

REVIEW FOR CH2

This review is not exhaustive, but it will cover key points for the exam. The second midterm will cover chapters 37-41.1, not including 40.2-40.4.

RELATIVITY

- Principle of Relativity:
- LAWS OF PHYSICS SAME IN ALL INERTIAL REF. FRAMES
 - SPEED OF LIGHT = c IN ALL INERTIAL REF. FRAMES

ΔT = Proper time = Time interval between two events occurring AT SAME POINT IN SPACE

$$= \sqrt{\Delta t^2 - \Delta x^2/c^2}$$

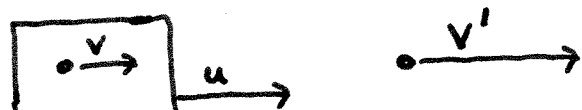
$$\Delta t = \Delta T \gamma = \frac{\Delta T}{\sqrt{1 - u^2/c^2}} \quad \text{Time dilation}$$

$$\underset{\substack{\uparrow \\ \text{moving}}}{L} = L_0 \sqrt{1 - u^2/c^2} \quad \text{Length contraction}$$

\uparrow
object at rest

$$\left. \begin{aligned} x' &= \gamma(x - ut) \\ t' &= \gamma(t - ux/c^2) \end{aligned} \right\} \text{LORENTZ}$$

$$v' = \frac{u + v}{1 + uv/c^2}$$

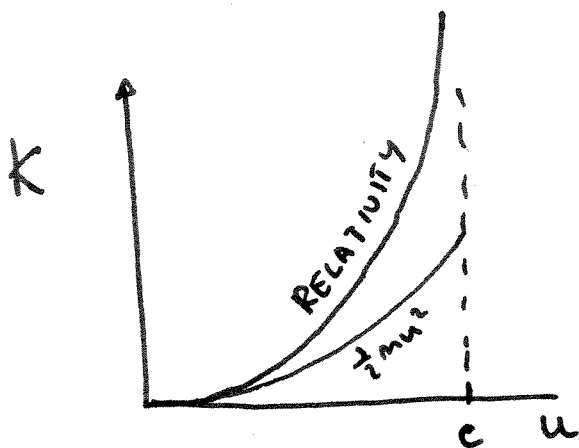


$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1-u^2/c^2}} \quad \gamma = \frac{1}{\sqrt{1-u^2/c^2}}$$

$$p = \gamma mu$$

$$K = \text{kinetic energy} = \gamma mc^2 - mc^2$$

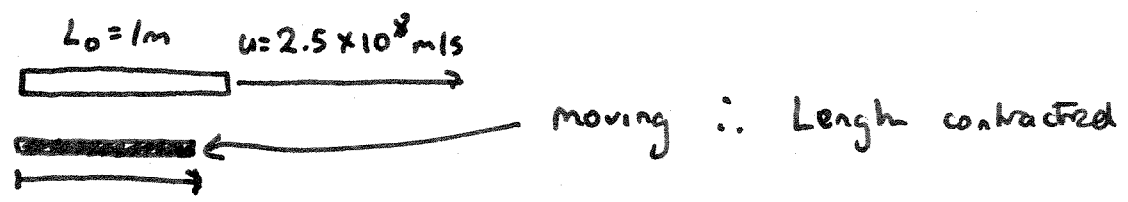
\uparrow total energy. \uparrow rest energy



Note also:

$$E^2 = p^2 c^2 + m^2 c^4$$

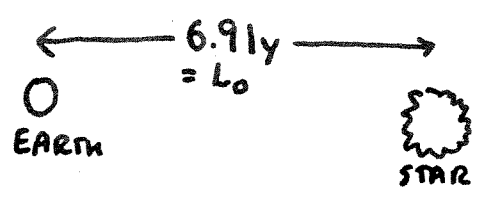
2. A meter stick comes flying past you at 2.50×10^8 m/s, pointing in the direction it is travelling. How long does it take between the time the front of the meter stick passes you and the time the back passes you. Express your answer in seconds.



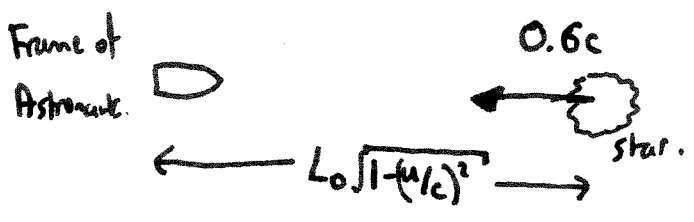
$$L = L_0 \sqrt{1 - (u/c)^2} = 0.55 \text{ m}$$

$$t = \frac{L}{u} = \frac{0.55}{2.5 \times 10^8} = \underline{2.21 \times 10^{-9} \text{ s}}$$

3. From the earth an astronaut travels to a star at a speed of $0.600 c$ relative to Earth and turns around and returns at the same speed. The star is 6.90 light years from Earth as measured by an observer at rest on Earth. When the astronaut returns to Earth, how many years will she have aged?



In the frame of the rocket, distance to star is length contracted



$$L = 6.9 \times \sqrt{1 - (0.6)^2}$$

$$= 6.9 \times (0.8) = 5.52 \text{ lyrs}$$

$$\Delta t = \frac{5.52}{0.6} = 9.2 \text{ yrs}$$

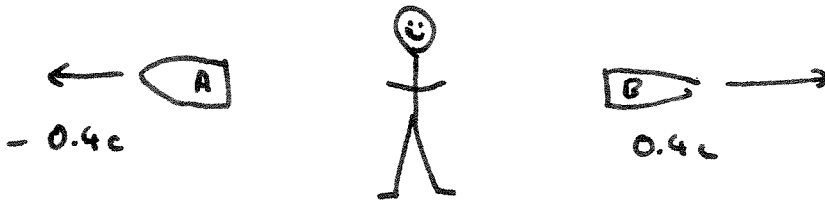
Total time for astronaut = $2 \times \Delta t = 18.4 \text{ yrs}$.

19. Two rockets each move with speed $0.4c$ in opposite directions in the laboratory frame of reference. What is the speed of one rocket, as determined by an observer in the other?

- a) $.69c$
 b) $.57c$
 c) $.44c$
 d) $.75c$
 e) $.8c$

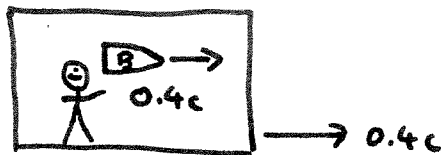
$$\frac{0.4 + 0.4}{1 + (0.4)^2} c = .690c$$

In our frame rocket A & B move at velocities $\pm 0.4c$



In A's frame, we are moving to the right at $0.4c$

A's view:



$$\therefore V'_B = \frac{0.4c + 0.4c}{1 + (0.4)^2} = 0.69c$$

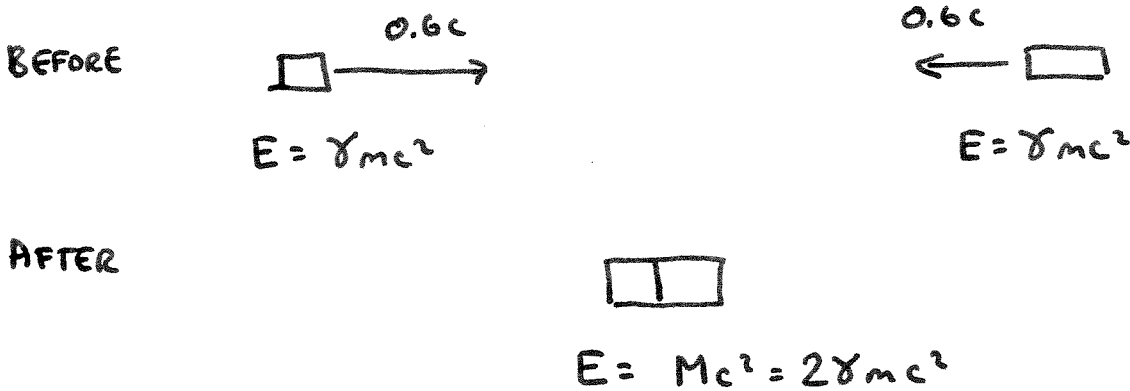
Kinematics

$$E = \gamma mc^2$$

$$\gamma = \frac{1}{\sqrt{1-(u/c)^2}}$$

20. Two lumps of clay, each having a mass of 100 grams and a speed of $0.6c$, collide head-on and stick together. Assuming that no radiation is emitted in the collision, what is the mass of the composite?

- a) 250 grams
- b) 125 grams
- c) None of the other answers
- d) 160 grams
- e) 200 grams

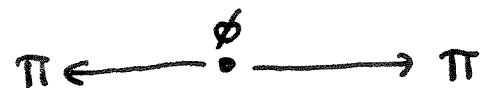


$$M = 2\gamma m = \frac{2}{\sqrt{1-(0.6)^2}} \times 100 \text{ g} = 2.5 \times 100 = 250 \text{ g}.$$

2. The ϕ is an unstable particle of mass of $mc^2 = 1020 \text{ MeV}$. It decays into two π mesons 7.30×10^{-3} percent of the time. For the π mesons, $mc^2 = 140 \text{ MeV}$. Suppose the ϕ is at rest when it decays. What is the speed of each π meson? Give your answer as a multiple of c , to three significant figures. Each π has

$$E = 510 \text{ MeV} = m_{\pi} c^2 \gamma, \text{ so } \gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{510}{140}$$

$$v = .962 c$$



$$2\gamma m_{\pi} c^2 = m_{\phi} c^2$$

$$\gamma = \frac{m_{\phi}}{2m_{\pi}} = \frac{1020}{280}$$

$$\Rightarrow u = \sqrt{1 - \frac{1}{\gamma^2}} = 0.962$$

Breakdown of Classical Mechanics

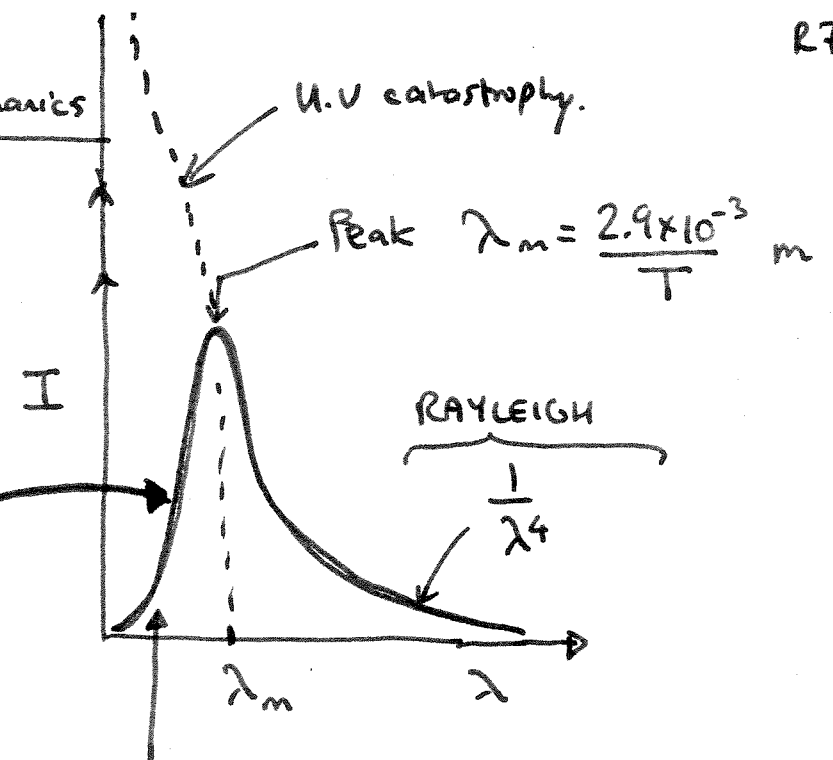
• BLACKBODY RADIATION

PLANCK

$$I(\lambda) \propto \frac{T}{\lambda^5 \left(e^{\frac{hc}{\lambda k_B T}} - 1 \right)}$$

$$I_{TOT} = \sigma T^4$$

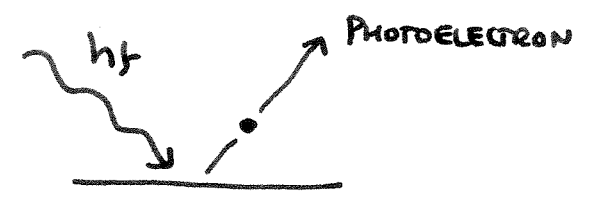
STEFAN BOLTZMANN



Planck: Avoids ultraviolet catastrophe.
 by suppressing short wavelength photons.

• PHOTOELECTRIC EFFECT

$$K = hf - \phi$$



- Energy of e^- INDEPENDENT of INTENSITY.
- $hf < \phi = \text{work fn} \Rightarrow$ NO ELECTRONS
- $K = e V_{\text{stopping}}$.

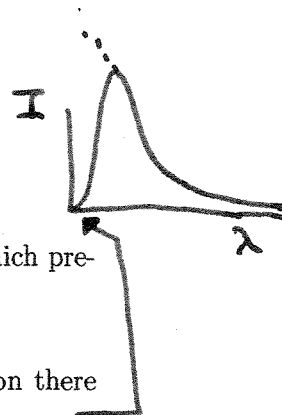
$$hf = \frac{hc}{\lambda} = \frac{1240 \text{ nm eV}}{\lambda (\text{nm})}$$

$$h = 4.14 \text{ eV-s} = 6.6 \times 10^{-34} \text{ Js}$$

$\times 10^{-15}$

22. Planck's blackbody radiation formula

$$I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5 (e^{hc/\lambda k_B T} - 1)}$$



differs radically from the Rayleigh-Jeans law, which preceded it,

- for all wavelengths.
- for short wavelengths, giving less radiation there and avoiding the ultraviolet catastrophe.
- for short wavelengths, giving more radiation there and producing the ultraviolet catastrophe.
- for long wavelengths, giving less radiation there and avoiding the infrared catastrophe.
- for intermediate wavelengths, giving less visible light from black bodies.

3. The work function of a certain metal is 2.24 electron-volts. The longest wavelength (in nm) for which light is able to produce photoelectrons is:

$$\frac{hc}{\lambda} = 2.24 \text{ eV}, \quad \lambda = \frac{1240}{2.24} \text{ nm} \\ = 554 \text{ nm}$$

$$hf = \frac{hc}{\lambda} > \phi = 2.24 \text{ eV}$$

$$\Rightarrow \lambda < \frac{hc}{\phi} = \frac{1240 \text{ nm-eV}}{2.24} \\ = 554 \text{ eV}$$

p.s $hc \equiv 1240 \text{ nm-eV}$

4. A photoelectric effect experiment is done using light of some wavelength λ_1 . The photoelectrons are observed to have a maximum kinetic energy of 7.00 eV. If the experiment is repeated using light of twice that wavelength, i.e. $\lambda_2 = 2.00\lambda_1$, on the same metal, it is found that the maximum electron kinetic energy is 2.00 eV. What is the work function of the metal? Express your answer in electron-Volts (eV).

$$hf_1 - \phi = 7\text{eV} \quad (1)$$

$$hf_2 - \phi = 2\text{eV}$$

$$\text{But } \lambda_2 = 2\lambda_1 \Rightarrow f_2 = \frac{1}{2}f_1$$

$$\frac{hf_1}{2} - \phi = 2\text{eV}$$

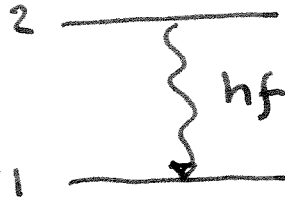
$$\Rightarrow hf_1 - 2\phi = 4\text{eV} \quad (2)$$

$$(1) - (2) \quad \phi = 7 - 4 = \underline{\underline{3\text{eV}}}$$

Bohr Atom, Energy levels, Compton Effect

- $$hf = E_2 - E_1$$

$$= \frac{hc}{\lambda}$$



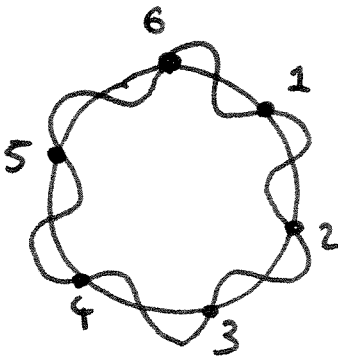
- Bohr Atom

$$E_n = -13.6 \left(\frac{Z^2}{n^2} \right) \text{ eV}$$

$$a_n = a_0 (n^2) \quad (a_0 \approx 0.5 \text{ \AA})$$

$$2\pi r = n\lambda$$

$$L = mvr = pr = \frac{nh}{2\pi}$$



$n=6$

- de Broglie

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

- Compton



$$(\lambda_f - \lambda_i) = (2.42 \times 10^{-12} \text{ m}) (1 - \cos\theta)$$

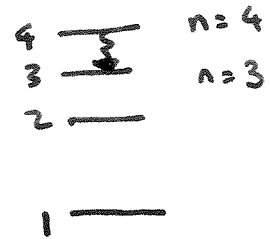
Outgoing photon has less E, larger λ .

16. A photon moving at a speed c makes a Compton collision with a free electron at rest. After the collision, the scattered photon travels at an angle θ relative to the direction of the incident photon. The speed of the scattered photon is

- a) $c \cos \theta$
 b) $c(1 - \cos \theta)$
 c) c
 d) $c \sin \theta$
 e) $c(1 - \sin \theta)$

$c = c'$ Photons always move @ speed of light!

4. The energies of the hydrogen levels are $-(13.6\text{eV})/n^2$. Singly ionized helium differs from hydrogen by having a nucleus of charge $+2e$. What is the wavelength of the photon emitted by singly ionized helium when it makes a transition from the $n=4$ state to the $n=3$ state? Give your answer in nm, to three significant figures.



$$13.6 Z^2 \left(\frac{1}{3^2} - \frac{1}{4^2} \right) = \frac{hc}{\lambda} = 2.644\text{eV} \quad \lambda = 469\text{nm}$$

$$E_n (Z=+2e) = -13.6 \left(\frac{Z}{n} \right)^2 = -13.6 \times \frac{2^2}{n^2} = -\frac{54.4}{n^2} \text{eV}$$

$$E_4 = -\frac{54.4}{16} = -3.4\text{eV}$$

$$E_3 = -\frac{54.4}{9} = -6.04\text{eV}$$

$$E_4 - E_3 = \underline{2.64\text{eV}}$$

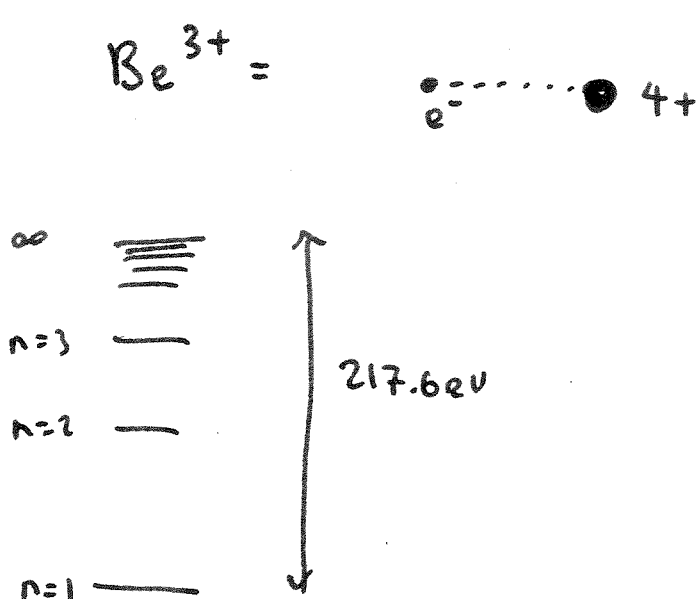
$$\frac{hc}{\lambda} = 2.64\text{eV}$$

$$\lambda = \frac{hc}{2.64\text{eV}} = \frac{1240\text{nm-eV}}{2.64}$$

$$= \underline{469\text{nm}}$$

21. The Beryllium nucleus contains 4 protons. If a Beryllium atom is triply ionized with the one remaining electron in the ground state, the energy required to remove that electron is:

- a) 217.6 eV
- b) 10.2 eV
- c) 13.6 eV
- d) 40.8 eV
- e) 54.4 eV

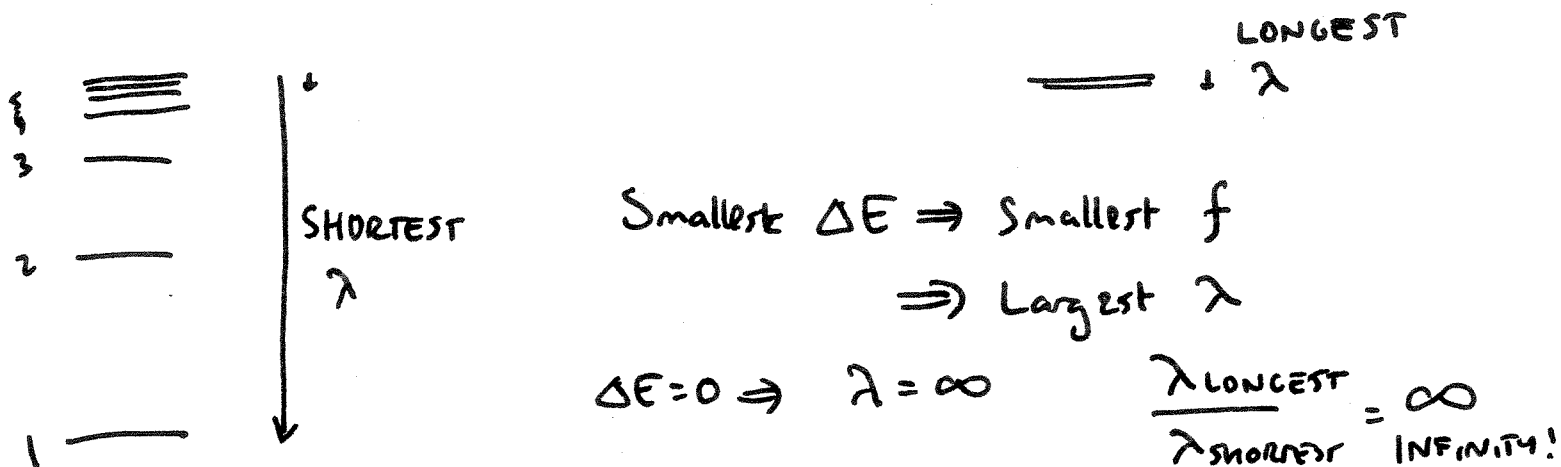


$$E_n = - \frac{13.6 \times (4)^2}{n^2}$$

$$= - \frac{217.6}{n^2} \text{ eV}$$

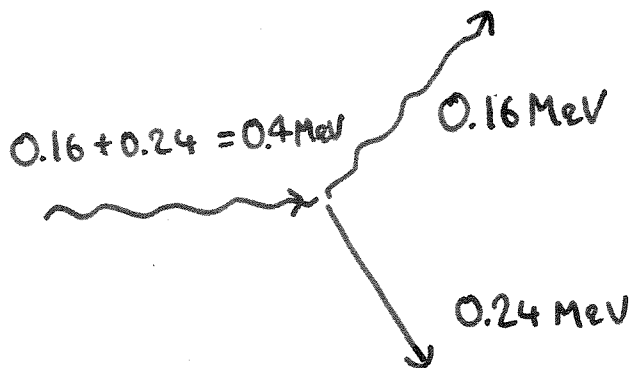
25. The Lyman series corresponds to transitions to the hydrogen ground state ($n = 1$). What is the ratio of the longest wavelength in the Lyman series to the shortest?

- a) 2
- b) 3
- c) 4
- d) $3/2$
- e) None of the other answers



26. A photon undergoes Compton scattering off a free electron at rest. The scattered photon has an energy of 0.160 MeV, and the recoiling electron has a kinetic energy of 0.240 MeV. In which of the following ranges does the photon's scattering angle θ lie?

- a) $\theta < 30^\circ$
- b) $30^\circ \leq \theta < 60^\circ$
- c) $60^\circ \leq \theta < 90^\circ$
- d) $90^\circ \leq \theta < 140^\circ$
- e) $\theta \geq 140^\circ$



$$E_f = 0.16 \times 10^6 \text{ eV} = \frac{hc}{\lambda_f} \Rightarrow \lambda_f = \frac{hc}{0.16 \times 10^6} = \frac{1240 \text{ nm}}{0.16 \times 10^6}$$

$$= 7.75 \times 10^{-3} \text{ nm}$$

$$= 7.75 \times 10^{-12} \text{ m}$$

$$E_i = 0.4 \text{ MeV} = \frac{hc}{\lambda_i} \Rightarrow \lambda_i = \frac{1240}{0.4 \times 10^6} = 3.1 \times 10^{-12} \text{ m}$$

$$\lambda_f - \lambda_i = 4.65 \times 10^{-12} \text{ m} = \frac{h}{m_e c} (1 - \cos \theta) = 2.42 \times 10^{-12} (1 - \cos \theta)$$

$$\Rightarrow 1 - \cos \theta = \frac{4.65 \times 10^{-12}}{2.42 \times 10^{-12}} = 1.92 \Rightarrow \cos \theta = -0.92$$

$$\theta \sim 157^\circ$$

(e)

Uncertainty

$$\Delta p \Delta x \gtrsim \frac{\hbar}{2} \quad \text{officially}$$

BUT FOR EXAM WE FOLLOW

$$\begin{array}{l} \Delta p \Delta x \gtrsim \hbar \\ \Delta E \Delta t \gtrsim \hbar \end{array}$$

$$\begin{aligned} \hbar &= 1.054 \times 10^{-34} \text{ Js} \\ &= 6.59 \times 10^{-16} \text{ eVs} \end{aligned}$$

22. An electron will remain in the 3p level of hydrogen for about 10^{-12} s before it makes a transition to the 2s level, emitting a photon. The the uncertainty in the energy of the corresponding spectral line is at least:

- a) 13.6 eV
- b) 1.9 eV
- c) 2.3 eV
- d) (1.3×10^{-11}) eV
- e) (3.3×10^{-4}) eV

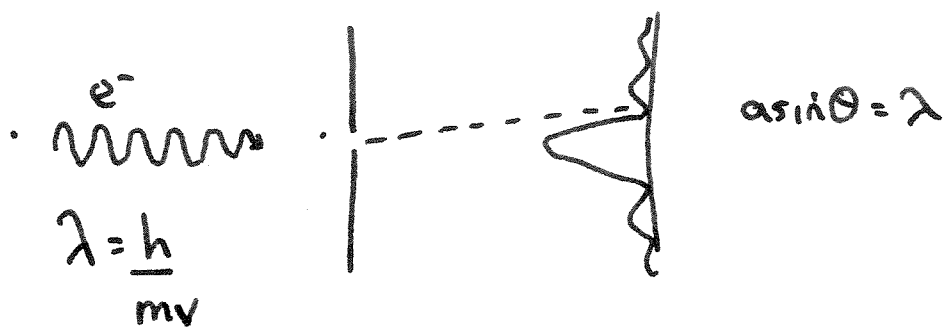
$$\Delta t = 10^{-12} \text{ s}$$

$$\Delta E = \frac{\hbar}{\Delta t} = \frac{6.59 \times 10^{-16} \text{ eVs}}{10^{-12} \text{ s}} = 6.6 \times 10^{-4} \text{ eV}$$

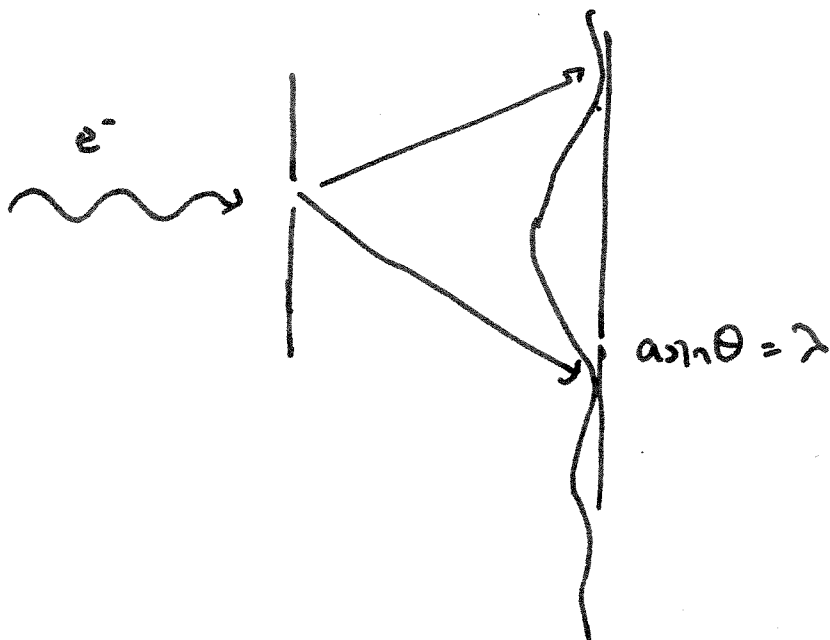
(Factor of 2 from (e) because we use \hbar rather than $\hbar/2$).

24. A beam of electrons passes through a hole in a screen and generates an intensity pattern beyond the hole. How will the pattern change if the velocity of the electrons is decreased?

- The pattern will spread out.
- The pattern will squeeze together.
- When the electron wavelength becomes bigger than the size of the hole, the electrons will no longer be able to get through.
- The pattern will stay the same but will get brighter.
- The pattern will stay the same but will get dimmer.



Decrease $v \Rightarrow \lambda$ increases \Rightarrow pattern gets more spread out.



Schrödinger Equation

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V\psi = E\psi$$

Free particle, $V=0$

$$\psi = e^{ikx}$$

$$k = \frac{2\pi}{\lambda}$$

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} = \frac{\hbar^2 k^2}{2m} \psi$$

 \Rightarrow

$$E = \frac{\hbar^2 k^2}{2m}$$

$$V\psi = 0$$

$$V \neq 0 \quad E = \frac{\hbar^2 k^2}{2m} + V$$

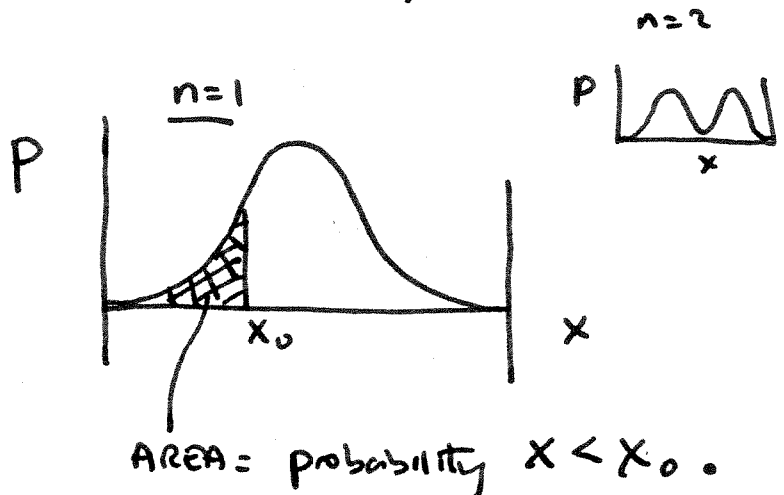
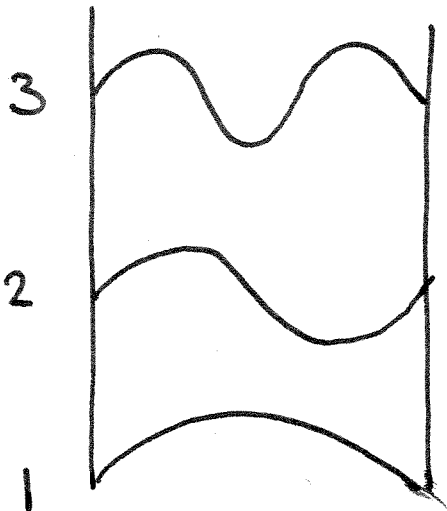
$$\left(k = \frac{\pi}{L} \times n \right)$$

$$p(x) = |\psi(x)|^2$$

$$\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi}{L} nx\right)$$

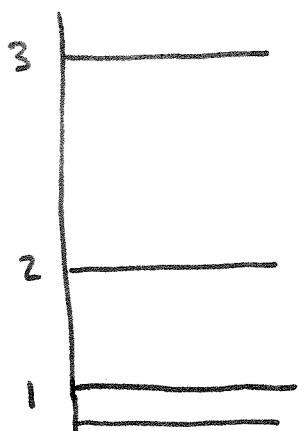
$$E_n = \frac{\hbar^2}{2m} \left(\frac{\pi}{L}\right)^2 n^2$$

Particle in box



19. A particle in a one-dimensional box (with $V = 0$ between infinitely high side walls) has a ground-state energy of 2 eV. In the first two excited states, the particle's energy would be respectively

- a) 8 eV and 18 eV
- b) None of the other answers
- c) 4 eV and 8 eV
- d) 3 eV and 4 eV
- e) 4 eV and 6 eV



$$E_1 \times (3)^2 = 18 \text{ eV}$$

$$E_1 \times (2)^2 = 8 \text{ eV}$$

$$E_1 = 2 \text{ eV}$$

(a).

25. A particle has a normalized wave function $\psi(x) = (2x/L^2)^{1/2}$ in the region $0 < x < L$. What is the probability that at any instant the particle lies between $x = 0$ and $x = L/3$?

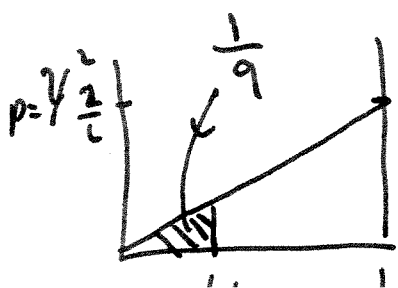
- a) Zero
- b) About 33%
- c) About 67%
- d) About 82%
- e) About 11%

$$P(0 \leq x \leq L/3) = \int_0^{L/3} \psi^2(x) dx$$

$$= \int_0^{L/3} \frac{2x}{L^2} dx = \left. \frac{x^2}{L^2} \right|_0^{L/3} = \frac{1}{9}$$

electrons

$$\psi^2(x) = p(x) = \left(\frac{2x}{L^2} \right)$$



$$P(0 \leq x \leq L/3) = \int_0^{L/3} \psi^2(x) dx$$

$$= \int_0^{L/3} \frac{2x}{L^2} dx = \left. \left[\frac{x^2}{L^2} \right] \right|_0^{L/3} = \frac{1}{9}$$

H ATOM

Quantum #s

$$(n, l, m_l, m_s)$$

not needed for exam.

$$n > l$$

$$L = \hbar \sqrt{l(l+1)}$$

$$-l \leq m_l \leq l$$

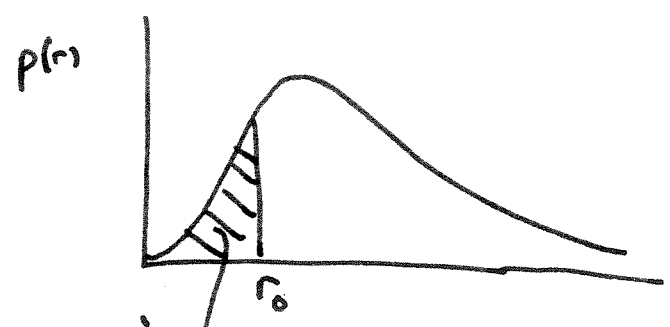
$$E_n = -13.6 \left(\frac{Z^2}{n^2} \right)$$

l	symbol
0	s
1	p
2	d
3	f
4	g

e.g. $n=3$ $l=2$

$$\equiv \boxed{3d}$$

$$p(r) = |\psi(r)|^2 4\pi r^2$$



$$P(r < r_0) = \int_0^{r_0} dr |\psi|^2 4\pi r^2$$

e.g. since $n > l$
 3d allowed ($l=2 < 3$)
 3f NOT ALLOWED ($l=3 \text{ not } < n=3$).

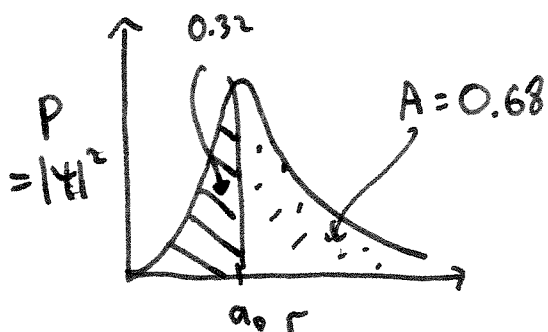
25. In a hydrogen atom, the ground state wave function is given by

$$\psi = (\pi a_0^3)^{-1/2} e^{-r/a_0}.$$

What, approximately, is the ratio of the probability that the electron will be found beyond the Bohr radius, $r > a_0$, to the probability that it is inside that distance ($r < a_0$)?

- a) 1.0
- b) 0.5
- c) 2.1
- d) 1.5
- e) 2.5

Hint: $4 \int_0^r x^2 e^{-2x} dx = 1 - (2r^2 + 2r + 1)e^{-2r}$



$$\begin{aligned} p(r) &= \psi^2 4\pi r^2 \\ &= \frac{1}{\pi a_0^3} e^{-2r/a_0} 4\pi r^2 \\ &= \frac{4r^2}{a_0^3} e^{-2r/a_0} \end{aligned}$$

$$\begin{aligned} P(r < a_0) &= \int_0^{a_0} dr \frac{4r^2}{a_0^3} e^{-2r/a_0} = 4 \int_0^1 dx x^2 e^{-2x} \\ &= \left[1 - (2r^2 + 2r + 1)e^{-2r} \right]_{r=1} \\ &= (1 - (2 + 2 + 1)e^{-2}) = 1 - 5e^{-2} = 0.32 \end{aligned}$$

$$p(r > a_0) = 1 - 0.32 = 0.68$$

$$p(r > a_0) / p(r < a_0) \approx \underline{2.1}$$

21. A single beam of neutral atoms splits into 7 distinct beams when it is sent through an inhomogeneous magnetic field (Stern-Gerlach experiment). We can conclude that the magnitude of the total angular momentum of the neutral atom is:

- a) $7\hbar$
 b) $6\hbar$
 c) $\sqrt{56}\hbar$
 → d) $\sqrt{12}\hbar$
 e) $3\hbar$
- splitting of m_l , so number of different $m_l \rightarrow 7$
 There are $2l+1$ m_l 's, so $l=3$
 $|L| = \sqrt{3(3+1)}\hbar = \sqrt{12}\hbar$

This goes a little beyond our exam. However you should know that if $l=3$, then

$$m_l = -3, -2, -1, 0, 1, 2, 3 \Rightarrow 7 \text{ states.}$$

$$L = \hbar \sqrt{l(l+1)} = \hbar \sqrt{3(4)} = \hbar \sqrt{12}$$

17. A hydrogen atom is in the 4d state ($n=4, l=2$). How many values of the z-component of the electron's orbital angular momentum are possible?

- a) 2
 b) 4
 (c) 5
 d) 8
 e) 9

$$l=2 \Rightarrow m_l = -2, -1, 0, 1, 2$$

$$L_z = \hbar m_l \quad 5 \text{ values.}$$