**Physics 228 - Spring 2004 - Final Exam Solutions**

1. Snell's law gives

$$\sin\theta_1 = n\sin\theta_2$$
$$\theta_2 = \sin^{-1}\left(\frac{\sin\theta_1}{n}\right)$$
$$\theta_1 = 35^o \Rightarrow \theta_2 = 18.6^o$$

2. Light reflecting from the first surface of the film undergoes a 180° phase change, but light reflecting from the second surface does not. Thus to have constructive interference we need

$$2nt = (m+\frac{1}{2})\lambda,\ m = 0,1,2,...$$
$$t_{\min} = \frac{\lambda}{4n} = 85.9 \text{ nm for } \lambda = 550 \text{ nm}$$

3. The energy levels of hydrogen are

| $n$ | E |
|---|---|
| 1 | −13.6 |
| 2 | −3.40 |
| 3 | −1.51 |
| 4 | −0.85 |
| 5 | −0.544 |
| 6 | −0.378 |

From n=4 to n=2, for example, the photon energy is 2.55 eV, and $\lambda$=486.3 nm.

4. The Fermi-Dirac distribution is

$$P(E) = \frac{1}{e^{(E-E_F)/kT}+1}$$
$$= 6.57\times10^{-13} \text{ at } 310 \text{ K}$$

5. The decay constant $\lambda=0.693/T_{1/2}$ is 0.0562 $y^{-1}$. After 18 years therefore the fraction of tritium remaining is

$$\frac{N}{N_o} = e^{-\lambda t} = 0.363$$

6. The energy released in the reaction is

$$m(n) + m(^{16}O) - m(^{17}O) = 4.448 \text{ u} = 4.14 \text{ MeV}$$

7. The Hubble relation is $v=HR$. Thus we have

$$H = v/R = (11\times10^6 \text{ m/s})/(500\times10^6 \text{ly})$$
$$= 0.022 \text{ m/s}\cdot ly$$

8. From the relativistic expression for total energy we have

$$m_{Z'}c^2 = 2\sqrt{p^2c^2 + m_W^2c^4}$$
$$p = \sqrt{\frac{m_{Z'}^2c^2}{4} - m_W^2c^2} = 59.9 \text{ GeV/c}$$

for a 200 GeV/$c^2$ $Z'$ particle.

16. For the object at infinity we have that the lens-image distance $q=f$=40 mm. For an object at 0.54 m, then we have

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$q = \left(\frac{1}{f}-\frac{1}{p}\right)^{-1} = 0.0432 \text{ m}$$

Thus the lens moves 3.2 mm away from the film plane.

17. From the lens equation we find

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$q = \left(\frac{1}{f}-\frac{1}{p}\right)^{-1}$$

If $p<f$ then $q<0$ and the image is virtual. Since the magnification $M=-q/p$ we have $M>0$ and the image is upright. Furthermore

$$M = -\frac{q}{p} = \left(1-\frac{p}{f}\right)^{-1} > 1 \text{ for } p < f$$

and thus the image is enlarged.

18. Destructive interference occurs at those values of $x$ such that the path difference d from A and B is a half-integer multiple of the wavelength:

$$d = x - \sqrt{x^2 + 9\lambda^2} = (m + \frac{1}{2})\lambda$$

The maximum path length difference is clearly 3λ and occurs at $x$=0. The path length difference goes to zero as $x$ goes to infinity. Thus the next to last destructive interference corresponds to m=1. We find

$$x - \sqrt{x^2 + 9\lambda^2} = \frac{3}{2}\lambda$$
$$x^2 + 3\lambda x + \frac{9}{4}\lambda^2 = x^2 + 9\lambda^2$$
$$x = 3\lambda - \frac{3}{4}\lambda = \frac{9}{4}\lambda$$

19. For the observer above the film we have constructive interference when

$$2t = (m + \frac{1}{2})\lambda$$

Therefore only in I can we have constructive interference. For the observer below the film the condition for constructive interference is

$$2t = m\lambda$$

which is satisfied at III.

20. For a circular aperture the resolution condition is

$$\theta_{\min} = 1.22\frac{\lambda}{D}$$
$$D = \frac{1.22\lambda}{\theta_{\min}} = 2.2 \text{ m}$$

21. The width is inversely proportional to the slit width, so the factor is 1/2.

22. At 0.8$c$ the time dilation (length contraction) factor is 1.67 so the ship appears 30 meters long and the trip takes 10 hours.

23. The total energy of the proton is greater than that of the electron since it has more mass. For a given kinetic energy we can find the momentum from (we drop the factors of $c$ here)

$$E = K + m = \sqrt{p^2 + m^2}$$
$$p^2 = K^2 + 2mK$$

Clearly for the same kinetic energy the more massive particle has more momentum.

24. Since $E=pc$ for photons the ratio is 2.

25. The power and efficiency are irrelevant; the momentum of the photon is

$$p = \frac{E}{c} = \frac{hc}{\lambda c} = \frac{1240 \text{ eV}\cdot\text{nm}}{(620 \text{ nm})c} = 2.0 \text{ eV}/c$$

26. The photon energy is 1240/435.8 eV= 2.84 eV.

27. In gold the force is about the same since the inner electrons shield the nucleus.

28. In a metal at absolute zero temperature electrons occupy all available states up to the Fermi energy, and no states above that. Thus the average kinetic energy differs significantly from zero.

29. State A is the ground state. B, C, and D are in the vibrational ground state but have non-zero angular momentum. State E is an excited vibrational state.

30. The rotational energy levels are given by

$$E_J = \frac{\hbar^2}{2I}J(J+1)$$
$$E_1 - E_0 = \frac{\hbar^2}{I}$$
$$I = 1.7\times10^{-46} \text{ kg}\cdot\text{m}^2$$

31. From the data given the decay constant $\lambda = 10^{-11}$ $s^{-1}$. The half life is then

$$T_{1/2} = 0.693/\lambda = 2200 \text{ y}$$

32. The incorrect reaction is

$$^{66}_{29}\text{Cu} \rightarrow ^{66}_{30}\text{Zn} + \gamma$$

because charge is not conserved.

33. Binding energy per nucleon for heavy elements is near 8 MeV. But we can calculate it from the data given:

$$\begin{aligned} E_b &= (41m_p + 41m_e + 52m_n - m(^{93}_{41}Nb))c^2 \\ &= 8.664 \text{ MeV} \end{aligned}$$

34. If only 25% of the $^{14}$C is remaining, we have that

$$\frac{N}{N_o} = 0.25 = e^{-\lambda t}$$

$$\Rightarrow t = \frac{1.386}{\lambda} = \frac{1.386}{0.693} T_{1/2} = 11400 \text{ y}$$

35. The time independent Schrodinger equation gives

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}kx^2\psi = E\psi$$

36. High temperatures are necessary to overcome the Coulomb barrier.

37. The antineutron decays according to

$$\bar{n} \rightarrow \bar{p} + e^{+} + \nu_e$$

38. The sun derives its energy from fusion of hydrogen nuclei (protons).

39. The universe is 13.5 billion years old. So $10^{10}$ years is closest.