L7. Relativity

Exerpt from "Great ideas that Shaped our world"

by Pete Moore (Friedman + Fairfax, 2002).

Imagine working as an editor at an academic journal, the Annalen der Physik. It is March, 1905 and you open an envelope and tip out a handwritten draft of paper with a note asking for it to be published "if there is room", signed by an unknown person calling himself Albert Einstein.

The paper elegantly updates Max Planck's theory of radiation. Two months later another letter arrives. This time the paper defines the way gas particles bounce off each other. June arrives, and so does a third paper. This one makes the rest look simple, in that it purports to modify the theory of space and time. You would surely spend some time trying to work out whether these were hoaxes, coming from a crackpot, or the work of genius. History tells us that the latter was true.

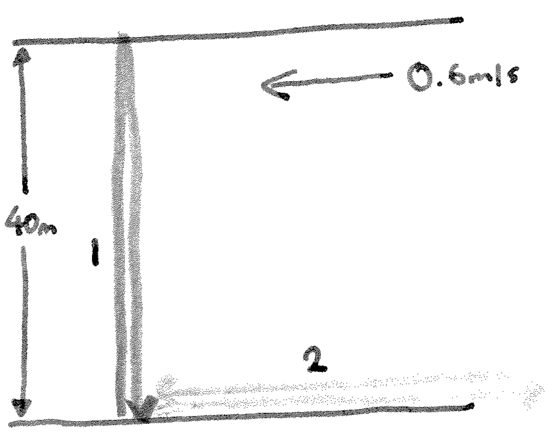
In the next three lectures, we are going to study Einstein's ideas of relativity. We're going to ask as he did: is there any way of measuring one's absolute velocity? At the end of the 19th century scientists still clung to the idea that yes, one can measure absolute velocity. After all - electromagnetism propogates as a wave moving at a vast, but finite velocity $c = 300{,}000$ km/s. Like other waves, such as sound, or water waves, they reasoned that to propogate, light had to move through a medium, a medium that was called the "ether".

But in 1887, Albert Michelson & Edward Morley failed to detect this motion through the ether, Their plan was to measure the speed of light in the direction the earth was moving, and compare it with the speed of light travelling in the opposite direction. To their surprise, whatever way they looked, light always travelled at the same speed.

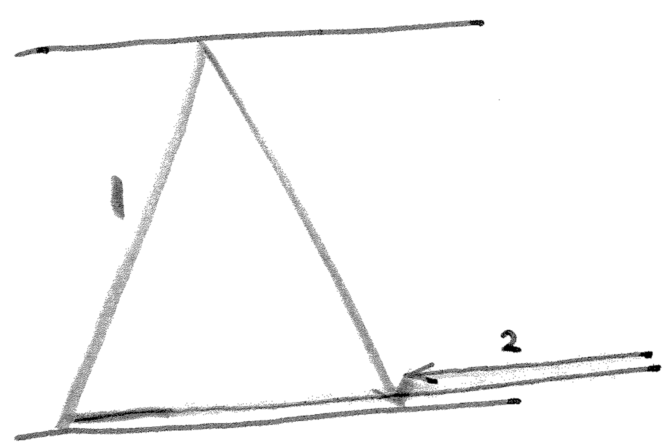
Michelson-Morley Experiment

(See: http://www.aip.org/history/gap)

Imagine two swimmers who take a bet. Both swim at 1m/s. Swimmer 1 swims across a 40m wide river & back again. Swimmer 2 swims upstream 40m & back again. The river flows at 0.6m/s. First swimmer back wins the bet. Who will win?



Viewed from the bank



Viewed from a boat drifting with the river

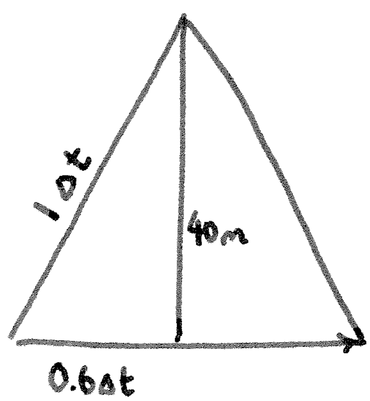
Lets first do it with numbers

$$t_2 = \frac{L}{c-v} + \frac{L}{c+v} = \frac{40}{0.4} + \frac{40}{1.6} = 100 + 25 = \underline{\underline{125s}}$$

$c = 1 m/s$

$v = 0.6 m/s$

Now for t_1, take $t_1 = 2\Delta t$



$40m^2 + (0.6\Delta t)^2 = (1\Delta t)^2$

$$40m = \sqrt{(1\Delta t)^2 - (0.6\Delta t)^2}$$

$$= 0.8\Delta t$$

$$\Delta t = \frac{40}{0.8} = 50s \qquad t_1 = 100s$$

Swimmer 1 will arrive back 25s earlier than swimmer 2.

Viewed from the water, swimmer 2 has swum 25m further than swimmer 1.

Lets now do it with symbols

$$t_2 = \frac{L}{c-v} + \frac{L}{c+v} = \frac{2cL}{c^2-v^2} \Rightarrow c\,t_2 = \frac{2c^2L}{c^2-v^2} = \frac{2L}{1-\frac{v^2}{c^2}}$$

$$L = \sqrt{c^2\Delta t^2 - v^2\Delta t^2} = \sqrt{c^2-v^2}\,\Delta t$$

$$t_1 = 2\Delta t = \frac{2L}{\sqrt{c^2-v^2}} \Rightarrow ct_1 = \frac{2cL}{\sqrt{c^2-v^2}} = \frac{2L}{\sqrt{1-\frac{v^2}{c^2}}}$$

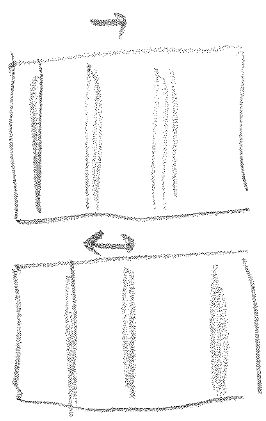
For light, we expect $v/c \ll 1$. (e.g $v_{earth} \sim 30\text{km/s}$, $c \sim 3\times10^5\text{km/s}$
$v/c \sim 10^{-4}$)

so $\left(1-\frac{v^2}{c^2}\right)^n \approx 1 - \frac{nv^2}{c^2}$, and

$$\left.\begin{aligned} \ell_1 &\approx 2L\left(1+\frac{v^2}{2c^2}\right) \\ \ell_2 &\approx 2L\left(1+\frac{v^2}{c^2}\right) \end{aligned}\right\} \quad \ell_2 - \ell_1 = L\left(\frac{v^2}{c^2}\right)$$

so that the fractional shift of the interference fringes would be $\alpha = \frac{L}{\lambda}\left(\frac{v^2}{c^2}\right)$. Michelson & Morley had their experiment on a bath of mercury & could rotate it through $90°$, reversing the shift to $-\alpha = -\frac{L}{\lambda}\left(\frac{v^2}{c^2}\right)$ so the difference in fringe locations would be

$$2\alpha = 2\left(\frac{L}{\lambda}\right)\left(\frac{v}{c}\right)^2$$



If $v \sim v_{earth} \sim 30\,\mathrm{km/s}$

$$\left(v/c\right) \sim \frac{30}{3\times 10^5} = 10^{-4}.$$

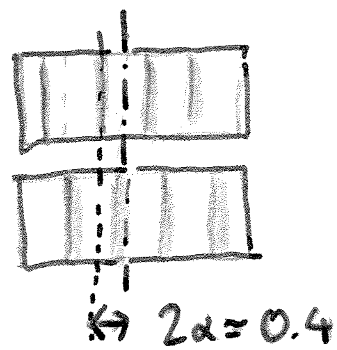
They had an effective path of $L \sim 11\mathrm{m}$ & $\lambda \sim 550\,\mathrm{nm}$ (yellow) so $L/\lambda \sim 2\times 10^7$. This then predicts $\alpha \sim 2\times 2\times 10^7 \times 10^{-8}$
~ 0.4 fringe.

Michelson & Morley knew that for light, a path difference

$$l_2 - l_1 \approx L\left(\frac{v^2}{c^2}\right)$$

would translate into a fractional movement of the fringe $\alpha = (l_2 - l_1)/\lambda$. By turning the apparatus through 90° they should detect a net movement of the fringe of

$$2\alpha = \frac{2L}{\lambda}\left(\frac{v^2}{c^2}\right)$$



In their experiment $L \sim 11\,\text{m}$, $\lambda \sim 550\,\text{nm}$ (yellow).

$$2\alpha \approx 2 \times (2 \times 10^{-7}) \times 10^{-8} \approx 0.4 \text{ fringe}$$

This shift has never been seen.

37.1 EINSTEINS POSTULATES OF RELATIVITY

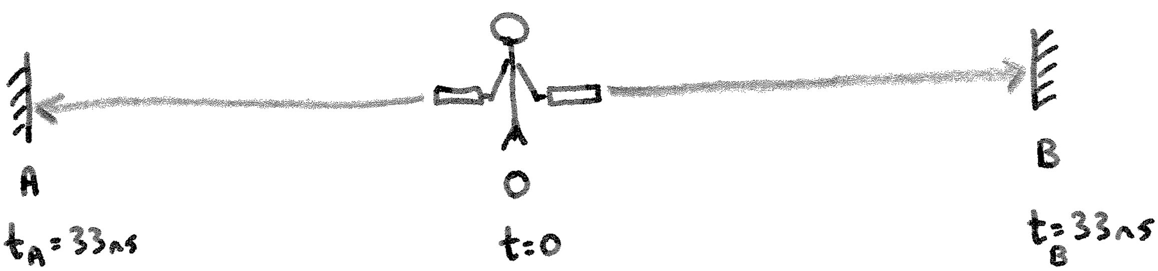
(Inertial reference frame ≡ SPECIAL RELATIVITY)

I	The laws of physics are the same in every inertial reference frame.
II	The speed of light in vacuum is the same in all inertial frames of reference and is independent of the motion of the source

- missile launched from plane moving at 1000 m/s picks up a speed $V = V_{missile} + 1000\ m/s$.
- light launched from a plane does <u>not</u> pick up an additional velocity.
- It is impossible to move at the speed of light. (Or would contradict II).

37.2 RELATIVITY OF SIMULTANEITY.

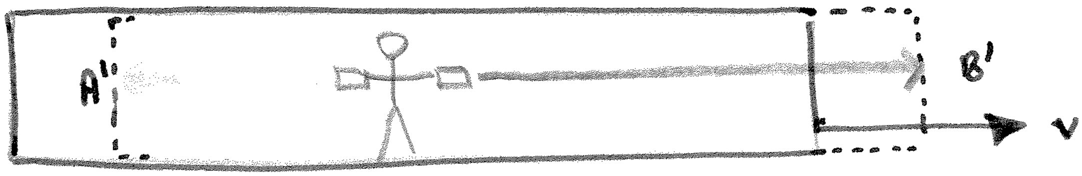
Two events which are simultaneous in one reference frame, are not simultaneous in another inertial reference frame. Suppose I am at the center of a 20m long room & I fire a laser beam at each wall. Suppose I press the trigger on both laser pointers simultaneously, then after a time $t = 10m/3\times10^8 \approx 33ns$ the light reaches each opposite wall.



The arrival of the beams at A & B is, for me, <u>simultaneous</u>

Now suppose I do the same experiment on a train — for me, the results are identical — I still say that the laser beam arrives simultaneously at opposite walls. But suppose you are on the platform watching me.

You see me passing by as follows:



You see me fire off the two laser beams at the same time, and you will be able to start your stop-watch. However — you will see the beam going oppositely to the direction of the train arrive earlier at A' than it arrives at B', i.e whereas

$t_A = t_B$ in the carriage - simultaneous

$t_A' < t_B'$ events not simultaneous.

Suppose $v \approx 1\% c$, then $t_{A'} = 33.3 \left(1-0.01\right) = 33 ns$

$t_{B'} = 33.3 \left(1+0.01\right) \approx 33.6\dot{6} ns$.

THE TWO EVENTS ARE 2/3 ns APART FROM ONE ANOTHER.