Physics 227 SECOND COMMON HOUR EXAM
Thursday, October 31, 2002
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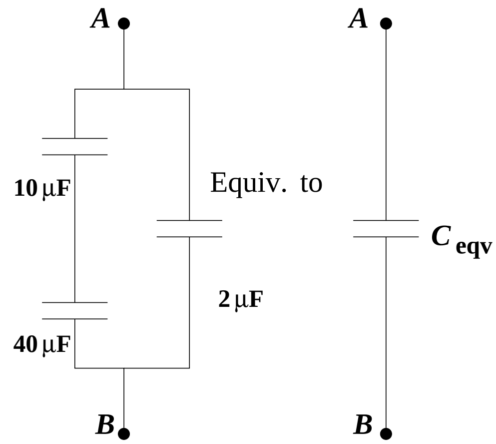
Exam with Solutions

1. Three capacitors are connected by conducting wires as shown. If any other circuitry were connected to the points A and B, the three together would act as an equivalent capacitor of capacitance C_{eqv}. What is C_{eqv}?



a) 10 μF
b) 52 μF
c) 1.6 μF
d) 1.92 μF
e) 2.13 μF

Solution: The 10 μF and 40 μF capacitors are in series, so they can be reduced to a single equivalent capacitor C_2 with

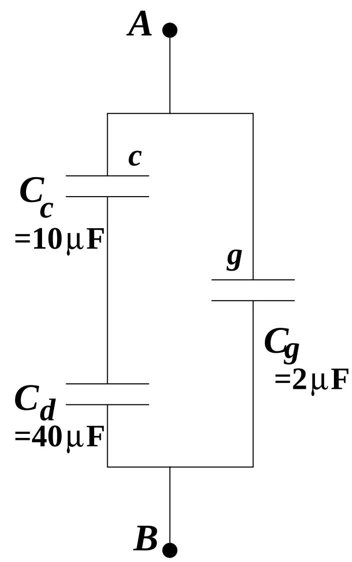
$$\frac{1}{C_2} = \frac{1}{10\ \mu\text{ F}} + \frac{1}{40\ \mu\text{F}} = \frac{5}{40\ \mu\text{F}} = \frac{1}{8\ \mu\text{F}},$$

so $C_2 = 8\ \mu$F. Then C_2 and the 2 μF capacitors are in parallel, giving an equivalent capacitor

$$C_{\text{eqv}} = C_2 + 2\ \mu\text{F} = 10\ \mu\text{F}.$$

Answer is (a).

2. Three capacitors are connected by conducting wires as shown. Suppose A is connected to the positive terminal of a 12 V battery and B is connected to the negative terminal of that battery. What are the charges present at the plates c and g of the two capacitors, as shown



a) $Q_c = +96\,\mu\text{C}$, $Q_g = +24\,\mu\text{C}$
b) $Q_c = +120\,\mu\text{C}$, $Q_g = +24\,\mu\text{C}$
c) $Q_c = +120\,\mu\text{C}$, $Q_g = -24\,\mu\text{C}$
d) $Q_c = -24\,\mu\text{C}$, $Q_g = +24\,\mu\text{C}$
e) $Q_c = 0$, $Q_g = -24\,\mu\text{C}$

Solution: There are $V_0 = 12$ V across C_g, so the charge on g is $Q_g = C_g V_0 = 24\ \mu$C. There is also 12 V across the series pair of C_c together with C_d. As above, these act as an equivalent capacitor of 8 μF, so the charge on c is $Q_c = C_2 V_0 = 8\ \mu\text{F} \cdot 12\ \text{V} = 96\ \mu\text{C}$.
The answer is (a).

3. A parallel plate capacitor is charged by placing 90 V across the plates, and then isolated electrically. A dielectric is then inserted between the plates, and as a result the potential across the plates drops to 30 V. The energy of the system changes, due to the introduction of the dielectric

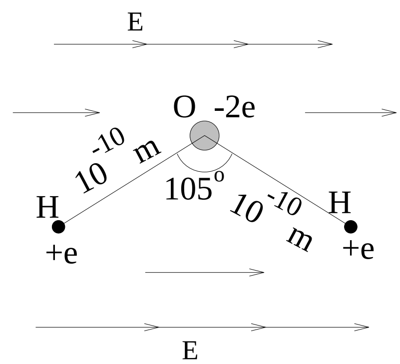
a) by decreasing by a factor of 9
b) not at all — it stays the same
c) by increasing by a factor of 3
d) by decreasing by a factor of 3
e) by increasing by a factor of 9

Solution: As the charge on the plates is unchanged but the voltage difference dropped by a factor of 3, the capacitance went up by a factor of 3, $C = 3C_0$. The energy is

$$U = \frac{Q^2}{2C} = \frac{Q^2}{2 \cdot 3C_0} = 1/3\frac{Q^2}{2C_0} = \frac{1}{3}U_0,$$

so the energy decreased by a factor of 3. The correct answer is (d).

4. A water molecule as shown is in a region of uniform electric field $\mathbf{E} = 1000\hat{\imath}$ V/m. This molecule experiences



a) A counterclockwise torque
b) A clockwise torque
c) A net force to the right
d) A net force to the left
e) None of the above

Solution: The positively charged hydrogens feel a force to the right, and the negatively charged oxygen a force to the left, so there is a counterclockwise torque. A dipole in a uniform field does not experience any net force. The answer is (a).

5. The working element of a toaster is a nichrome wire which acts as a resistor. All other wires have negligible resistance. The toaster heats up the bread by consuming 1000 W of power when plugged into a DC voltage source of 110 V. What is the resistance of the nichrome wire?

a) $8.3 \times 10^{-2}\,\Omega$
b) The information given is not sufficient — we need to be given the current drawn as well.
c) $9.1\ \Omega$
d) $12.1\ \Omega$
e) $1.1 \times 10^{5}\,\Omega$

Solution: The power dissipated by a resistor is $\mathcal{P} = \frac{(\Delta V)^2}{R}$ so

$$R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(110\text{ V})^2}{1000\text{ W}} = 12.1\ \Omega.$$

The correct answer is (d)

6. A cylindrical copper wire (resistivity $= 1.70 \times 10^{-8}\,\Omega \cdot \text{m}$) with a diameter of 1×10^{-3} m and 0.01 m long is used to connect two points on a printed circuit board at room temperature. If the current in the wire is 10 mA, what is the potential drop across the wire?

   a) 0.54 μV
   b) 2.16 μV
   c) 5.4 μV
   d) 2.16 mV
   e) 5.4 mV

   Solution: The resistance of a length ℓ of wire is $R = \rho\ell/A$, where the area $A = \pi r^2 = \pi d^2/4 = \pi(0.001)^2/4 = 7.85 \times 10^{-7}$ m^2, $\ell = 0.01$ m, so

$$R = \frac{1.70 \times 10^{-8} \times 0.01}{7.85 \times 10^{-7}}\Omega = 2.16 \times 10^{-4}\ \Omega,$$

   and the voltage drop is

$$\Delta V = IR = 0.01 \times 2.16 \times 10^{-4} = 2.16 \times 10^{-6}\ \text{V} = 2.16\ \mu\text{V}.$$

   The correcct answer is (b).

7. A potential difference V is applied to a copper wire of diameter d. If d is doubled (keeping ℓ and the applied potential difference V unchanged) the drift velocity of the electrons is multiplied by

   a) 4
   b) 2
   c) 0.5
   d) 0.25
   e) 1

   Solution: If V and the length of the wire are unchanged, the electric field in the wire is unchanged, so the current density is unchanged, and the drift velocity is unchanged. So the correct answer is (e).

   It is true that the area is quadripuled, so the resistance will fall by a factor of 4, the current will go up by a factor of 4, but the formula $I = nqv_dA$ will then tell us that nqv_d is unchanged, and as n and q are unchanged properties of the material, v_d is unchanged.

8. A toaster with a nichrome heating element draws an initial current of 1.5 A at T = 20°C from a constant 120 V source. When the heating element reaches it's final temperature, it draws 1.13 A. What is the final temperature of the heating element? (note α_{nichrome}=0.4×10^{-3} K^{-1}).

   a) 515°C
   b) 35°C
   c) 840°C
   d) 110°C
   e) 1100°C

Solution: From $\Delta V = IR$ and ΔV is unchanged,

$$\frac{R(T)}{R(20^\circ)} = \frac{I(20^\circ)}{I(T)} = \frac{1.50}{1.13} = 1.327.$$

The resistance at temperature T is $R(T) = R(20^\circ)(1 + \alpha(T - 20^\circ) = R(20^\circ)[1 + 0.4 \times 10^{-3} \times (T - 20^\circ)]$, so

$$\frac{R(T)}{R(20^\circ)} = 1.327 = 1 + 0.4 \times 10^{-3} \times (T - 20^\circ)$$

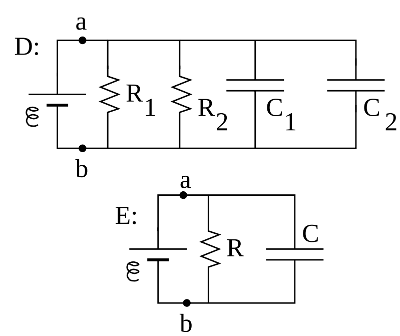
or

$$T - 20^\circ = 0.327/(0.4 \times 10^{-3}) = 819^\circ, \quad \text{so } T = 839^\circ \text{ C}.$$

Correct answer is (c)

9. The circuit D will be equivalent to the circuit E if

   a) $R = R_1 + R_2$, $C = C_1 + C_2$
   b) $R = \dfrac{R_1 R_2}{R_1 + R_2}$, $C = C_1 + C_2$
   c) $R = R_1 + R_2$, $C = \dfrac{C_1 C_2}{C_1 + C_2}$
   d) $R = \dfrac{R_1 R_2}{R_1 + R_2}$, $C = \dfrac{C_1 C_2}{C_1 + C_2}$
   e) $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{C_1} + \dfrac{1}{C_2}$, $C = R_1 + R_2 + C_1 + C_2$

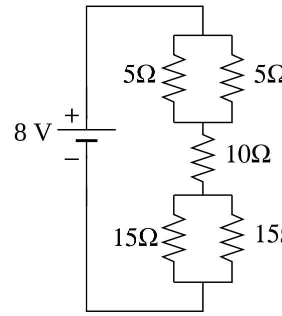


Solution: For resistors in parallel, as is true here,

$$\frac{1}{R_{\text{eqv}}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{or } R_{\text{eqv}} = \frac{R_1 R_2}{R_1 + R_2}.$$

For capacitors in parallel, as here, the capacitances add, $C_{\text{eqv}} = C_1 + C_2$, so the correct answer is (b)

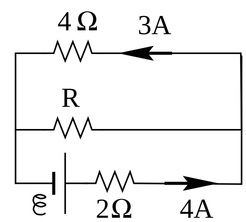
10. In the circuit shown, what is the current through the 10 Ω resistor?



a) 10A
b) 20A
c) 0.8A
d) 0.4A
e) 0.2A

Solution: The two 5 Ω resistors in parallel are equivalent to one 2.5 Ω resistor, the two 15 Ω resistors are equivalent to one 7.5 Ω, which leaves us with a series combination of 2.5 + 10 + 7.5 Ω which gives a total equivalent resistance of 20 Ω, a current $I = \Delta V/R_{\rm eqv} = 0.4$ A, all of which flows through the 10 Ω resistor. So the correct answer is (d).

11. What is the EMF of the battery? Assume zero internal resistance for the battery.



a) 4 V
b) 12 V
c) 8 V
d) 20 V
e) None of the other answers.

Solution: In the bottom of this circuit, the voltage on the right is $\mathcal{E} - 2\times 4$V above the voltage on the left, but from the top of the circuit we see that the voltage on the right is $3 \times 4V$ above the voltage on the left, so

$$\mathcal{E} - 2 \times 4 \text{ V} = 12 \text{ V},$$

so $\mathcal{E} = 20$ V, and (d) is correct

12. The wattage rating of a light bulb indicates the power dissipated by the bulb if it is place across a 110V DC potential difference. If a 50W bulb and 100W bulb are connected in series to a 110V DC source, how much power will be dissipated in the 50W bulb?

a) 50W b) 100W c) 22W d) 11W e) 0

Solution: From $\mathcal{P} = (\Delta V)^2/R$ we see that the 50 W bulb has twice the resistance of the 100 W bulb, $R_{50} = 2R_{100}$, and the two bulbs is series has an equivalent resistance $R_{\text{eqv}} = R_{50} + R_{100} = 1.5R_{50}$. The current in this series configuration is $I_s = \Delta V/R_{\text{eqv}} = (2/3)\Delta V/R_{50}$, or 2/3 the current we would normally have. From $\mathcal{P} = I^2 R$ we see that we use $(2/3)^2$ times the normal power, or $(4/9) \times 50$ W. So the correct answer is 22 W or (c).

13. A Cs$^-$ ion ($q = -e$) with velocity $\vec{v} = v_0\hat{\imath}$ encounters a uniform magnetic field given by $\vec{B} = B_0\hat{\jmath}$. The force exerted on this ion when it first encounters the field is:

    a) $\vec{F} = -(ev_0B_0)\hat{k}$
    b) $\vec{F} = (v_0B_0/e)\hat{k}$
    c) $\vec{F} = -(ev_0B_0)\hat{\jmath}$
    d) $\vec{F} = (ev_0B_0)\hat{\jmath}$
    e) $\vec{F} = -(ev_0B_0)\hat{\imath}$

    Note: $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ are unit vectors in the x, y, and z directions, respectively.

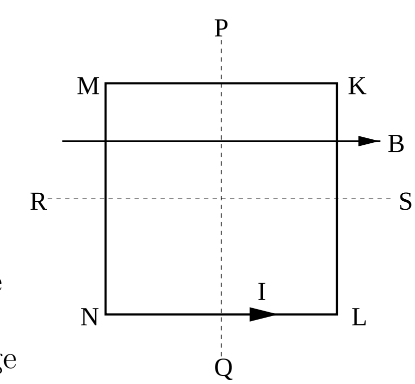
    Solution:

$$\vec{F} = q\vec{v} \times \vec{B} = -ev_0\hat{\imath} \times B_0\hat{\jmath} = -ev_0B_0\ \hat{\imath} \times \hat{\jmath} = -ev_0B_0\hat{k},$$

    so (a) is the right answer.

14. A square loop of wire lies in the plane of the page and carries a current I as shown. There is a uniform magnetic field $\vec{B}$ parallel to the side MK as indicated. The loop will tend to rotate:

    a) about PQ with KL coming out of the page
    b) about PQ with KL going into the page
    c) about RS with MK coming out of the page
    d) about RS with MK going into the page
    e) about an axis perpendicular to the page



Solution: The forces are $I\vec{L} \times \vec{B}$, which is zero on MK, zero on NL, into the page on KL and out of the page on MN. So the torque is about PQ with KL pushed into the page and MN out, or (b) is the right answer.

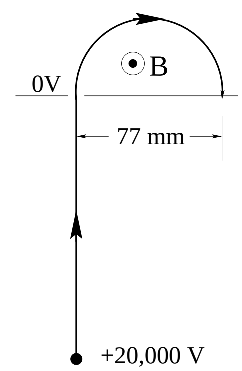
15. An electron and a proton in the same uniform magnetic field trace out circles of the same radius, one clockwise and the other counterclockwise. What is the ratio of electron to proton momentum?

a) 1
b) 1840
c) (5.4×10^{-4})
d) It depends on the radius of the circle.
e) It depends on the magnetic field.

Solution: From $r = mv/qB$ and the fact that the electron and proton have charges of the same magnitude (though opposite signs), we see that the momenta are the same, in magnitude. So (a) is the right answer.

16. An ion is produced at rest from the filament of a vacuum tube which is at an electric potential of +20,000 V, relative to ground. It accelerates in the y direction towards a cathode, which is grounded, and then passes through the cathode into a region with a uniform magnetic field of 0.75 T in the z direction. In this region it makes a semicircular path of diameter 0.077 m. The ion is

a) a proton, mass m_p, charge $+e$
b) a deuteron, mass $\sim 2m_p$, charge $+e$
c) a singly ionized Helium atom, mass $\sim 4m_p$, charge $+e$
d) a singly ionized Beryllium atom, mass $\sim 8m_p$, charge $+e$
e) a doubly ionized Beryllium atom, mass $\sim 8m_p$, charge $+2e$



Solution: After acceleration we have $\frac{1}{2}mv^2 = q\Delta V$, and from the bending in the field $r = mv/qB$, we have $v = qBr/m$, so

$$\frac{1}{2}m\left(\frac{qBr}{m}\right)^2 = q\Delta V = \frac{q^2B^2r^2}{2m},$$

so

$$\frac{q}{m} = \frac{2\Delta V}{B^2r^2} = \frac{2 \times 20000}{(0.75)^2(0.077/2)^2} = 4.80 \times 10^7 \text{C/kg}.$$

For a proton,

$$\frac{q}{m} = \frac{1.602 \times 10^{-19}}{1.67 \times 10^{-27}} = 9.60 \times 10^7 \text{C/kg},$$

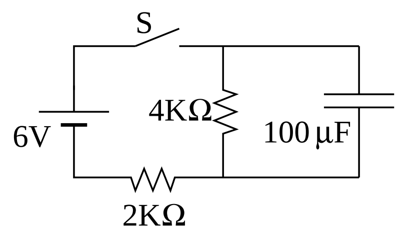
so our particle needs twice the mass to charge ratio as that of a proton. A deuteron satisfies that requirement, but none of the other choices do. So (b) is correct.

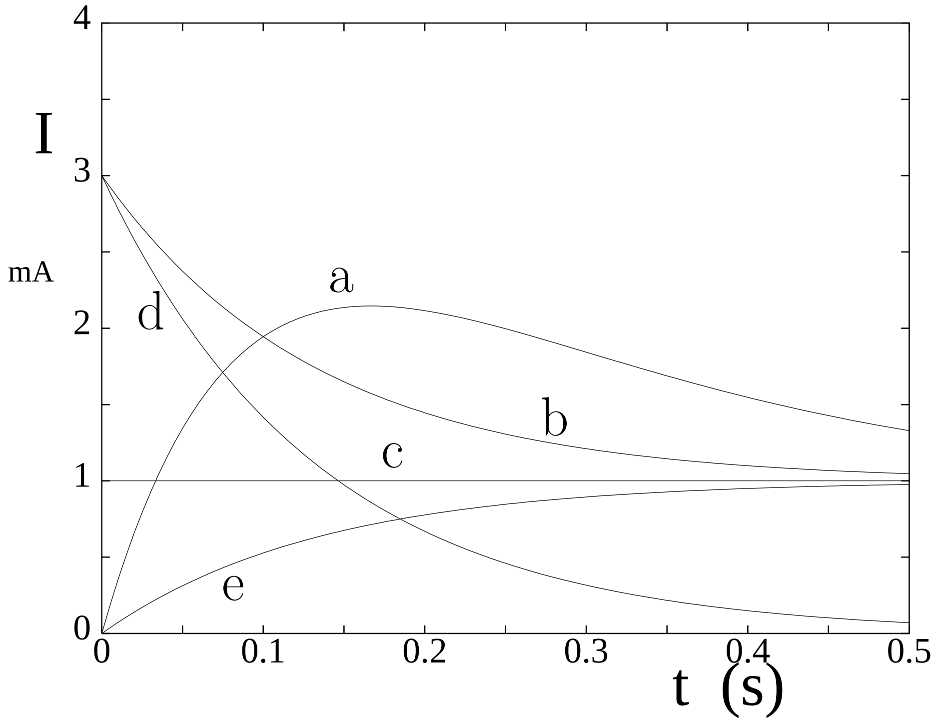
17. In the Hall effect, we have a current flowing in the presence of a uniform magnetic field, and we get a potential difference across the conductor. The reason is

    a) a changing magnetic field produces an induced EMF.
    b) charges are bent from their paths until an electric field can be built up transverse to the wire, to stop them from bending.
    c) the charges in the wire are moving, so the electric fields are changing with time.
    d) the conductor is not a perfect one, so there must be an electric field proportional to the current and the resistivity.
    e) the charges produced by the current repell each other, and produce a build up of charge on the surface of the conductor.

    Solution: (b) is correct, as explained in lecture.

18. In the circuit shown, the switch had been open a long time, and there was no current flowing and no charge on the capacitor. Then, at time $t = 0$, the switch S was closed. Which graph correctly gives the current through the 2 KΩ resistor as a function of time after the switch is closed.





Solution: The moment the switch is closed, there is still no charge on the capacitor, so no voltage drop across it, and therefore the voltage across the 2 KΩ resistor is 6 V, and the current 3 mA. After a very long time no current will be flowing across the capacitor, so the current will see an effective resistance of 4 + 2 KΩ, and the current will be 1 mA. Only curve (b) satisfies those requirements.