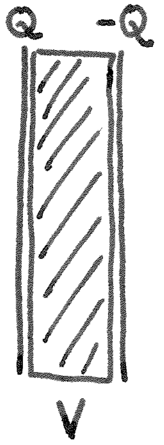
L8

The practical realization of capacitors requires a method of enhancing the capacitance, a method of developing higher internal fields without dielectric breakdown. The key to this is the use of dielectrics. Along the way we will learn that the effect of a field on matter is to produce a polarization of charges in the atoms & molecules of the material. It is this polarization that enhances the capacitance. We call the factor of enhancement, the dielectric constant "k".

$$\boxed{k = \frac{C}{C_0}}$$



$C = C_0$

$V_0 = \frac{Q}{C_0}$

$C = k C_0$

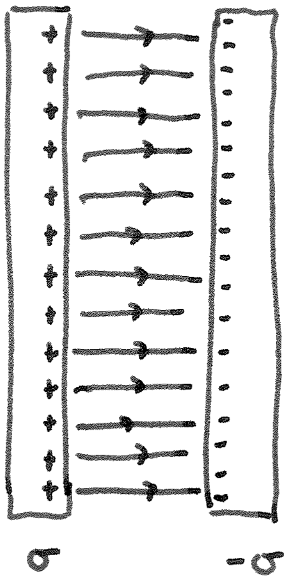
$$\Rightarrow \underline{V = \frac{Q}{kC_0} = \frac{V_0}{k}}$$

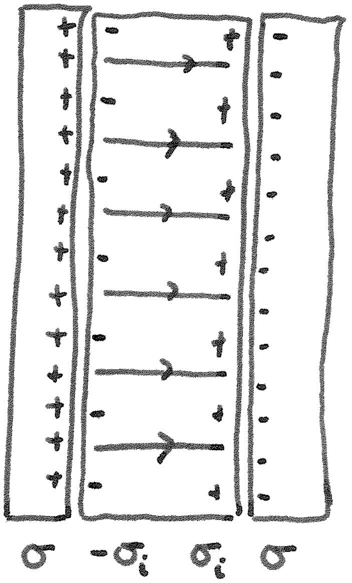
For a given charge, the dielectric reduces the voltage and the internal field by a factor k.

Material	k
Air	1.00059
Glass	5 − 10
Water	80.4

Highly polarizable.

In practice – always a certain leakage current.





$E_0 = V_0/d$

$E = E_0/K$ (when Q is constant)

$$E_0 = \frac{\sigma}{\epsilon_0}$$

$$E = \frac{\sigma - \sigma_i}{\epsilon_0} = \frac{\sigma}{K\epsilon_0}$$

$$\sigma_i = \sigma\left(1 - \frac{1}{K}\right)$$

induced surface charge density.

- Vacuum $k = 1$ $\sigma_i = 0$
- metal $k = \infty$ $\sigma_i = \sigma$

$\epsilon = k\epsilon_0$ "<u>permittivity</u>" of matter.

$$E = \frac{\sigma}{\epsilon_0}$$

$$C = \kappa C_0 = \kappa \epsilon_0 \frac{A}{d} = \epsilon \frac{A}{d}$$

$$u = \left(\frac{C V^2}{2}\right) \frac{1}{Ad} = \frac{\epsilon A}{2d} \frac{(Ed)^2}{Ad}$$

$$\boxed{u = \frac{1}{2} \epsilon E^2} \quad \left(= \frac{1}{2} \kappa \epsilon_0 E^2\right)$$

e.g. Capacitor with area $A = 0.5\,m^2$, $d = 1cm$ charged to 5kV and then disconnected from voltage source. Voltage reduces to 1kV after a dielectric is inserted.

a) What was original C_0 ?

b) What is the magnitude of the charge on each plate, Q ?

c) What is the new capacitance, C ?

d) What is the dielectric constant ? What is ϵ ?

e) What is the induced charge Q_i on the face of the dielectric ?

f) What are the initial & final electric fields ?

a) $$C_0 = \frac{\epsilon A}{d} = 8.85 \times 10^{-12}\,F/m \times \frac{0.5}{0.01} = 443 \times 10^{-12}\,F$$
$$= \underline{443\,pF}$$

b) $$Q_0 = CV_0 = 443 \times 10^{-12} \times 5 \times 10^3 = 2.21 \times 10^{-6}\,C$$
$$= 2.21\,\mu C$$

c) $$C = KC_0 \qquad V = V_0/K \qquad K = V_0/V = 5kV/1kV = 5$$
$$C = 5 \times 443 \times 10^{-12} = 2.21 \times 10^{-9}\,F$$

d) $k = 5$

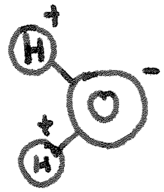
e) $Q_i = Q\left(1 - \frac{1}{5}\right) = \frac{4}{5} Q = 0.8 \times 2.21 \mu C = 1.77 \mu C$

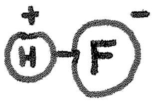
f) $E_0 = \frac{5 \times 10^3}{0.01} = 5 \times 10^5 \ V/m$

$E = E_0/k = 10^5 \ V/m.$

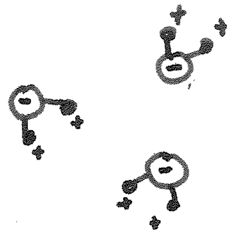
Molecular Model of Induced Charge.

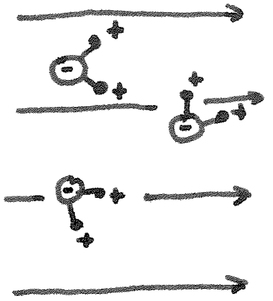
Polar molecule



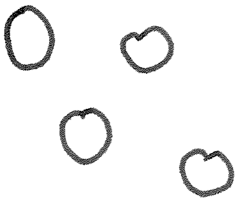


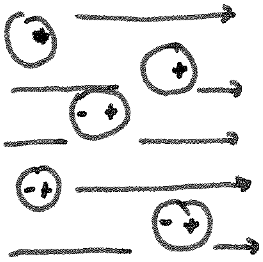
In a field they align





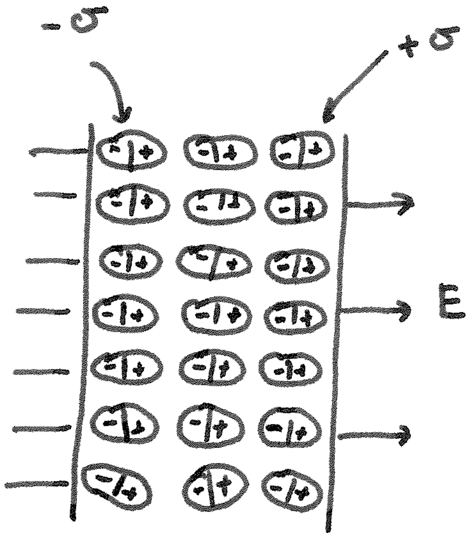
Non polar
e.g He.

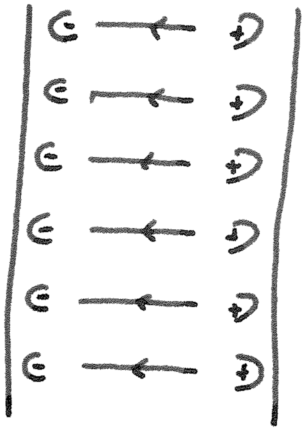




Bound charges redistribute

"polarizatión"





induced field
opposite to
applied field

$E_0\left(1-\frac{1}{k}\right)$

$$E_0 - E_0\left(1-\frac{1}{k}\right) = \underline{E_0/k}$$