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Review for Exam 1 (Optics)

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Note: there is no guarantee that everything on the test is mentioned in this review.

Geometrical Optics (Ray Approximation)

- Light travels at a finite speed depending on the medium. For vacuum it is fastest and independent of frequency, and is called c , which is $\approx 3.00 \times 10^8$ m/s.
- In media other than the vacuum, light travels with a velocity v which is less than c and generally depends on frequency. The **index of refraction** of the medium is defined by

$$n = c/v.$$

- An approximation for the behavior of light is the ray approximation, which is suitable for describing things on a scale large compared to the wavelength.

- At a surface, some light can be reflected, with the outgoing rays emerging at an **angle of reflection** relative to the normal to the surface which is equal to the **angle of incidence** between the incoming rays and the normal.
- If the medium behind the surface is transparent, some light may also be refracted, or passed into the medium, forming rays which make an **angle of refraction** θ_2 with the normal to the surface. The angle is determined by Snell's law

$$n_1 \sin \theta_1 = n_2 \sin \theta_2.$$

- In the denser medium there is a maximal angle, called the **critical angle**, for which Snell's law can be satisfied. Light hitting the surface at a greater angle will undergo **total internal reflection**. The critical angle is given by

$$\sin \theta_c = n_2/n_1.$$

- Fermat's principle says light takes a path which minimizes the time taken to get from an initial point to the final point, going at the appropriate speed in whatever medium it is travelling through.
- Optical components form images of objects. They deflect light so that to an observer tracing back the path the light is taking entering the eye, the light seems to come from the image. Images may be real or virtual, depending on whether the light rays actually pass through the image point or not.
- Images can be magnified by a ratio M , which is considered positive if transverse dimensions are not inverted, and negative if they are.
- The object distance p is the distance the object is away from the optical element, and the image distance q is the distance of the image from the same point. But each of these may be considered negative if they are

not on the “natural” side of the optical element. That is, q is positive on the near side of a mirror and the far side of a refractive surface or lens, and negative if the image is on the opposite side. If $p > 0$, the image is real if $q > 0$.

- $M = -q/p$.
- The rays from the object, after deflection, appear to come from the image, but that is only true, in general, for paraxial rays, which make small angles with the principal axis. For larger angles spherical aberration describes the fact that the rays do not come from precisely the point of the image they ought to.
- For spherical mirrors, $\frac{1}{q} + \frac{1}{p} = \frac{2}{R}$.
- For spherical interfaces between transparent media, $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$.
- q , p , R , f , and M can be positive or nega-

tive, with sign conventions given in the tables, one for mirrors, one for an interface, one for thin lenses.

- Multiple active optical elements (including interfaces) can be treated by successively calculating each image and making that the object of the next element.
- For thin lenses (in air), $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ with the focal length given by

$$\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right).$$

A tricky point is that in this formula R 's are positive if their centers of curvature are to the right (considering rays coming from the left).

- Ray tracing is useful, especially for combinations of thin lenses. Convenient rays to trace are 1) parallel to the principle axis on one side and passing through (or away from)

a focal point on the other, and 2) passing through the center of lens undeflected.

Wave Optics

- Interference is the effect that coherent sources cannot be treated by adding their intensities, but instead the relative phases must be considered. When multiple sources are in phase they interfere constructively, giving an intensity greater than the sum, but when they are 180° out of phase they destructively interfere, giving a minimum.

- For a double slit, with $d \ll L$,

$$d \sin \theta = \left\{ \begin{array}{ll} m\lambda & \text{constructive interference} \\ (m + \frac{1}{2})\lambda & \text{destructive interference} \end{array} \right\}$$

with $m = 0, \pm 1, \pm 2, \dots$

- If also $y \ll L$,

$$y \approx \frac{L}{d} d \sin \theta = \left\{ \begin{array}{ll} \frac{\lambda L}{d} m & \text{for maximum} \\ \frac{\lambda L}{d} (m + \frac{1}{2}) & \text{for minimum} \end{array} \right\} .$$

- The intensity received at an angle θ from a double slit varies as

$$I = I_{\max} \cos^2 \left(\frac{\pi d \sin \theta}{\lambda} \right).$$

- The intensity for double or multiple slits is effectively calculated by using phasors.
- Light reflected from a higher n material or a conductor undergoes an extra 180° phase shift.
- Thin film interference comes from interference between the light reflected from the front and back surfaces. If the film is surrounded by material of lower n , there is constructive interference for $2nt = (m + \frac{1}{2})\lambda$ and destructive for $2nt = m\lambda$ and destructive for
- A Michelson interferometer splits a beam into two and recombines it, in a way that provides very accurate measurements of relative distances and of wavelengths.

- Waves do not actually move in straight lines, but have some spreading out when they pass boundaries, called **diffraction**.
- Fraunhofer diffraction occurs when the interfering light can be considered parallel as it passes through the aperture.
- Each portion of the aperture can be considered a source interfering with all the other portions.
- For a single slit of width a , the intensity at angle θ is

$$I = I_{\max} \left(\frac{\sin(\pi a \sin \theta / \lambda)}{\pi a \sin \theta / \lambda} \right)^2 .$$

- For multiple slits each with finite width a , the intensity is the product of the formula for a single slit of width a with that for multiple infinitesimal slits.
- Rayleigh's criterion for being able to resolve two images is that the central maximum of the diffraction of one lie at the minimum of

the central maximum of the other. If they are closer than that, it is hard to distinguish the images.

- For a slit of width a , this gives $\theta_{\min} = \frac{\lambda}{a}$. For a circular aperture $\theta_{\min} = 1.22\frac{\lambda}{D}$, where D is the diameter.
- Diffraction gratings with N “slits” a distance d apart will produce maxima for $d \sin \theta = m\lambda$ for integer m . The resolving power of such a grating, using its m 'th order maxima, is

$$R := \frac{\lambda}{\Delta\lambda} = Nm.$$

- Multiple planes of atoms can act as a reflection grating, giving bright spots when there is a plane of atoms at angle θ with respect to the beam, where $2d \sin \theta = m\lambda$. Notice that this θ is, confusingly, not the angle of incidence but its complement.
- Light, being a *transverse* wave, can be po-

larized with two independent linear polarizations. Polaroid filters, reflection at an angle, and scattering of light can cause one polarization to behave differently than the other. This can also happen in birefringent materials and in optically active materials.

- If linearly polarized light is passed through a polarizer with transmission axis at an angle θ with respect to the initial polarization direction, the intensity of the transmitted light is multiplied by $\cos^2 \theta$.
- Light incident on an interface (between normal transparent materials) at the Brewster angle will have one polarization, that with \vec{E} in the plane including the normal to the surface, entirely refracted with no reflection, so the reflected beam will be 100% polarized along the opposite axis.