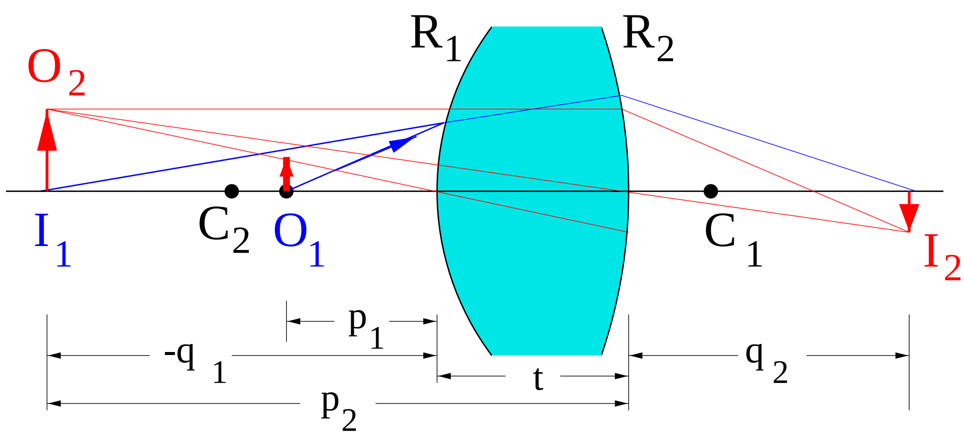
Physics 228, Lecture 3
Thursday, Jan. 27, 2005

Lenses;interference. Ch 34:4; 35:1-2

1 Thin Lenses

Last time we considered a single interface between two transparent media, but most of the time we are interested in more complex systems, if only because the light leaves the optical device as well as entering. Consider a lens. It will have two surfaces through which the light passes. Most lenses are round and focus light in both transverse directions, but we have some cylindrical lenses useful for displaying focussing in only one transverse direction ("blackboard optics"). If light comes in parallel rays from a point object very far away, the rays may converge to a focus, as we can see with this converging lens. We will assume each surface is spherical[1], with radii R_1 and R_2 respectively. Light leaves the real object O_1, which is in air with index of refraction ≈ 1 and enters the glass, with index of refraction $n \sim 1.5$.



There is an image I_1 of the object as seen from within the glass, although that image may well be, as shown, a virtual image. That virtual image which describes the divergence of the light within the glass serves as the object, O_2, for the second interface, and that object has an object distance

[1]Circular for blackboard optics lenses

$p_2 = |q_1| + t = -q_1 + t$ for the case drawn, where t is the thickness of the lens. That object forms an image in the air, I_2, a distance q_2 behind the second surface. The equation for the first and second refractions are

$$\frac{1}{p_1} + \frac{n}{q_1} = \frac{n-1}{R_1}, \qquad \frac{n}{p_2} + \frac{1}{q_2} = \frac{1-n}{R_2},$$

where we note that for the second surface, $n_1 = n$ and $n_2 = 1$, and R_2 is negative the way we have drawn it.

Often the lens we are interested in is thin, and the thickness t can be neglected in comparison to the p's and q's. Then we can replace p_2 with $-q_1$ and add the two equations to get

$$\frac{1}{p_1} + \frac{1}{q_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

If we forget about the intermediate image and object, we can drop the subscripts on the p and q, and get

$$\frac{1}{p} + \frac{1}{q} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

The right hand side depends only on the lens, while the left involves the object and image distances. If we define the focal length f of the lens as the image distance q when the object is infinitely far away, the left side is then $0 + 1/f$, so we have

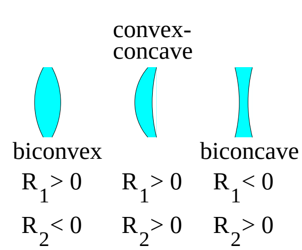
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

which is known as the **lensmaker's equation**. We can then write our lens equation in the same form as worked for the mirror:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}.$$

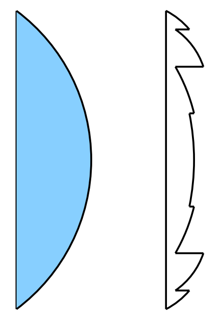
In this context this is called the **thin lens equation**.

The sign conventions are as for the interface, but it may be confusing that the radii are considered positive if the center of curvature is in back of the surface for both surfaces. Thus a biconvex lens such as the one I have displayed has a front surface with positive R_1 and a back surface with negative R_2. Your glasses probably have positive R_1 and R_2, but if the two R's were the same they would not have any function.



Note also that the focal length is positive if the lens is converging, producing a real image from light from infinity, and negative if the lens is diverging. Nearsighted people have eyes that focus naturally on things a foot away from them, so they need glasses which image objects far away with a virtual image with $q \sim -30$ cm. Optometrists talk in diopters, which is the inverse of the focal length in meters, so such a person needs glasses of -3.3 diopters.

Notice that the thing that mattered in determining how the light was bent and therefore focussed was the angle of the surface. In our overhead projector we have a bulb which is only a few inches from the light source, but we want the light to be in a beam focussed a long way off. So we need a collimating lens with a focal length of a few inches. According to our formula, with glass of $n \approx 1.6$, we would need a lens with radii of curvature of a few inches but more than a foot in diameter! This would make a very thick lens, if it were not for Fresnel's idea, that one can chop the lens into pieces and remove the flat parts, and make a much thinner lens.



1.1 Ray tracing for thin lenses

Because the image forming properties of the lens are entirely determined by its focal length, we can examine the imaging properties by tracing a few rays from the tip of the object, somewhat off the principal axis. One ray to look at leaves the object parallel to the principle axis. It is taking a path appropriate for an object at infinity on the axis, so that ray will be bent to pass through the focal point (if converging) or to appear to leave the focal point (if diverging). Another ray to look at is the one that passes through the center of the lens. As the front and back surfaces are parallel at that point the ray will be undeflected. Finally, we note that the thin lens formula is symmetric between object and image, so there is another focal point on the opposite side of the lens appropriate for light going to infinity on the right, as shown in purple. The ray from the object passing through that focal point will pass through the lens and emerge parallel to the principal axis.

Three diagrams, S&BV5 36.27 biconvex, real and virtual, biconcave

5" × 6"

For a converging lens, if the object is further than the focal point the image will be real and behind the lens, but if the object is closer to the lens than the focal point, the converging properties of the lens are not enough to make the rays converge. Thus the image is a virtual one in front of the lens.

Let us see a converging lens in action. Here is a bright filament bulb with some arrows marked on the outside, facing the lens and screen. The arrows are up and to the left. The light passes though this converging lens, and if I adjust the object distance correctly I can make a real image on the screen. Notice the arrows are down and to the right so the image in inverted in both transverse directions. Notice the filament and the arrows are not perfectly focussed together - the region of object distance in reasonable focus is called the **depth of field**.

What will happen to the image if I cover the bottom half of the lens with an opaque piece of cardboard?.

What will happen if I place this cardboard with only a small hole next to the lens?.

[Display Fresnel lens in place of other.]

Finally, we note that for a diverging lens, the focal point for incoming rays is on the left, so rays coming in parallel to the principal axis are diverged so they appear to come from the focal point in front of the lens, as for the purple ray in Fig 34.33b. For all **real objects**, the image for a diverging lens will always be virtual and upright.

Note that the magnification is given by the similar triangles formed by the ray that passes unbent through the center of the lens, and the principal axis, together with the image and objects respectively. Thus

$$M = \frac{h'}{h} = -\frac{q}{p}.$$

What happens if part of the lens is covered?

Lenses are often used in combinations. If thin lenses are placed right next to each other, their diopters add. That is, the combined lens has a focal length f given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}.$$

Of much more interest is what happens when they are separated by distances comparable to their focal lengths. That is the way to form the compound microscope or telescope. The focusing and magnifying properties can be worked out by following the intermediate image and objects one lens at a time. Although this is a very useful technological field, we will not cover such instruments in this course. We will also not cover the others of sections 5–8 in this chapter.

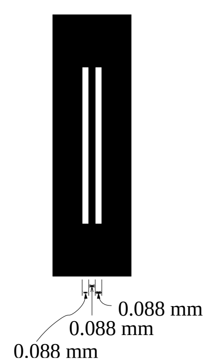
2 Interference and Coherence

Thus far we have been dealing with optics without any direct effects of the wave nature of light, but it is time to turn to that now. The most dramatic and convincing indication of the wave nature of light is in interference. When light can reach some point from two sources, its intensity is not the sum of the intensity of the two sources, but rather the values of the wave displacement are added. As the wave displacement can be positive or negative, adding waves can cause both constructive and destructive interference.

Visible light waves oscillate so rapidly that we never observe the instantaneous value of the fields, but rather the intensity, proportional to the time average of the square of the fields. Interference, which refers to the intensity being different from the sum of the intensities of the interference waves, is only observable when the sources are **coherent**. Being coherent means they are of the same frequency so precisely that their relative phases remain fixed in time. Independently generated light sources will not interfere because they cannot be made coherent. But if the two sources are being fed by a common source of light, then they can be coherent. The simplest example is Young's double slit.

3 Young's double slit

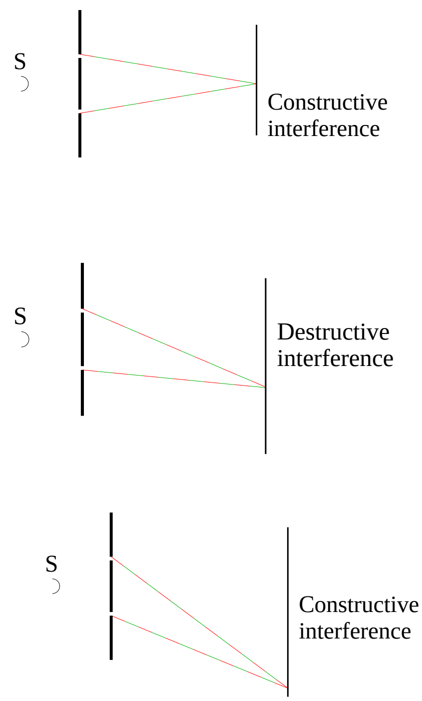
In the back of the room there is a laser which is passing through two very narrow slits, which are each 0.088 mm wide and are separated by only 0.088 mm. You see the results if I turn off the lights. Let's figure out why.



Consider a source of light S_0 which illuminates two narrow slits S_1 and S_2. The light emerging from those two slits is coherent, because the phase is determined by the geometry and the fact that all the light is coming originally from the same source. Each source has wave fronts emerging and spreading out, so that at points behind the slits, there is an part of the wave coming from one and part from the other. What will happen? A given point will generally be different distances from the two slits, so it will take these two waves different amounts of time to reach the point in question, and they need not be in phase with each other. For any point p equidistance from S_1 and S_2, at any instant the two waves reaching p will have originated from S_1 and S_2 at the same instant, so they will be in phase and will add constructively. The wave amplitude will be twice the amplitude we would have from one slit alone, so the intensity will be four times what we would have had from one slit alone. But at another point R which is further from S_1 than it is from S_2 by one-half wavelength, it takes the wave from S_1 one half period longer to reach R than it takes the wave from S_2. Thus the waves reaching R at any instant emerged from the two slits half a period apart, with opposite values of the fields, so the two waves are 180° out of phase, and interfere destructively, cancelling one another out, or nearly so. If we go further down, the distance to the top slit may be a full wavelength more than the distance to the bottom slit, and the waves will again be in phase, and interfere constructively.

S&BV5 Fig 37.1a waves from double slit

4 1/4" × 4 5/8"



When we are a distance from the two slits much larger than their separation, the points which are half a wavelength further from one slit than the other form a line, shown by the dotted line with the empty blue dots, and no light will be observed at those points. This is true also if the difference in distances from the two slits is $3\lambda/2$ or $5\lambda/2$, *etc.*. There will be constructive inteference not only when the distances are the same but also if they differ by an integer number of wavelengths, $n\lambda$, for n an integer.

Often we will observe the interference pattern on a screen, as in the figure. We call d the separation between the two slits. and consider a point P on a screen a distance L away from the plane of the slits, making an angle θ with the normal from the point Q halfway between the slits. The distances r_1 and r_2 that P is away from the slits is what determines the brightness at P. If $\delta := r_2 - r_1$ is an integer number of wavelengths, $\delta = m\lambda$, we will have a bright fringe and constructive interference. If $\delta = (m + \frac{1}{2})\lambda$ for some integer m, there will be destructive interference and a dark fringe. m is called the **order number**.

S&BV5 Fig 37.4a Δr from double slit

5 3/4" × 3 3/4"

If we measure off a distance r_1 on r_2 so the triangle is isoceles, the difference $r_2 - r_1 = \delta$. If L is much greater than the distance between the slits, the triangle will be very narrow, the line PQ very nearly the bisector of the angle and short side, so it is nearly perpendicular to the short side and the two angles marked θ are very nearly the same. Furthermore the yellow triangle is very nearly a right triangle, so $\delta = d \sin\theta$. Thus

$$d\sin\theta = \left\{ \begin{array}{cc} m\lambda & \text{constructive interference} \\ (m+\frac{1}{2})\lambda & \text{destructive interference} \end{array} \right\} \text{with } m = 0, \pm 1, \pm 2, ...$$

Notice that the point O perpendicular to the plane of the slits and halfway between them is a maximum, for it is equidistance from the two slits, and corresponds to $m = 0$ constructive interference. Measuring distance y from this point, we have $y = L\tan\theta$. Often, we are interested only in a range $|y| \ll L$, so θ is small and we can approximate $\tan\theta = \sin\theta$, so

$$y \approx \frac{L}{d} d\sin\theta = \begin{cases} \frac{\lambda L}{d} m & \text{for maximum} \\ \frac{\lambda L}{d}(m + \frac{1}{2}) & \text{for minimum} \end{cases},$$

approximately when $\lambda \ll d$, $y \ll L$, $\theta \ll 1$.

4 Summary

- Multiple active optical elements (including interfaces) can be treated by successively calculating each image and making that the object of the next element.

- For thin lenses, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ with the focal length given by

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

  A tricky point is that in this formula R's are positive if their centers of curvature are to the right (considering rays coming from the left).

- Ray tracing is useful, especially for combinations of thin lenses. Convenient rays to trace are 1) parallel to the principle axis on one side and passing through (or away from) a focal point on the other, and 2) passing through the center of lens undeflected.

- Interference is the effect that coherent sources cannot be treated by adding their intensities, but instead the relative phases must be considered. When multiple sources are in phase they interfere constructively, giving an intensity greater than the sum, but when they are 180° out of phase they destructively interfere, giving a minimum.

- For a double slit, with $d \ll L$,

$$d\sin\theta = \left\{\begin{array}{cc} m\lambda & \text{constructive interference} \\ (m+\frac{1}{2})\lambda & \text{destructive interference} \end{array}\right\} \text{with } m = 0, \pm 1, \pm 2, ...$$

- If also $y \ll L$,

$$y \approx \frac{L}{d} d\sin\theta = \begin{cases} \frac{\lambda L}{d} m & \text{for maximum} \\ \frac{\lambda L}{d}(m+\frac{1}{2}) & \text{for minimum} \end{cases}.$$