Physics 228, Lecture 1
Thursday, Jan. 20, 2005

Introduction to Optics, Ch. 33:1-4,7

Intro to Physics 228, Analytical Physics IIB

This is Physics 228, the fourth semester of our introductory physics sequence designed for engineers. In this semester we will deal with optics and with modern physics, which, roughly speaking, is anything since 1900, as physics has a long history. We will deal with some things which have immense technological usefulness and some things which do not but which are mind-blowing philosophically.

Nearly all of you were in Physics 227 last term, so I am not going to give you the introductory pep-talk I did last term, but I will post one on the web. Those of you who were not in 227 should read it, as there are some hints there which some of those who heard me give later regretted not paying attention to. I will just mention here the differences from last term.

The most important difference from last term is that homeworks will now be due on Monday nights, just before the recitations which discuss that material.

The PRS will work as it did last term, except that we will start counting your PRS grades on January 24, so anyone who does not have a PRS unit will have to get one by then. We will discard the lowest four PRS grades, so only your best 20 will count. This should cover any absences due to illness or other excuses. Don't forget to bring your transmitters and to check that your transmissions are being received. This is very important — it is not unusual for a button press not to register, but we have never had a case where one which showed up on the screen was not recorded. If you need to, press the button again.

All the important information about this course is available through the course home page, which is now
`http://www.physics.rutgers.edu/ugrad/228/`
In particular you can find the

- Course Information and Requirements

- Syllabus
- Instructors and recitations
- lecture notes
- homework solutions, old exams, etc
- links to WebAssign and the PRS registration form and grades.

What is Light?

For the next three weeks we will be looking at light and optics. While some ancients believed vision involved something emanating from the eyes and touching the viewed object, it was clear by the time science considered the subject that it was the other way around — light comes from the viewed object to the eyes, which detect it. Newton thought light consisted of small particles. The idea that light might be waves came up at about the same time, but was not generally accepted until Young, in 1801, demonstrated **interference**. Together with other experiments showing **diffraction**, these effects convinced everyone that light is a wave, with the subject of debate becoming the question of what medium was doing the oscillating. We will discuss interference and diffraction soon. We will also see how efforts to understand the medium led to the first really modern theory of physics, the special theory of relativity. It is ironic that after the wave theory conclusively won the battle, quantum mechanical effects, in the form of the photoelectric effect, showed that the question itself was bogus, that light and elementary "particles" like electrons are both simultaneously particles and waves. This led to the second major modern theory of physics, quantum mechanics.

The speed of light

As we now know, light in vacuum travels at 2.998×10^8 m/s, which is very fast. Galileo tried to measure the speed of light by sending two people up mountaintops with lanterns at night. The first opened his lantern and timed how long it took for his light to reach the other, have him open his lantern, and then the light to return to the first person. He realized from the results only that light travelled faster than he could measure by that procedure. (As the light travel time was only about 10 microseconds, that was the best he could do.) The first real measurement was due to Romer in 1675, who used the motion of Io, one of the moons of Jupiter, as a clock. While the eclipses occur with great regularity, it takes longer for the light to reach Earth when the Earth is on the opposite side of the Sun, from Jupiter, than when it is closer. Romer measured that delay, and knew roughly the size of the Earth's orbit, so he was able to calculate the speed of light to be roughly 2.1×10^8 m/s, and definitely not infinite. Over a hundred years later, in 1849, the first terrestrial experiment was done by Fizeau, using a toothed wheel. Shining light through the teeth in a gear, bouncing it off a mirror, and examining the light that returns through the same spot, we will see light when there is no tooth in the way on either half of the trip. If the wheel is rotating, sometimes light will get through, and when it gets back it will pass on to the eye if there is still, or again, a gap at the crucial point. So if the wheel is rotating fast enough, light will get through one gap on its way to the mirror and the next gap on its way back. Knowing how fast the mirror is rotating, we can determine how long it took to get to the mirror and back.

Of course these days we have devices capable of directly measuring times of microseconds or even nanoseconds, so we have much more elegant ways to measure the speed of light.

In fact, with good apparatus we can measure speeds with higher accuracy than we can measure distances, so now the speed of light is **defined** to be 299,792,458 m/s, and a meter is defined by the distance light travels in a second divided by that number.

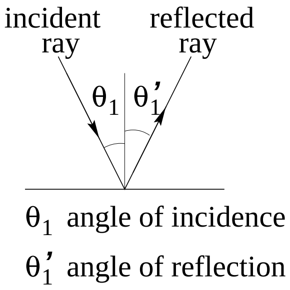
Geometrical Optics; Ray Approximation

Even though we know that light is, for most purposes, best described as a wave, it is a wave with a very short wavelength. When the wavelength is much shorter than other distances in question, the wavelets travel in straight lines much as if they were free particles. The **ray approximation** consists of treating a beam of light as a collection of rays moving in straight lines except when moving from one transparent medium to another or reflecting off a surface. If a beam of sunlight passes through a triangular hole in the wall, it will light up a triangular patch on the opposite wall or floor. Approximately. We will find the corrections next week.

Reflection

When light hits a surface, it might pass into a new medium or it might be bounced back into the original one. Actually, if both media are transparent, it does both, that is, part of the light is **reflected**, and part is **refracted**. Of course, some of the light can also be **absorbed** — that is, energy is lost from the electromagnetic fields and converted to heat in the medium. First we consider reflection.

When a ray of light hits a smooth surface, the angle between the incoming ray and the **normal to the surface** is called the **angle of incidence**. The law of reflection says that the reflected beam will consist of rays making an angle with the normal, called the **angle of reflection**, equal to the angle of incidence (and in the same plane with the incident ray and the normal, perpendicular to the surface).



demo?

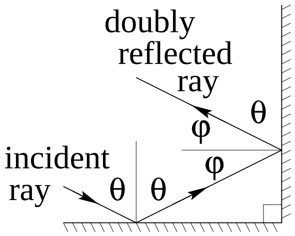
$$\theta_1 = \theta_1'.$$

Note that this is what one would expect if the light were particles colliding elastically with a much heavier surface which exerted a force normal to the surface.

There is always some reflection off any surface, but some surfaces are a lot more reflective than others. Shiny metal surfaces are good reflectors, and when that is deliberate they are called mirrors. Consider two mirrors at right angles to each other, and an incident ray hitting one with an angle of incidence θ. By the law of reflection the once reflected ray also makes an angle of θ with the vertical, so it is at an angle $\phi = 90° - \theta$ with respect to the normal to the second mirror. It bounces off at an angle of (second) reflection ϕ with respect to the normal, which means it is at an angle

$$90° - \phi = 90° - (90° - \theta) = \theta$$

with respect to the vertical. In other words, it goes back out at whatever angle it came in at.



If you ever see two vertical mirrors at right angles to each other, your face will be where they join, no matter how you move around.

We assumed the surface doing the reflection was smooth and flat, so there is a well defined normal direction. This is called **specular reflection**. A rough surface will have the different rays within the beam incident on regions which are locally flat with different normals, and each ray will be scattered in different directions. This is called **diffuse reflection**. It is important, for if all surfaces were perfectly specularly reflecting, they would appear dark even in bright sunlight except when they blinded you with the direct image of the sun from the spot where the angles to your eye and to the sun are equal. A road at night appears less lit up by your headlights if it is wet because the moisture causes more specular reflection from you headlights, so the road does not return the light in your direction but shows a reflection of your car to oncoming vehicles.

Refraction

Light passes through transparent media, so another possibility when light is incident on the surface of glass, say, is that the light will continue to travel through the glass. But the rays of light do not continue along the line that they were going in the air. If we examine a ray entering the glass in this demo, we see that the incoming ray is bent towards the normal to the surface. Each material m has an **index of refraction** n_m which determines how much the ray will bend, according to Snell's law (1621).

Demo with rect block, reproduce Fig 35.9

For the vacuum, n is exactly 1, and for all other media it is greater than 1. When light moves from one medium, say medium 1, onto an interface with medium 2, some of the light is reflected, with $\theta'_1 = \theta_1$, but what goes into medium 2 travels at an **angle of refraction** θ_2 to the normal. Snell's law states that

show fig 35.9 7" × 3 1/4"

$$n_1 \sin\theta_1 = n_2 \sin\theta_2.$$

Notice that Snell's law doesn't distinguish between the incoming and outgoing medium, so light travelling the reverse direction will take the same path backwards. In passing from a lower to a higher index of refraction, the light is bent towards the normal, and in travelling from a higher to a lower n, it is bent away from the normal. We see this in the light on the right in the picture. We also see examples of reflection, which can happen at any interface between media with different indices of refraction. Beam 4 has been reflected (with $\theta_2' = \theta_2$) from the bottom surface of the glass, and so beam 5 has the same angle with the normal as the incoming beam 1 and the reflected beam 2 do.

Observe that reflections occur from surfaces we would not ordinarily think of as mirrors. You can see your reflection in a clear glass window, especially if you are standing in the light and the area in back of the window is dark. The strength of the reflection has to do with the change in index of refraction. Here we have two pyrex beakers, one inside the other. Look at the inner beaker, and watch what happens when we first fill it, then cover it with oil. Whoops — it disappeared. What we were looking at before was light relected from the surfaces of the beaker, but the oil and the plastic have the same index of refraction, so under the oil no light is reflected from those surfaces, and we can't see the beaker!

demo disappearing beaker.

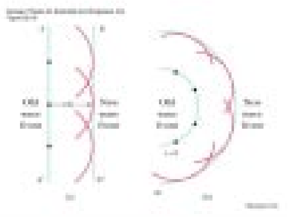
Refraction has some curious effects. Because we "see" objects along the line that the ray of light enters our eye, if the light has bent in the process, we see it somewhere other than where it really is. For example, if I take this straight rod and immerse part of it in water, the rod doesn't actually bend, but it sure looks like it does.

Demo rod in water

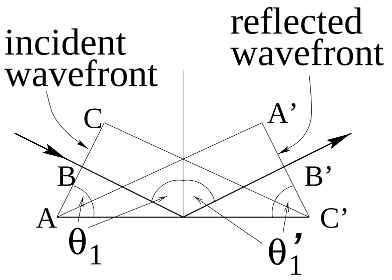
Notice that Snell's law is older than any measurement of the velocity of light, even before people knew it wasn't infinite. Newton's particle picture could explain Snell's law if we assume the surface exerts a force perpendicular to itself on the light, and that the light then travels with a velocity proportional to n. But measurements of the velocity in water and glass showed, in the mid 1800's, that light travels more slowly in media other than the vacuum. For other reasons the wave theory had already dominated the particle theory, and as we will see, in the wave theory we can understand Snell's law by assuming the index of refraction is **inversely** proportional to the speed of light v_i in the medium:

$$n_i = c/v_i.$$

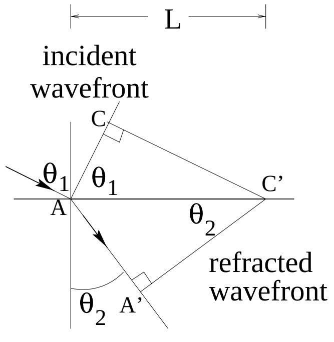
The way to understand that is by Huygens' principle. This is a method of understand the propagation of wave fronts. A wave front is the surface in space (or curve on a surface) which represents all the points at which the wave is at a common argument. For example, the points at which the wave is at its maximum at a single time is a wave front. Huygens said to find where the wave front will be at a time Δt later, let each point of the wave front at t propagate a circular (spherical) wave through a distance $v\Delta t$. These circles will overlap each other, but there is an **envelope**, and that envelope forms the new wave front. The implication is that the time to travel from one wave front to the next along a (locally) minimum path is the same for all points on the new wave front.



Consider reflection. If the beam is incident at an angle θ_1, the angle the wavefront A–B–C makes with the surface is also θ_1. The time between that wavefront and another, A'–B'–C' is the time it takes the light to get from A to A', which is $L\sin(\theta_1')/v_1$. But it is also the time it takes for the light to get from C to C', which is $L\sin(\theta_1)/v_1$. As the L and velocities v_1 are the same, $\sin\theta = \sin\theta'$, or $\theta = \theta'$, giving the law of reflection.



Now consider refraction. Again the beam is incident at an angle θ_1, and the angle the wavefront A–C makes with the surface is also θ_1. Call the angle the refracted wavefront makes θ_2, where the subscript refers to the medium in which the light is travelling. The time between one wavefront and the other is the time it takes the light to get from A to A', which is $L\sin(\theta_2)/v_2$, because the light is travelling through medium 2 at speed v_2. But this time needs to be the same as the time it takes for the light to get from C to C', which is $L\sin(\theta_1)/v_1$. Equating these times,



$$\frac{L\sin(\theta_2)}{v_2} = \frac{L\sin(\theta_1)}{v_1} \quad\longrightarrow\quad \frac{\sin(\theta_1)}{v_1} = \frac{\sin(\theta_2)}{v_2}.$$

Multiply by the speed of light in vacuum, and then define n_i for each medium as

$$n_i = \frac{c}{v_i},$$

and we have

$$\frac{c}{v_1}\sin(\theta_1) = \frac{c}{v_2}\sin(\theta_2) \quad \longrightarrow \quad n_1 \sin(\theta_1) = n_2 \sin(\theta_2).$$

This is Snell's law explained.

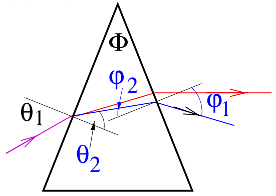
So we see that the velocity of light in water or glass is less than it is in vacuum. At the interface between two media, the electric field oscillating at a given frequency will be oscillating at the same frequency on both sides of the interface. Thus $f_1 = f_2$. But then the wavelengths must differ, as $v_i = f_i \lambda_i$, so we see that the wavelength changes when changing media, proportional to the velocity and therefore inversely proportional to the index of refraction.

Dispersion

We have spoken of the velocity of light, as if it were the same for all waves. When we examined Maxwell's laws for an electromagnetic field in the absence of matter, we found that the velocity of the waves **is the same** for all waves. But in matter the velocity is not the same for all frequencies, but rather the velocity in a given medium is slowly dependent on frequency. This dependence of the index of refraction on wavelength is called **dispersion**. For visible light in uncolored transparent materials, the velocity gets smaller and the index of refraction larger as the frequency increases and the wavelength decreases. This behavior is called normal dispersion, and it tells us that violet light will be bent more than red.

One way of seeing this dispersion dramatically is to pass a beam of white light through a prism, or a triangular cylinder. Consider a beam of light incident at angle θ_1 on one face of a prism, from air, which is nearly vacuum with $n_1 \approx 1$. Thus $\sin\theta_2 = \sin\theta_1 / n_2$, and the larger n_2, the smaller the angle θ_2 will be. Thus for blue light, which has a higher n_2 than red light does, θ_2 will be smaller

demo



than for red. Now the sum of θ_2 and ϕ_2 is fixed by geometry[1], so the blue ϕ_2 is larger than the red one, and $\sin\phi_2$ for blue is larger than the red. But the blue n_2 is also larger than the red, so taken together, $n_2 \sin\phi_2$ is definitely larger for blue than for red. But

$$n_2 \sin\phi_2 = \sin\phi_1,$$

so we see that the blue light is bent more than the red. If the light then travels some distance to a screen, we see the white light dispersed into its different colors, with blue at the bottom and red on the top.

Total internal reflection

When light is incident on a boundary between two transparent media, we have the possibility that some will be reflected and some refracted. But if the light is coming from a higher n region to a lower n, there is an interesting effect. Suppose the angle of incidence is such that

$$\sin\theta_1 > \frac{n_2}{n_1}.$$

Then Snell's law would say

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 > 1.$$

But there are no angles with sine greater than 1, so there is no possible direction for the refracted light to go. What happens in this case is **total internal reflection** — all the light is reflected. High quality binoculars work that way. They have a prism with a high index of refraction.

Demo with semicircular block.

The angle which separates incident rays which will be partially refracted from those which will undergo total relection is called the **critical angle** θ_c, and we see that it is the angle

$$\sin\theta_c = \frac{n_2}{n_1}.$$

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[1] The top triangle has one angle which is Φ, one is $90^\circ - \theta_2$, and one is $90^\circ - \phi_2$, which must sum up to 180°, so $\theta_2 + \phi_2 = \Phi$.

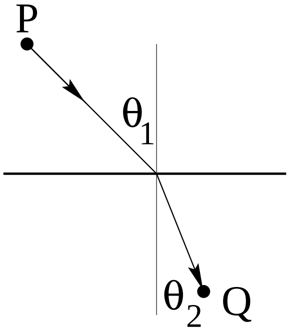
When a fish looks up at the sky on a calm day, he sees the whole thing inside the horizon, which is a cone with "half-angle" of 48°. Outside that circle he sees only reflections of other fish and the bottom of the lake.

Total internal reflection is the principle which makes fiber optics possible. For light running approximately in the direction of the fiber, it will hit the walls always at angles greater than the critical angle, and will be reflected back inside the fiber.

Demo fiber optic pipe.

Fermat's Principle

We have seen that we can follow wave fronts by using Huygen's principle, which says that the new wavefront is the envelope of wavelets emitted from each point on the old wavefront. Consider a point on the new wavefront and the point from which that wavelet was generated. Fermat says the path taken by a light ray is the path which takes the least amount of time. For example, consider a point P in air and Q under the water. The shortest distance between them is a straight line, but as the light travels faster in the air, it will actually take a shorter time to go by the route shown. You can minimize the time taken as a function of the position of entry into the water, and you will find you derive Snell's law!



Fermat's principle is connected to Huygens'. He said the new wavefront is the envelope of wavelets. If the path taken were not the minimal time path, there would be another path for which the light could have gone beyond the point on the wave front in the time allotted. Thus our point would be behind the wavefront rather than on it.

Summary

- Light travels at a finite speed depending on the medium. For vacuum it is fastest and independent of frequency, and is called c, which is $\approx 3.00 \times 10^8$ m/s.

- An approximation for the behavior of light is the ray approximation, which is suitable for describing things on a scale large compared to the wavelength.

- At a surface, some light can be reflected, with the outgoing rays emerging at an **angle of reflection** relative to the normal to the surface which is equal to the **angle of incidence** between the incoming rays and the normal.

- In media other than the vacuum, light travels with a velocity v which is less than c and generally depends on frequency. The **index of refraction** of the medium is defined by

  $$n = c/v.$$

- If the medium behind the surface is transparent, some light may also be refracted, or passed into the medium, forming rays which make an **angle of refraction** θ_2 with the normal to the surface. The angle is determined by Snell's law

  $$n_1 \sin\theta_1 = n_2 \sin\theta_2.$$

- In the denser medium there is a maximal angle, called the **critical angle**, for which Snell's law can be satisfied. Light hitting the surface at a greater angle will undergo **total internal reflection**. The critical angle is given by

  $$\sin\theta_c = n_2/n_1.$$

- Fermat's principle says light takes a path which minimizes the time taken to get from an initial point to the final point, going at the appropriate speed in whatever medium it is travelling through.