

Particles

			S
	k^0	k^+	1
π^-	π^0	π^+	0
	k^-	\bar{k}^0	-1

	$\bar{s}d$	$\bar{s}u$
$\bar{u}d$	$\bar{u}u$	$u\bar{d}$
	$\bar{u}s$	$\bar{d}s$

			S
	n^0	p^+	0
Σ^-	Σ^0, Λ^0	Σ^+	-1
	Ξ^-	Ξ^0	-2

	udd	und
$sdd.$	sdu	suu
	ssd	ssu

	L_e	L_μ
e^-	1	
ν_e	1	
μ^-		1
ν_μ		1

u	$Q = \frac{2}{3}$	$S = 0$
d	$Q = -\frac{1}{3}$	$S = 0$
s	$Q = -\frac{1}{3}$	$S = -1$

34. If a beam of K^- particles collides with protons in a target, a possible outcome of the collision, due to the strong interactions, is $K^- + p \rightarrow$?

- a) $\Lambda^0 + p$
- b) $\Lambda^0 + K^0$
- \rightarrow c) $\Lambda^0 + \pi^0$
- d) $p + \pi^0$
- e) $p + \pi^-$

X

	K^-	+	p	\longrightarrow	$\Lambda^0 + K^0$ (b)
Q	-1		+1		
S	-1		0		
B	0		1		

$\Lambda^0 + p$	Q=+1	no X
$\Lambda^0 + K^0$	S=-2	X
	Q=0	X
	B=1	

(c)	$\Lambda^0 + \pi^0$	S=-1	✓
	$p + \pi^0$	S=0	X
	$p + \pi^-$	S=0	X

39. The following decay and reaction each involve a neutrino.

$K^+ \rightarrow \mu^+ + ?$

$? + n \rightarrow p^+ + e^-$

What are the identities of the missing neutrinos in the first and second equations, respectively?

- a) ν_μ, ν_μ
- b) $\bar{\nu}_\mu, \bar{\nu}_e$
- \rightarrow c) $\nu_\mu, \bar{\nu}_e$ X
- d) $\nu_\tau, \bar{\nu}_e$
- e) ν_μ, ν_e

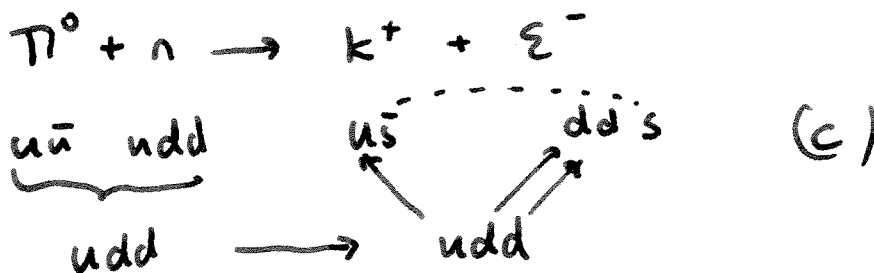
	K^+	\longrightarrow	μ^+	+	ν_μ	
L_μ	0		-1		+1	
	$\bar{\nu}_e$	\longrightarrow	p^+	+	e^-	
L_e	-1		0		-1	

(c)

29. The following reaction can occur by the strong interaction: $\pi^0 + n \rightarrow K^+ + \Sigma^-$. If the quark composition of the n is (udd) , the π^0 is $(u\bar{u})$, and the K^+ is $(u\bar{s})$, find the quark composition of the Σ^- .

X

- a) $\bar{u}s$
- b) sss
- c) dds
- d) $dd\bar{s}$
- e) uds



27. The following decay is forbidden:

$$\pi^+ \rightarrow \mu^+ + \nu_e + \nu_\mu$$

X because it is not possible to conserve:

- a) charge
- b) electron lepton number
- c) baryon number
- d) muon lepton number
- e) energy

	π^+	\rightarrow	μ^+	ν_e	ν_μ
Q	+1		+1		
L_e	0		0	1	
			-1	0	1

$\Delta L_e = +1$ No! b)

35. The Δ^{++} particle is a baryon with electric charge $+2e$ and strangeness, charm, bottomness and topness all equal to zero. Which of the following is a possible quark combination for this baryon?

- a) $\bar{c}u$
- b) ddd
- c) $\bar{c}d$
- d) uuu
- e) $s\bar{s}u$

$$S = C = B = T = 0$$

$\bar{c} \bar{c} u$	$\frac{2}{3}$	
ddd	-1	(d).
$c \bar{c} d$	$\frac{2}{3}$	
uuu	2	*
$s \bar{s} u$	$\frac{2}{3}$.

24. A meson has strangeness -1 and charge -1. The quark content of the meson is:

- a) ssd
- b) $s\bar{u}$
- c) $s\bar{d}$
- d) sdd
- e) $d\bar{u}$

$$\begin{array}{l}
 S = -1 \Rightarrow s \leftarrow q_s = -\frac{1}{3} \\
 Q = -1 \Rightarrow \bar{u} \quad q_{\bar{u}} = -\frac{2}{3}
 \end{array}$$

(b)

26. The reaction ${}^2\text{H} + {}^2\text{H} \rightarrow {}^3\text{H} + {}^1\text{H}$ produces protons. The Q of the reaction is

- a) 4.03 MeV
- X b) 1.04 MeV
- c) 7.06 MeV
- d) 3.27 MeV
- e) 17.59 MeV

The needed masses are hydrogen 1.007825 u, deuterium (d) 2.014102 u, ${}^3\text{H}$ 3.016049 u, ${}^3\text{He}$ 3.016029 u, ${}^4\text{He}$ 4.002603 u, neutron 1.008665 u.

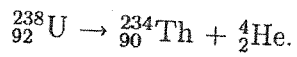
Recall $Q = (M_{\text{initial}} - M_{\text{final}}) \times 931.5 \text{ MeV}$

$$M_{\text{initial}} = 2(2.014102) = 2({}_1^2\text{M}) = 4.0282$$

$$M_{\text{final}} = 3.016049 + 1.007825 = 4.02387$$

$$Q = 4.03 \text{ MeV} \quad (\text{a})$$

35. How much energy is released in the α -decay of ${}_{92}^{238}\text{U}$:



Where:

$$M({}_{92}^{238}\text{U}) = 238.050786u$$

$$M({}_{90}^{234}\text{Th}) = 234.043583u$$

$$M({}_2^4\text{He}) = 4.002603u$$

a) 0.05 MeV

b) 52.75 MeV

c) 4.29 MeV

d) 9.73 MeV

e) 1.06 MeV

$$Q = (M_i - M_f) 931.5$$

$$= (238.050786 - 234.043583 - 4.002603)$$

$$\times 931.5 = 4.28 \text{ MeV} \quad (\text{c}).$$

25. A beam of protons of kinetic energy K collides with a hydrogen target (which contains protons essentially at rest). Pions are produced by the reaction



Denote the rest energy of a proton by Mc^2 , and of a pion by mc^2 . The minimum kinetic energy K that will produce the reaction is:

- a) $K = mc^2[2 + 4(m/M)]$
- b) $K = mc^2[4 + 4(m/M)]$
- c) $K = mc^2[2 + 3(m/M)]$
- d) $K = mc^2[4 + 2(m/M)]$ *
- e) $K = mc^2[2 + 2(m/M)]$

$$E_{cm}^2 = 2(E + Mc^2)Mc^2$$

$$E_{cm}^2 = 2(E + Mc^2)Mc^2 = 2(2Mc^2 + K)Mc^2 = (2Mc^2 + 2mc^2)^2$$

$$\Rightarrow (2 + K/Mc^2) = 2(1 + m/M)^2 \Rightarrow K = mc^2(4 + 2m/M)$$

MOLECULE

30. The ground state vibrational energies of the CO and N₂ molecules are 0.125 eV and 0.100 eV, respectively. The ratio of their effective spring constants (i.e. K_{CO} / K_{N_2}) is about:
(Note: $A_C = 12$; $A_N = 14$; $A_O = 16$)

- a) 1.25 b) 1.53 c) 0.75 d) 1.0 e) 2.35

$$E_{gs} = \frac{\hbar\omega}{2}$$

$$\omega = \sqrt{\frac{K}{\mu_{red}}}$$

$$\frac{E_{CO}}{E_{N_2}} = 1.25 = \frac{\omega_{CO}}{\omega_{N_2}} = \sqrt{\frac{K_{CO}}{K_{N_2}} \left(\frac{\mu_{N_2}}{\mu_{CO}} \right)}$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$\mu_{CO} = \frac{12 \cdot 16}{28} = 6.86$$

$$\mu_{N_2} = \frac{14 \cdot 14}{28} = 0.5(14) = 7$$

$$\frac{K_{CO}}{K_{N_2}} = (1.25)^2 \times$$

$$\left(\frac{\mu_{CO}}{\mu_{N_2}} \right)$$

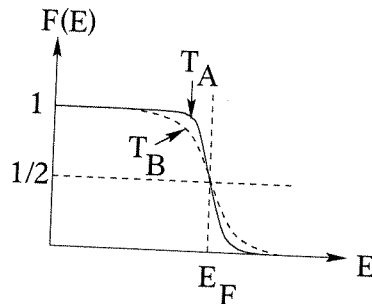
$$= 1.56 \times \left(\frac{6.86}{7} \right) = 1.53 \quad (b)$$

p.s putting $\mu_{CO} / \mu_{N_2} = 1$ gives close to (b).

BANDS.

22. The figure shows the Fermi function for a solid at two different temperatures T_A and T_B . From the shape of these curves we can tell that:

- a) the solid is an insulator.
 → b) $T_A < T_B$
 c) $T_B = 0 \text{ K}$
 d) states above the Fermi energy are more likely to be occupied at T_A than at T_B .
 e) $T_A > T_B$



$$T_B > T_A \quad (b)$$

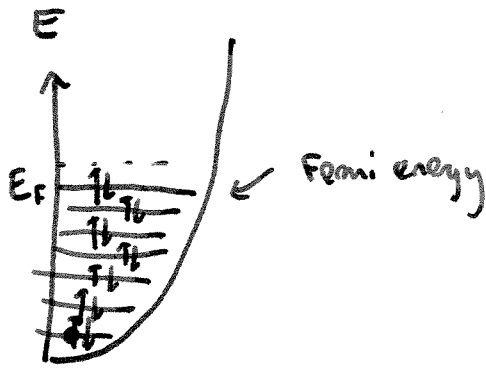
32. At room temperature ($T = 300\text{K}$), the probability that a valence electron in Al has kinetic energy 10% greater than the Fermi energy ($E_F = 11.7 \text{ eV}$) is about:

- ⊗ a) 2.1×10^{-2}
 → b) 2.0×10^{-20}
 c) 9.2×10^{-16}
 d) 0.94
 e) 0.11

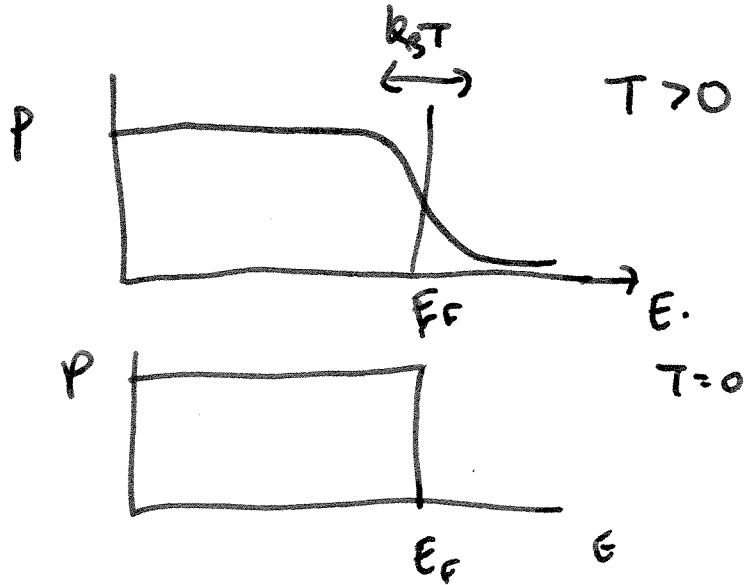
$$E - E_F = 0.1 \times 11.7 \text{ eV} = 1.17 \text{ eV} = 1.872 \times 10^{-19} \text{ J}$$

$$k_B T = 1.38 \times 10^{-23} \times 300 = 4.14 \times 10^{-21} \text{ J}$$

$$P = \frac{1}{e^{(E - E_F) / (k_B T)}} \approx 2.2 \times 10^{-20} \quad (b)$$



$$P(\text{occupied}) = \frac{1}{e^{(E-E_F)/k_B T} + 1}$$



31. In the Fermi gas model for valence electrons in a metal, the Fermi energy is:




- a) The minimum kinetic energy of an electron at $T = 0$.
- b) The maximum kinetic energy of an electron at $T = 0$.
- c) The maximum kinetic energy of an electron at high temperatures.
- d) The average kinetic energy of the electrons.
- e) The kinetic energy of all electrons at $T = 0$.

← (b)

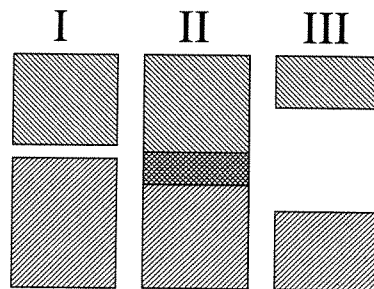
33. Which one of the following statements concerning electron energy bands in solids is true?

- a) the bands occur as a direct consequence of the Fermi-Dirac distribution function
- b) an insulator has a large energy separation between the highest filled band and the lowest empty band
- × c) electrical conduction arises from the motion of electrons in completely filled bands
- d) within a given band, all electron energy levels are equal to each other
- e) only insulators have energy bands

b)

23. In the energy level diagrams shown, the  region represents the conduction band, the  region represents the valence band, and the  region represents overlap of the two bands, of three different solids. Which best describes solids (I), (II), and (III)?

- a) (I) insulator, (II) metal, (III) metal
- b) (I) metal, (II) metal, (III) semiconductor
- c) (I) metal, (II) insulator, (III) semiconductor
- d) (I) semiconductor, (II) metal, (III) insulator
- e) (I) insulator, (II) metal, (III) semiconductor



III = insulator

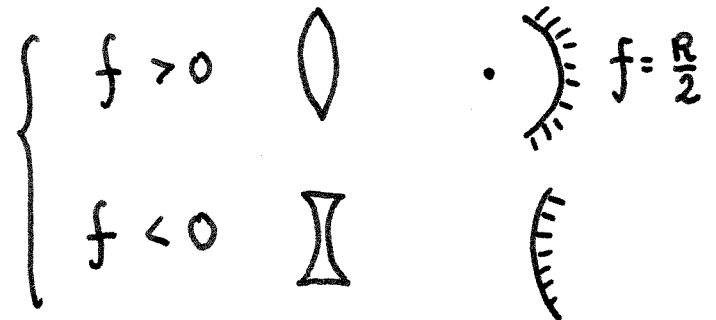
I = semi

II = metal

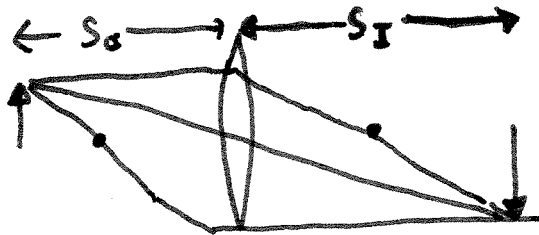
(d)

Lenses

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$



$$m = -\frac{s_i}{s_o}$$



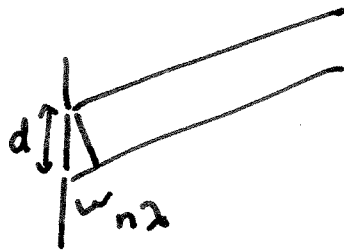
Diffraction Grating



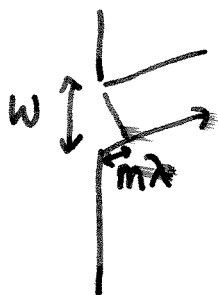
$$d \sin \theta = n \lambda \quad \text{MAXIMA.}$$

$$\sin \theta = n \frac{\lambda}{d} = n \lambda \left(\frac{N_{\text{slits}}}{L_{\text{slit}}} \right)$$

Double slit



Single slit



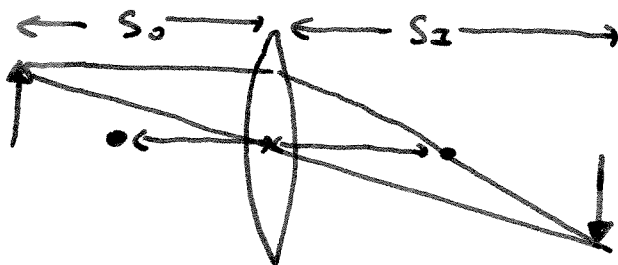
$$w \sin \theta = m \lambda \quad \text{MINIMA}$$

2. A convergent lens has a focal length of 0.359 meters. How far in front of the lens must we place an object so it forms an inverted image of size 0.579 times the size of the object? Give your answer in meters to three significant figures.

$$m = -\frac{q}{p} = -0.579 \text{ so } q = 0.579p, \quad \frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{1}{0.359}$$

$$\frac{1}{p} + \frac{1}{0.579p} = \frac{2.727}{p} = \frac{1}{0.359}$$

$$p = 2.727 \times 0.359 = 0.979 \text{ m}$$



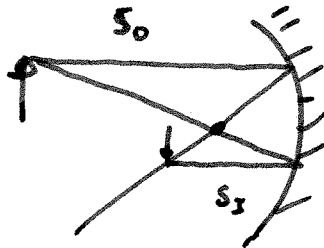
$$M = -0.579$$

$$-\frac{S_i}{S_o} = M \Rightarrow S_i = |M|S_o = 0.579S_o$$

$$\frac{1}{S_o} + \frac{1}{0.579S_o} = \frac{1}{0.359} \Rightarrow \frac{1}{S_o} \left(1 + \frac{1}{0.579} \right) = \frac{1}{0.359}$$

$$S_o = \frac{0.359}{\left(1 + \frac{1}{0.579} \right)}$$

$$= \underline{\underline{0.979 \text{ m}}}$$



$$f = \frac{R}{2} = 3 \text{ cm}$$

16. An object is placed a distance s in front of the concave mirror of focal length $+3$ cm. The image is on the same side of the mirror as the object, and lies 8 cm closer to the mirror than the object, i.e. the distance between the object and the image is 8 cm. In which of the following ranges does the value of s lie?

X

- a) $14 < s \leq 17$ cm
- b) $8 < s \leq 11$ cm
- c) $11 < s \leq 14$ cm
- d) $s > 20$ cm
- e) $17 < s \leq 20$ cm

$$\frac{1}{s} + \frac{1}{s-8} = \frac{1}{3}$$

$$s-8 > 3 \text{ cm} \Rightarrow s > 11 \text{ cm} \Rightarrow (c)$$

More carefully $3(2s-8) = s(s-8)$

$$s^2 - 14s + 24 = 0$$

$$(s-7)^2 - 25 = 0$$

$$s = 7 \pm 5 = \begin{cases} 12 \\ 2 \end{cases} \leftarrow s = 12$$

17. Light from an incandescent bulb is passed through a filter which transmits yellow light, and then serves as the source for a Young's double slit experiment. Which of the following changes would cause the interference fringes to be more closely spaced?

- a) move the light source farther away from the slits
- b) use a light source of higher intensity
- c) use slits that are closer together
- d) use a filter which transmits blue instead of yellow
- e) use a light source of lower intensity

$$d \sin \theta = n \lambda$$

1. The first order spectral line produced by a diffraction grating occurs at an angle of 11.31° . The grating has 3,660 slits per centimeter. What is the wavelength of the light? Express your answer in **nanometers**, to three significant figures.

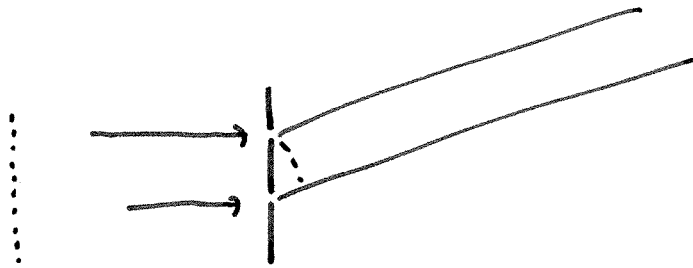
$$d \sin \theta = n \lambda \quad n=1 \quad d = \frac{0.01}{3660} \quad \lambda = 536 \text{ nm}$$

18. A light beam consists of two components, one of unknown wavelength and the other of 504 nm wavelength. When this light is incident on a single slit, diffraction pattern shows that the third minimum of the unknown coincides with the fourth minimum of the 504 nm light. What is the unknown wavelength.

- a) 2016 nm
- b) 168 nm
- c) 378 nm
- d) 672 nm
- e) Cannot be determined without knowing the width of the slit

$$d \sin \theta = n \lambda$$

17.



$$\sin \theta = \frac{n \lambda}{d}$$

More closely spaced $\Rightarrow \sin \theta \downarrow \quad \frac{\lambda}{d} \downarrow \quad d \uparrow \text{ or } \lambda \downarrow \quad \underline{\underline{(d)}}$

18.

$$d \sin \theta = n \lambda \quad n=1$$

$$d = \left(\frac{0.01}{3660} \right)$$

$$\lambda = d \sin \theta$$

$$= \left(\frac{0.01}{3660} \right) \sin 11.31$$

$$= 536 \times 10^{-9}$$

$$= \underline{\underline{536 \text{ nm.}}}$$

19.

$$d \sin \theta = m \lambda \quad \text{minima.}$$

$$\left. \begin{aligned} d \sin \theta &= 4 \lambda_k \\ &= 3 \lambda_u \end{aligned} \right\}$$

$$3 \lambda_u = 4 \lambda_k$$

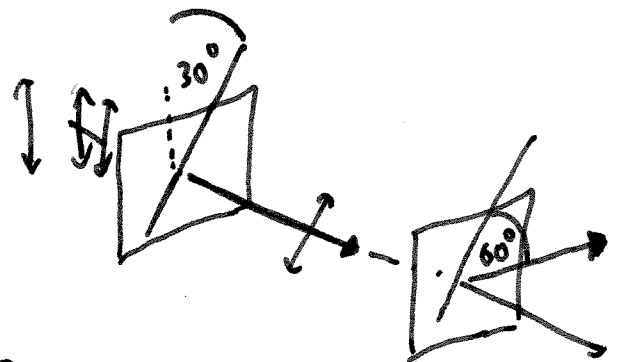
$$\lambda_u = \frac{4}{3} \lambda_k = 672$$

19. A beam of linearly polarized light strikes two polarizing sheets. The characteristic (i.e. polarizing) direction of the second sheet is oriented at 90° with respect to the initial polarization. The characteristic direction of the first sheet is at angle θ with respect to the initial polarization. If the $\theta = 30^\circ$, what fraction of the incident beam intensity is transmitted through the two polarizers?

- a) .50
b) .43
c) .19
d) .87
e) .75

$\frac{I}{I_0} = \cos^2 \theta$

angle between
polarizer &
direction
of light
polarization



$$I_f = I_0 \times \cos^2 30 \times \cos^2 60$$

$$= I_0 \times \left(\frac{3}{4}\right) \times \left(\frac{1}{4}\right) = I_0 \times \frac{3}{16} = 0.1875$$

$$\approx 0.19 \quad (c)$$

Relativity

$$\left. \begin{aligned} t &= t_0 \gamma = \frac{t_0}{\sqrt{1-v^2/c^2}} \\ L &= L_0 \sqrt{1-v^2/c^2} \end{aligned} \right\}$$

$$x' = \gamma(x - vt)$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$y' = y$$

LORENTZ

Energy momentum

$$E = \gamma mc^2$$

$$p = \gamma mv$$

$$E^2 = (mc^2)^2 + (pc)^2$$

$$K + mc^2 = \gamma mc^2$$

Collision



$$E_{CM}^2 = [(m+M)c^2]^2 + 2kMc^2$$

if $m=M$

$$E_{CM}^2 = 2Mc^2 (k + 2Mc^2)$$

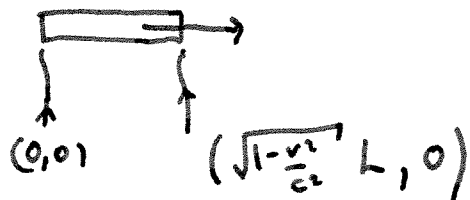
20. A rocket of proper length 20 m is moving past the earth at a speed of 0.6 c. Lights are mounted at each end of the rocket. According to observers on earth, the lights flash simultaneously. According to observers in the rocket, what is the time interval between flashes? (Just find the absolute value; don't worry about the sign.)

- a) 1.1×10^{-7} s
- b) 5.0×10^{-8} s
- c) 1.4×10^{-7} s
- d) 3.2×10^{-8} s
- e) 4.0×10^{-8} s

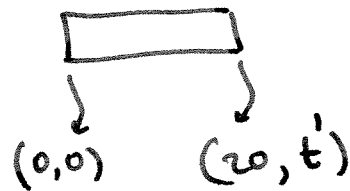
$$x' = \gamma(x - vt)$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

EARTH



REST FRAME OF ROCKET



$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(0 - \frac{v}{c^2} L \sqrt{1 - \frac{v^2}{c^2}} \right) = -\frac{vL}{c^2}$$

$$= - (0.6) \times \frac{20}{3 \times 10^8}$$

$$= -4 \times 10^{-8} \text{ s}$$

$$|t'| = 4 \times 10^{-8} \text{ s}$$

20. The mass of a particle is m . In order for its total energy to be three times its rest mass energy, its momentum must be:

- a) $\sqrt{3}mc$
 → b) $\sqrt{8}mc$
 c) $\sqrt{2}mc$
 d) $\sqrt{5}mc$
 e) $\sqrt{6}mc$

$$E = \gamma mc^2 = 3mc^2$$

$$p = \gamma mv$$

$$\gamma = 3 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \gamma = \frac{1}{1 - \frac{v^2}{c^2}} \Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{9}$$

$$\frac{v}{c} = \sqrt{\frac{8}{9}} = \frac{\sqrt{8}}{3}$$

$$p = 3 \left(\frac{\sqrt{8}}{3} c \right) m = \underline{\underline{\sqrt{8} mc}}$$

$$v = \frac{\sqrt{8}}{3} c$$

Alternatively

$$E^2 = (mc^2)^2 + (pc)^2 = 9(mc^2)^2$$

$$\Rightarrow (pc)^2 = (9 - 1)(mc^2)^2 = 8(mc^2)^2$$

$$pc = \sqrt{8} mc^2$$

$$p = \underline{\underline{\sqrt{8} mc}}$$

3. The *kinetic energy* of a particle is 0.215 times its *rest energy*. What is the speed of the particle? Express your answer as a fraction of the speed of light, to three significant figures.

$$\gamma = \frac{E}{mc^2} = \frac{K}{mc^2} + 1 = 1.215 = \frac{1}{\sqrt{1-v^2/c^2}}, \quad v = 0.568c$$

$$K + mc^2 = E = \gamma mc^2$$

$$K = 0.215 mc^2$$

$$\Rightarrow \gamma mc^2 = 1.215 mc^2 \quad \gamma = 1.215 = \frac{1}{\sqrt{1-(v/c)^2}}$$


$$\Rightarrow 1 - \left(\frac{v}{c}\right)^2 = \frac{1}{(1.215)^2} \Rightarrow \left(\frac{v}{c}\right)^2 = 1 - \frac{1}{(1.215)^2} = \frac{0.3225}{0.3225}$$

$$\Rightarrow v = \underline{\underline{0.568c}}$$

16. A photon moving at a speed c makes a Compton collision with a free electron at rest. After the collision, the scattered photon travels at an angle θ relative to the direction of the incident photon. The speed of the scattered photon is

- a) $c \cos \theta$
- b) $c(1 - \cos \theta)$
- c) c
- d) $c \sin \theta$
- e) $c(1 - \sin \theta)$

Compton Effect



$$(\lambda' - \lambda)$$

$$= \frac{h}{m_e c} (1 - \cos \theta)$$

$$\theta = 0 \quad \lambda' = \lambda$$

$$\theta = 180^\circ \quad \lambda' = \frac{2h}{mc} + \lambda$$

$$\frac{h}{m_e c} = \text{COMPTON WAVELENGTH}$$

$$= 2.426 \times 10^{-12} \text{ m}$$

Energy imparted to electron: -

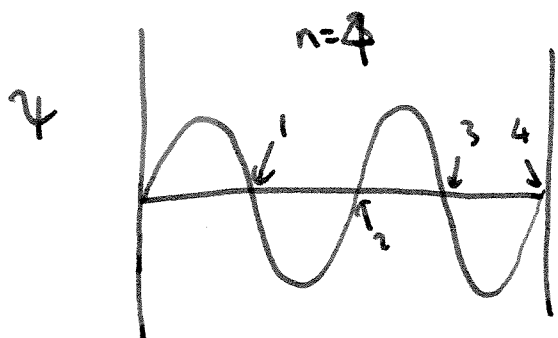
$$= hf - hf' = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = 0 \quad \cos \theta = 1$$

$$= \text{maximum} \quad \cos \theta = -1.$$

$$c' = c !$$

21. A particle is in the ground state in a one dimensional infinite square well of width L . The probability, P , that the particle is in the left quarter of the well ($0 \leq x \leq L/4$) is:

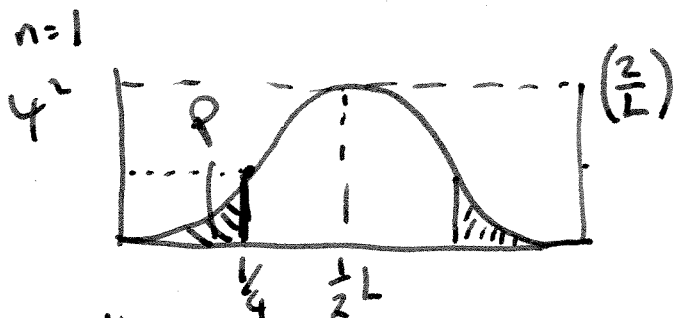
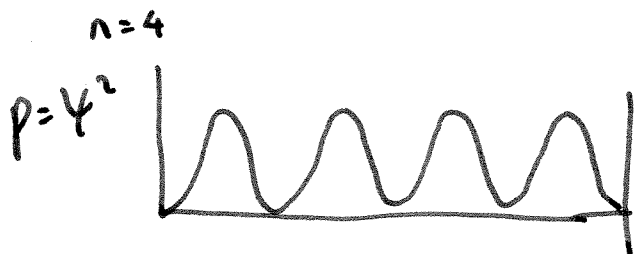
- a) $0.6 \leq P$
- X b) $0.01 < P \leq 0.2$
- c) $0 \leq P \leq 0.01$
- d) $0.2 < P \leq 0.3$
- e) $0.3 < P < 0.6$



$n = \#$ of half wavelengths

$$\psi = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

$$k = \frac{n\pi}{L} \quad \frac{n\lambda}{2} = L$$



$P = \text{Area beneath curve}$

$$< \frac{1}{2} \times \left(\frac{1}{4} L \times \frac{1}{L} \right)$$

$$= \frac{1}{2} \times \frac{1}{4} = \frac{1}{8} = 0.125$$

Exactly: $\int_0^{1/4} dx 2 \sin^2 \pi x = 0.091$

(b)

17. A hydrogen atom is in the 4d state ($n = 4, \ell = 2$). How many values of the z -component of the electron's orbital angular momentum are possible?

- a) 2
- b) 4
- c) 5
- d) 8
- e) 9

3

$$L_z = m\hbar$$

$$m = \{-2, -1, 0, 1, 2\} \quad 5 \text{ values.}$$

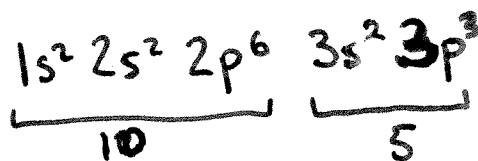
	s	p	d	f
ℓ	0	1	2	3

$$L = \hbar \sqrt{\ell(\ell+1)}$$

26. Phosphorus has atomic number $Z = 15$. In the ground state, what is the configuration of its outermost electrons?



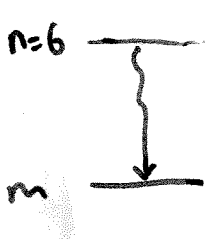
- a) $3p^1 4s^2$
- b) $2p^{11}$
- c) $3p^5$
- d) $3p^3$
- e) $3p^2 4s$



In general: $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 \dots$

22. A hydrogen atom is in its fifth excited state. The atom emits a 1,090 nm wavelength photon. What is the maximum possible orbital angular momentum quantum number of the electron after emission?

- a) 1
- b) 0
- c) 3
- d) 2
- e) No maximum. Orbital angular momentum can be arbitrarily high.



$$E_{\text{photon}} = hf = \frac{hc}{\lambda} = \frac{1240\text{eV}}{\lambda} = 1.14\text{eV}$$

$$\Rightarrow E_i - E_f = E_{\text{photon}} = \left(-\frac{13.6}{6^2} + \frac{13.6}{m^2} \right)$$

$$\Rightarrow 1.14 = -0.375 + \frac{13.6}{m^2}$$

$$\Rightarrow 13.6/m^2 = 1.52\text{eV} \Rightarrow m = \sqrt{\frac{13.6}{1.52}} = 3$$

$$l = 0, 1, 2 \quad (d)$$