Physics 228, Lecture 22
Monday, April 18, 2005

Elementary Particle Physics. Ch. 44:1, 44:3

We have been pursuing a reductionist approach to understanding nature, explaining the behavior of materials by breaking it into pieces, which we hope will be simpler to understand than is the whole. Thus chemistry is explained by the behaviour of molecules, which in turn are explained by the behavior of the atoms of which they are composed, which are in turn understood by the motion of electrons orbiting atomic nuclei. Then we found that atomic nuclei can be understood as consisting of protons and neutrons. We have learned that quantum mechanics provides the framework for understanding behavior at small scales, and electromagnetism the mechanics of electrons, so it is natural that we must turn to understanding the mechanics of "subnuclear" particles.

This reductionist approach tries to understand things in terms of the "fundamental physics" which governs the "elementary particles" of which more complex things are built. This is not the only way to gain understanding of more complex systems. While no one doubts that the action of DNA is governed by the laws of quantum mechanics and Maxwell's electromagnetism, biologists will understand mitosis more in terms of classical structures than wave functions, and condensed matter physicists speak in terms of phonons and effective electron masses rather than in terms of the elementary particles. Certainly no weather forecasters are doing quantum mechanical calculations.

Thus different approaches to understanding are useful, but there is a certain satisfaction in a "fundamental" approach which deals with the "basic laws" of physics. In the next few lectures we will discuss the basic laws as we now understand them.

We have already mentioned the four basic interactions which we believe govern all of nature at the fundamental level. They are

- the gravitational force
- the electromagnetic force
- the strong interactions

- the weak interactions

The gravitational and electromagnetic forces differ from the other two in that they are long range forces, seeming to act over large distances, with forces that can fall off as slowly as $1/r^2$ with distance r. Though Newton's law of gravitation and Coulomb's law have the same form, in which the force between two particles is proportional to the product of their "charges" and inversely to r^2, they have somewhat different manifestations because while the charges relevant to electromagnetism come in either sign, the "charges" that come into Newton's laws are not the electric charges but the masses, and they are always positive, and the force always attractive. As a consequence, the gravitational forces cannot be shielded, so large objects cannot be neutral to gravity, as they usually are to electrical forces. Thus the gravitational force dominates the interactions between planets and moons in the solar system, between stars in the galaxy, and between galaxies in the universe as a whole.

On an elementary particle level, however, the gravitational force is much weaker than the Coulomb force. As both are proportional to r^{-2}, we can compare the forces between two protons, for example, without worrying about how far apart they are:

$$\begin{aligned}\frac{F_{\text{grav}}}{F_{\text{Coul}}} &= \left(G\frac{m_p^2}{r^2}\right)\Big/\left(k_e\frac{e^2}{r^2}\right) = \frac{Gm_p^2}{k_e e^2} = \frac{6.67\times 10^{-11}(1.67\times 10^{-27})^2}{8.99\times 10^9(1.60\times 10^{-19})^2} \\ &= 8.1\times 10^{-37}.\end{aligned}$$

So the gravitational force is nearly undetectible on individual elementary particles.

The strong interactions, as we have discussed in nuclear binding, are very strong but only at very short distances. That is, unless two particles are separated by distances of order 10^{-15} m or less, there is essentially no strong force between them. The weak force is also very short ranged — in fact its weakness can be understood as being due to operating over roughly 10^{-18} m.

We will return to comparing the four forces, and whether they may not actually be different aspects of a unified force, after we have learned how to deal better with subatomic physics, which is inherently quantum mechanical and relativistic. For as we try to investigate behavior of particles at ever smaller distances, the Heisenberg uncertainty principle forces us to consider higher momenta, high enough so that we need to use Einsteinian relativity rather than Newtonian mechanics.

1 Dirac, positrons, and other antiparticles

We saw that the Schrödinger equation can be thought of as simply a rephrasing of

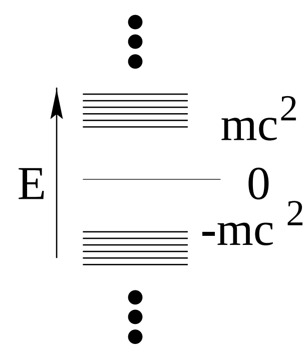
$$E = \vec{p}^2/2m + U(r)$$

in terms of de Broglie ideas, as a differential equation determining the wave function. In the absence of a potential, this is simply $E = \vec{p}^2/2m$. Every solution of this equation represents a possible state of a free non-relativistic particle.

But if we are to consider relativistic particles, we need to deal with the relativistic equation instead:

$$E^2 = \vec{p}^2c^2 + m^2c^4,$$

which has for each possible value of $\vec{p}$, *two* solutions for the energy, one positive and one negative. If there were absolutely no interactions, perhaps we could by fiat declare that only positive energy states are possible, but as for atoms, the electromagnetic field always provides opportunities for an electron to transition from a higher energy state to an unoccupied lower energy state by emitting a photon.



In 1928 Dirac took a very imaginative approach to try to avoid getting negative energy solutions. He thought if he had two wave functions for the electron, he could get an equation linear instead of quadratic in the energy. It turned out he had to have four wave functions instead, which meant that he was describing two different states, the spin up and spin down states, and this is how spin was discovered theoretically. To his chagrin, however, his trick wound up not curing the negative energy problem!

So Dirac then took another way out. Suppose there are all those negative energy levels, *but they are all full!*. Thus empty space is a **Dirac sea** of filled negative energy electrons. As the sea is uniform, we can't detect the uniform charge, but we could detect it if one of those states should become unoccupied, perhaps by a high energy gamma ray ($hf > 2 \times 0.511\,\mathrm{MeV}$) exciting an electron from a negative energy state to a positive energy state. This would leave a vacancy in the negative energy sea which would appear as a positive charge, as well as creating an ordinary electron with positive energy, just as a photon in a semiconductor can move an electron from the valence band to the conduction band, producing two charge carriers, one of them a positively charged hole.

At first physicists hoped the holes could be protons, but it was soon understood that the hole in the full sea behaved *exactly* like an electron, except for having the opposite charge. That is, the hole, which we now call a positron, has exactly the same mass and magnetic moment as an electron. It is our first example of an **antiparticle**.

This turned out to be more general than the Dirac electron. For every particle, there is an antiparticle with all conserved "charges" reversed, with the same mass and same spin. Neutral particles, such as the photon, can be their own antiparticles, but protons cannot, and there are antiprotons with charge $-e$ and baryon number -1. Even the neutron has a separate antiparticle, for while the neutron has no electric charge it does have baryon number 1, while the antineutron has baryon number -1.

Positrons were experimentally found by Anderson in 1932 by observing positively charged tracks bending in a magnetic field in a bubble chamber, induced by cosmic rays. As we mentioned, positrons can also be made by a gamma ray of enough energy interacting with a nucleus, with the energy of the γ going into the $2m_ec^2$ rest masses and whatever additional kinetic energy the electron and positron emerge with.

As antiparticles have the opposite charges of their particle, a particle and its antiparticle, if they get together, can always annihilate. One possible result will be two high energy gamma rays. One use of this is **positron emission tomography** a medical diagnostic procedure known as a PET scan, in which a nucleus unstable to β^+ decay within glucose is injected and distributed by blood flow to places glucose is needed. When it decays, the positron emitted finds a nearby electron and annihilates, producing two photons of 511 keV each. By detecting the gamma rays the glucose-burning activity can be monitored.

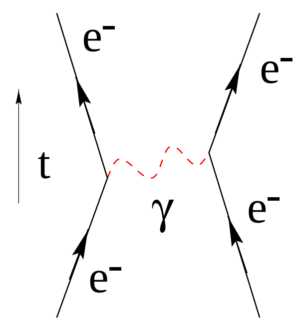
2 Mesons

The Dirac equation was immensely successful in understanding the properties of electrons, but the same equation applied to protons was not. A field of experimental research arose in studying the interactions of protons scattered at energies of hundreds of MeV. From the scattering, it is possible to find a potential energy function describing the force between two protons to be

$$U(r) = -A\frac{e^{-\mu rc/\hbar}}{r}.$$

Where can such a potential come from?

Actually the idea of a potential energy $U(r)$ for two particles interacting with each other at a distance r is one which doesn't sit well with relativity, because it means that one particle responds to a force that depends where the other particle is *right now*, even though the particle is a finite distance away and there is no way for the first particle to know *where the second particle is right now*. In relativistic particle physics we don't think of particles interacting in terms of a potential, but instead they interact by exchanging **virtual particles**. For example, the Coulomb repulsion between two electrons can be understood by giving a history of the particle paths through time. Plotting the world line of the electrons, we can imaging that at some time one electron emits a photon, and at another time the other electron absorbs the photon. In the process, momentum can be transferred from one electron to the other, which is just what we mean by a force acting[1]. In 1931 Møller showed that this photon exchange leads to the same scattering between electrons as the coulomb force.

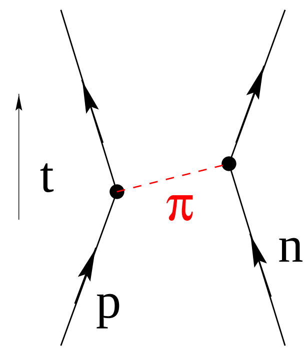


Now you might object that the first electron cannot emit a photon and conserve both energy and momentum. This is most easily seen in the rest frame of the electron, where initially it has only the rest energy $m_e c^2$ and therefore cannot give up energy to the photon. But due to the Heisenberg uncertainty principle it is okay to cheat a bit on energy conservation, provided the cheat is covered up within time Δt, where $\Delta t \Delta E < \hbar/2$, where ΔE is the amount by which energy conservation is violated.

For photons, the energy cheating can be arbitrarily small, because the massless photon can carry off arbitrarily little energy. Thus the time interval can be long provided the momentum transferred is small. But now let's

[1]Actually a classical force is represented by exchanging an arbitrary number of virtual photons. The exact connection between virtual particle exchange and the potential energy is taught in graduate courses in quantum field theory.

consider what would happen if particles, for example a proton and a neutron, tried to exchange a particle with mass, which we will call a pion. The minimum energy the system of a proton and a pion can have is $(m_p + m_\pi)c^2$. In the rest frame of the initial proton, this must violate energy conservation by at least $\Delta E = m_\pi c^2$, which is undetectable as long as this system only stays around a time $\Delta t \le \hbar/2\Delta E$. In that time, the pion certainly can't travel a distance more than



$$r = c\Delta t \le \frac{\hbar}{2m_\pi c}.$$

So virtual pion exchange can produce a force which can act only over a short distance, inversely proportional to the pion mass. The exact mathematical consequences are to produce a potential with an exponential falling off with distance, as mentioned earlier.

Our discussion of the pi meson as producing the **Yukawa potential**

$$U(r) = -A\frac{e^{-\mu rc/\hbar}}{r}$$

is reversed from the historical development — the forces between nucleons were known from scattering, and Yukawa explained that that should be the result of a particle which had not been observed, but which he predicted, and called a meson, intermediate in mass between a proton and an electron. Soon thereafter a particle of roughly the right mass was found and dubbed the **mu meson**, and at first it was believed to be Yukawa's particle. But it was soon observed to have the wrong properties, not having strong interactions with nucleons and in fact behaving just like an electron except for being 200 times heavier. As I. I. Rabi said when learning about these discrepancies, of the mu meson: "Who ordered that?" Later, the pi mesons were discovered, and that turned out to be the particles giving the Yukawa force.

Notice that if the exchanged particle is massless, the exponential suppression of large distances disappears. So long range ($F \propto 1/r^2$) forces are due to the exchange of massless particles. We know about the photon, but what about gravity? The massless particle that conveys the gravitational force, never actually detected individually but firmly believed in, is called the **graviton**.

This way of looking at interactions, as due to the virtual exchange of particles rather than potentials between particles, is the essence of **quantum field theory**. In quantum field theory particles are constantly being created or destroyed, but in ways that conserve what should be conserved.

3 Classification of particles

The fundamental particles may be classified into groups in several ways.

First, all particles are classified into fermions, which obey Fermi-Dirac statistics (and the Fermi-Dirac distribution function if you can get a box full of them), and bosons, which is everything else. Actually they are distinguished by spin — fermions have spin 1/2 or 3/2 or some other integer added to 1/2, while bosons have integer spin (all in units of $\hbar$). All the *fundamental* fermions have spin 1/2. Electrons and nucleons are fermions with spin 1/2. The fundamental bosons are mostly spin 1, which includes the photon. The pion, which is not now considered fundamental, has spin 0, while the graviton has spin 2. There are also three particles, the W^+, W^- and Z^0 bosons, which are spin 1 and are the carriers of the weak interactions.

The electron and the neutrino are members of a family of **leptons**. Originally leptons meant "light particles", as opposed to **baryons**, or heavy particles, which referred initially to the proton and neutron and some others we will learn about shortly. The pion, or pi meson, and another particle called the muon or mu-meson, were called mesons, or medium-weight particles, because their masses, a few hundred times heavier than the electron but 6 times lighter than a proton, were in the middle. But that distinction turned out not to be very useful — we now recognize the muon to be almost the same as an electron, and the leptons now consist of three "generations" of pairs of particles:

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix}, \qquad \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}, \qquad \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix},$$

with the heaviest of these, the tau lepton τ^-, weighing almost twice as much as the proton!

The leptons are distinguished from other particles called **hadrons** in that leptons do not participate in strong interactions. The bottom lepton in each of the three "doublets" shown above is not only neutral (has no electric charge) but also has very little mass, if any. There is a great deal of interest right now in whether or not these three neutrinos have mass.

The baryons are a class of fermions, including the proton and neutron, and other particles which can turn into a proton by emitting pions. Like the proton, they are all hadrons, *i.e.* they feel the strong interaction force.

The mesons, not including the muon which really doesn't fit in this category, are bosons which are also hadrons. In addition to the pion, there are other spin 0 particles, the four kaons and two eta (η) mesons, and a number of spin one hadrons, including the three rho (ρ) mesons, which like the pion come in charges ± 1 and 0.