L19: NUCLEAR PHYSICS

Over the past half century or so, our civilization has been profoundly influenced by nuclear physics – the ending of the war in the Pacific by the two atom bombs on Hiroshima and Nagasaki, and the ensuing cold war that lasted almost forty years were events in history tied up with nuclear physics. Today that influence continues ~ MRI scanners routinely probe the nuclear spins in our body ~ with global warming Nuclear Energy is once again in the news as a possible alternative to fossil fuels –

we still live under the threat of nuclear weapons — and the sun — our local "fusion" reactor — continues to shine and power the photosynthesis that keeps our world alive.

In short, nuclear physics is something the informed citizen, especially the scientist and engineer, needs to know about.

43.1 PROPERTIES OF NUCLEII

The atom is mostly empty space.

$r_{\text{atom}} \sim 1\text{Å} \equiv 10^{-10}\,\text{m}$

$r_{\text{nucleus}} \sim 1\text{fm} \equiv 10^{-15}\,\text{m}.$

The nucleus contains almost all of the atoms mass. We can approximately measure the size of a nucleus in a Rutherford scattering experiment. From a detailed analysis of the way α-particles & protons are scattered off the nucleus we know that

$$\boxed{R = \underset{\underset{1.2\,\text{fm}}{\uparrow}}{R_0}\, \underset{\underset{\text{nucleon number}}{\uparrow}}{A}^{1/3}}$$

Each "nucleon" inside a nucleus has a mass approximately equal to

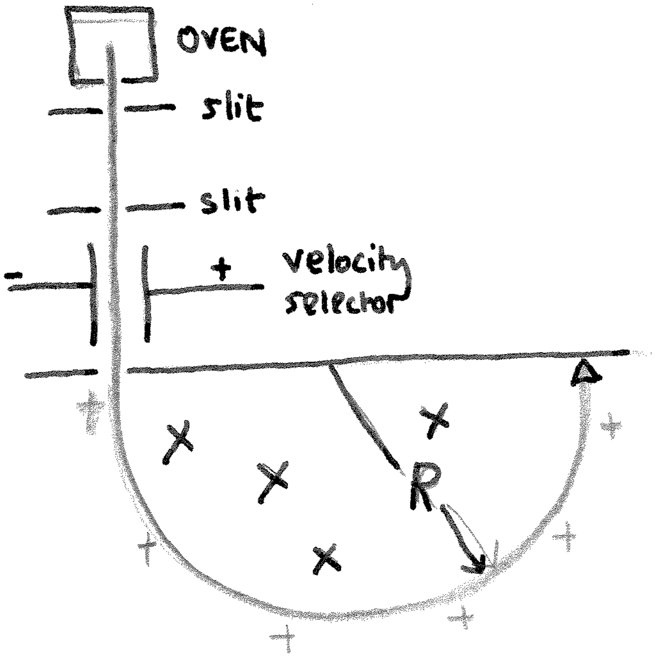
$$1u = 1.66\ldots \times 10^{-27}\,\text{kg} \equiv 931.5\,\text{MeV}$$

(Carbon atoms are by definition 12u), so that

$$\boxed{m_{\text{nucleus}} \approx A\,u}$$

MASS SPECTROMETER

Nuclear masses are measured by passing ions through a mass spectrometer.



$$\frac{mv^2}{R} = evB$$

$$\Rightarrow m = \frac{eBR}{v}$$

RADIUS DETERMINES MASS

The relationship $R = R_0 A^{1/3}$ implies that the nuclear volume

$$V = \frac{4}{3}\pi R^3 = \left(\frac{4}{3}\pi R_0^3\right) \times A$$

is proportional to the number of nucleons. Each nucleon has a volume $\frac{4}{3}\pi R_0^3$.

e.g calculate the radius and matter density inside an iron nucleus with $A = 56$.

a) $R = R_0 A^{1/3} = 1.2\,\text{fm}\,(56)^{1/3} = 4.6\,\text{fm} \equiv 4.6 \times 10^{-15}\,\text{m}$

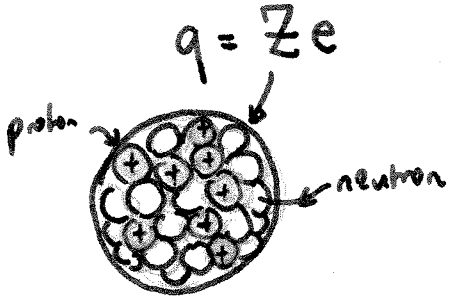
b) $\text{Nuclear density} = \frac{\text{mass}}{\text{volume}} = \frac{m}{\frac{4}{3}\pi R^3} = \frac{uA}{\frac{4}{3}\pi R_0^3 A} = \frac{u}{\frac{4}{3}\pi R_0^3}$

$\boxed{\text{does not depend on } A}$

$$\frac{\text{mass}}{\text{volume}} = \frac{1.66 \times 10^{-27}\,\text{kg}}{\frac{4}{3}\pi \times (1.2 \times 10^{-15})^3} = \underline{2.3 \times 10^{17}\,\text{kg/m}^3}$$

(10^{13} × higher than iron metal)

Nuclides & Isotopes



A nucleons

Z protons

There are

$$N = A - Z$$

neutrons.

Q. Is a neutron an electrical bound state of an electron and proton?

A. This was a great controversy for many years. However from the uncertainty principle we know that if an electron is confined within $\Delta x \sim 10^{-15}$ fm its energy is

$$KE \sim \frac{\hbar^2}{2m\Delta x^2} \approx \frac{(10^{-34})^2}{(10^{-30}\,\text{kg})\,(10^{-30}\,\text{m}^2)} \sim 10^{-8}\,\text{J}$$

$$\sim 10^{11}\,\text{eV}\ !$$

This is much larger than electrical, or even nuclear binding energies_

The neutron was discovered by James Chadwick in 1935.

It is a distinct, neutral particle.

$$\left\{\begin{array}{llll} \text{proton} & m_p & = 1.0073\,u & = 1.673\times10^{-27}\,\text{kg} \\ \text{neutron} & m_n & = 1.0087\,u & = 1.675\times10^{-27}\,\text{kg} \\ \text{electron} & m_e & = 0.000549\,u & = 9.109\times10^{-31}\,\text{kg} \end{array}\right\} \begin{array}{l} m_p, m_n \\ \approx 2000\,m_e \end{array}$$

Nuclide = nucleus with specific $A, Z, \underset{\substack{\uparrow \\ \text{neutron} \\ \text{number.}}}{N} = A - Z$.

$${}^{\overset{\text{nucleon number}\searrow}{A}}_{\underset{\nearrow\ \text{atomic \#}}{Z}}\underset{\substack{\uparrow \\ \text{element}}}{X}$$

		A	Z	N
${}^{1}_{1}H$		1	1	0
${}^{2}_{1}H$ = deuterium ≡ ${}^{2}_{1}D$		2	1	1
${}^{12}_{6}C$	(99%)	12	6	6
${}^{13}_{6}C$	(1%)	13	6	7
${}^{238}_{92}U$	(99.27%)	238	92	143
${}^{235}_{92}U$	(0.72%)	235	92	140

Isotopes ≡ nuclides with same # protons Z

≡ atoms with same chemistry, but different masses.

CAN NOT BE SEPARATED CHEMICALLY.

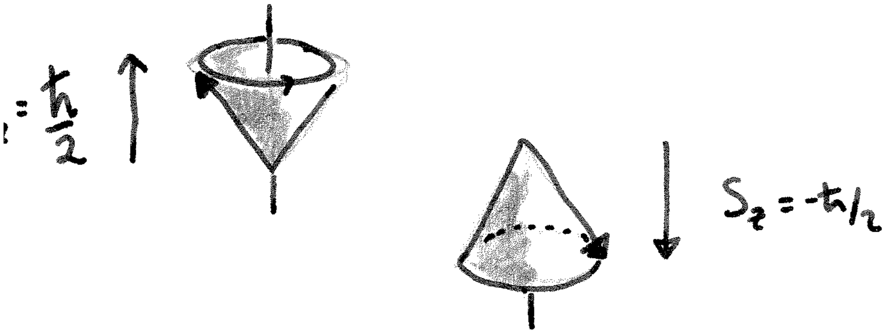
Nuclear fuel rods	3% U-235
Weapons grade U	> 90% U-235

(Can be purified isotopically using centrifuges).

NUCLEAR SPIN + MAGNETIC MOMENTS

$$\boxed{\begin{array}{l} S = \sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)}\,\hbar = \sqrt{\frac{3}{4}}\,\hbar \\ S_z = \pm \hbar \end{array}}$$

like the electron, protons & neutrons have spin-$\frac{1}{2}$



When combined inside the nucleus, the <u>total</u> angular momentum $\vec{J}$ has magnitude

(OFTEN CALLED "NUCLEAR SPIN")

$$J = \sqrt{j(j+1)}\,\hbar$$

$$J_z = m_j \hbar \qquad (m_j = -j, \dots, +j)$$

A even $\Longrightarrow$ j an integer 0, 1, 2 ....

A odd $\Longrightarrow$ j a half integer $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ ...

Like electrons, protons & neutrons have a magnetic moment.

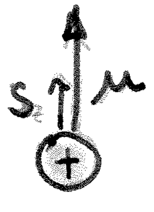
Since their mass is much greater we no longer use $\mu_B = e\hbar/(2m_e)$. Now we use the nuclear magneton

$$\mu_n = \frac{e\hbar}{2m_p} = 5.05\times10^{-27}\,\mathrm{J/T} = 3.15\times10^{-8}\,\mathrm{eV/T}$$

↑

proton mass ≃ 1836 m_e. $\mu_n \sim \frac{1}{1836}\mu_B$.

$$|\mu_{sz}|_{\text{proton}} = 2.79\,\mu_n$$



proton

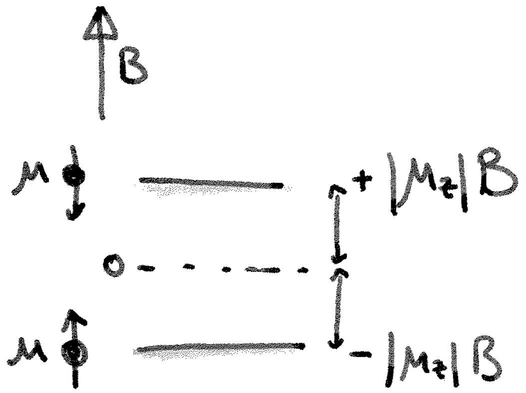
$$|\mu_{sz}|_{\text{neutron}} = 1.91\,\mu_n$$

neutron.

(as if negatively charged.)

Anomalous moments are due to the fact that nucleons are made up of quarks.

In a field



$$U = -\vec{\mu}\cdot\vec{B} = -\mu_z B$$

e.g. How much energy is required to flip a proton spin in a 2 Tesla field?

$$\Delta U = 2|\mu_z|B = 2\times\left(2.79\times 3.152\times10^{-8}\,\text{eV/T}\right)\times 2\,\text{T}$$

$$= 3.52\times10^{-7}\,\text{eV}$$

What frequency radiation can produce this spin flip?

$$hf = \Delta U \Rightarrow f = \frac{\Delta U}{h} = \frac{3.52\times10^{-7}\,\text{eV}}{4.14\times10^{-15}\,\text{eV s}}$$

$$= 8.5\times10^{7}\,\text{Hz}$$

$$= 85\,\text{MHz}.$$

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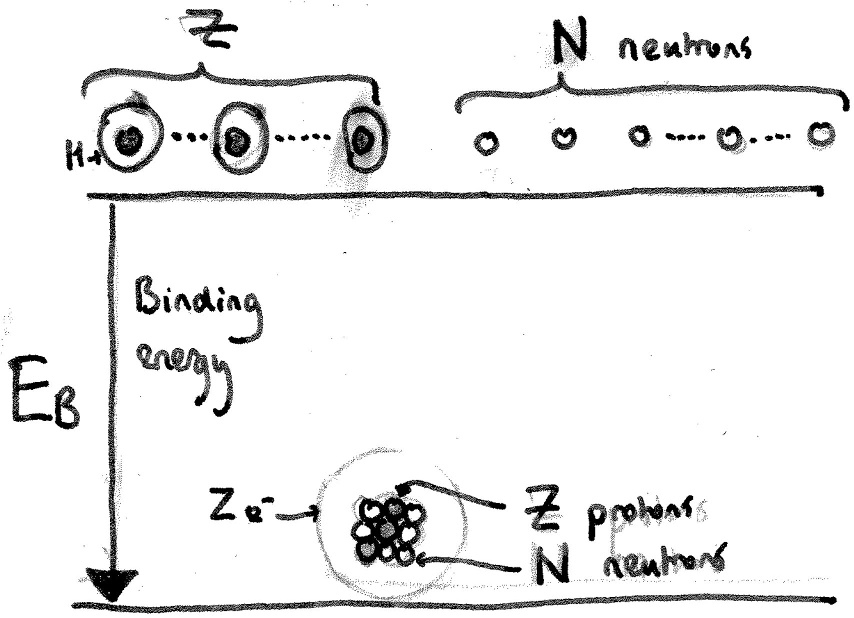
By measuring the absorption frequency & magnetic field one can measure the nuclear magnetic moment.

The measurement of nuclear spins is the basis of magnetic resonance imaging.

43.2 NUCLEAR BINDING & NUCLEAR STRUCTURE



$$\boxed{E_B = \left(Z M_H + N m_n - {}^{A}_{Z}M\right)c^2}$$

nuclear binding energy.

$$1u = 1.66 \times 10^{-27} \text{ kg}$$

$$uc^2 = 1.66 \times 10^{-27} \times (3 \times 10^8)^2 = 1.49 \times 10^{-10} \text{ J}$$

$$= \frac{1.49 \times 10^{-10}}{1.6 \times 10^{-19}} = 931.5 \times 10^6$$

$$\equiv \underline{\underline{931.5 \text{ MeV}}}$$

$$E_B = \left(Z(1.007825) + N(1.008665) - {}^{A}_{Z}\overset{\text{in u}}{M}\right) \times 931.5 \text{ MeV}$$

e.g $^{235}_{92}U$ has a mass of 235.0439299 u. What is its nuclear binding energy?

$$E_B = \left[92 \times (1.007825) + 143 \times (1.008665\,u) - 235.0439299\,u\right] \times 931.5$$

$= 1783.88$ MeV

$\equiv 7.59$ MeV/nucleon.