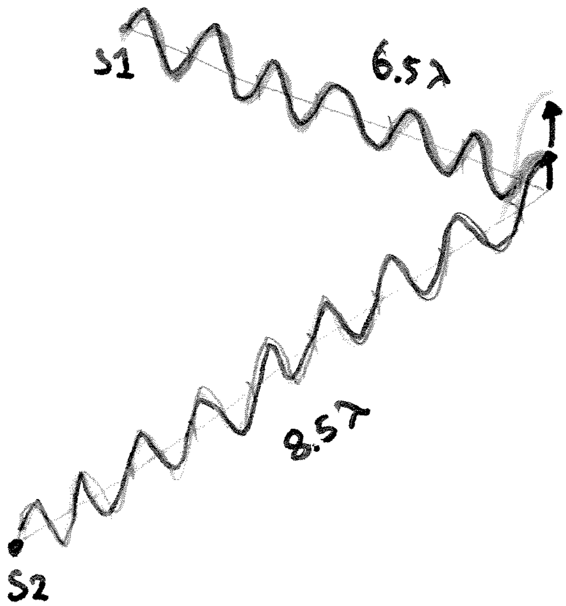
INTERFERENCE

When we look at oil on water — in a puddle, or in a soap bubble as we do the washing up — we think nothing of the rainbow colors. Yet if we pause for a moment — we should wonder indeed — for beneath our very eyes we are seeing dramatic indication of the wave nature of light. The phenomenon we are witnessing in a colored soap bubble — is the phenomenon of interference.

Today we'll study interference in light — but what I want you to remember is that interference is fundamental to <u>all</u> waves – sound waves, water waves, light waves & absolutely most important of all – to <u>matter</u> waves. Indeed the entire structure of atoms, molecules & you yourself, is due to the delicate interference of matter waves down on the atomic scale. So interference is fundamentally important to all of nature – which is why we must now concentrate on this fascinating phenomenon.

35.1 Constructive + Destructive INTERFERENCE

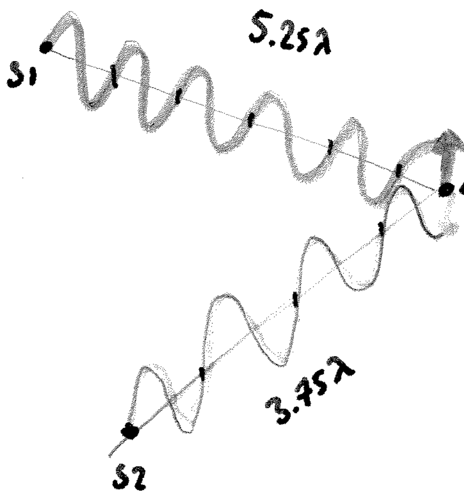


Constructive.

$$\boxed{r_2 - r_1 = m\lambda}$$

$m = 0, \pm 1 \ldots$

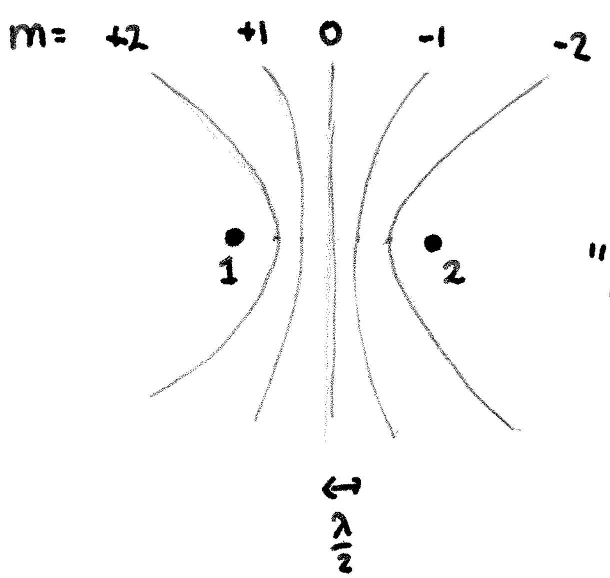
"ANTINODE"



no net disturbance

$$\boxed{r_2 - r_1 = \left(m + \tfrac{1}{2}\right)\lambda}$$

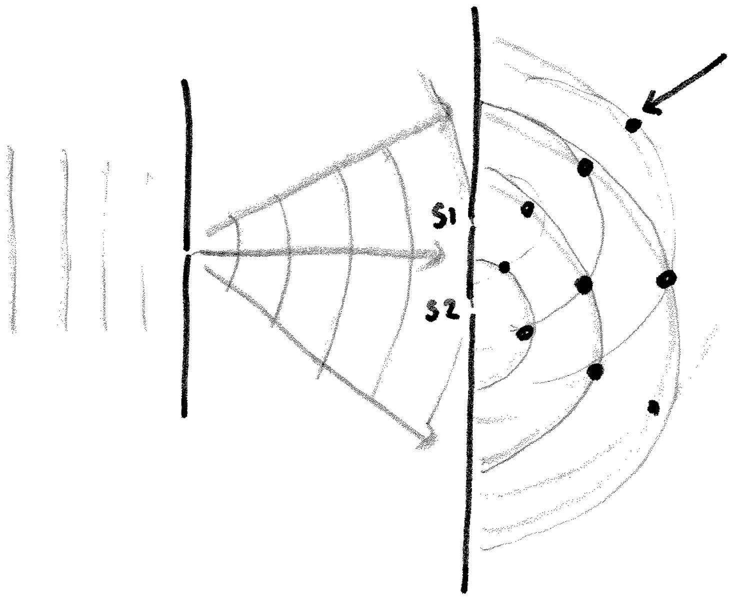
DESTRUCTIVE INTERFERENCE

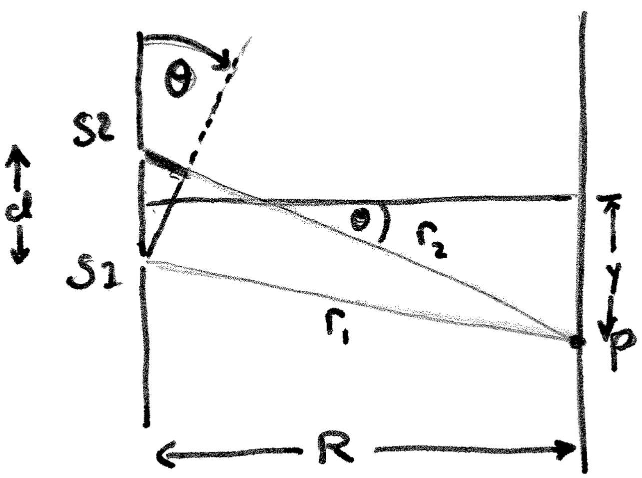


"antinodal lines"

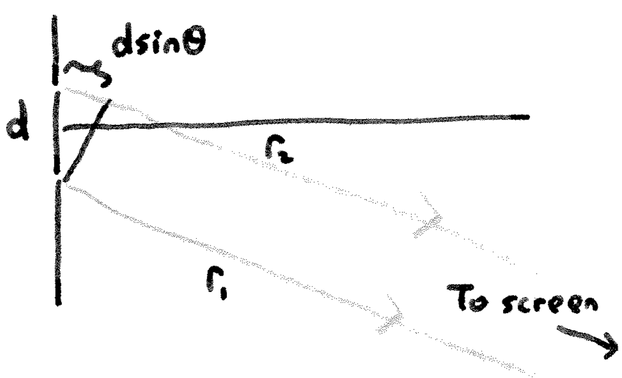
35.2 TWO SLIT INTERFERENCE

Requires two synchronized or "coherent" sources.





$$r_2 - r_1 = d\sin\theta$$



$$\boxed{d\sin\theta = m\lambda}$$
CONSTRUCTIVE

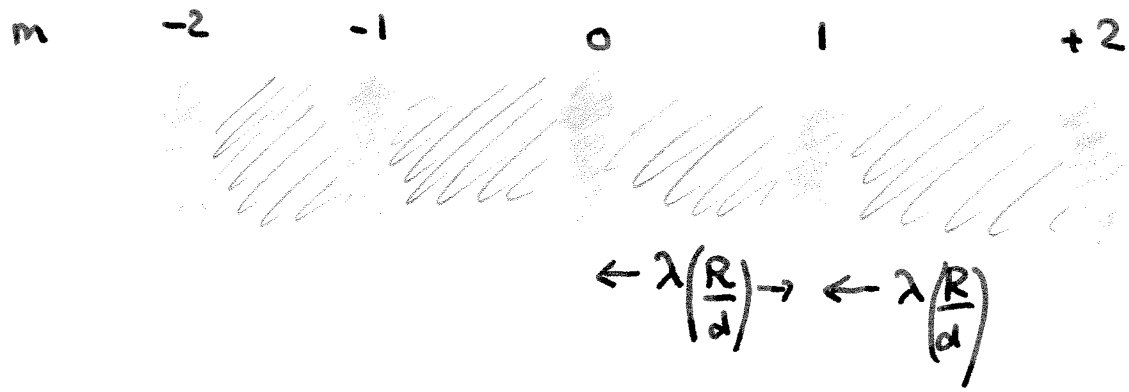
$$\boxed{d\sin\theta = \left(m+\tfrac{1}{2}\right)\lambda}$$
DESTRUCTIVE

$$y_m = R\tan\theta_m$$

For small θ $\quad \sin\theta_m \sim \theta_m \sim \tan\theta_m \;. \Rightarrow \theta_m = y_m / R$

$$\boxed{y_m = R\left(\frac{m\lambda}{d}\right)}$$

Youngs Fringes



Ex 1

e.g 2 slit expt, slits 0.2mm apart, screen @ a distance of 1.m. If the third bright fringe is 7.5mm from the central fringe, what is the wavelength of light?

$d = 0.2\,mm$

$R = 1m$

$$\longleftarrow 3\lambda\left(\frac{R}{d}\right) \longrightarrow$$

$$y_3 = m\lambda\left(\frac{R}{d}\right) = 7.5\,mm$$

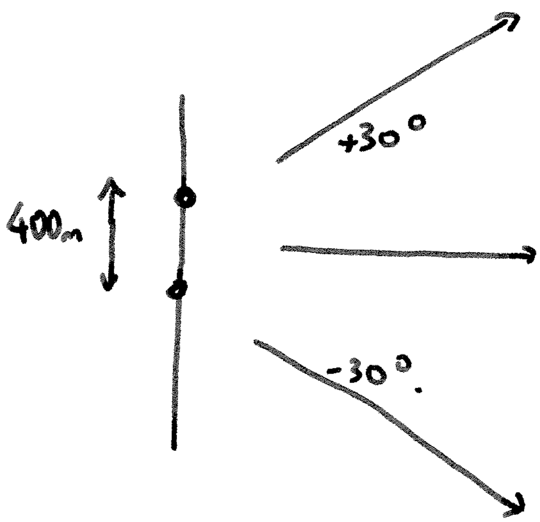
$$\Rightarrow \lambda = \frac{y_3}{3}\left(\frac{d}{R}\right) = \frac{7.5\times10^{-3}}{3}\left(\frac{0.2\times10^{-3}}{1m}\right) = 0.5\times10^{-6}\,m$$

$$= \underline{500\,nm}$$

EX 2

Radio station operating at $f = 1.5\,MHz$ has two dipole antennae, $d = 400m$ apart.

Far away, what directions have the greatest intensity?



$$\sin\theta_m = m\left(\frac{\lambda}{d}\right)$$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{1.5\times10^6\,Hz} = 200m$$

$$\sin\theta_m = m \times \left(\frac{200}{400}\right) = \frac{m}{2} = 0, \pm\frac{1}{2}$$

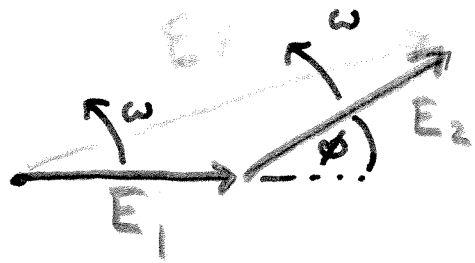
$$\theta_m = 0, \pm 30°$$

35.3 Intensity in Interference Patterns

$E_1 = E \cos \omega t$

$E_2 = E \cos(\omega t + \phi)$

↑
PHASE
SHIFT



$\vec{E}_p = \vec{E}_1 + \vec{E}_2$

$$E_p^2 = (E_1 + E_2)\cdot(E_1 + E_2) = (E_1)^2 + (E_2)^2 + 2\vec{E}_1\cdot\vec{E}_2$$

$$= 2E^2 + 2E^2 \cos\phi$$

$$= 2E^2(1 + \cos\phi)$$

$$= 4E^2 \cos\phi/2$$

$$\boxed{E_p = 2E\left|\cos(\phi/2)\right|}$$

amplitude in two-source interference.

Intensity $= \quad S_{AV} = c\left(\frac{\epsilon_0 E_p^2}{2}\right)$

$$I = \frac{1}{2}\epsilon_0 c E_p^2 = 2\epsilon_0 c E^2 \cos^2\left(\frac{\phi}{2}\right)$$

$$\boxed{I = I_0 \cos^2 \phi/2}$$

$$I_0 = 2\epsilon_0 c E^2 = 4 \times \left(\frac{\epsilon_0 c E^2}{2}\right)$$

When the phase difference is zero, the intensity is QUADRUPLED, amplitude DOUBLED

15.3b PHASE DIFFERENCE, PATH DIFFERENCE

$$\frac{\phi}{2\pi} = \frac{(r_2 - r_1)}{\lambda}$$

PHASE DIFFERENCE BETWEEN WAVES.

$$\boxed{\phi = \frac{2\pi}{\lambda}(r_2 - r_1) = k(r_2 - r_1)}$$

PHASE DIFFERENCE IN TERMS OF WAVENUMBER.

Wavenumber. $k = \frac{2\pi}{\lambda}$

In a medium $\begin{cases} \lambda = v/f = c/nf = \lambda_0/n \\ k = k_0 n \end{cases}$

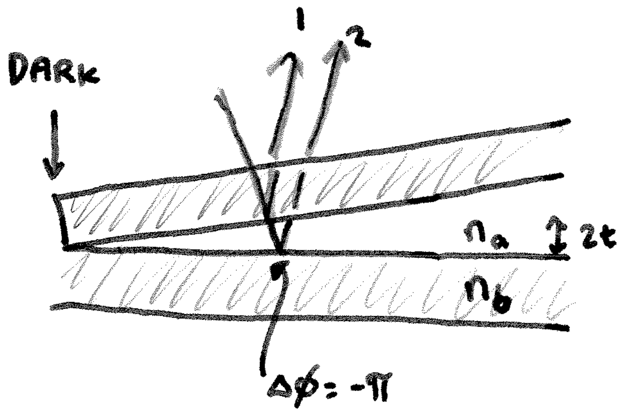
$r_2 - r_1 = d\sin\theta$

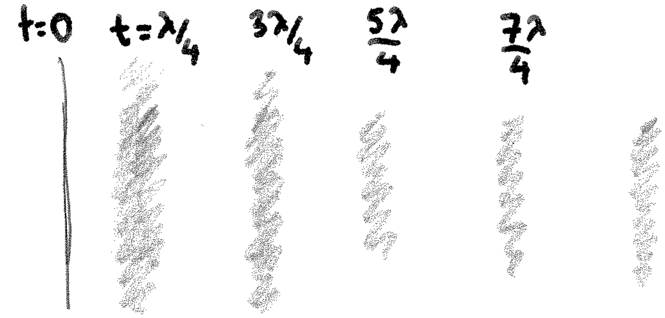
$$\phi = k(r_2 - r_1) = kd\sin\theta = \frac{2\pi}{\lambda} d\sin\theta$$

$$\boxed{I = I_0 \cos^2\left(\frac{1}{2} kd\sin\theta\right) = I_0 \cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right)}$$

max intensity when $\pi d/\lambda \sin\theta = m\pi \Rightarrow d\sin\theta = m\lambda$.

35.4 Thin Film Interference





$$E_r = \left(\frac{n_a - n_b}{n_a + n_b}\right) E_r$$

		$\Delta\phi$
$n_a > n_b$	no sign change	0
$n_a = n_b$	no reflection	
$n_a < n_b$	sign change	$\pm\pi$

Reflected beam 2 picks up a phase shift

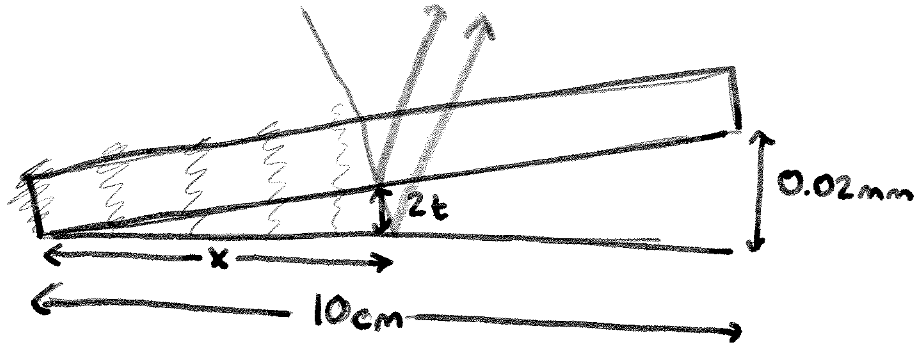
$$\phi = -\pi + 2\pi\left(\frac{2t}{\lambda}\right)$$

$$\phi = 2\pi m \quad \Rightarrow \quad 2\pi\left(m + \tfrac{1}{2}\right) = 2\pi\left(\frac{2t}{\lambda}\right)$$

$$\Rightarrow \quad \boxed{2t = \lambda\left(m + \tfrac{1}{2}\right)}$$ CONSTRUCTIVE INTERFERENCE.

$2t = \frac{\lambda}{2} \Rightarrow t = \frac{\lambda}{4}$ First bright fringe

35.4 ex 1



WHAT IS THE SPACING BETWEEN FRINGES

$\lambda_0 = 500\text{nm}.$

$$2t = \lambda_0\left(m+\tfrac{1}{2}\right) \Rightarrow t_m = \frac{\lambda_0}{2}\left(m+\tfrac{1}{2}\right) = (250\text{nm})\times\left(m+\tfrac{1}{2}\right)$$

$$\frac{t}{x} = \frac{0.02\text{mm}}{100\text{mm}} \Rightarrow x = t\times\frac{(100)}{0.02} = 5000t$$

$$= 5\times10^3\times2.5\times10^{-7}\times\left(m+\tfrac{1}{2}\right)$$

$$= 1.25\times10^{-3}\times\left(m+\tfrac{1}{2}\right)$$

$$x_m = \left(m+\tfrac{1}{2}\right)\times(1.25\text{ mm})$$

$$\underline{\Delta x = 1.25\text{mm}}$$