8 Time Dilation, Length Contraction and the Lorentz Transformation

In the last lecture, we saw that the principle of relativity forces us to abandon the idea of simultaneity for all observers. This means that time can no longer be considered as something absolute, but rather - the time co-ordinate of an event now depends on the observer.

For example - although we can synchronize the clocks across the USA, viewed from a passing satellite, the clocks in San Fransisco & New York do not chime exactly at the same instant.

We'll see today that we are forced to acknowledge that space & time are intimately intertwined. That clocks tick more slowly in moving objects, & that a meter stick becomes shorter & shorter, the closer it moves to the speed of light. We call the slower passage of time in a moving object "time dilation", & the shrinking length of a moving object is called "length contraction".

Lastly we'll have to revise our ideas about about the spatial & temporal separation of two events. In Newtonian physics, we say that if two events are seperated by Δx, Δy & Δz in their spatial co-ordinates, that they lie a distance

$$s = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

apart in space & they are separated by a time interval

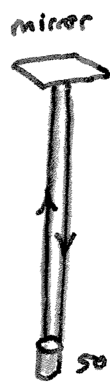
$$\Delta t.$$

We'll have to replace these ideas with a new kind of Pythagoras' theorem! The distance between two events is now the space-time separation

$$\tilde{S}^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2$$

37.3 Time Dilation

To understand the relativity of time, we shall consider a primitive clock, formed by a light ray, or laser beam bouncing between two mirrors. The elementary unit of time for the light to bounce back off a mirror a distance d away is



M

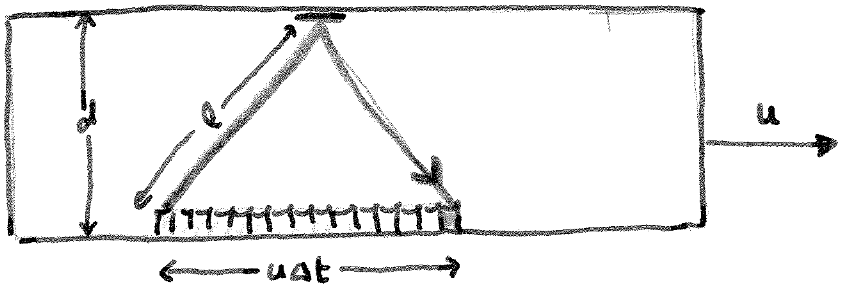
Primitive clock.

$$\Delta t_0 = \frac{2d}{c}$$

TIME BETWEEN "TICKS"

So how fast does this clock "tick" when its moving? Lets put it on a train. The observer on the train will say nothing has changed – the laws of physics are the same in all inertial reference frames. But what do <u>we</u> see when we look at the clock from the station platform?

S



We'll see the clock tick every Δt seconds, and during this time, the train moves forwards a distance $u\Delta t$. The distance travelled by the light is $2l$, where

$$l = \sqrt{d^2 + \left(\frac{u\Delta t}{2}\right)^2}$$

so the round trip time measured on the platform is

$$\Delta t = \frac{2l}{c} = \frac{2}{c}\sqrt{d^2 + \left(\frac{u\Delta t}{2}\right)^2}$$

But $d = c\Delta t_0/2$, so

$$\Delta t = \frac{2}{c}\sqrt{\left(\frac{c\Delta t_0}{2}\right)^2 + \left(\frac{u\Delta t}{2}\right)^2} = \sqrt{\Delta t_0^2 + \left(\frac{u}{c}\right)^2 \Delta t^2}$$

i.e

$$\Delta t^2 = (\Delta t_0)^2 + \left(\frac{u}{c}\right)^2 \Delta t^2$$

$$\boxed{\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{u^2}{c^2}}}}$$

Time dilation

(Δt bigger $\therefore$ clock ticks more slowly)

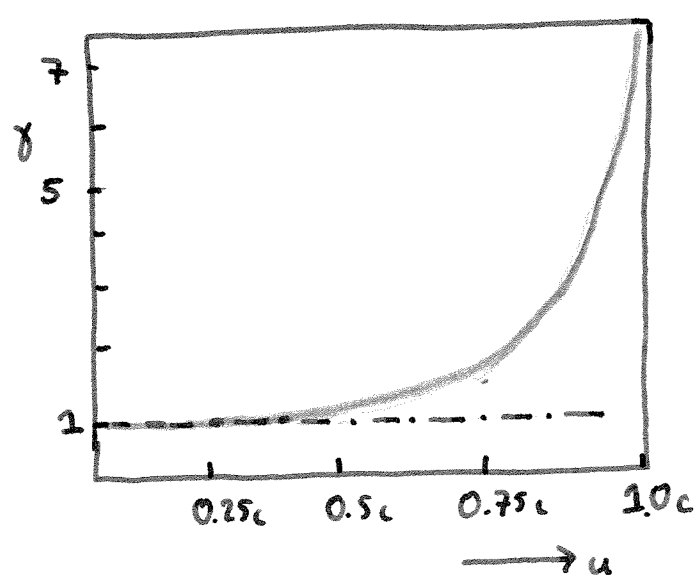
We will often use the notation

$$\boxed{\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}}$$

(gamma)

so in this notation

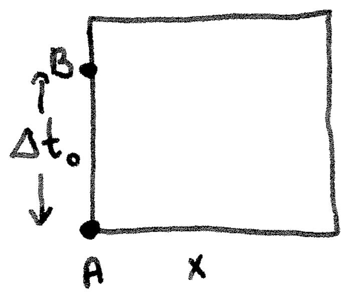
$$\Delta t = \gamma \Delta t_0$$



γ diverges as $u \to c$

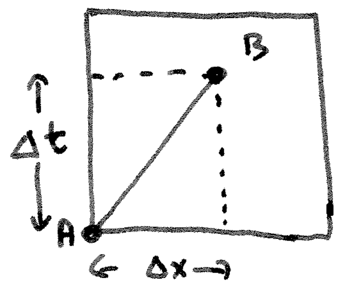
37.3b PROPER TIME.

Δt_0 = time interval between two events at the same point in space = proper time.



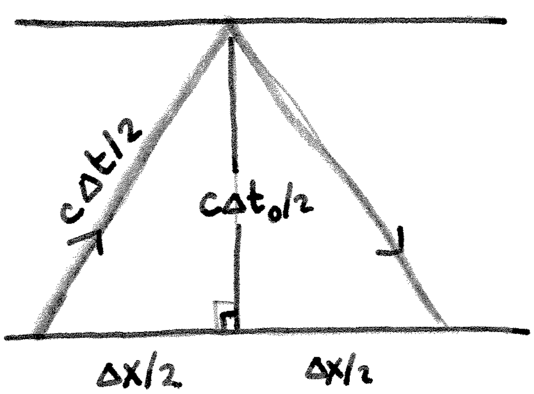
PROPER TIME

A & B SAME LOCATION.



Measured time. (Not Proper).

A & B DIFFERENT LOCATION



$$\left(\frac{c\Delta t}{2}\right)^2 = \left(\frac{c\Delta t_0}{2}\right)^2 + \left(\frac{\Delta x}{2}\right)^2$$

$$\boxed{\Delta t_0^2 = \Delta t^2 - (\Delta x/c)^2}$$

PROPER TIME.

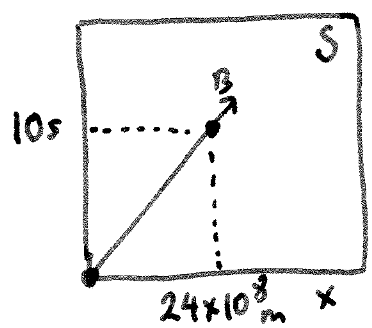
37.3 I

M = Mans

S = Stanley

M passes S at a relative speed $u = 0.8c$. At the instant M passes S, both start their timers.

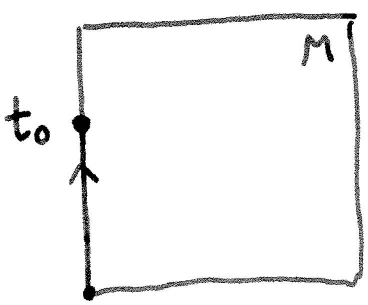
a) At the instant when S measures M to be 24×10^8 m away, what does M's time read?



$$t_s = \frac{l}{0.8c} = \frac{24 \times 10^8}{0.8 \times 3 \times 10^8} = 10\,s$$

$\boxed{\text{Event} = \text{M reads her timer}}$

t_o = M's time ≡ PROPER TIME

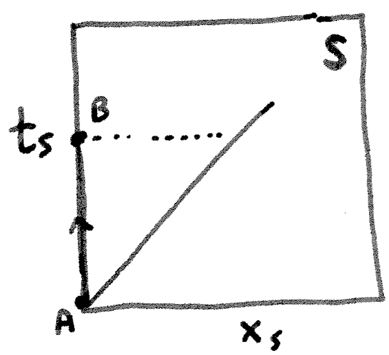


$$t_s / t_o = \gamma \qquad t_o = \frac{t_s}{\gamma} = t_s\sqrt{1-(0.8)^2}$$

$$= 10 \times 0.6$$

$$= \underline{6\,s}$$

b) What time does Stanley's timer read "when" M's reads 6s ?



$\boxed{\text{Event = S reads his timer}}$

$t_S \equiv t_0$ proper time

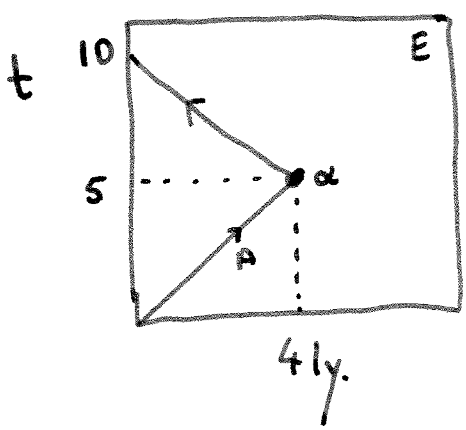
$$t_M = \frac{t_S}{\sqrt{1-(0.8)^2}}$$

$t_S = 0.6\, t_M$

$= 3.6s$

37.3 II TWIN PARADOX

Two twins start out on earth, aged 20 years. E stays on earth. A journeys at 0.8c to a nearby star, 4 light years away, then promptly turns around & returns home. What are their ages when they meet again?

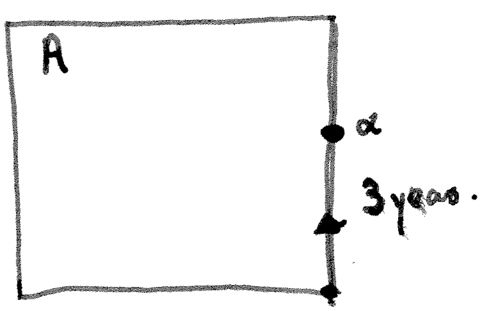


From E's vantage, A takes

$$\frac{4 \text{ ly}}{0.8 \text{ ly/year}} = 5 \text{ years}$$ to reach distant star.

∴ 10 years for round trip. $t = 5$

E is 30 years old. when A returns.

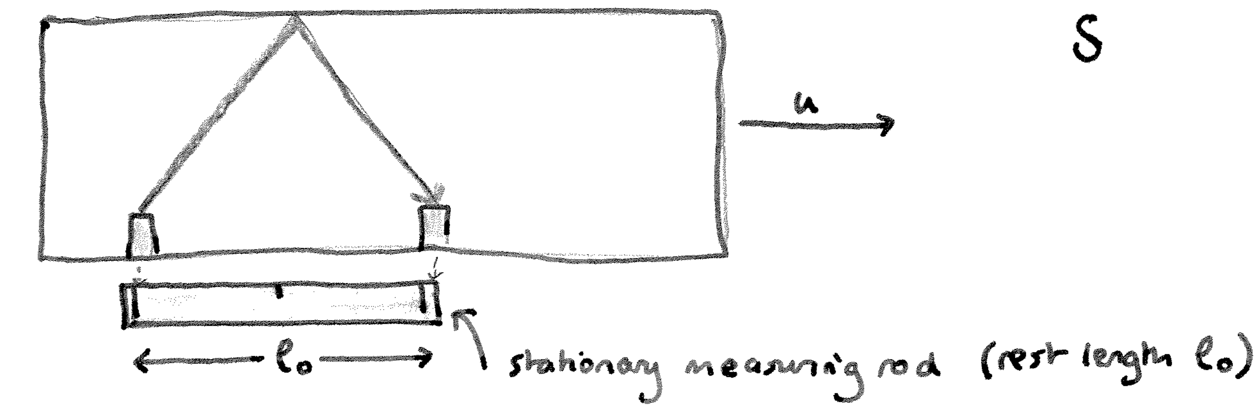


A's take-off & arrival at star occur at the same point in space ∴ PROPER time $t_0 \gamma = t$

$$t_0 = 5\sqrt{1-(0.8)^2} = 5 \times 0.6 = 3 \text{ years}.$$

⇒ A is 26 years old when she returns

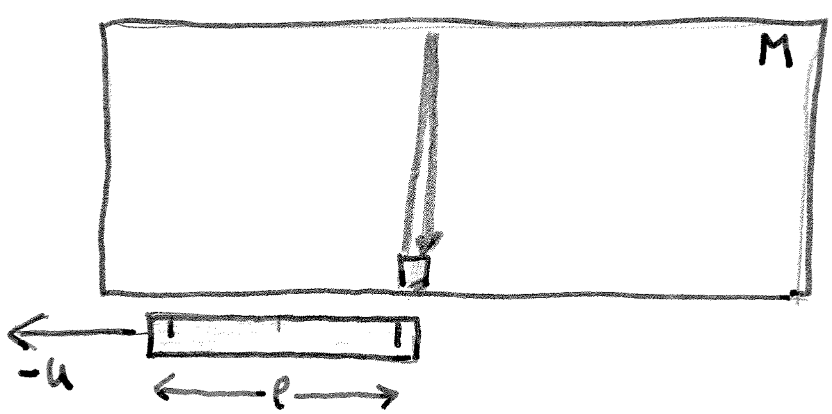
37.4. Relativity of Length



Suppose, every time M's clock ticks, it makes a mark on the platform. On the platform, the marks are separated by a distance

$$\ell_0 = u \Delta t$$

We say this is the rest-length, because the platform is at rest. Viewed in M's frame, the platform is moving @ $-u$



and the distance between marks is

$$\ell = u \Delta t_0$$

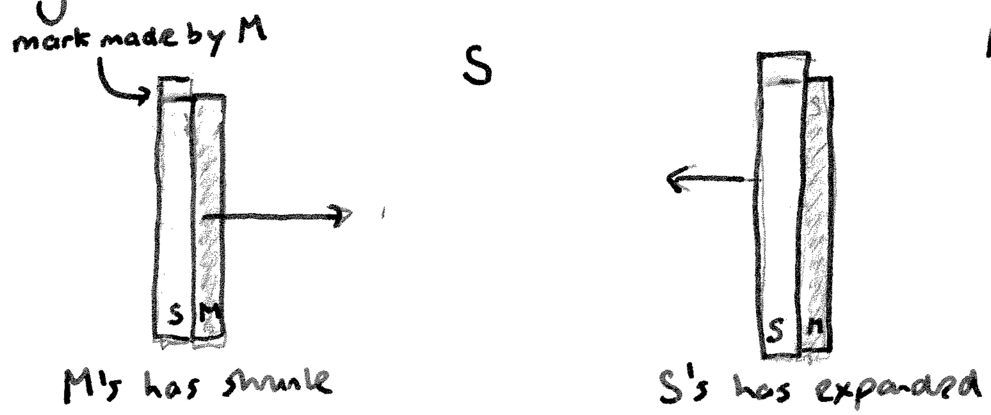
So, since $\Delta t = \Delta t_0 / \sqrt{1-(u/c)^2}$, $\ell_0 = \ell / \sqrt{1-(u/c)^2}$

or

$$\boxed{\ell = \ell_0 \sqrt{1-(u/c)^2}}$$

<u>Length contraction</u>.

Length's $\perp$ to direction of motion do not change.



Suppose S measures M's rod to have shrunk in the transverse direction, then M will measure S's rod to have expanded.

This violates the principle of relativity.

37.5 I

e.g a) A spaceship flies past at 0.9c. On earth it is measured to have a length of 300m. What is its rest-length ?

$$l = 300m = l_0\sqrt{1-(0.9)^2} = l_0\sqrt{0.19}$$

$$l_0 = \frac{300}{\sqrt{0.19}} = 688\,m = \text{proper length.}$$

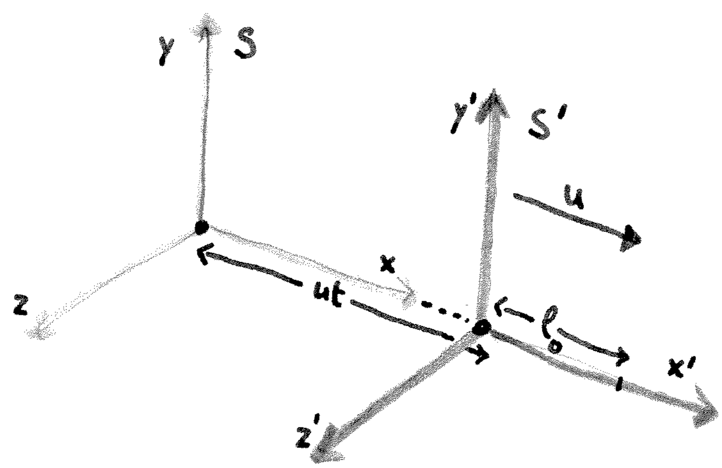
b) The two observers on earth measuring the length of the spaceship are 300m apart. How far apart are they measured to be by the crew on the spacecraft.

300m = proper separation of observers on earth = l_0

$$l = 300\sqrt{1-(0.9)^2} = \underline{130.8\,m}$$

37.5 Lorentz Transformation

S' is moving relative to S.



$\ell_0 = x'$

$\ell = x'\sqrt{1-(u/c)^2}$

$x = ut + \ell$

$= ut + x'\sqrt{1-(u/c)^2}$

$$\Longrightarrow \boxed{x' = \frac{x - ut}{\sqrt{1-(u/c)^2}}}$$

$$\boxed{\begin{aligned} y' &= y \\ z' &= z \end{aligned}}$$

Reverse transform

$x' = -ut' + x\sqrt{1-(u/c)^2}$

$\dfrac{(x-ut)}{\sqrt{1-(u/c)^2}} = -ut' + x\sqrt{1-(u/c)^2}$

$t' = \dfrac{t}{\sqrt{}} - \dfrac{x}{u}\left(\dfrac{1}{\sqrt{}} - (1-(u/c)^2)\right)$

$$\Longrightarrow \boxed{t' = \frac{t - ux/c^2}{\sqrt{1-(u/c)^2}}}$$

- Like a "rotation" in space time.

  Conventional rotations preserve $\Delta x^2 + \Delta y^2 + \Delta z^2$.

  Lorentz tranformations preserve $\Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2$.

- Two events at the same time t in S, but different locations x_1 & x_2, occur at times

$$t'_{1,2} = \frac{t - \frac{v x_{1,2}}{c^2}}{\sqrt{1-(v/c)^2}}$$

e.g. 37.5I Two events occur at $t = 0$ & $x = \pm 150\text{m}$. What are their co-ordinates in a frame S' moving at speed $u = 0.6c$.

$$\gamma = \frac{1}{\sqrt{1-(0.6)^2}} = 1.25$$

$$x' = \pm(150)\gamma = \pm 187.5\text{m}$$

$$t' = -\frac{0.6(\pm 150)\gamma}{3\times10^8} = \mp 0.78\times10^{-6}\text{s}.$$

$$\left.\begin{aligned}(x',t') &= (187.5\text{m}, -0.78\mu\text{s})\\ (x',t') &= (187.5\text{m}, +0.78\mu\text{s})\end{aligned}\right\}$$