Physics 228 - Second Common Hour Exam
Solution

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1. Find the speed of a particle whose total energy is twice its rest energy.
   - a) 0.925 c
   - b) **0.866 c**
   - c) 0.500 c
   - d) 0.250 c
   - e) 0.792 c

   Solution:

$$E = \gamma mc^2 \qquad \gamma = 2 = \frac{1}{\sqrt{1-\beta^2}} \qquad 1-\beta^2 = 1/4$$

$$\beta^2 = 3/4 \qquad v \underline{= .866} c$$

2. Which of the following statements about the photoelectric effect is <u>FALSE</u>:
   - a) The photocurrent increases with increasing light intensity above the cut off frequency.
   - b) The cut-off frequency is independent of photon intensity.
   - c) The maximum photoelectron kinetic energy increases with decreasing photon wavelength.
   - d) **The maximum photoelectron kinetic energy increases with increasing light intensity.**
   - e) The stopping potential increases with increasing photon frequency.

   Solution:

   The maximum photoelectron kinetic energy does NOT depend on the intensity of the light.

3. An astronaut travels from the earth to a star 4.8 light years away (as seen by an observer on the earth). Then he returns to the earth. His outbound and return trip are all at the same constant speed of 0.80 c relative to the earth. How much has the astronaut aged during his trip? (Assume that times for acceleration, deceleration and his stay at the distant star are all negligible.)

   a) 16 years
   b) 12 years
   c) 9.6 years
   d) **7.2 years**
   e) 5.8 years

Solution:
The astronaut will see the distance between the earth and the star contracted:

$$d_{astro} = d_{Earth}\sqrt{1-(u/c)^2}$$

The distance he travels is twice this (since he goes out and returns). So the amount the astronaut ages is the time he spends traveling in his own reference frame, which is

$$t = \frac{2d_{astro}}{u} = \frac{2 \cdot 4.8 \text{ ly}}{0.8c}\sqrt{1-(0.8)^2} = 7.2 \text{ years}$$

(Note: since 1 light-year is $c * 1$ year, we don't have to do any conversions!)

4. An observer measures a time interval of 0.60 μs between two events that occur 100 m apart. What is the speed of a reference frame in which the two events occur at the same point in space?

   a) 0
   b) 0.25c
   c) **0.56c**
   d) 1.1c
   e) 1.8c

Solution:
There are two ways to think about this one: the naive way and the sophisticated way, and they both give the same answer. The naive way says that if the two events are caused by a single object moving at the speed u, then the speed is

$$u = D/t = \frac{100 \text{ m}}{0.6\ \mu s} = 0.56\ c$$

You can make them appear at the same point by moving in the same direction at the same speed. The sophisticated method is to perform a Lorentz transformation:

$$D' = 0 = \frac{D - ut}{\sqrt{1-(u/c)^2}}$$

The γ factor doesn't change anything: we can divide it out, which leaves us with $D - ut = 0$, which yields the same answer that the naive approach gives!

5. The Rayleigh and the Planck formulae for intensity of blackbody radiation are

$$I_{Rayleigh}(\lambda, T) = \frac{2\pi c k_B T}{\lambda^4}$$

$$I_{Planck}(\lambda, T) = \frac{2\pi h c^2}{\lambda^5\left(e^{hc/\lambda k_B T} - 1\right)}.$$

How is the Planck formula better?

a) It avoids the infrared catastrophe at long wavelengths.
b) **It avoids the ultraviolet catastrophe at short wavelenghs.**
c) It avoids the ultraviolet catastrophe at long wavelengths.
d) It avoids the infrared catastrophe at short wavelengths.
e) It explains the spectrum of hydrogen atoms.

Solution:

Rayleigh predicted $I(\lambda) = \frac{2\pi c k T}{\lambda^4}$. If we compare this formula with Planck's blackbody radiation formula, we find the two agree in the limit of large wavelengths, but not at small wavelengths, where Rayleigh's formula gives infinitely large intensity.

The correct answer is "for short wavelengths, giving less radiation there and avoiding the ultraviolet catastrophe."

6. The metals lithium, beryllium and mercury have work functions of 2.3 eV, 3.9 eV and 4.5 eV respectively. If light of wavelength 400 nm is incident on each of these metals, which of them exhibit the photoelectric effect?

a) **lithium only**
b) lithium and beryllium only
c) lithium, beryllium and mercury
d) mercury only
e) beryllium and mercury

Solution:

$$E_{photon} = \frac{hc}{\lambda} = \frac{(4.136\times10^{-15}\,eV\cdot s)(3\times10^{8}\,m/s)}{4\times10^{-7}\,m}$$

$$= 3.10\ eV$$

Thus $E_{photon} > \phi_{Li}$; $E_{photon} < \phi_{Be}$ and ϕ_{Hg}

Only Lithium will exhibit photoeffect

7. In the He^+ ion, one electron orbits a nucleus which contains two protons. What is the ground state energy for the electron in He^+?

a) −13.6 eV
b) −27.2 e V
c) −40.8 eV
d) −3.4 eV
e) **−54.4 eV**

Solution:

The ground state energy of an ion having one electron and a nucleus of charge Ze is $-13.6Z^2$ eV

8. The wavelength of the matter wave associated with a 10-eV free electron is:

   a) $1.9\text{x}10^{-7}$ m
   b) $1.3\text{x}10^{-34}$ m
   c) $1.2\text{x}10^{-9}$ m
   d) **$3.9\text{x}10^{-10}$ m**
   e) $1.9\text{x}10^{-10}$ m

Solution:

The energy of a particle is

$$E = \frac{p^2}{2m}$$

where $p = h/\lambda$ is the momentum. This means

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = 3.9 \times 10^{-10}\ \text{m}$$

9. A neutron is confined within a nucleus of diameter 4×10^{-14} m. Assuming that the nuclear potential is a one- dimensional infinite potential well of width $4{\times}10^{-14}$ m, estimate the ground state energy of the neutron.

   a) 130 MeV
   b) 2.1×10^{-14} eV
   c) 3.7×10^{-44} eV
   d) **130 keV**
   e) 7.7×10^{23} eV

Solution:

The energy of a particle in an infinite square well is

$$E_n = \frac{n^2h^2}{8mL^2}$$

so here $n = 1$, m is the neutron mass, and L is the diameter of the potential well.

10. An electron is confined in an infinite, one dimensional, square potential well of width 0.200 nm.

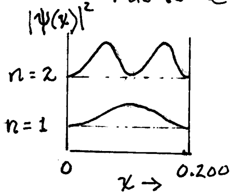
$$V = \begin{cases} \infty \text{ for } x < 0 \text{ and } x > 0.200\text{nm} \\ 0 \quad \text{for } 0 < x < 0.200\text{nm} \end{cases}$$

The electron is in state n = 2. The ratio of the probability that it will be found at the point $x = 0.050$ nm to the probability that it will be found at the point $x = 0.025$ nm is:

   a) 0.707
   b) 1.414
   c) **2.000**
   d) 0.500
   e) 2.828

Solution:

This is a "particle in a box".



$$\psi(x) = C \sin\left(\frac{n\pi x}{L}\right)$$

For n=2, L=0.200,

$$\psi(x) = C \sin\left(\frac{2\pi x}{0.200}\right) = C \sin(10.0\pi x)$$

$$|\psi(0.025)|^2 = c^2 \sin^2(10\pi \times 0.025)$$
$$= C^2 \sin^2 0.785\text{rad}$$
$$= c^2 \sin^2 45°$$
$$= 0.50\, c^2$$

Similarly, $|\psi(0.050)|^2 = c^2 \sin^2(10\pi \times 0.050)$
$$= c^2 \sin^2 90°$$
$$= 1.00\, c^2$$

$$\frac{|\psi(0.050)|^2}{|\psi(0.025)|^2} = \frac{1.00}{0.50} = 2.00$$

11. Which of the following (n, ℓ, m_ℓ) combinations is impossible for an electron in an atom?

a) 3, 1, 1
b) **3, 1, −2**
c) 3, 2, −2
d) 6, 2, 0
e) 1, 0, 0

Solution:
The rules for atomic quantum numbers say that n can be any positive integer, $0 \leq \ell < n$, and $m_\ell = 0, \pm 1, \pm 2, \ldots \pm \ell$. This means that $m_\ell = -2$ isn't possible if $\ell = 1$.

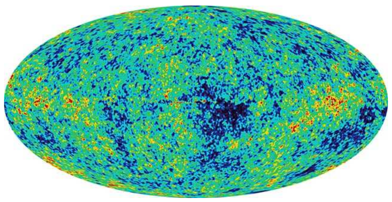
12. Which of the following statements about relativity is *false*?

a) Kinetic energy at relativistic speeds is greater than the classical value
b) $E = mc^2$ describes the rest energy of a massive particle
c) Blueshifting of light occurs when the emitter is moving towards the receiver
d) **Events occuring simultaneously in one frame will always appear to be simultaneous in any frame**
e) The quantity γ can never be less than 1.

Solution:
Since the time measured between two events depends on the reference frame, two events are not necessarily simultaneous from the point of view of different frames of reference.

13. Early in the history of the universe, free electrons joined with free protons, releasing a huge amount of radiation. This radiation, called the cosmic microwave background (CMB), follows the blackbody law and peaks today at a wavelength of 1.06 mm. What is the temperature of the radiation?

a) 0 K
b) 2.73×10^{-3} K
c) 0.366 K
d) **2.73 K**
e) 366 K



Solution:

$$\lambda_M T = 2.90 \times 10^{-3} \ \text{m} \cdot \text{K}$$

$$T = \frac{2.90 \times 10^{-3}}{1.06 \times 10^{-3}} \approx 2.74 \ \text{K}$$

14. A particle moves in one dimension at a constant potential energy V, so that in a stationary state, its wavefunction satisfies the one dimensional Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V\psi(x) = E\psi(x)$$

Suppose its wavefunction is given by

$$\psi(x) = A(e^{i\alpha x} + e^{-i\alpha x})$$

What is the energy of the particle?

a) $\frac{\hbar^2\alpha^2}{2m}$.
b) Undefined, because the particle is not in a stationary state.
c) $\boldsymbol{\frac{\hbar^2\alpha^2}{2m} + V}$.
d) $-\frac{\hbar^2\alpha^2}{2m}$.
e) $-\frac{\hbar^2\alpha^2}{2m} + V$.

Solution:
Using the form of $\psi(x)$ given,

$$d\psi(x)/dx = (i\alpha)A(e^{i\alpha x} - e^{-i\alpha x}).$$

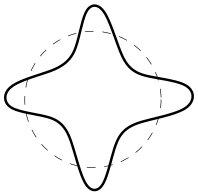
Then

$$d^2\psi(x)/dx^2 = (i\alpha)^2 A(e^{i\alpha x} + e^{-i\alpha x}) = -\alpha^2\psi(x).$$

The left side of Schrödinger's equation is therefore a multiple a $\psi(x)$, the multiple being the expression given by answer c). $\psi(x)$ is a solution of Schrödinger's equation with E given by answer c).

15. In the Bohr model of the atom, a stationary state is represented by a standing wave that circulates around the orbit of the electron. Consider an electron in the stationary state represented by the figure. What is the energy of the electron in this stationary state?

a) $-13.6/8^2$ eV.
b) $13.6/4^2$ eV.
c) $\mathbf{-13.6/4^2}$ **eV.**
d) $13.6/8^2$ eV.
e) -13.6 eV.



Solution:

There are 4 maxima in this standing wave, so four wavelengths fit around the circular orbit and the stationary state corresponds to $n=4$. For the Bohr model,

$$E_n = -\frac{13.6\,\text{eV}}{n^2} = -\frac{13.6\,\text{eV}}{4^2}$$

(cf. Fig. 39.2 and accompanying discussion on p. 1491 of text).