L10. BREAKDOWN OF CLASSICAL PHYSICS

At the dawn of the 20th century, classical physics was in trouble. The most elementary property of matter – that as it gets hotter it changes color from red, to white hot – the observation of a myriad of sharp emission lines in the spectra of elements – and the discovery of X-rays – none of these phenomena could be explained in terms of the classical theory of radiation.

The great discovery of this time was that radiation — electromagnetic radiation, including light — is not smooth and continuous, as assumed in Maxwells theory — but rather — it is "grainy" and made up of tiny packets of energy called "quanta of energy".

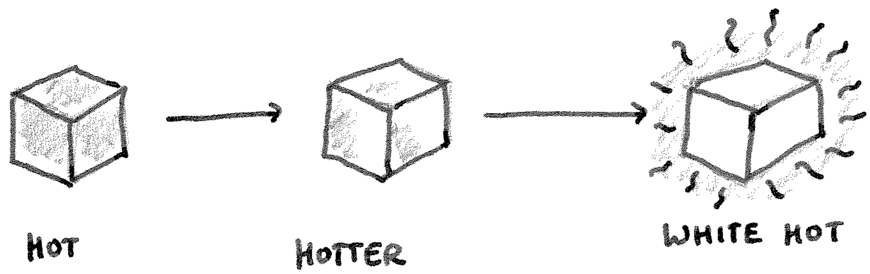
It was Max Planck who first observed that the changing color of hot bodies can be understood if the energy of the radiation splits up into quanta of energy ϵ, where

$$E = hf$$

$$h = 6.626 \times 10^{-34}\ Js$$

It was Einstein who made the radical leap and concluded that light must be made up of particles – which we call photons, each carrying energy hf. Einstein made this proposal in 1905, but it took about twenty years before it was widely accepted.

38.8 BLACKBODY RADIATION



HOT MATTER EMITS A CONTINUOUS SPECTRUM OF "BLACKBODY" RADIATION.

$I(\lambda)\,d\lambda$ = intensity of radiation with wavelength between λ & $\lambda + d\lambda$

- $\boxed{\text{TOTAL INTENSITY} = I = \int_0^\infty I(\lambda)\,d\lambda = \sigma T^4}$

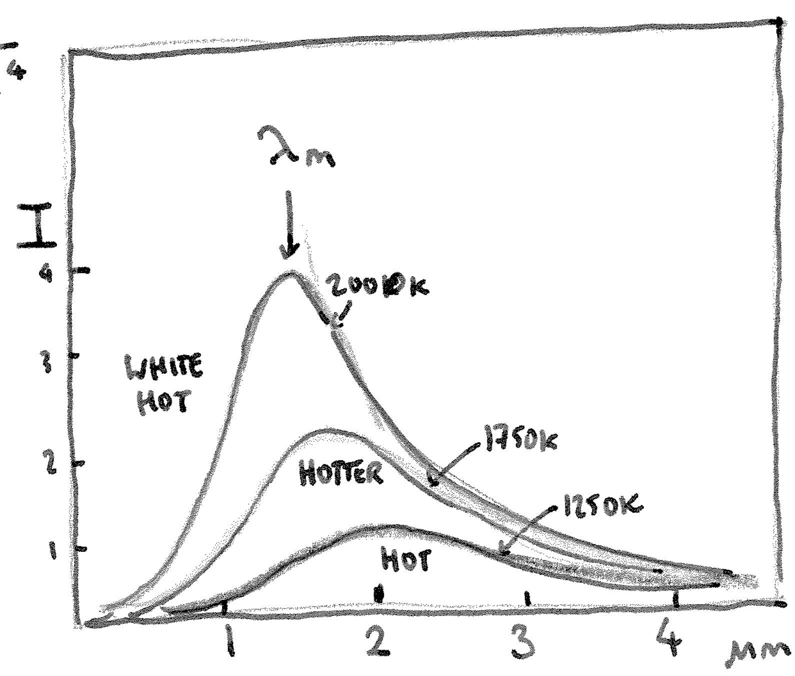
STEFAN-BOLTZMANN

$$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$$

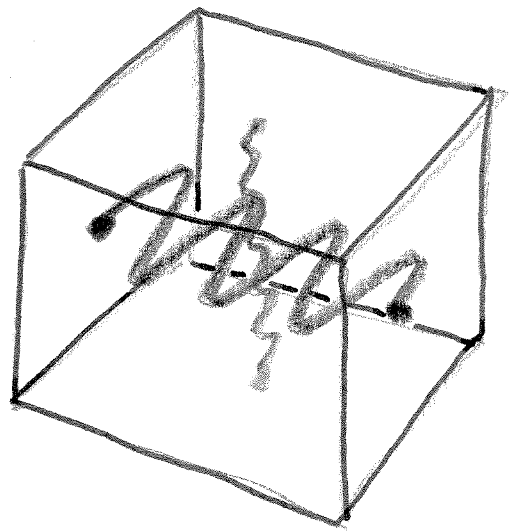
- $\boxed{\lambda_m T = 2.9 \times 10^{-3}\ m \cdot K}$

WIEN DISPLACEMENT LAW

Higher $T \rightarrow$ Smaller λ_m



Fitting waves into a box.



Rayleigh:

$$I(\lambda) = \text{number of modes with wavelength } \lambda \times k_B T$$

$$= \frac{2\pi c}{\lambda^4} k_B T$$

- Can't explain $\lambda_m \propto 1/T$
- Leads to an ultraviolet catastrophy $I = \# \int_0^\infty \frac{d\lambda}{\lambda^4} = \infty$

CLASSICAL DILEMMA.

Planck: energy in each mode is $P \times hf$

$= 0, hf, 2hf, 3hf, 4hf \ldots$

He then deduced

$$\boxed{I(\lambda) = \frac{2\pi c}{\lambda^4} \times \frac{\left(\frac{hc}{\lambda}\right)}{e^{(hc/\lambda k_B T)} - 1}}$$

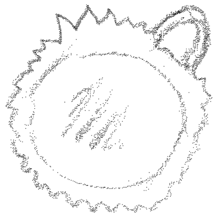
PLANCK RADIATION LAW

- $\lambda_m = \frac{hc}{kT} \times \frac{1}{4.97}$
- $I = \int I(\lambda)\, d\lambda = \sigma T^4 \qquad \sigma = \frac{2\pi^5 k_B^4}{15 c^2 h^3}$

Photon energies

$hf \sim k_B T$

38.8 EX



Sun $T_{surface} \sim 5800K$

a) Peak intensity @ $\lambda_m = \dfrac{2.9\times10^{-3}\,mK}{5800K} = 0.5\times10^{-6} = 500nm$

b) Intensity $I = \sigma T^4 = \left(5.67\times10^{-8}\,\dfrac{W}{m^2K^4}\right)\times\left(5.8\times10^{3}\right)^4$

$= 6.42\times10^{7}\,W/m^2 = 64.2MW/m^2$

c) How much energy radiated between 622 & 630 nm ?

$$\frac{hc}{\lambda k_B T} = \frac{6.63\times10^{-34} \times 3\times10^{8}}{622\times10^{-9}m \times 1.38\times10^{-23}\,J/k \times 5800} = 4.0$$

$$I(\lambda) = \left(\frac{2\pi \times 3\times10^{8}}{\lambda^4}\right)\times k_B T \times \left(\frac{\frac{hc}{\lambda k_B T}}{e^{hc/\lambda k_B T}-1}\right)$$

$$= \frac{2\pi\times3\times10^{8}}{(622\times10^{-9})^4}\times(1.38\times10^{-23}\times5800)\times\left(\frac{4}{e^4-1}\right)$$

$$= 1.26\times10^{34}\,m^{-3}s^{-1}\times\left(8.0\times10^{-20}J\right)\times 0.746$$

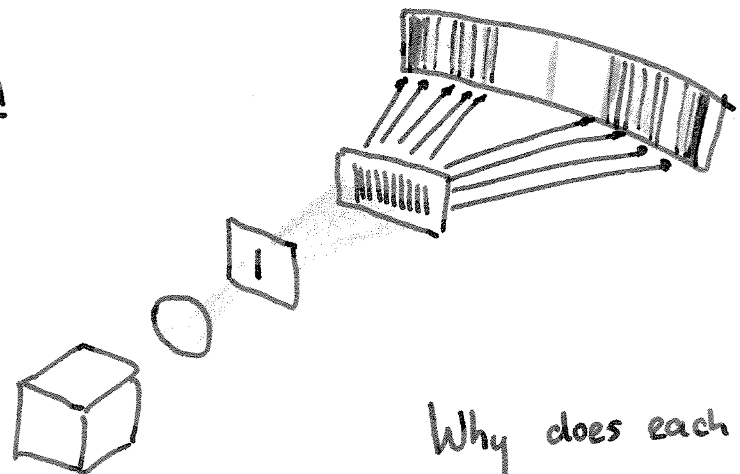
$$= 7.53\times10^{13}\,W/m^3$$

$I(\lambda)\,\Delta\lambda$

$= 7.53\times10^{13}\times 8\times10^{-9} = 6.0\times10^{5}$

38.1 MYSTERIES RESOLVED BY QUANTUM HYPOTHESIS

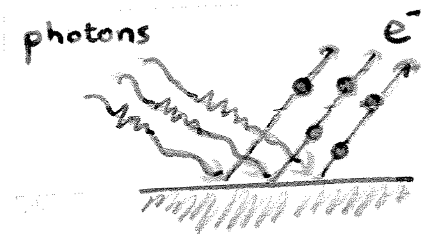
• <u>LINE SPECTRA</u>



Why does each gas produce a discrete set of lines?

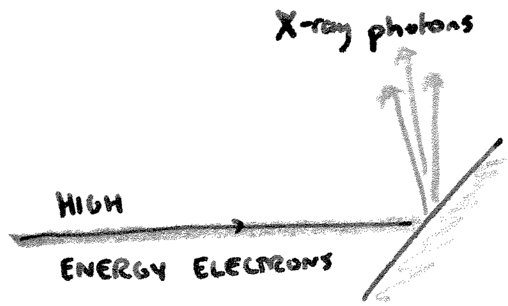
• <u>PHOTO-ELECTRIC EFFECT</u>

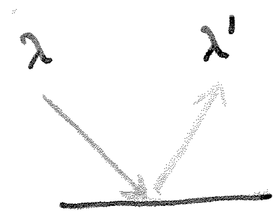
(photon in, e^- out)



• <u>X-RAYS</u>.

(e⁻ in, photon out)

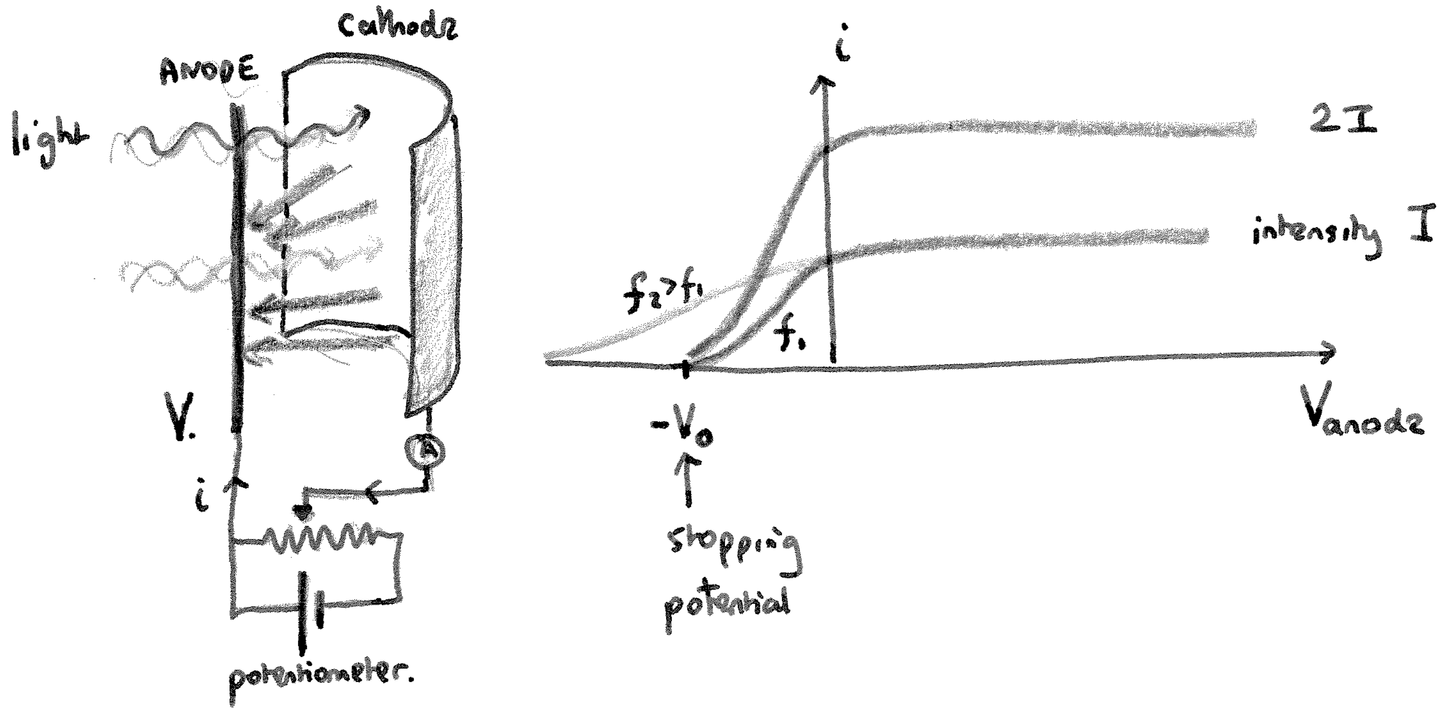




$\lambda' > \lambda$ on scattering

<u>incre</u>

38.2 PHOTOELECTRIC EFFECT



$$eV_0 = K.E_{max} = \frac{1}{2} m v_{max}^2$$

does not depend on INTENSITY

$$\boxed{\begin{aligned} K.E_{max} &= \text{photon energy} - \text{work fn} \\ &= hf - \phi \end{aligned}}$$

EINSTEIN, 1905.

The <u>independence</u> of the electron energy on intensity & the dependence of $K.E_{max}$ on frequency are accounted for by the photon concept.

- Photons have no rest mass but they are never at rest $m = m_0 \gamma$ is finite even though $m_0 = 0$.

- $E = mc^2$, $p = mc$ $\Bigg\}$ $\boxed{E = pc}$

- $p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$

- Physicists often use the electron volt

  $1 eV = 1.602 \times 10^{-19} J$

$h = 6.626 \times 10^{-34} Js = 4.136 \times 10^{-15} eV s$

38.2 ex

Sodium has a work function $\phi = 2.7 eV$.

a) What is the minimum energy photon required to produce a photocurrent?

b) What is the maximum electron energy produced by light of wavelength 400nm?

---

a) $hf_{min} = \phi = 2.7 eV$

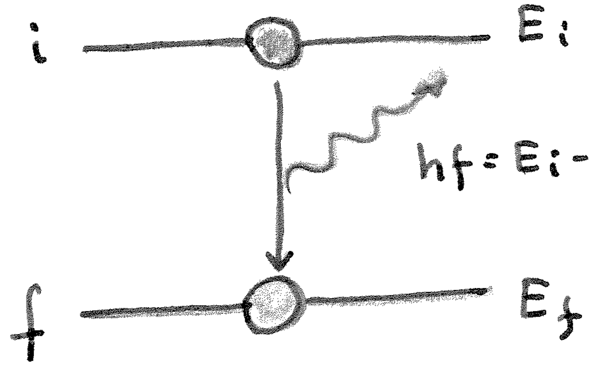
b) $\frac{1}{2}mv^2_{max} = hf - \phi$

$$f = \frac{c}{\lambda} = \frac{3\times10^8 m/s}{4\times10^{-7} m} = 0.75\times10^{15} Hz$$

$$hf = 4.14\times10^{-15} eVs \times 0.75\times10^{15} = 3.11 eV$$

$$\frac{1}{2}mv^2_{max} = 3.11 - 2.7 = \underline{0.41 eV}$$

38.3 ATOMIC LINE SPECTRA + ENERGY LEVELS



$hf = E_i - E_f = \frac{hc}{\lambda}$ energy of emitted photon.

The origin of the line spectra derives from the EMISSION OF A PHOTON in making a transition between two energy levels (BOHR, 1913).

e.g. Orange light emitted by krypton. $\lambda = 606\,nm$

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{606 \times 10^{-9}\,m} = 3.28 \times 10^{-19}\,J$$

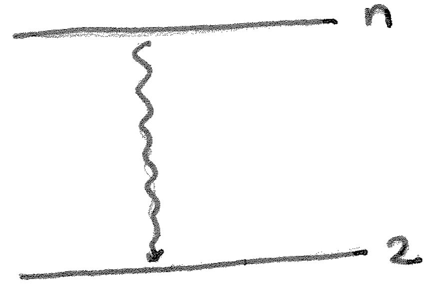
or

$$E = \frac{hc}{\lambda} = \frac{4.14 \times 10^{-15}\,eVs \times 3 \times 10^8}{606 \times 10^{-9}\,m} = 2.05\,eV \quad \left(= \frac{3.28 \times 10^{-19}}{1.6 \times 10^{-19}}\right.$$

HYDROGEN SPECTRUM

BALMER SERIES (VISIBLE)

$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \qquad R = 1.097 \times 10^7\ m^{-1}}$$



e.g. $n = 3$

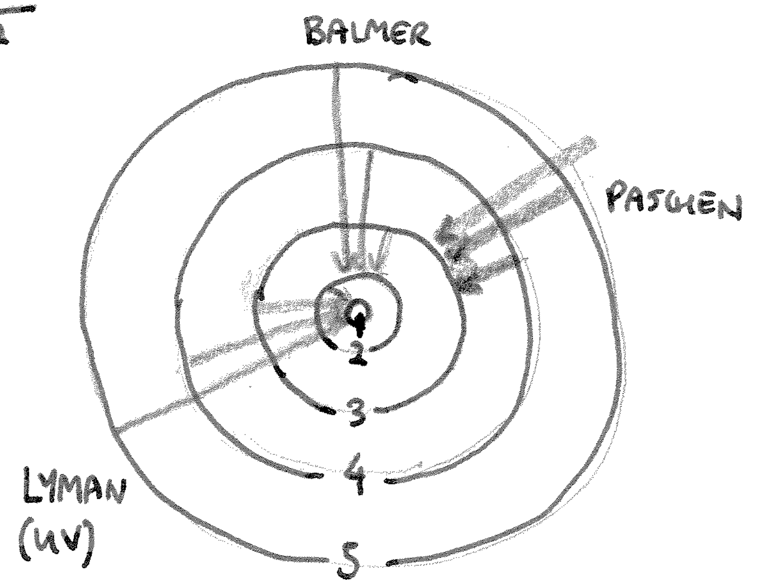
$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{9}\right) = 1.52 \times 10^6\ m^{-1}$$

$$\lambda = 656\,nm$$

$$E = \frac{hc}{\lambda} = \frac{hcR}{2^2} - \frac{hcR}{n^2}$$

$$\boxed{E_n = -\frac{hcR}{n^2} = -\frac{13.6}{n^2}\,eV}$$

BOHR HYPOTHESIS.



$$\frac{hc}{\lambda} = hcR\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

38.7 COMPTON SCATTERING + X-RAYS

X-rays are produced by the absorption of high energy electrons. Called "braking radiation"

$$\boxed{eV = \text{energy of incoming } e^- = hf_{max} = \frac{hc}{\lambda_{min}}}$$

or "brehmstrahlung".

When X-rays scatter off e^-, they lose momentum and energy, which increases their wavelength. This process is called the "Compton Effect".

Compton Scattering.

$$\boxed{(\lambda' - \lambda) = \frac{h}{mc}(1 - \cos\phi)}$$

$$\frac{h}{mc} = \frac{6.626 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^{8}} = 2.43 \times 10^{-12}\ \text{m}$$