Physics 228 - Final Exam Solutions
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1. Which of the following quantities remains *unchanged* when light passes from a vacuum into a slab of glass with a 45° angle of incidence?

   a) **its frequency**
   b) its wavelength
   c) its speed
   d) its direction of travel
   e) none of these

   Solution:
   The frequency remains unchanged. The speed changes because the index of refraction of glass is not 1.0, like the vacuum. Since the speed changes and the frequency remains fixed, the wavelength changes. The direction of travel changes according to Snell's law.

2. A converging and a diverging lens, each with a focal length of 30 cm, are arranged so that they are separated by 60 cm. If a candle is placed 90 cm to the left of the converging lens, where is the image produced by the diverging lens?
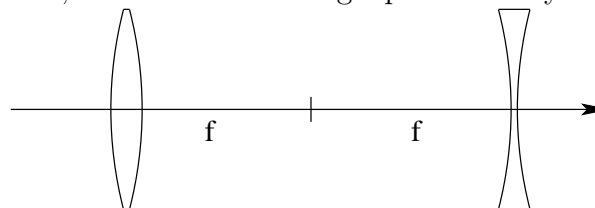
   a) 30 cm to the right of the diverging lens
   b) **10 cm to the left of the diverging lens**
   c) 18 cm to the left of the diverging lens
   d) 30 cm to the left of the diverging lens
   e) 90 cm to the right of the diverging lens

   Solution:
   For the converging and diverging lenses,

   $$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{30 \text{ cm}}, \qquad \frac{1}{\bar{d}_o} + \frac{1}{\bar{d}_i} = -\frac{1}{30 \text{ cm}}.$$

   $d_o = 90$ cm, so $d_i = 45$ cm. This image is 15 cm to the left of the diverging lens, so $\bar{d}_o = 15$ cm. Then $\bar{d}_i = $ -10 cm. The final image is 10 cm to the left of the diverging lens.

3. In a Young's double-slit experiment, light of wavelength 500 nm illuminates two slits which are separated by 1 mm. The separation between adjacent bright fringes on a screen 5 m from the slits is:

   a) 0.10 cm   b) 0.50 cm   c) 1.0 cm
   d) 0.05 cm   e) **0.25 cm**

   Solution:
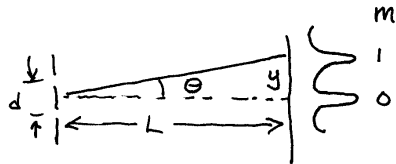
$$d\sin\theta = m\lambda$$
$$d = 1\,mm = 1.0\times10^{-3}\,m$$
$$L = 5.0\,m$$
$$\lambda = 500\,nm = 5\times10^{-7}\,m$$
$$\sin\theta \sim \tan\theta = \frac{y}{5.0\,m} = \frac{m\lambda}{d}$$
$$y = \frac{5.0\,m \times 1 \times 5.0\times10^{-7}\,m}{1.0\times10^{-3}\,m} = 2.5\times10^{-3}\,m$$
$$\underline{y = 0.25\,cm}$$

4. A glass (n = 1.6) lens is coated with a thin film (n = 1.3) to minimize reflection of certain incident light. If $\lambda_{air} = 500$ nm is the wavelength of the light in air, the least film thickness is:

   a) 78 nm   b) **96 nm**   c) 162 nm
   d) 200 nm   e) 250 nm

   Solution:
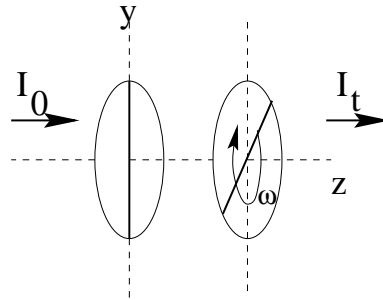   We want to find the solution for the first *destructive* interference fringe. Since $n_{air} < n_{film} < n_{glass}$, there will be a phase shift at each reflection, so the relative phase shift between the light reflected from the film and the light reflected from the glass will be zero. This means we can use

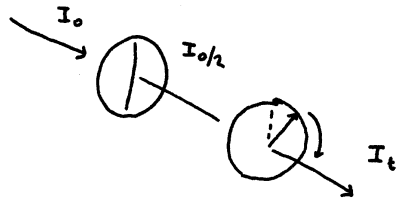 $$2t = \frac{1}{2}\lambda_{film} = \frac{\lambda_{air}}{2\,n_{film}}$$

   which gives us $t = \lambda_{air}/(4\,n_{film}) = 96$ nm.

5. Consider two polarizers as shown. Light traveling along the z-axis passes through the pair. One of the polarizer is rotated with an angular frequency ω. The intensity of light transmitted through the pair, I_t, is periodic with frequency:

a) ω
b) $\omega/2$
c) 2ω
d) 3ω
e) $\omega/3$

Solution:

$I_t : I_o \times \frac{1}{2} \times \cos^2\theta$

$= \frac{I_o}{2} \cos^2\omega t$

$= \frac{I_o}{4} \left(\cos 2\omega t + 1\right)$

∴ Light is transmitted with intensity that is periodic with frequency 2ω.

6. A spacecraft in its own rest frame is shaped like an ellipse, with one axis twice as long as the other axis. How fast and in what direction should you move to make the craft appear circular?

a) **0.866 c in the y-direction**
b) 0.866 c in the x-direction
c) 1.73 c in the x-direction
d) 0.750 c in the x-direction
e) 0.750 c in the y-direction
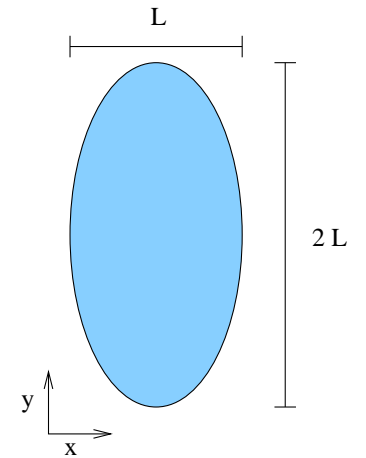
Solution:
An observer traveling with speed v in the y direction sees a contracted spacecraft in that direction. To make it appear circular, the contraction should be 50%. Thus γ must be 2.0, and $\beta = v/c$ must be 0.866. There is no contraction in the x direction.

7. A nucleus of mass M is at rest in the center-of-mass frame of reference. It spontaneously fissions into two pieces of equal mass m, each moving at speed $0.8c$ in opposite directions in the same center-of-mass system. What is the mass m of either fragment, in terms of M?

   a) 0.18 M
   b) 0.83 M
   c) 0.79 M
   d) **0.30 M**
   e) 0.50 M

Solution:

Use conservation of relativistic energy in the center-of-mass system to determine m. We have

$$Mc^2 = 2\frac{mc^2}{\sqrt{1-(0.8)^2}} = 2\frac{mc^2}{0.6} = \frac{10}{3}mc^2.$$

Thus $m = 0.30\ M$.

8. The figure shows a schematic plot of intensity I of blackbody radiation versus wavelength λ at temperature T_0. When the temperature increases above T_0 the wavelength corresponding to the maximum intensity will:

   a) increase
   b) **decrease**
   c) increase initially, and eventually decrease
   d) decrease initially and eventually increase
   e) remains the same
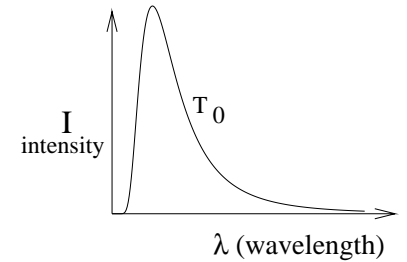
Solution:

Wien displacement law:
$$\lambda_m T = 2.90 \times 10^{-3} m\cdot K$$
As T increases above T_0, the wavelength λ_m corresponding to the maximum intensity will <u>decrease</u>.

9. The stopping potential for electrons ejected by 6.8×10^{14}-Hz electromagnetic radiation incident on a certain sample is 1.8 V. The kinetic energy, K, of the most energetic electrons ejected and the work function, ϕ, of the sample, respectively, are:

    a) $K = 1.8$ eV, $\phi = 2.8$ eV
    b)   **$K = 1.8$ eV, $\phi = 1.0$ eV**
    c) $K = 1.8$ eV, $\phi = 4.6$ eV
    d) $K = 2.8$ eV, $\phi = 1.0$ eV
    e) $K = 1.0$ eV, $\phi = 4.6$ eV

Solution:
The stopping potential and the maximum kinetic energy are related as $K_{max} = eV$, so that $K_{max} = 1.8$ eV. Using the photoelectric equation
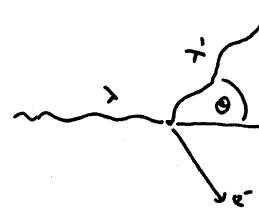
$$eV = hf - \phi \quad \rightarrow \quad \phi = hf - K_{max}$$

gives us $\phi = 1.0$ eV.

10. In Compton scattering from stationary electrons, the largest amount of energy will be imparted to the electron when the photon is scattered through:

    a) $0°$ b) $45°$ c) $90°$ d) **$180°$**   e) $270°$

Solution:

Wavelength shift $= \lambda' - \lambda = \dfrac{h}{mc}\left(1 - \cos\theta\right)$

Largest energy transfer occurs when $\lambda' - \lambda$ is maximized

$\Rightarrow \cos\theta = -1, \quad \theta = 180°$.

11. The binding energy of an electron in the n=2 state in a hydrogen atom is about:

    a) **3.4 eV**   b) 13.6 eV   c) 10.2 eV
    d) 1.0 eV   e) 27.2 eV

Solution:
The total energy of the hydrogen atom in the $n = 2$ state is -3.4 eV (ie, -13.6/4). Thus it takes 3.4 eV to separate the electron in this state from the proton in the H atom, ie, its binding energy is 3.4 eV.

12. A ruby laser delivers a 1-ns pulse of 1.0 MW average power. If the light has a wavelength of 694.3 nm, how many photons are contained in the pulse?

    a) 3.5×10^{24}
    b) 5.5×10^{14}
    c) **3.5×10^{15}**
    d) 7.3×10^{15}
    e) 1.7×10^{24}

Solution:
The energy in the pulse in $(1.0 \times 10^6$ J/s$) \times (1.0 \times 10^{-9}s) =$ 1.0×10^{-3} J. The energy carried by each photon is $hf = hc/\lambda$ $= 2.87 \times 10^{-19}$ J/photon. Thus the number of photons is $(1.0 \times 10^{-3}$ J$)/(2.87 \times 10^{-19}$ J/photon$) = 3.49 \times 10^{15}$ photons.

13. An electron is confined in an infinite, one dimensional, square potential well of width 0.200 nm.
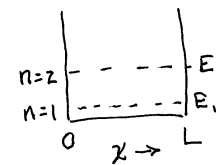
$$V = \begin{cases} \infty & \text{for } x < 0 \text{ and } x > 0.200\text{nm} \\ 0 & \text{for } 0 < x < 0.200\text{nm} \end{cases}$$

The energy of the ground state is:

    a) 0.142 eV
    b) 1.50 eV
    c) **9.40 eV**
    d) 13.6 eV
    e) 54.4 eV

Solution:

Particle (electron) in a box of width $L = 0.200$ nm

$$E_n = \frac{n^2 \hbar^2 \pi^2}{2mL^2}$$
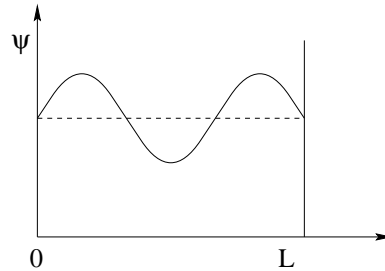
$$E_1 = 1 \times \frac{\left(\frac{6.63 \times 10^{-34} \text{ J.s}}{2\pi}\right)^2 \times \pi^2}{2 \times 9.11 \times 10^{-31} \text{ kg} \times (0.200 \times 10^{-9} \text{ m})^2}$$

$$E_1 = 1.51 \times 10^{-18} \text{ J} \times \frac{\text{eV}}{1.60 \times 10^{-19} \text{ J}}$$

$$\underline{E_1 = 9.44 \text{ eV}}$$

14. An electron is in a one-dimensional potential well of width L with zero potential energy in the interior and infinite potential energy at the walls. A graph of its wave function $\psi(x)$ versus x is shown. The value of quantum number n is:

a) 0
b) 1
c) 2
d) **3**
e) 4

Solution:

Since the wavefunction for a particle in a box is

$$\Psi_n(x) \propto \sin\left(\frac{n\pi}{L}x\right)$$

for $n = 1, 2, 3, \ldots$, we can determine n out by looking at the picture. One and one-half wavelengths are fit into the box, so the wavelength is $\lambda = 2L/3$. Since each half-wavelength corresponds to one energy level, this means the quantum energy number is 3.

15. If we think of an electron in a hydrogen atom as being a standing wave, then in the ground state of radius, a_0, what is the electron's de Broglie wavelength?

a) $2\pi a_0$
b) $h/(m^2v^2)$
c) $a_0/(\hbar c)$
d) a_0
e) $\hbar/(mv)$

Solution:

In the nth state $\quad n\lambda = 2\pi r, \quad r = n^2 a_0$
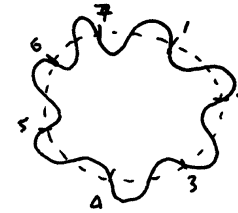
$n = 1 \Rightarrow \quad \underline{\lambda = 2\pi a_0}$

e.g $\quad n = 7$

16. The maximum number of electrons that can be accommodated in an orbital with quantum number $\ell = 3$ is:

a) 2   b) 3   c) 7   d) 9   e) **14**

Solution:
For $\ell = 3$, m_ℓ can take on the seven integer values from -3 to +3. For each of these values of m_ℓ, m_s can be + or -1/2.

17. Sodium has an unpaired electron in its outer $4s$ subshell. If a large collection of sodium atoms are subjected to a 1 T magnetic field, what is the size of the split in the energy level of this electron?

    a) 0 eV
    b) 5.8×10^{-5} eV
    c) **1.2×10^{-4} eV**
    d) 2.3×10^{-4} eV
    e) 2.9×10^{-5} eV

    Solution:
    The energy of a spin magnetic moment in a magnetic field is $E(m_s) = g(e\hbar)/(2m)Bm_s$. The energy splitting is $\Delta E = E(1/2) - E(-1/2)$ To two significant figures, g/2=1.00, so $\Delta E = (ehB)/(2\pi m) = (1.85 \times 10^{-23}$ J$)/(1.60 \times 10^{-19}$ J/eV$)=1.16 \times 10^{-4}$ **eV**.

18. What is the correct ground-state electron configuration of Mg (Z = 12)?

    a) **$1s^2 2s^2 2p^6 3s^2$**
    b) $1s^2 2p^6 2d^4$
    c) $1s^2 2s^2 2p^6 2d^2$
    d) $1s^2 2s^2 3s^2 3p^6$
    e) $1s^2 2s^2 3s^2 4s^2 2p^4$

19. The energy gap for silicon at 300 K is 1.14 eV. What is the wavelength of the lowest energy photon that will promote an electron from the valence band to the conduction band?

    a) 1.14 nm
    b) 263 nm
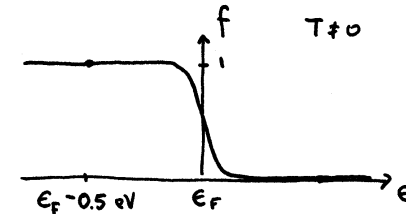   c) **1.09 μm**
    d) 1.24 μm
    e) 342 nm

    Solution:
    The photon energy required to bridge the gap is just $\Delta E = hc/\lambda$. This means that
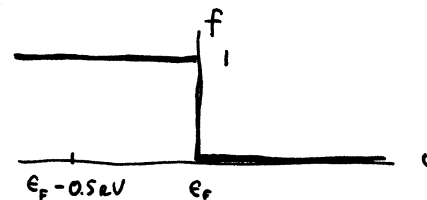
    $$\lambda = \frac{hc}{\Delta E} = 1.09 \ \mu m$$

20. For a metal at T = 0 K, the probability that a state 0.50 eV below the Fermi level is occupied is:

    a) 0
    b) 5.0×10^{-9}
    c) 5.0×10^{-6}
    d) 5.0×10^{-3}
    e) **1**

    Solution:

21. Consider the nuclear reaction: $^{9}_{4}\text{Be} + \alpha \rightarrow n + ^{12}_{6}\text{C}$

(Note: $M(^{9}_{4}\text{Be}) = 9.012183u$; $M(\alpha) = 4.002603u$; $M(n) = 1.008665u$; $M(^{12}_{6}\text{C}) = 12.000000u$)

This reaction:

a) cannot occur because it violates charge conservation.
b) will not proceed unless the reactants have a total kinetic energy of 6.3 MeV.
c) **will release energy.**
d) is used in carbon dating.
e) cannot proceed because it violates conservation of baryon number.

Solution:
Total mass before the reaction is

$$9.012183 + 4.002603 = 13.014786u$$

Total mass after the reaction is

$$1.008665 + 12.000000 = 13.008665u$$

The extra mass before the reaction is converted into kinetic energy of the outgoing particles.

22. The half-life of radium is about 1600 years. If a rock initially contains 1 gram of radium, the amount left after 8000 years will be about:

a) 200 mg
b) 63 mg
c) **31 mg**
d) 16 mg
e) less than 1 mg

Solution:
8000 years is 5 half lives. The amount of radium remaining after 8000 years is $2^{-5} \times 1$ g $= 31$ mg.

23. A large collection of nuclei is undergoing alpha decay. The rate of decay at any instant is:

a) **proportional to the number of undecayed nuclei present at that instant**
b) proportional to the time since the decays started
c) proportional to the time remaining before all have decayed
d) proportional to the half-life of the decay
e) a universal constant

Solution:
Given that the number of particles at a given time is

$$N(t) = N(0)\text{e}^{-\lambda t}$$

the rate is

$$\frac{\text{d}N(t)}{\text{d}t} = -\lambda N(0)\text{e}^{-\lambda t} = -\lambda N(t)$$

the rate of decay is proportional to the number of nuclei present.

24. In a certain nuclear fission process,

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^{1}_{0}n$$

Where:
$m(^{235}_{92}U) = 235.043924$ u
$m(^{141}_{56}Ba) = 140.9139$ u
$m(^{92}_{36}Kr) = 91.8973$ u
$m(^{1}_{0}n) = 1.008665$ u

The energy released in this process is about:

a) 86 MeV
b) **200 MeV**
c) 2.19 GeV
d) 79 MeV
e) 120 GeV

Solution:
The energy released is going to be due to the mass difference between the original nucleus (and the catalyst neutron) and the final products:

$$
\begin{aligned}
Q &= M_{LHS}c^2 - M_{RHS}c^2 \\
&= (m(^{235}_{92}U) + m(n))c^2 - (m(^{141}_{56}Ba) + m(^{92}_{36}Kr) + 2\,m(n))c^2 \\
&= (m(^{235}_{92}U) - m(^{141}_{56}Ba) + m(^{92}_{36}Kr) + 2\,m(n))c^2 \\
&= 200 \text{ MeV}
\end{aligned}
$$

25. Each of the following reactions is forbidden. Determine a conservation law that is violated for each reaction.
I. $\pi^- + p^+ \rightarrow p^+ + \pi^+$
II. $p^+ + p^+ \rightarrow p^+ + \pi^+$
III. $p^+ + \mu^+ \rightarrow p^+ + p^+ + \bar{\nu}_\mu$
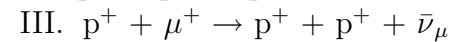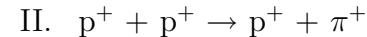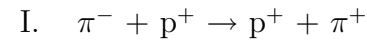
a) **I. charge, II. baryon number, III. baryon number**
b) I. charge, II. lepton number, III. baryon number
c) I. charge, II. baryon number, III. lepton number
d) I. charge, II. lepton number, III. lepton number
e) I. lepton number, II. baryon number, III. baryon number

Solution:

26. Consider the following reaction: $\pi^- + p \rightarrow K^o + (?)$; which in terms of quarks is: $(\bar{u}d) + (uud) \rightarrow (d\bar{s}) + (?)$ Which of the following is a candidate for the unknown product?

a) $\bar{p} = (\bar{u}\bar{u}\bar{d})$
b) $\bar{K}^o = (\bar{d}s)$
c) $\Xi^o = (uss)$
d) $\Sigma^- = (dds)$
e) $\Lambda^o = (uds)$

Solution:

π^- + P $\longrightarrow$ k^o + ?

$\bar{u}d$ uud   d$\bar{s}$ + ?

udd   udd
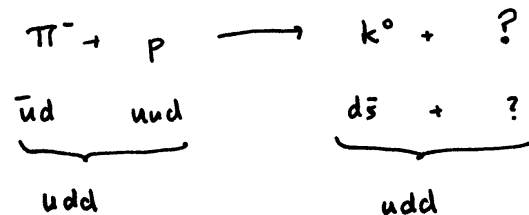
To cancel the $\bar{s}$ in the final state, ? must

contain 1 s quark. It must also contain an up

& a down quark.

? $\equiv$ uds $\equiv$ Λ^o.

27. A telescope has a diffraction grating with 750 slits per centimeter. Two different wavelengths of radiation, $\lambda_1 = 900$ nm and $\lambda_2 = 700$ nm fall on the grating. How far apart are their first maxima, in degrees?

a) $0°$ b) $(1.5 \times 10^{-10})°$ c) $0.0086°$
d) $0.015°$ e) **$0.86°$**

Solution:

Grating with $\frac{750 \text{ slits}}{\text{cm}}$ => distance between slits $d = \frac{1}{750 \text{ slits/cm}}$

$d = 1.33 \times 10^{-3}$ cm
$= 1.33 \times 10^{-5}$ m

$d \sin\theta = m\lambda$
$\sin\theta = \frac{m\lambda}{d}$ $\theta = \arcsin\left(\frac{m\lambda}{d}\right)$

$\theta_1 = \arcsin\left(\frac{1 \times 9.00 \times 10^{-7}\text{m}}{1.33 \times 10^{-5}\text{m}}\right) = 3.88°$

$\theta_2 = \arcsin\left(\frac{1 \times 7.00 \times 10^{-7}\text{m}}{1.33 \times 10^{-5}\text{m}}\right) = 3.02°$

$\Delta\theta = \theta_1 - \theta_2 = 3.88° - 3.02°$
$\Delta\theta = 0.86°$

28. Which of the following statements about the "Standard Model" of particle physics is *false*?

a) **Bosons are the matter constituents. Fermions mediate the forces between particles.**
b) The weak force is short-ranged because of the large mass of the Z and W particles.
c) Strangeness is not conserved by the weak interaction.
d) There are three known generations of leptons and quarks.
e) Hadrons are bound states of quarks, or antiquarks.

Solution:
Bosons mediate the force between particles.

29. Protons are accelerated in a cyclotron with internal field of 0.2 T. If beam exits at a radius of 2 m from the center of the cyclotron, what is the energy of the protons?

    a) **7.67 MeV**   b) 14 GeV   c) 38.3 MeV
    d) 1.92 MeV   e) 15.3 MeV

Solution:

The cyclotron angular frequency is

$$\omega = \frac{eB}{m} = \frac{(1.60 \times 10^{-19} \text{ C})(0.200 \text{ T})}{1.67 \times 10^{-27} \text{ kg}} = 1.92 \times 10^7 \text{ rad/sec.}$$

The speed of the exiting protons is $v = \omega R = (1.92 \times 10^7$ rad/sec) (2.00 m)$= 3.84 \times 10^7$ m/sec. The kinetic energy of the protons is

$$\begin{aligned} K &= \frac{1}{2}mv^2 = \frac{1}{2}(1.67 \times 10^{-27} \text{ kg})(3.84 \times 10^7 \text{ m/sec})^2 \\ &= \frac{1.23 \times 10^{-12} \text{ J}}{1.60 \times 10^{-13} \text{ J/MeV}} = 7.67 \text{ MeV.} \end{aligned}$$

30. An electron in an atom has quantum numbers $n = 2$, $\ell = 1$, $m_\ell = -1$, and $m_s = +1/2$. What is the magnitude of the *orbital* angular momentum of this electron?

    a) 0   b) $-\hbar$ c) $\sqrt{3}\hbar/2$ d) $\hbar/2$ e) $\sqrt{2}\hbar$

Solution:

Electron in atom has quantum numbers
$n = 2$, $\ell = 1$, $m_\ell = -1$, $m_s = +\frac{1}{2}$

Orbital angular momentum $L = \sqrt{\ell(\ell+1)}\,\hbar$

Here, $L = \sqrt{1(1+1)}\,\hbar$

$\underline{L = \sqrt{2}\,\hbar}$