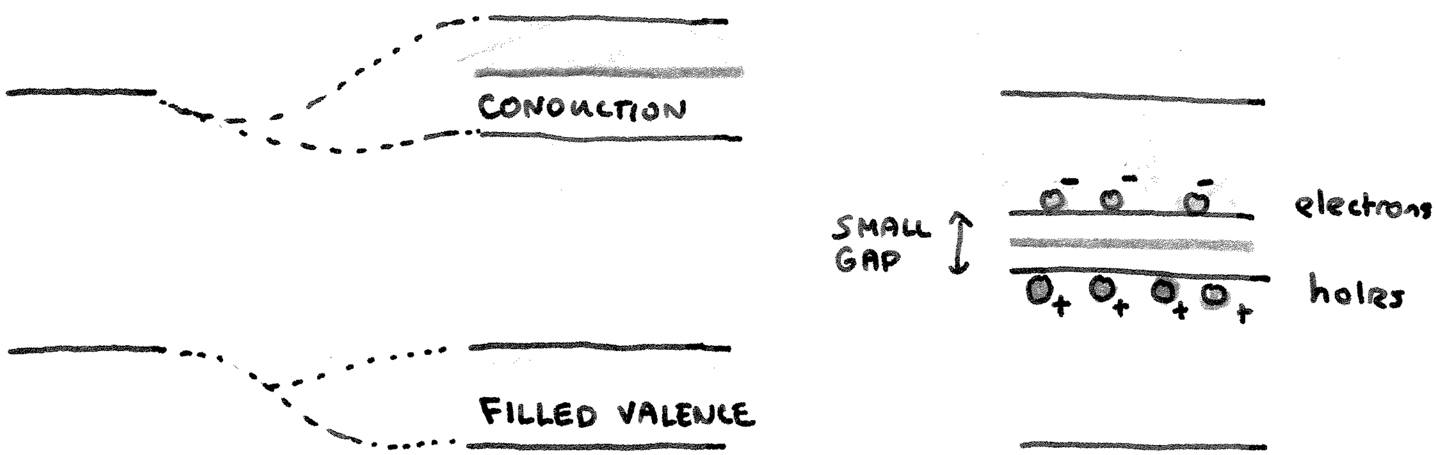
L18: Metals and Semiconductors

The invention of the transistor & the discovery of semi-conductor electronics was one of the transforming events of the 20th century, and it occured not twenty miles from here, at Bell Labs in Murray Hill. Today we're going to learn about two important consequences of the energy band model of electrons in matter — the free electron model of metals & the energy gap model of semiconductors.



ATOM

Discrete levels

METAL

High density

FAST ELECTRONS

SEMICONDUCTOR

Low density

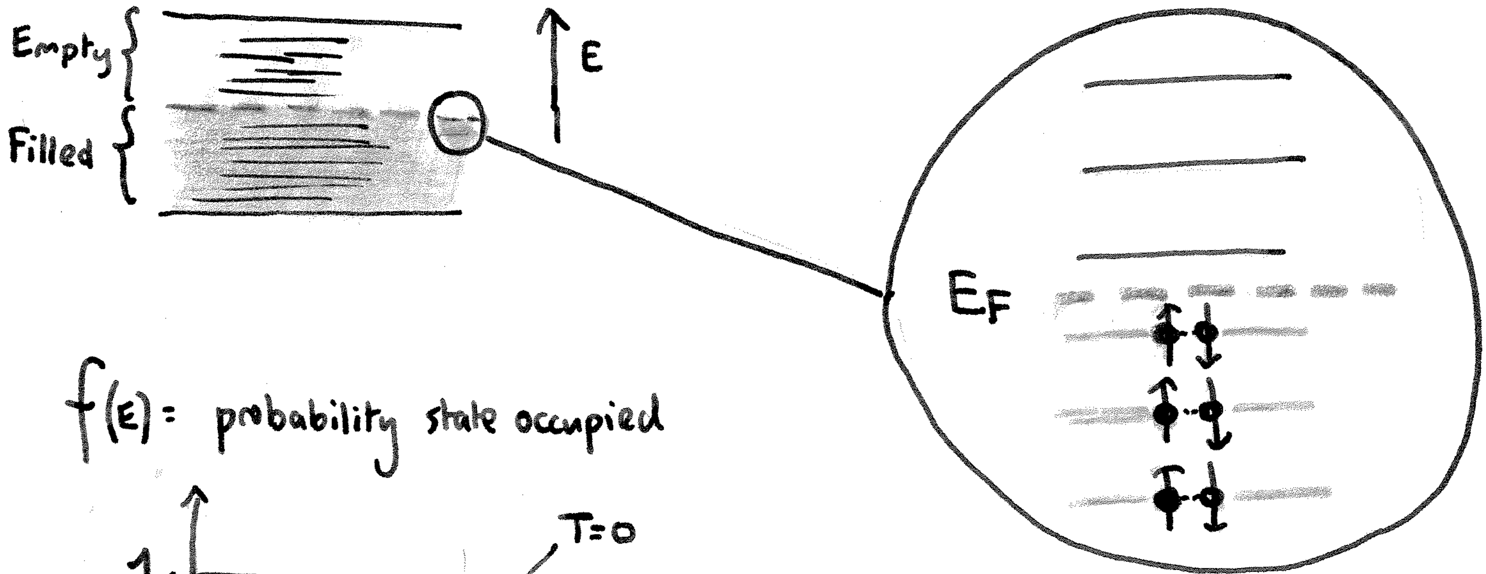
TUNABLE CONCN.

of electrons (n)

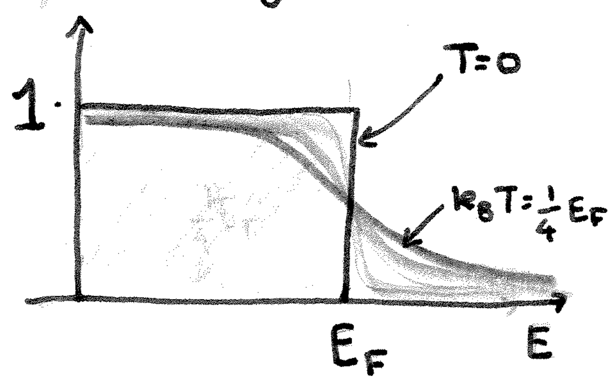
& holes (p)

42.5 Free Electron Model

A) Fermi Function



$f(E)$ = probability state occupied



"FERMI ENERGY"

$$\boxed{f(E) = \frac{1}{e^{(E-E_F)/k_BT} + 1}}$$

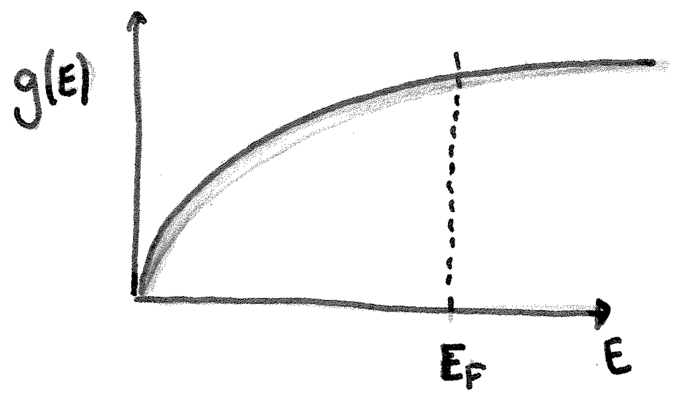
e.g. At 1000K, what is the probability that a state 0.1eV above the Fermi energy of a metal is occupied?

$$f = \frac{1}{e^{\frac{0.1 \times 1.6 \times 10^{-19}\,J}{1.38 \times 10^{-23} \times 1000}} + 1} = 0.24 \qquad \underline{\underline{24\%}}$$

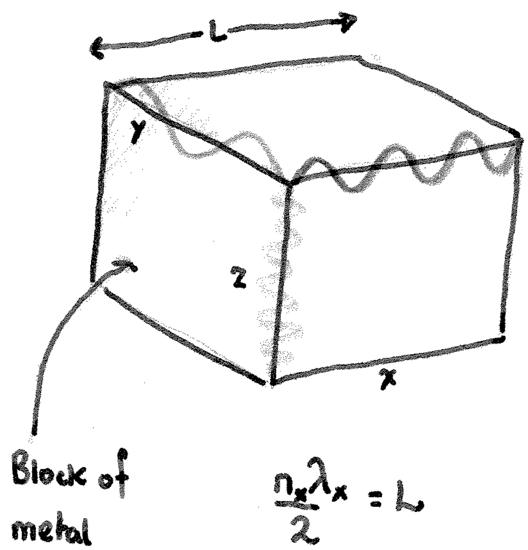
B) DENSITY OF STATES

The number of states / unit energy = DENSITY OF STATES

$= g(E)$



$$\boxed{g(E) = \frac{(2m)^{3/2}}{2\pi^2 \hbar^3} V E^{1/2}}$$



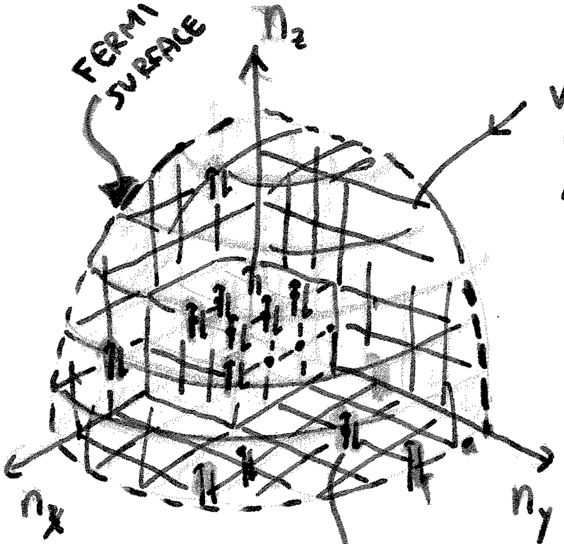
$$\frac{n_x \lambda_x}{2} = L$$

$$k_x = \frac{2\pi}{\lambda_x} = n_x \frac{\pi}{L}$$

$$k^2 = \frac{\pi^2}{L^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

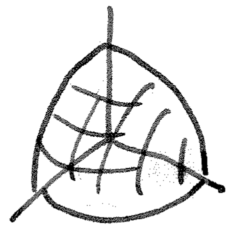
$$E = \frac{\hbar^2 \pi^2}{2mL^2} \times \underbrace{\left(n_x^2 + n_y^2 + n_z^2\right)}$$

$$\boxed{E = \frac{\hbar^2 \pi^2}{2mL^2} n_{rs}^2}$$



volume of $\frac{1}{8}$th sphere

$$= \frac{1}{8} \times \frac{4\pi}{3} n_{rs}^3 = \frac{\pi}{6} n_{rs}^3$$



Inside sphere : each state filled by two electrons

Outside : states empty

$$\# \text{ states} = \# \text{ up} + \# \text{ down}$$

$$= 2 \times \frac{\pi}{6} n_{rs}^3 = \frac{\pi}{3} n_{rs}^3$$

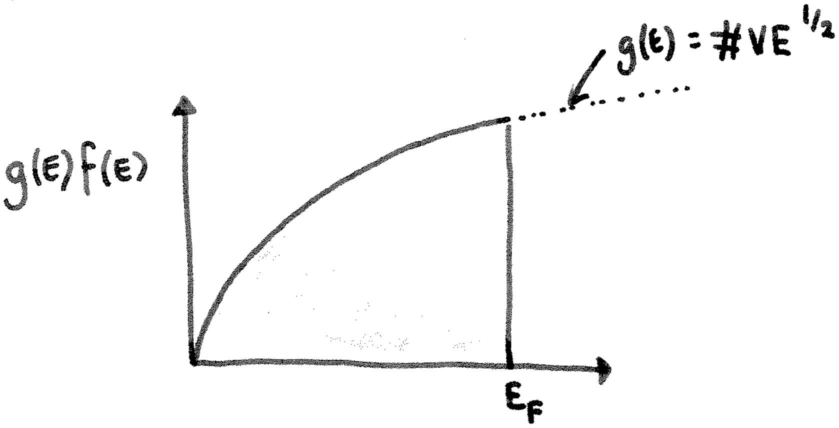
$$\boxed{n = \frac{\pi}{3} n_{rs}^3} = \frac{\pi}{3} \times \Bigg[\bigg(\overbrace{\frac{2mL^2}{\hbar^2 \pi^2} E}^{n_{rs}} \bigg)^{1/2} \Bigg]^3 = \frac{(2m)^{3/2} \overset{V}{L^3} E^{3/2}}{3\pi^2 \hbar^3}$$

= # of states with $E < E_{FERMI}$

$$= \left[\frac{(2m)^{3/2}}{3\pi^2 \hbar^3} \right] V E^{3/2}$$

$$\boxed{\frac{dn}{dE} = g(E) = \left(\frac{(2m)^{3/2}}{2\pi^2 \hbar^3} \right) V E^{1/2}}$$

C) ELECTRON DENSITY



$$dN = g(E)f(E)\,dE$$

$$= \underbrace{\#}_{\frac{(2m)^{3/2}}{2\pi^2\hbar^3}} VE^{1/2} \underbrace{\frac{1}{e^{(E-E_F)/k_BT}+1}}_{\substack{=1 \quad E<E_F \\ =0 \quad E>E_F}} dE$$

$$N = \int_0^{E_F} \# \, VE^{1/2}\,dE = \frac{2}{3}\# \, V E_F^{3/2} = \frac{(2m)^{3/2}}{3\pi^2\hbar^3} V E_{F0}^{3/2}$$

free electron density

$$E_{F0} = \left[\frac{3\pi^2\hbar^3}{(2m)^{3/2}}\right]^{2/3} \left(\frac{N}{V}\right)^{2/3}$$

$$\frac{N}{V} = \left[\frac{(2m)^{3/2}}{3\pi^2\hbar^3}\right] E_{F0}^{3/2}$$

= density of free electrons

≠ density of electrons.

e.g Copper. Density of free electrons is $\frac{N}{V} = 8.45 \times 10^{28} / m^3$.

What is the Fermi energy in electron volts?

$$\left[\frac{3\pi^2 \hbar^3}{(2m)^{3/2}} \right]^{2/3} = \left[\frac{3 \times \pi^2 \times (1.05 \times 10^{-34})^3}{(2 \times 9.1 \times 10^{-31})^{3/2}} \right]^{2/3} = 5.84 \times 10^{-38} \text{ J m}^2$$

$$E_{F0} = (5.84 \times 10^{-38}) \times (8.45 \times 10^{28})^{2/3} = 1.125 \times 10^{-18} \text{ J}$$

$$= \underline{7.03 \text{ eV}}$$

How fast is an electron with this energy moving?

$$\frac{1}{2} m v_F^2 = E_F$$

$$v_F = \sqrt{2E_F/m} = \sqrt{\frac{2 \times 1.125 \times 10^{-18} \text{ J}}{9.1 \times 10^{-31} \text{ kg}}} = 1.6 \times 10^6 \text{ m/s}$$

0.5% of the speed of light.

D) Average energy

$$E_{av} = \frac{E_{TOTAL}}{N} = \frac{\int_0^{E_{F0}} \# V E^{1/2} \times E \, dE}{\int_0^{E_{F0}} \# V E^{1/2} \, dE}$$

$$= \frac{\frac{2}{5} E_{F0}^{5/2}}{\frac{2}{3} E_{F0}^{3/2}} = \frac{3}{5} E_{F0}.$$

This average energy is much, much greater than the thermal energy $\frac{3}{2} k_B T$. At room temperature

$\frac{3}{2} k_B T = 0.04\,\text{eV}$.

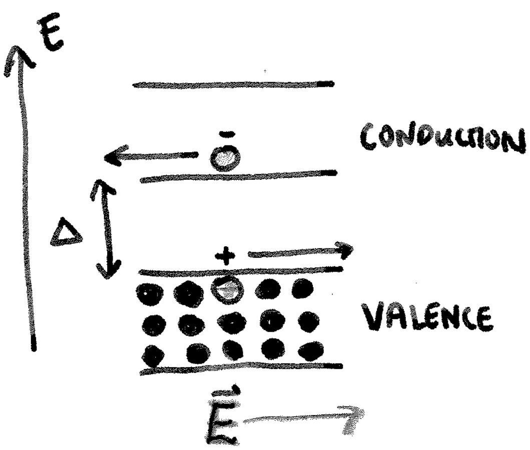
42.6 SEMICONDUCTORS

- Resistivity intermediate between metal & insulator.
- Small gap, typically less than 1eV.

Hole concept

"Electronic equivalent of antimatter" !

hole ≡ empty state in valence band

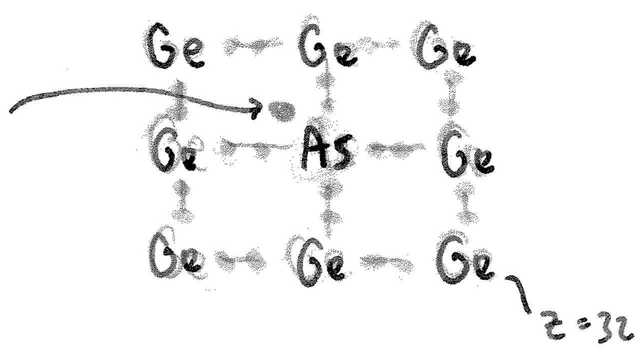


Impurities

donor = impurity with one more e^- $\quad Z' = Z+1$

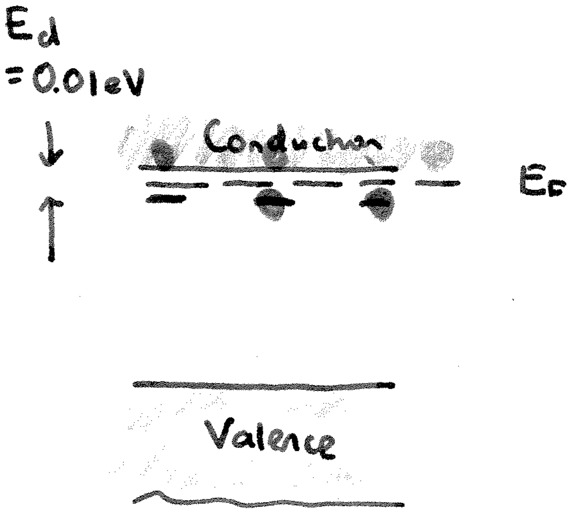
acceptor = impurity with one less e^- $\quad Z' = Z-1$

donor electron



Ge $4s^2\,4p$

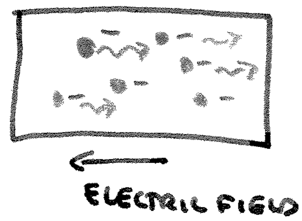
As $4s^2\,4p^2$ $\quad Z = 33$



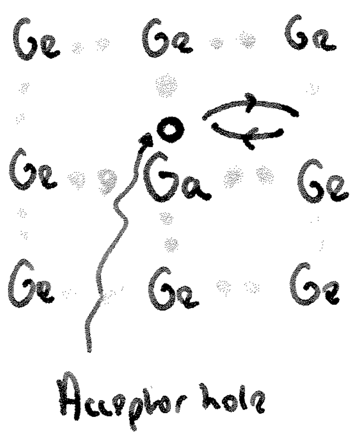
$$E_d \ll \frac{1}{4^2}(13.6) = 0.85$$

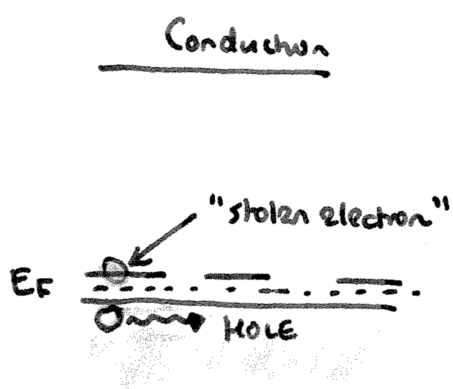
because of screening.

"n-type semiconductor"

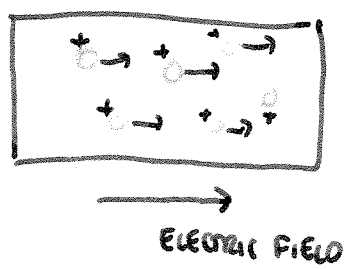


Whereas for a p-type e.g. Ga $4s^2$ impurity. ($Z = 31$)

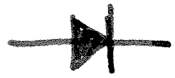


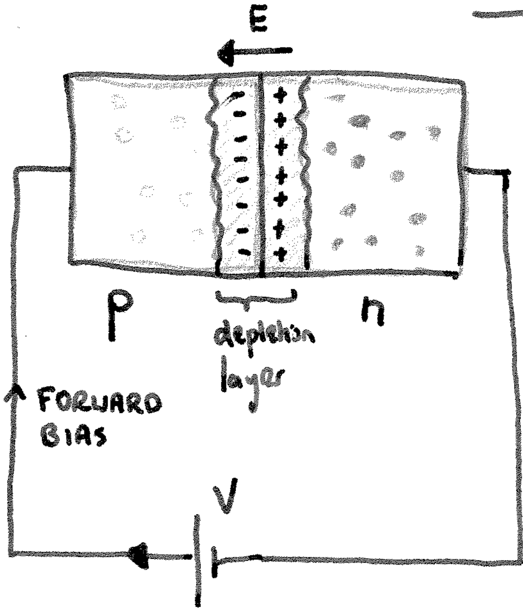


"p-type semiconductor"



42.7 p-n JUNCTIONS

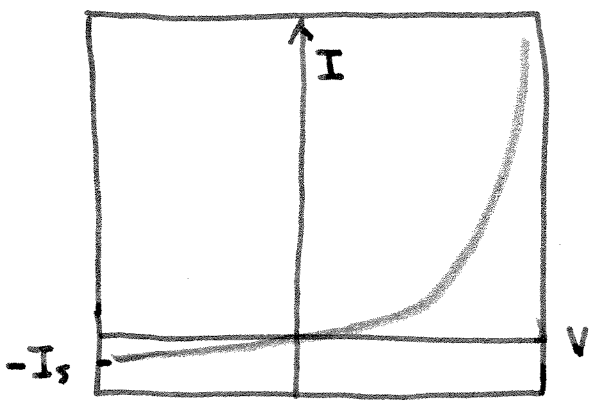


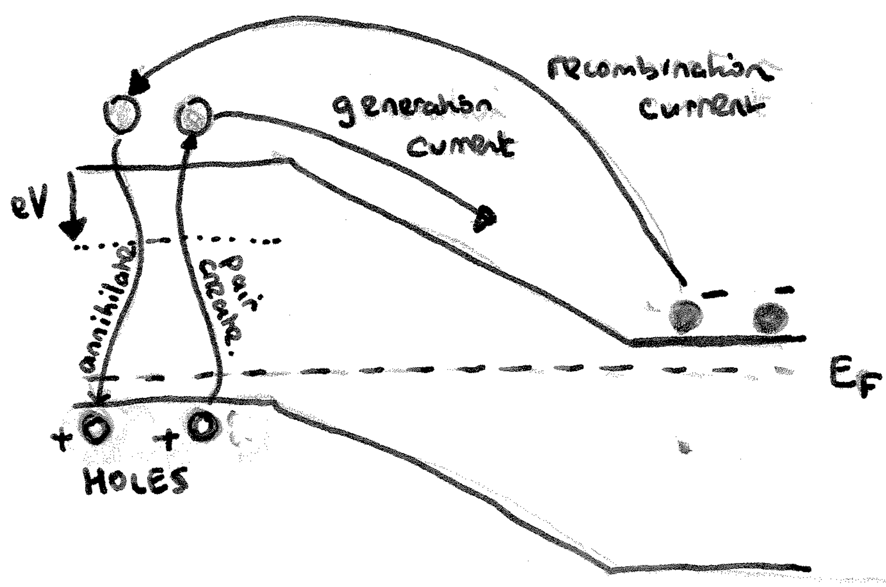


Basic work-horse of semiconductor electronics, leds, solar panels, semiconductor lasers.

$$\boxed{I = I_s\left(e^{eV/k_BT} - 1\right)}$$

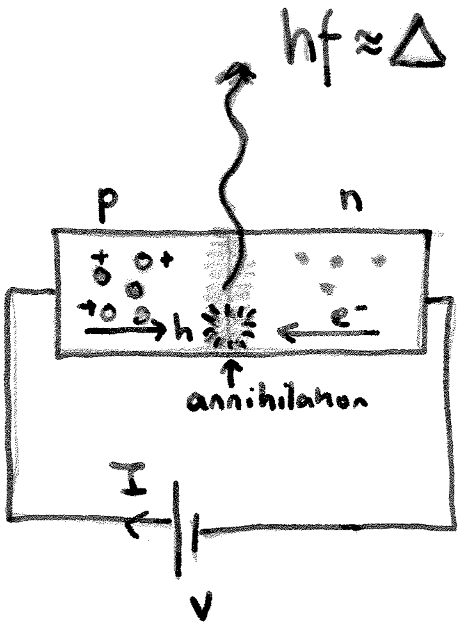
recombination current



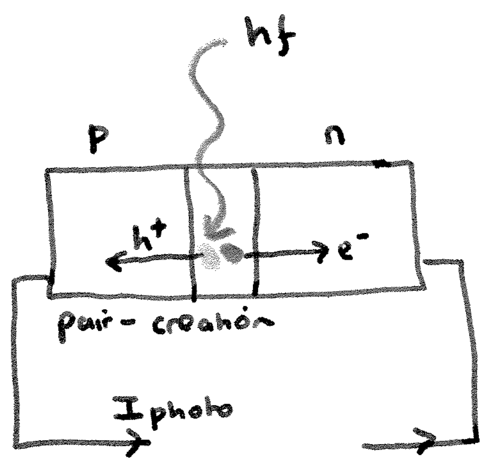


$$\begin{cases} I_{recomb} = I_s\, e^{eV/k_BT} \\ I_{generation} = -I_s \end{cases}$$

LED



SOLAR PANEL



THESE EVENTS ARE THE MINI ELECTRONIC ANALOG OF PROCESSES OCCURING INSIDE COLIDERS, AND IN COSMIC RAY EVENTS.