Physics 228 - First Exam Solutions

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1. A ray of light goes from air into water. Which of the following is true?

   a) Its wavelength stays the same
   b) Its frequency decreases
   c) Its speed stays the same
   d) Its frequency increases
   e) **Its speed decreases**

   Solution:

   $$n = \frac{c}{v}$$

   $n_w > n_{air}$

   $\Rightarrow v$ decreases

2. A concave mirror has focal length $f > 0$. A real object is placed a distance $2f$ from the mirror. Then the image is

   a) Real; inverted; smaller
   b) Virtual; upright; bigger
   c) **Real; inverted; same size as object**
   d) Real; inverted; bigger
   e) Virtual; upright; smaller

   Solution:
   The location of the image is determined from

   $$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{2f} + \frac{1}{d_i}.$$

   Thus $d_i = f$. The fact that $d_i > 0$ tells us the image is real. $m = -d_i/d_o = -f/f = -1$. The image is inverted and the same size as the object.

3. A fish is swimming 30 cm below the water surface, and an insect is flying 12 cm above the surface. If the index of refraction of water is 4/3, how far from the water surface will the fish see the insect's image? (Assume the line of sight makes a small angle to the normal).

   a) 9 cm
   b) **16 cm**
   c) 31.5 cm
   d) 56 cm
   e) 12 cm

   Solution:
   Use a coordinate system whose x-axis lies along the surface of the water, and whose y-axis is vertical upwards. Let the insect be at $(0, h)$. Consider the ray from the insect that meets the water surface at $(d, 0)$, with $d \ll h$. The angle this ray makes with the vertical is $\theta_i = d/h$. For this small angle, Snell's law states $\theta_i = n\theta_r$, where n is the index of refraction of the water, and θ_r is the angle the refracted ray in the water makes with the vertical. The fish thinks the insect is at height H, where $\theta_r = d/H$. Thus

   $$H = \frac{d}{\theta_r} = \frac{nd}{\theta_i} = nh = (4/3)(12 \text{ cm}) = 16 \text{ cm}.$$

4. A beam of red light of wave length 7000 Å shines on a diffraction grating which has 5000 lines per cm. The central maximum occurs on a wide, distant wall at $\theta = 0$. How many additional maxima can be observed on the wall for θ positive?

a) none
b) one
c) **two**
d) three
e) four

Solution:

$$d \sin\theta = m\lambda$$

$$\sin\theta_{max} = 1 = \frac{m\lambda}{d} = \frac{(m)(7\times10^{-5})\,\text{cm}}{\frac{1}{5\times10^{3}}\,\text{cm.}}$$

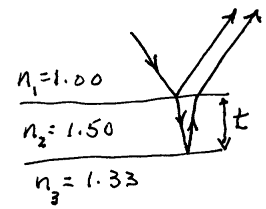
$$\Rightarrow \quad m = \frac{1.0}{35\times10^{-2}} = 2.85$$

So $m = 3$ is not possible.

5. The *thinnest* film of oil (n=1.50) floating on water (n=1.33) that would give maximum reflection of red light (λ_0=680 nm) is

a) 256 nm
b) **113 nm**
c) 170 nm
d) 128 nm
e) 227 nm

Solution:



Reflected wave has half-cycle phase shift at top interface, and no phase shift at bottom interface. For constructive interference,

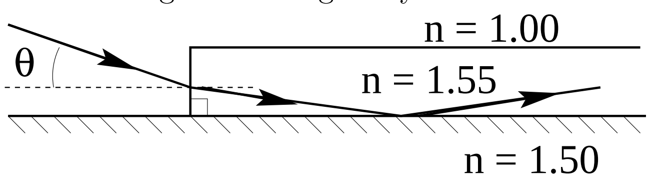
$$2t = (m + \tfrac{1}{2})\lambda$$

But $m = 0$ $\quad \lambda = \lambda_0/1.50$

$$2t = \frac{\lambda_0}{3.00}$$

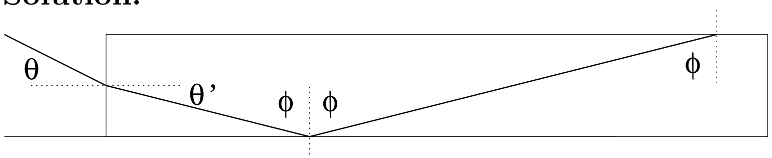
$$t = \frac{\lambda_0}{6.00} = \frac{680}{6.00} = \underline{113\text{ nm}}$$

6. A thin film of index $n = 1.55$ deposited on a glass slide ($n = 1.50$), as shown, is to be used as an optical waveguide. To operate properly, total internal reflection must occur at both the film-air and the film-glass interfaces. What is the maximum entrance angle θ that light ray can have for this to work?



a) 35° b) 47° c) 14.5° d) 75° e) **23°**

Solution:



The figure above shows a little more detail than the figure attached to the problem. Using trigonometry, we know that ϕ is related to the angle of refraction as the light enters the film $\theta' = 90° - \phi$, and that if total internal reflection occurs at both surfaces, ϕ must be greater than or equal to the critical angles at both surfaces. The film-air critical angle is

$$\phi_{f-a} = \sin^{-1}\left(\frac{1}{n_{film}}\right) = 40.2°$$

and the film-glass critical angle is

$$\phi_{f-g} = \sin^{-1}\left(\frac{n_{glass}}{n_{film}}\right) = 75.4°.$$

The corresponding refraction angles are then 49.8° and 14.6° respectively. Since the critical angle for the film-glass interface is larger (and thus the refraction angle is smaller), we'll reach internal reflection at that interface first. Using Snell's law,

$$\sin\theta = n_{film}\sin\left(90° - \phi_{f-g}\right) \rightarrow \theta = 23°$$

7. A radio station operating at frequency 99.5 MHz broadcasts from two towers. What is the farthest distance the towers can be apart to make sure there are no "dead zones", places where the radio signal is completely canceled out by destructive interference?

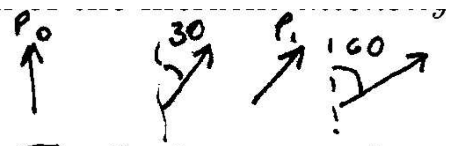
a) 0 m b) **1.5 m** c) 3.0 m d) 1500 km
e) 0.17 m

Solution:
Destructive interference occurs when the distances of the observer from the two towers differ by $(m + 1/2)\lambda$, where $m = 0, 1, 2, \cdots$. There will be no places where this happens as long as the distance between the towers does not exceed $\lambda/2$. The wavelength of the waves is $c/f = (3.00 \times 10^8 \text{ m/s})/(9.95 \times 10^7 \text{ /s}) = 3.02$ m. Therefore distructive interference will not occur as long as the towers are no more than 1.5 m apart.

8. An polarized beam of light is incident on a group of two polarizing sheets that are lined up so that the polarizing axis of the first sheet is rotated by 30° with respect to the polarization of the incident light, and the polarizing axis of the second sheet is rotated by 30° with respect to the polarizing axis of the first sheet. What fraction of the incident *intensity* is transmitted?

a) 1/4
b) 2/3
c) 3/4
d) **9/16**
e) 27/64

Solution:



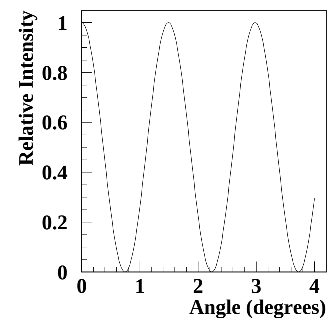
$I_0 \quad I_1 = I_0 \cos^2\varphi \quad I_2 = I_1 \cos^2(60-30)$

$\varphi = 30°$

$$I_2 = I_0 \cos^2 30 \cos^2 30$$

$$= \frac{9}{16} I_0$$

9. Light of wavelength 520 nm passes through a double slit, yielding the interference pattern of intensity I versus deflection angle θ shown. What is the separation, d, between the slits?



a) $\mathbf{2.0 \times 10^{-2}}$ **mm**
b) 1.1×10^{-2} mm
c) 6.7×10^{-3} mm
d) 4.5×10^{-3} mm
e) 5.0×10^{-2} mm

Solution:

Double slit, $\lambda = 520$ nm

$m = 0 \quad \theta = 0°$
$m = 1 \quad \theta = 1.5°$
$m = 2 \quad \theta = 3°$

$d \sin\theta = m\lambda$

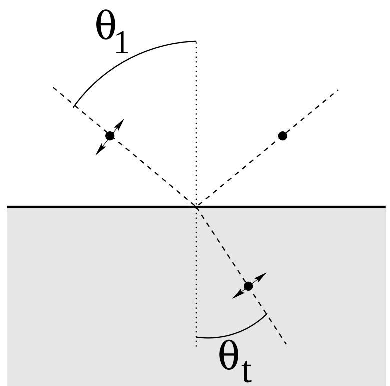
$$d = \frac{m\lambda}{\sin\theta} = \frac{1 \times 520\,\text{nm}}{\sin 1.5°} = 1.98 \times 10^{-4}\,\text{nm}$$

convert to mm:

$$d = 1.98 \times 10^{-4}\,\text{nm} \times \frac{1\,\text{mm}}{10^6\,\text{nm}}$$

$$\underline{d = 2.0 \times 10^{-2}\,\text{mm}}$$

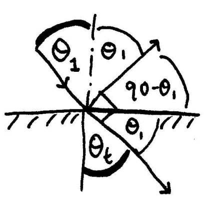
10. An unpolarized beam of light is incident in air onto the surface of a flat glass slab (n = 1.5). The reflected light is completely plane polarized. What is the angle of refraction (θ_t) for the light transmitted into the glass?



a) 28.1°
b) 21.7°
c) 45°
d) **33.7**°
e) 90°

Solution:

Perfect polarization implies refracted & reflected beams are perpendicular



$n \sin\theta_t = \sin\theta_1$

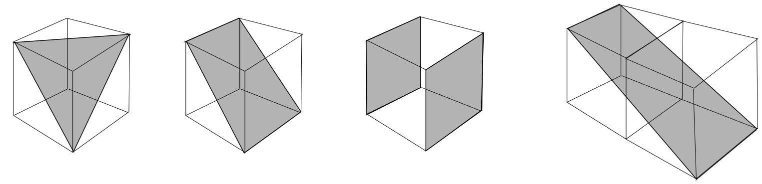
$\theta_1 = 90 - \theta_t$

$n \sin\theta_t = \sin(90-\theta_t) = \cos\theta_t$

$\tan\theta_t = \frac{1}{n} = \frac{1}{1.5} \Rightarrow \theta_t = 33.7°$

11. X-rays of wavelength 0.2 Å are diffracted off a crystal with a cubic unit cell side length $a = 1$ Å. One of the X-ray diffraction spots occurs when the beam makes an angle of 10 degrees to the planes. Which of the planes shown in the figure produces this reflection?

(I) $d = a/\sqrt{3}$ (II) $d = a/\sqrt{2}$ (III) $d = a$ (IV) $d = a/\sqrt{5}$



a) **(I)** b) (II) c) (III) d) (IV)
e) none of the above

Solution:

$2d\sin\theta = \lambda \Rightarrow d = \frac{\lambda}{2\sin\theta} = \frac{0.2\text{Å}}{2\sin 10°} = 0.576$

$(a/d)^2 = (1/0.576)^2 = 3.01 \simeq 3.00$

$\Rightarrow d = a/\sqrt{3}$.

12. The Rutgers radio telescope (on the roof of the physics building) is a dish 2.3 meters in diameter. If the light falling on it has a wavelength of 21 cm, what is the smallest angle the dish can see on the sky? (Hint: treat the dish like a circular aperture.)

a) 0.11°
b) 5.2°
c) **6.4°**
d) 9.1°
e) cannot solve the problem with the information given

Solution:

Radio telescope, $D = 2.3$ m, $\lambda = 21$ cm
Rayleigh's criterion:

$$\sin\theta_1 = 1.22 \frac{\lambda}{D}$$

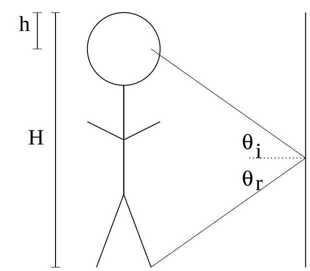
$$\sin\theta_1 = 1.22 \times \frac{21\text{ cm}}{230\text{ cm}} = 0.11$$

$$\underline{\theta_1 = 6.4^\circ}$$

13. A person 1.62 m tall wants to be able to see her full image in a flat mirror. What is the maximum height above the floor the bottom of the mirror be, assuming her eyes are 15.0 cm below the top of her head?

a) 0.150 m b) **0.735 m** c) 0.810 m
d) 1.62 m e) 0 m

Solution:

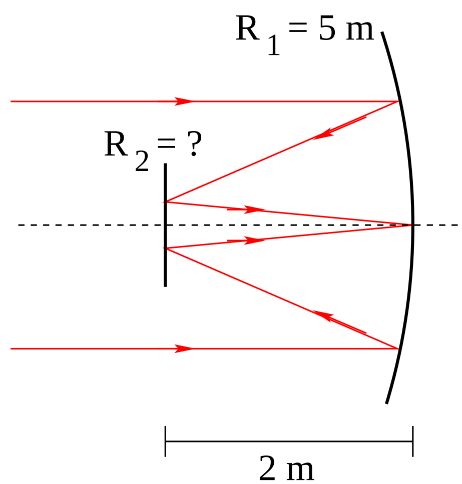


Since the angle of incidence equals the angle of reflection, the person's feet will be visible when the bottom of the mirror is *half* the distance between her eyes and the floor. (See the extraordinarily well-rendered figure above.) If her height is H meters and her eyes are h meters from the top of her head, this means that

$$d = \frac{H - h}{2} = 0.735 \text{ m}.$$

14. Parallel light from a distant object ($s \approx \infty$) strikes a large concave mirror (radius $R_1 = 5.00$ m), reflects off a smaller mirror of unknown radius, and focuses on the vertex of the larger mirror. The centers of the mirrors are 2 meters apart. What is the radius of curvature of the smaller mirror, and is it concave or convex?

a) $R_2 = +4.00$ m, mirror is concave
b) $R_2 = -2.00$ m, mirror is convex
c) $R_2 = -4.00$ m, mirror is convex
d) **$\mathbf{R_2 = -1.33}$ m, mirror is convex**
e) $R_2 = -0.66$ m, mirror is convex



Solution:
Keeping with the notation of the figure, the object for mirror 1 will be at infinity: $s_1 = \infty$. Recalling that the focal length is $f_1 = R_1/2$, we see that the image distance is at the focal length: $s'_1 = R_1/2 = 2.5$ meters. This is *behind* the second mirror, so instead of a real image serving as the object for the second mirror, it will be a *virtual* object. This means that $s_2 = 2 - s'_1 = -0.5$ m, and the image distance will be 2 meters, since it falls at the center of the first mirror. Using the thin lens equation again, we see that

$$\frac{2}{R_2} = \frac{1}{s_2} + \frac{1}{s'_2} = -\frac{1}{0.5\text{ m}} + \frac{1}{2\text{ m}}$$

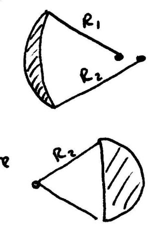
so that $R_2 = -4/3$ meters, or -1.33 m. (Note that we could reject the concave option outright, since the light would never focus. To see that, finish drawing the ray diagram.)

15. A lens made of glass of refractive index 1.6 is flat on one side, and convex, with curvature 40 cm on the other side. What is the focal length of this lens?

a) **67 cm** b) 25 cm c) 33 cm
d) -67 cm e) -25 cm

Solution:

Recall:



$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.6-1)\left(\frac{1}{\infty} - \frac{1}{-40}\right) = \frac{0.6}{40}$$

$$\Rightarrow \underline{\underline{f = 40/0.6 = 67\text{cm}}}$$

Here $R_1 = \infty$, $\underline{R_2 = -40\text{cm}}$ (opposite side to outgoing light)