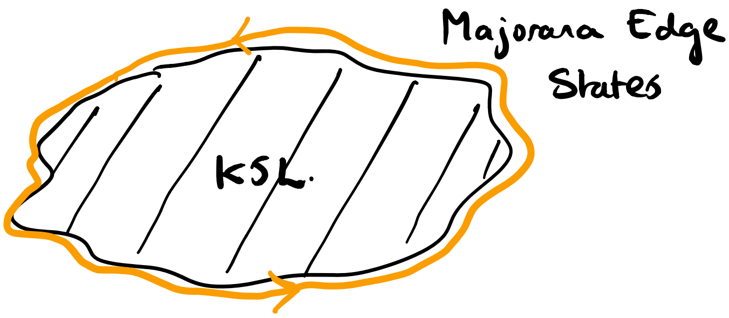
$d = \frac{c-c^\dagger}{\sqrt{2}\, i} = d^\dagger$

c - fermions form a neutral ~~metal~~ Majorana metal.

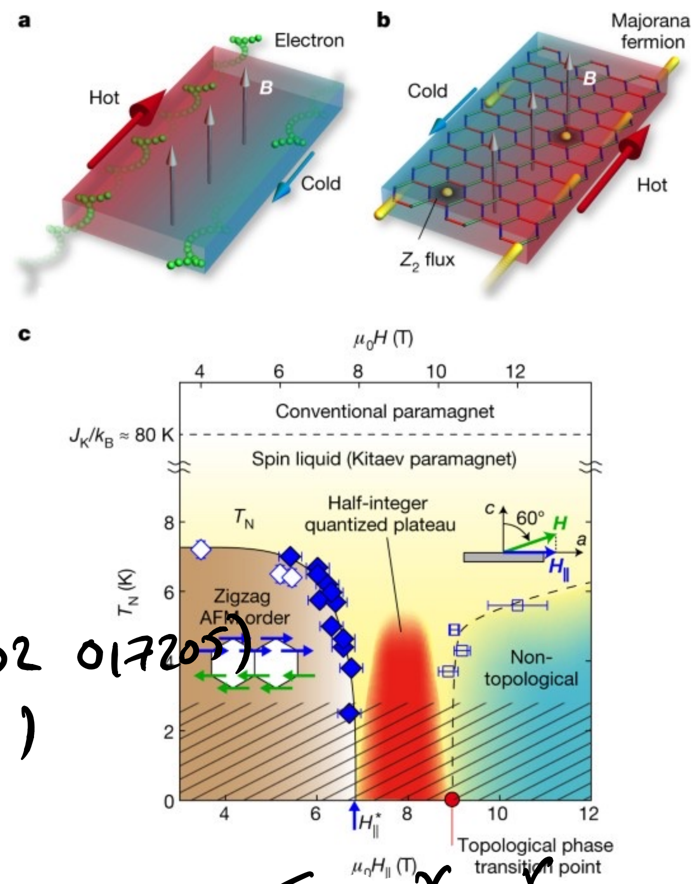


$c_{\vec{k}} = c^\dagger_{-\vec{k}}$

"Viton" excitation.



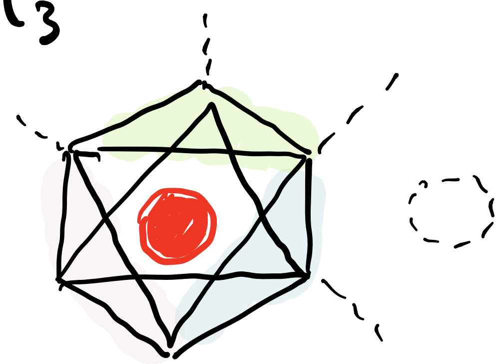
Fractional Thermal Hall Effect



Kitaev (2006).

Jakel + khaliunin (PRL, 102 017205)
edge sharing octahedra (2009)

$RuCl_3$



$$H = J_K \sum \sigma_i^\gamma \sigma_j^\gamma$$
$$+ J_H \sum \vec{\sigma}_i \cdot \vec{\sigma}_j$$
$$+ \Gamma \sum \left(\sigma_i^\alpha \sigma_j^\beta + \sigma_i^\beta \sigma_j^\alpha \right)$$

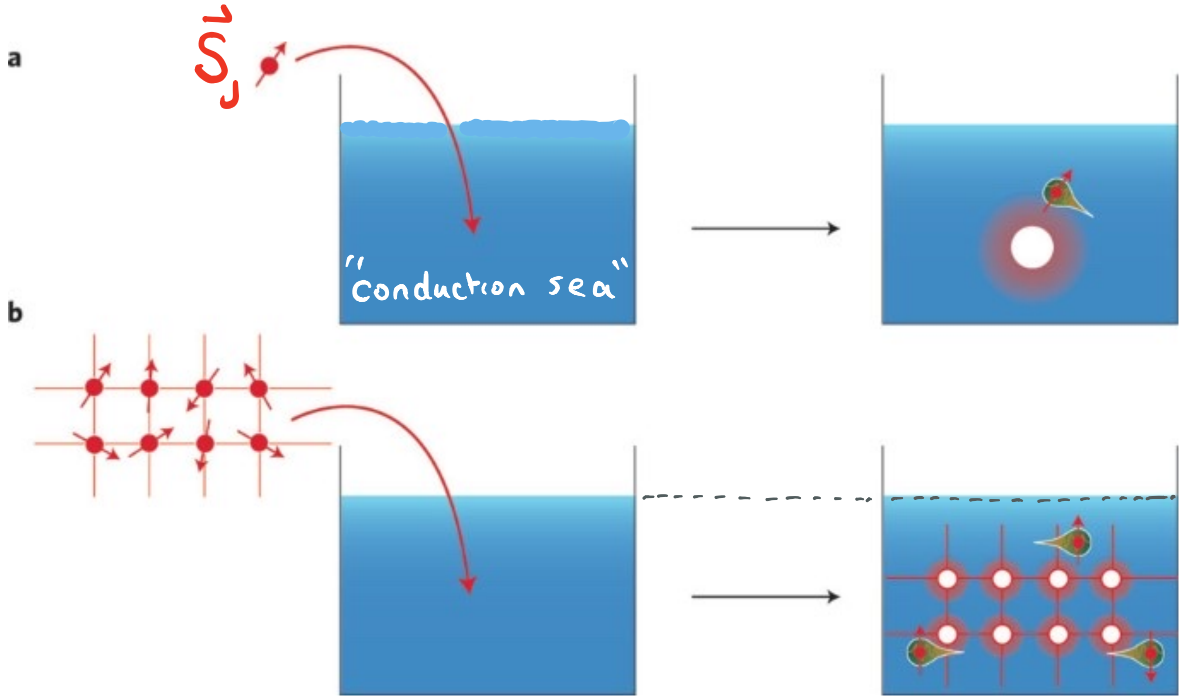
ArXiV 1901.05283

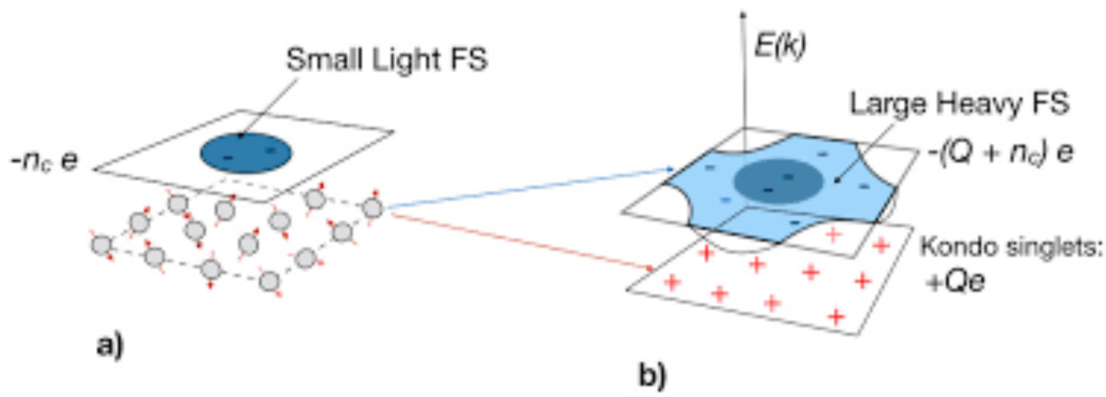
Maria Hermans.

HEAVY FERMIONS

IMPURITY

LATTICE





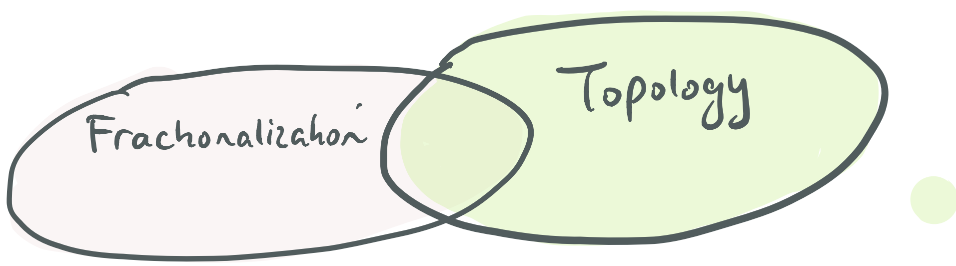
$$\vec{S} \longrightarrow f^{\dagger}_{\alpha} \left(\frac{\vec{\sigma}}{2} \right)_{\alpha\beta} f_{\beta}$$

FRACTIONALIZATION

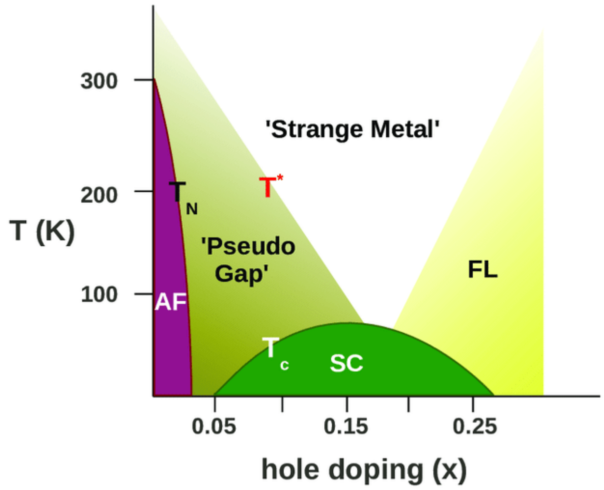
$$2 \frac{V_{FS}}{(2\pi)^d} = n_e + n_s$$

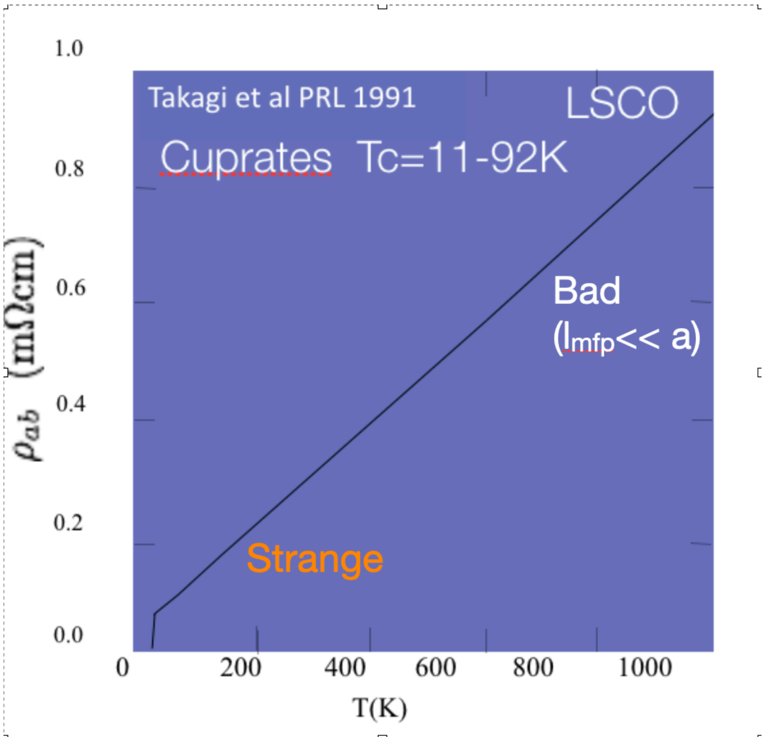
Oshikawa, (2000).

Fractionalized Excitation	Model/System	Charge/Spin	Microscopic Excitation	Topology?
Soliton	Polyacetylene	$Q=0$ $S=1/2$ $Q=\pm e$ $S=0$	electron	✓
Anyon	FQHE	$Q=e/m$	electron	✓
Spinon	Heisenberg AFM	$S=1/2$	magnon $\Delta S_z=1$	—
Majorana	Kitaev Honeycomb	$S=0$	spin $\Delta S=1$	✓
Heavy Fermion	Kondo Lattice	$S=1/2$ $q=e$	spin $\Delta S=1$	



STRANGE METALS





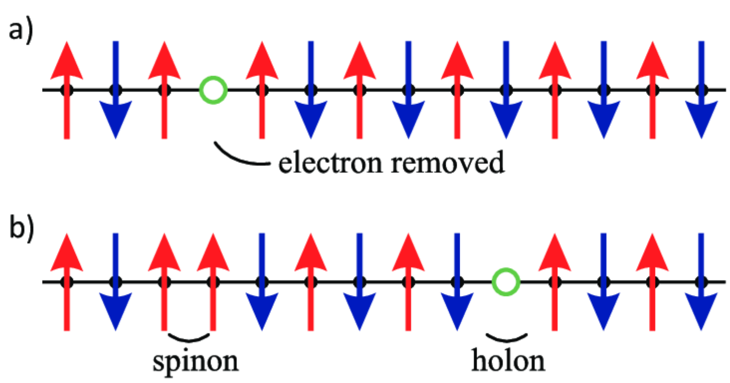
Drude

$$\tau = \left(\frac{\ell}{v_F}\right)$$

$$\tau = \frac{\hbar}{k_B T} \qquad \text{"PLANCKIAN DISSIPATION"}$$

$$\Rightarrow R = \frac{1}{\sigma} = \left(\frac{\hbar}{e^2}\frac{m}{n}\right) k_B T$$

In 1D electrons fractionalize into spinons & holons



Does something analagous happen in STRANGE METALS?