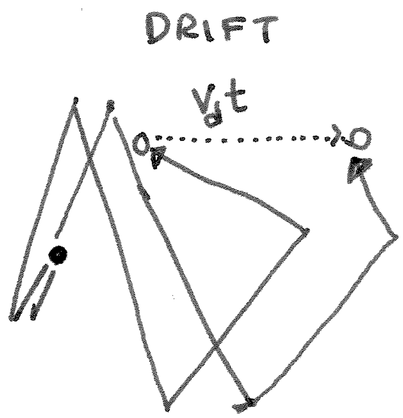
L9 Current, Resistance + E.M.F.

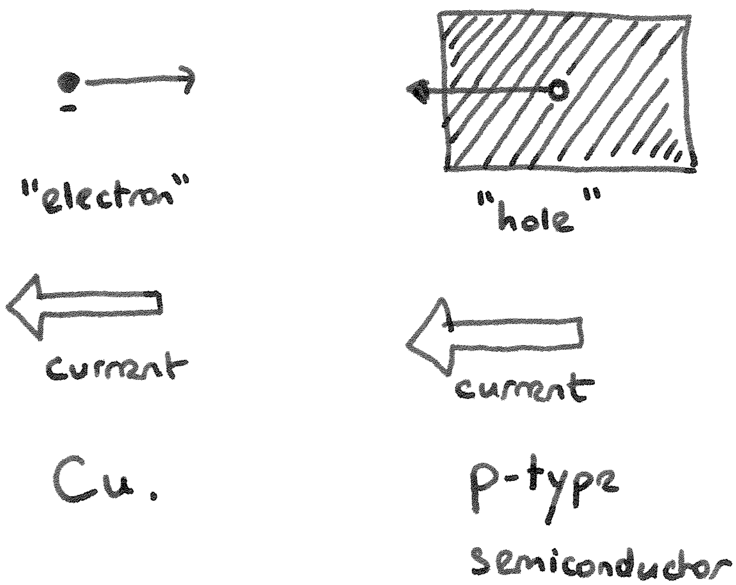
In today's lecture we begin the study of "charge in motion", or electric current. Almost every modern instrument depends on current — from your cell-phone to your electric toothbrush — all depend on current. To drive a current you need an <u>electric</u> field — and we shall shortly see that once you have a current, you produce a <u>magnetic</u> field.

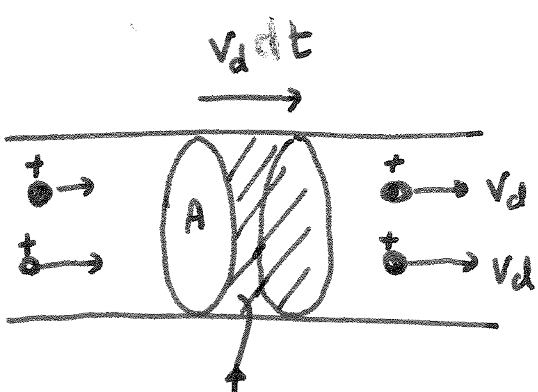
Inside a conductor electrons move at about $\frac{1}{300}$ th the speed of light $v_e \sim 10^6$ m/s.

However they collide with impurities and this causes random motion. In a field the electrons experience a constant force $\vec{F} = q\vec{E}$ which causes them to drift at a velocity $\vec{V}_d$. The drift velocity is tiny about $\frac{1}{10}$ th mm/s !



Depending on the material, the current carriers can be negatively charged electrons, or positively charged "holes"





$$dQ = q\,(n A v_d\, dt)$$

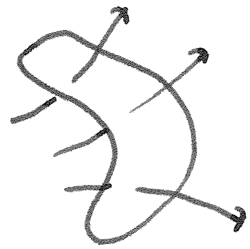
$$I = \frac{dQ}{dt} \quad \text{(Amperes)}$$

$$= n q v_d \times A$$

$$\boxed{J = \frac{I}{A} = n q v_d}$$ CURRENT DENSITY.

$$\boxed{\vec{J} = nq\vec{v}_d}$$

vector current density



$$I = \int \vec{J} \cdot d\vec{A}$$

like electric flux

e.g 18 gauge wire radius $r = 0.51\,mm$, $n_e = 8.5 \times 10^{28}\ e^-/m^3$

$I = 1.67\,A$.

a) What is the current density?

$$A = \pi r^2 = \pi \times (0.51 \times 10^{-3})^2 = 8.17 \times 10^{-7}\,m^2$$

$$J = \frac{1.67\,A}{8.17 \times 10^{-7}\,m^2} = 2.04 \times 10^6\ A/m^2$$

b) What is the drift velocity

$$n e v_d = j \Rightarrow \quad v_d = \frac{j}{ne} = \frac{2.04 \times 10^6}{8.5 \times 10^{28} \times (1.6 \times 10^{-19})}$$

$$= 1.5 \times 10^{-4}\,m/s$$

$$\equiv 0.15\,mm/s.$$

Resistivity

$$\rho = \frac{E}{J}$$ electric field required per unit current density.

Material	ρ (Ωm)
Copper	1.72×10^{-8}
Wood	$10^{8} - 10^{11}$

units $1V/A \equiv 1 \text{ ohm} \equiv 1\,\Omega$

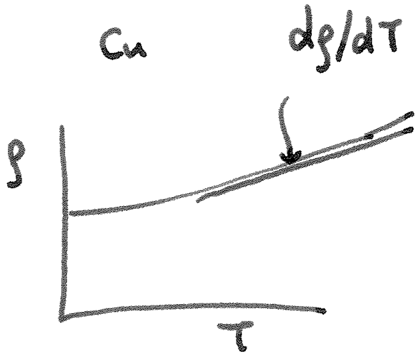
$$\left.\begin{array}{l}[E] = V/m \\ [J] = A/m^2\end{array}\right\} \quad [E/J] = \frac{V\,m}{A} = \Omega m.$$

Typically ρ is temperature dependent

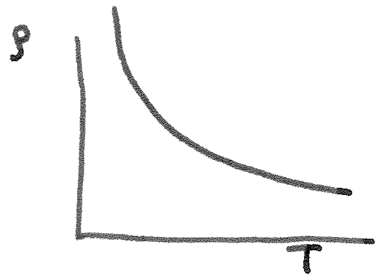
$$\rho = \rho_0 \left[1 + \alpha (T - T_0) \right]$$

e.g copper

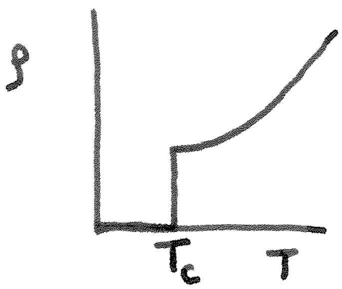
$\alpha = 0.00393.K^{-1}$



metal (Cu)



semiconductor (Si)



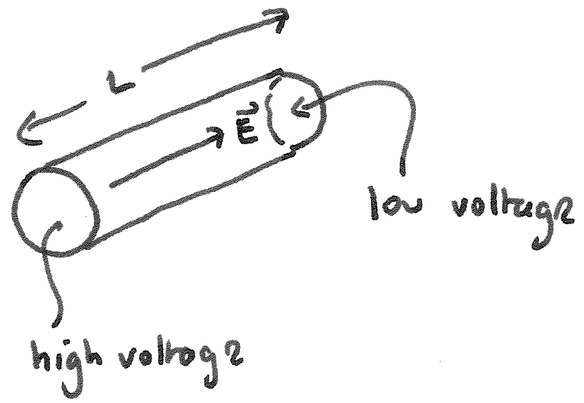
superconductor (Pb).

In conventional metals, resistivity goes up as you raise the temperature, because the atoms in the crystal vibrate more at higher temperatures, scattering the electrons more.

In semiconductors, temperature "shakes loose" more carriers, causing the resistance to <u>drop</u> with temperature.

In a superconductor, the resistance drops to <u>zero</u> below the transition temperature. This is because the electrons self-organize so that they all move together as a single "block" of charge or condensate — but this is another story.

Resistance



$$\vec{E} = \rho \vec{J}$$

$$\frac{V}{L} = \rho\left(\frac{I}{A}\right) \implies \boxed{V = \underbrace{\left(\frac{\rho L}{A}\right)}_{R} I}$$

$$\boxed{R = \frac{V}{I}}$$

Resistance. "Ohms"

(Ω = 1Volt/Ampere).

$$\boxed{R = \frac{\rho L}{A}}$$

relationship between resistance + resistivity.

$$\boxed{V = IR}$$

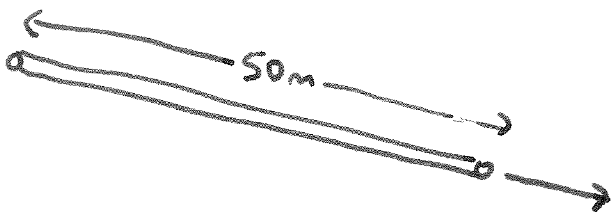
relationship between voltage & current.

"Ohm's law"

e.g. 18 gauge wire, cross section $A = 8.2 \times 10^{-7}\,m^2$

If it carries a current $I = 1.67$ Amperes

what is the potential drop across 50m of wire & what is the resistance of 50m of wire



$1.67A = I$

$$E = \rho J = \overbrace{\left(1.72 \times 10^{-8}\,\Omega m\right)}^{\rho} \times \overbrace{\left(\frac{1.67A}{8.2 \times 10^{-7}\,m^2}\right)}^{J}$$

$$= \underline{0.0350}\ V/m$$

a) $V = Ed = 0.035 \times 50 = \underline{1.75\,V}$

b) $R = V/I = \dfrac{1.75}{1.67} = \underline{1.05\,\Omega}$