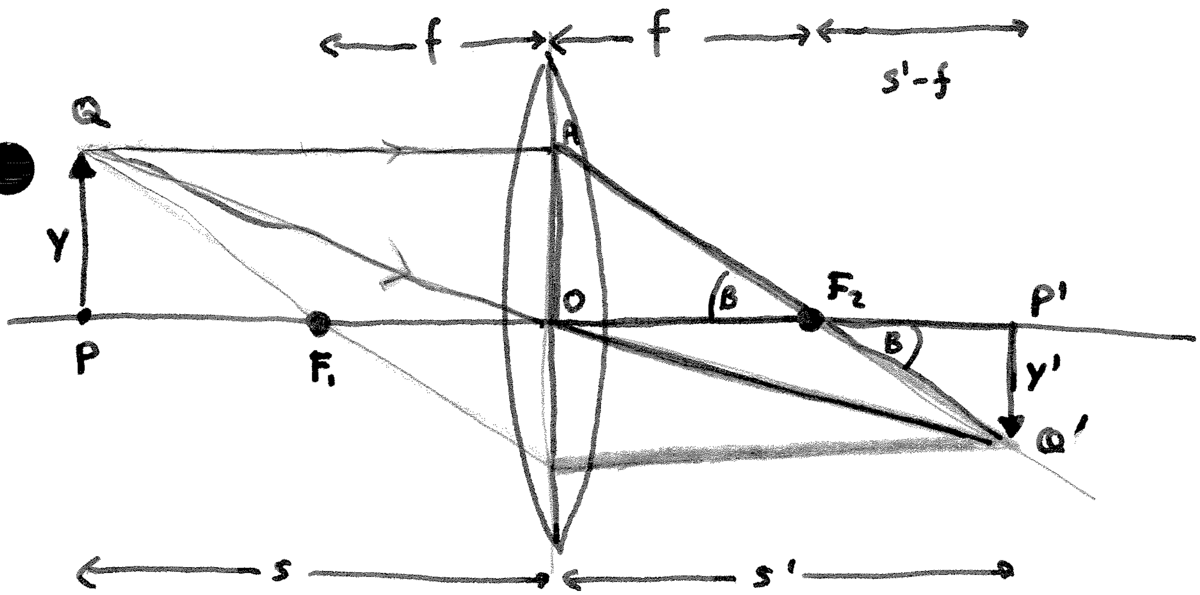
3. IMAGES FORMED BY REFRACTION:

THIN LENSES.

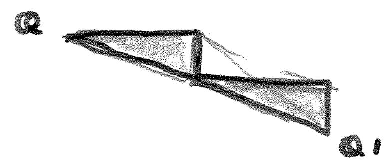
The lense is the most widdly used optical device.

On a hot sunny day, you have surely experimented with magnifying glasses & observed how a lens can be used to burn holes in paper, fry ants & even start a fire. When you do this, you are imaging the surface of the sun, on the paper, (or ant) that you are burning. What then is the relation between the image & object for a lens?

34.4a



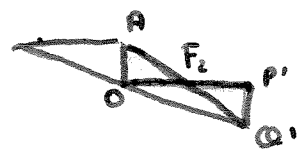
ΔQAO & $\Delta Q'OP'$ are similar



$$\frac{y}{s} = -\frac{y'}{s'} \Rightarrow \boxed{m = \frac{y'}{y} = \frac{-s'}{s}} \quad (1)$$

magnification - thin lens

Also $\Delta P'Q'F_2$ & ΔAF_2O are similar



$$\Rightarrow \quad y/f = -\frac{y'}{s'-f} \Rightarrow \frac{y'}{y} = -\frac{s'-f}{f} \quad (2)$$

Combining (1) & (2) gives $\quad \frac{s'}{s} = \frac{s'-f}{f} \Rightarrow \frac{f}{s} = 1 - \frac{f}{s'}$

$$\Rightarrow \boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}}$$

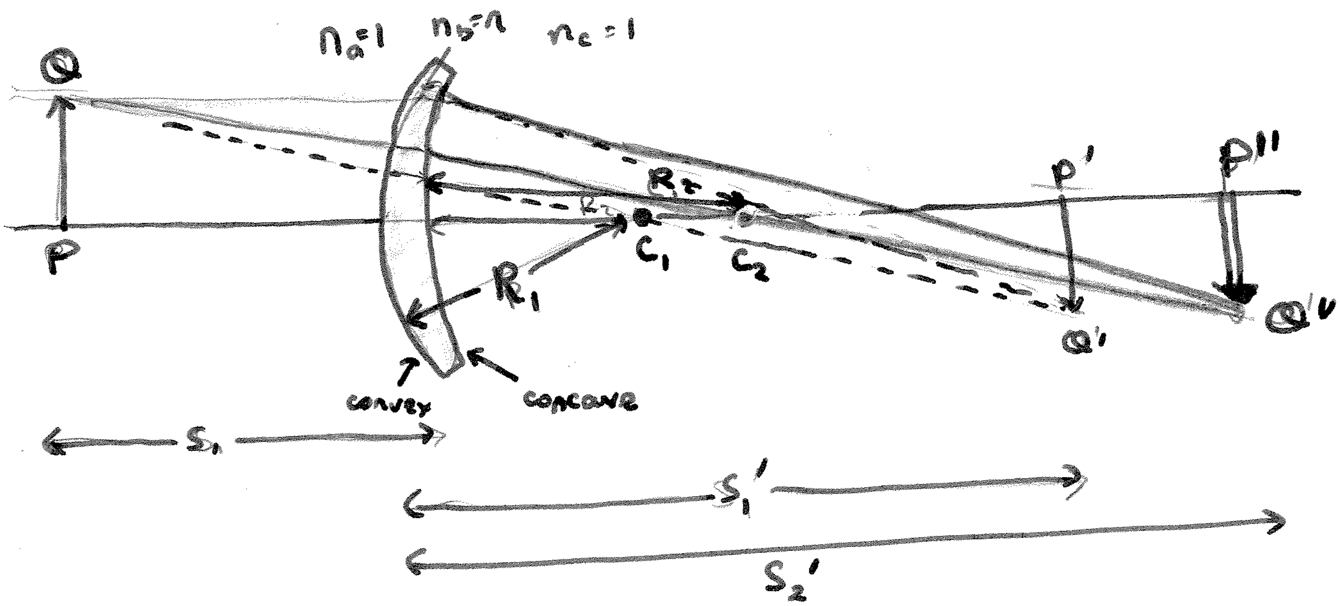
thin lens o-image reln.

34.4b Lensmaker Equation

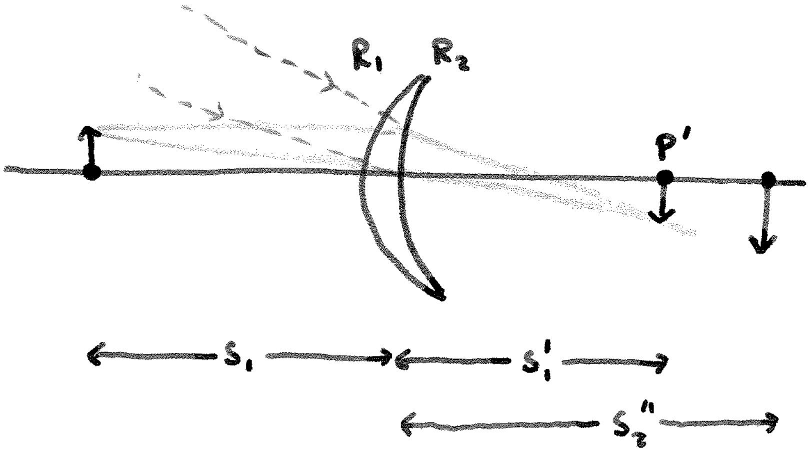
- But how is the focal length related to the dimensions of the lens? Here we can use our basic refraction equation

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

- but apply it to a lens which has two radii of curvature



- The light refracting off the second surface is focussed at a distance s_1' to the right of the surface, and behaves as an OBJECT at position $s_2 = -s_1'$ for the second surface.



$$\frac{1}{S_1} + \frac{n}{S_1'} = \frac{n-1}{R_1} \qquad (1)$$

The image at P_1' behaves as a virtual object for the second surface, so in

$$\frac{n}{S_2'} + \frac{1}{S_2''} = \frac{1-n}{R_2} \Rightarrow -\frac{n}{S_1'} + \frac{1}{S_2''} = \frac{1-n}{R_2} \qquad (2)$$

$$S_2' = -S_1'$$

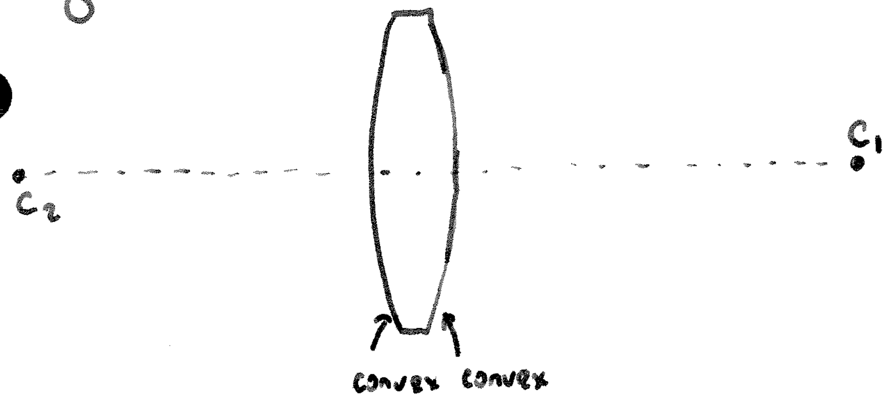
(1) & (2)

$$\boxed{\frac{1}{S_1} + \frac{1}{S_2''} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

Lensmaker equation

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

e.g



$|R_1| = |R_2| = 10\,cm$

$n = 1.52$

What is the focal length?

C_1 to right $\Rightarrow R_1 > 0$ $\quad R_1 = +10\,cm$

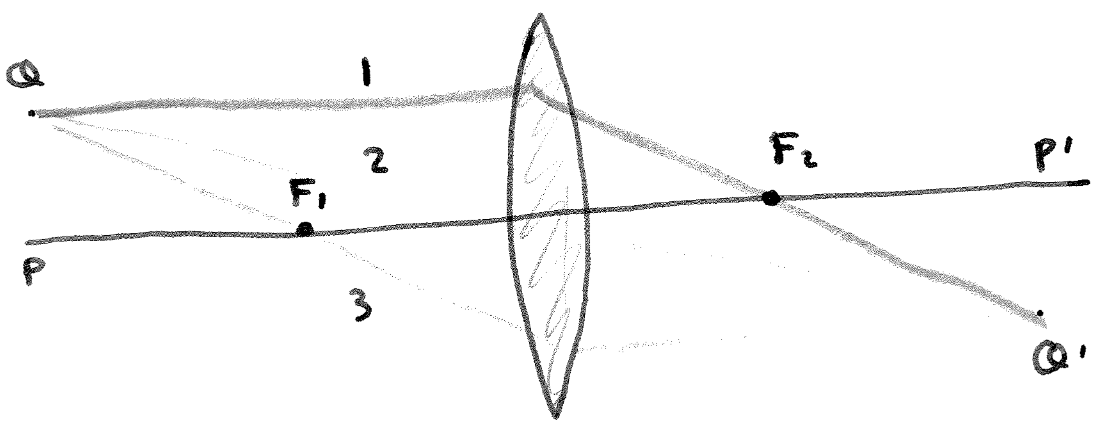
C_2 to left $\Rightarrow R_2 < 0$ $\quad R_2 = -|R_2| = -10\,cm$

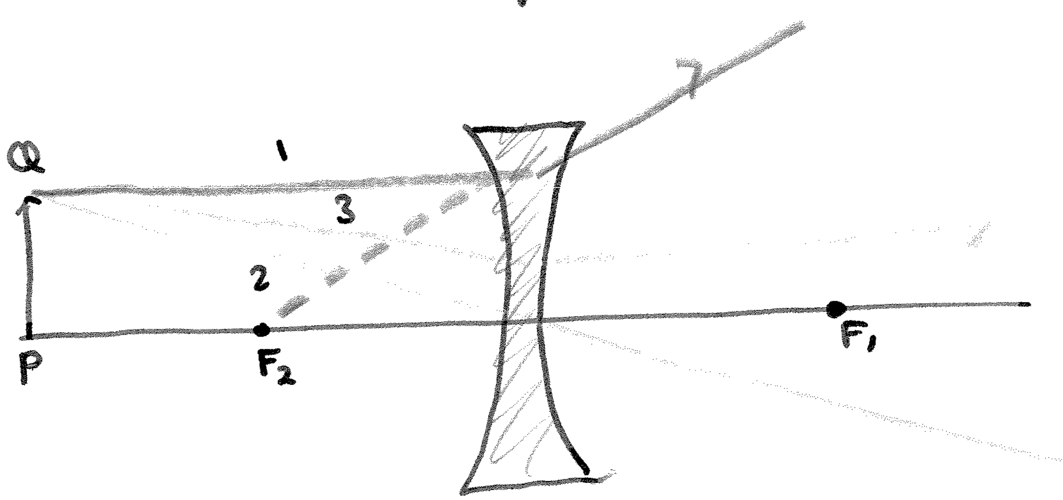
$$\frac{1}{f} = (1.52-1)\left(\frac{1}{10\,cm} - \frac{1}{-10\,cm}\right) = 0.52 \times \frac{2}{10\,cm} = \frac{0.52}{5\,cm}$$

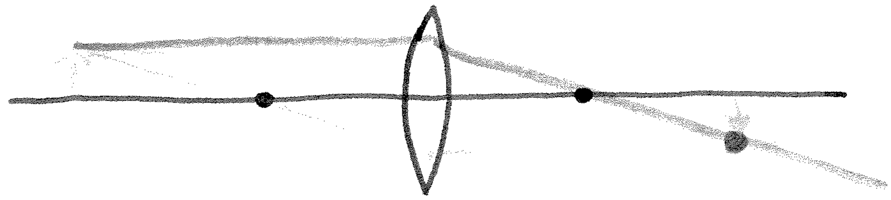
$\Rightarrow f = 9.6\,cm.$

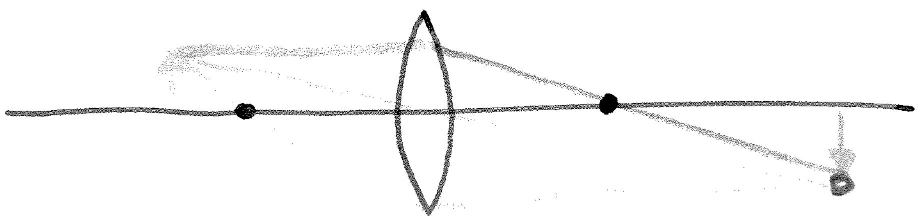
34.4c Graphical Methods for Lenses.

1. Parallel ray ⟶ F_2
2. Ray through center
3. Ray through F_1 ⟶ parallel

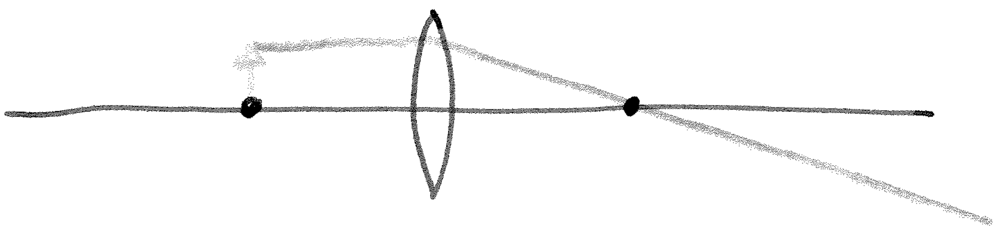








$\frac{s'}{s}$ bigger



$\frac{s'}{s} = \infty$

image at infinity

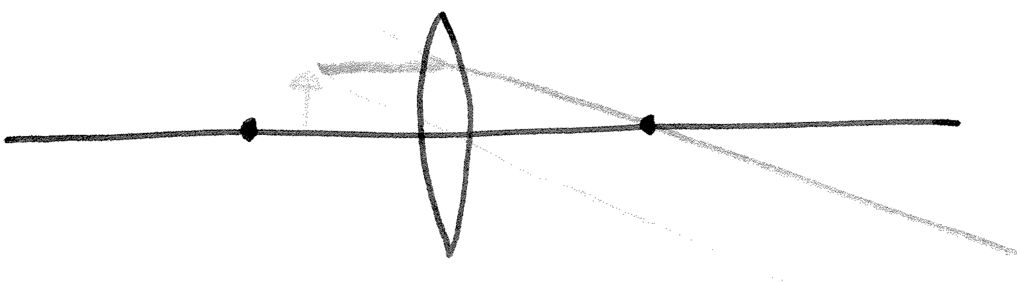
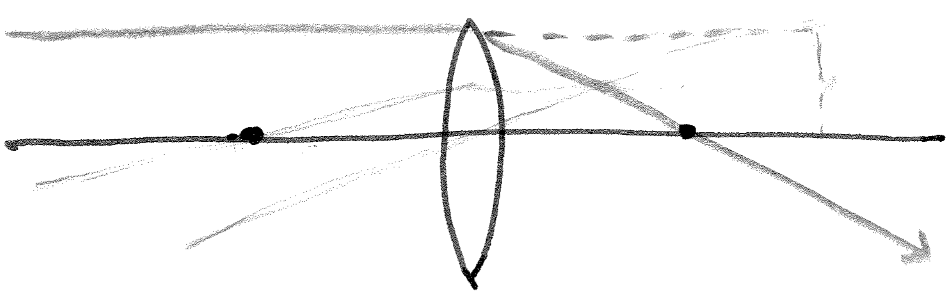


image virtual
& larger than object
(magnifying glass)



Light rays converging
on lens.

e.g. (A)

$f = 20\text{cm}$. Calculate m & s'

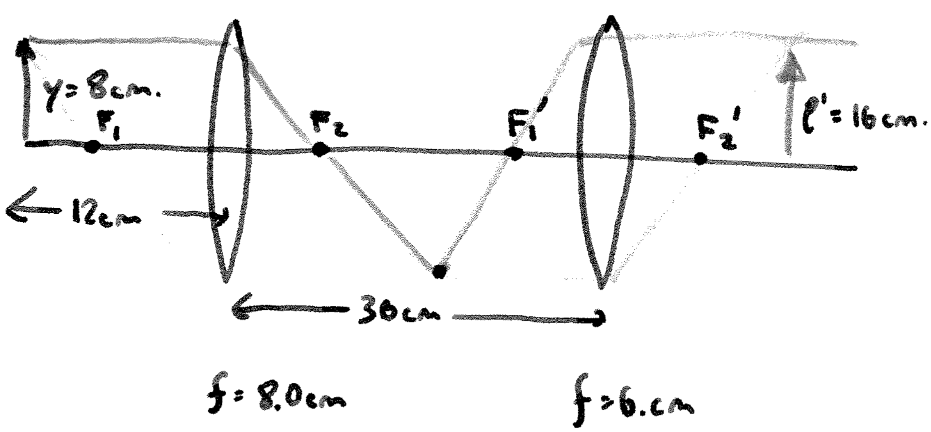
a) $s = 50\text{cm}$ $\quad \frac{1}{50} + \frac{1}{s'} = \frac{1}{20} \Rightarrow s' = 33.3\text{cm} \quad m = -\frac{33.3}{50} = -\frac{2}{3}$

b) $s = 20\text{cm}$ $\quad \frac{1}{20} + \frac{1}{s'} = \frac{1}{20} \Rightarrow s' = \pm\infty \quad m = \pm\infty$

c) $s = 15\text{cm}$ $\quad \frac{1}{15} + \frac{1}{s'} = \frac{1}{20} \Rightarrow s' = -60 \quad m = \frac{60}{15} = 4$

d) $s = -40\text{cm}$ $\quad \frac{1}{-40} + \frac{1}{s'} = \frac{1}{20} \Rightarrow s' = 13.3\text{cm} \quad m = -\frac{(13.3)}{-40.0} = \frac{1}{3}$.

(B) Image of an image.



Find position, size & orientation of image produced.

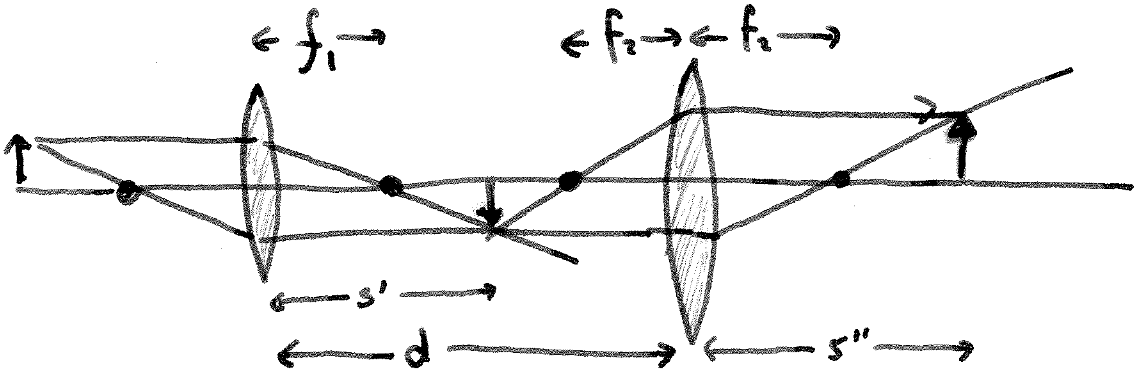
$\frac{1}{12} + \frac{1}{s_1'} = \frac{1}{8.0} \Rightarrow s_1' = +24\text{cm} \Rightarrow s_2 = 12\text{cm} \quad m_1 = -\frac{24}{12} = -2$

$\frac{1}{12} + \frac{1}{s_2'} = \frac{1}{6\text{cm}} \Rightarrow s_2' = +12\text{cm}. \quad m_2 = -\frac{12}{12} = -1$

$m_{TOT} = m_1 m_2 = +2$

- $$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Two lenses: image of second is object of first.



- e.g. $s = 20\text{cm}$ $\quad f_1 = 10\text{cm}$ $\quad f_2 = 5\text{cm}$

$d = 40\text{cm}$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$\frac{1}{20} + \frac{1}{s'} = \frac{1}{10} \qquad \frac{1}{s'} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20} \Rightarrow s' = 20\text{cm}$$

$m_1 = -1$

$$S_2 = d - s' = 40 - 20 = 20\text{cm}.$$

- $$\frac{1}{20} + \frac{1}{s''} = \frac{1}{5} \qquad \frac{1}{s''} = \frac{1}{5} - \frac{1}{20} = \frac{4-1}{20} = \frac{3}{20}$$

$$\Rightarrow s'' = \frac{20}{3} = 6.\dot{6}\text{ cm}$$

$m_2 = -\frac{1}{3}$ $\qquad m_1 m_2 = +\frac{1}{3}.$