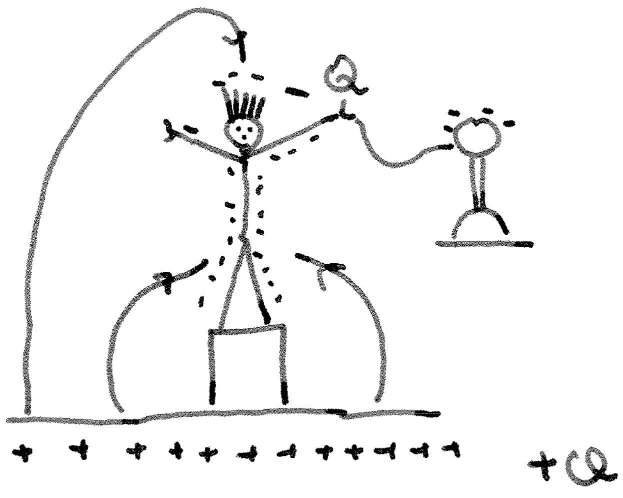
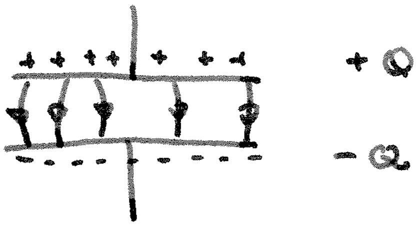
L7 CAPACITORS + STORED ENERGY

Capacitors are the devices we use to store energy in the electric field. Capacitors are used widely – for example – each "bit" in a random access memory is an individual capacitor – a 1GB Ram contains 8 billion capacitors! We also use capacitors to store energy, energy that can be quickly released, as in a camera flash. How do we calculate this energy – what exactly IS a capacitor? – These are the topics of our lectures this week.

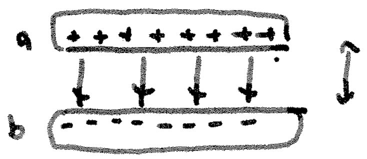




Any two bodies of charge separated by an insulator constitute a capacitor

$$C = \frac{Q}{V}$$

unit: Farad (F)



$$V_{ab} = V_a - V_b = Ed \equiv V$$

$$EA = \frac{\sigma A}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

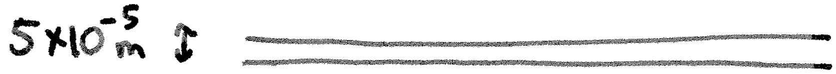
$$\boxed{\frac{Q}{V} = \frac{\epsilon_0 E A}{E d} = \frac{\epsilon_0 A}{d}}$$

$$\boxed{C = \frac{\epsilon_0 A}{d}}$$

$$1\,\mathrm{F} = 1\,\mathrm{C}^2/\mathrm{N}\cdot\mathrm{m} = 1\,\mathrm{C}^2/\mathrm{J}$$

$$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$$

e.g. I have a 400 μF capacitor, with plates separated by 50 μm. What is the area of the capacitor



$$C = \frac{\epsilon_0 A}{d} \Rightarrow A = \frac{C\,d}{\epsilon_0} = \frac{400\times10^{-6} \times 5\times10^{-5}}{8.85\times10^{-12}}$$

$$= \underline{2260\,\mathrm{m}^2}$$

e.g 24.2 Plates $A = 2m^2$; $d = 1\,mm$, $V = 10{,}000V$

a) what is field E ?

b) capacitance

c) charge.

a) $E = \frac{10{,}000}{10^{-3}} = 10^4 \times 10^3 = 10^7\ V/m$

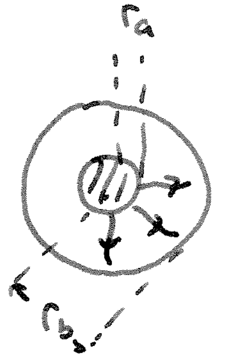
b) $C = \frac{\epsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 2m^2}{10^{-3}m} = 1.77 \times 10^{-8}\ F$

c) $Q = CV = 1.77 \times 10^{-8} \times 10{,}000$

$= 1.77 \times 10^{-4}C.$

(alternatively $\frac{Q}{\epsilon_0} = EA$ $Q = 10^7 \times 8.85 \times 10^{-12} \times 2, = 1.77 \times 10^{-4}C$

24.4 Cylindrical capacitor (coaxial cable



$$V_{ab} = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r_b}{r_a}$$

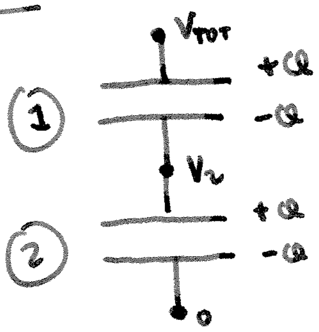
$$Q = \lambda L$$

$$\frac{C}{L} = \left(\frac{Q}{V}\right)\frac{1}{L} = \frac{\lambda \cancel{L}}{\left(\frac{\lambda}{2\pi\epsilon_0} \ln \frac{r_b}{r_a}\right)} \frac{1}{\cancel{L}} = \frac{2\pi\epsilon_0}{\ln\left(\frac{r_b}{r_a}\right)}$$

$$= \frac{55.6 \times 10^{-12}}{\ln r_b/r_a} \text{ Farad/m.}$$

24.2 Series + parallel capacitors

Series

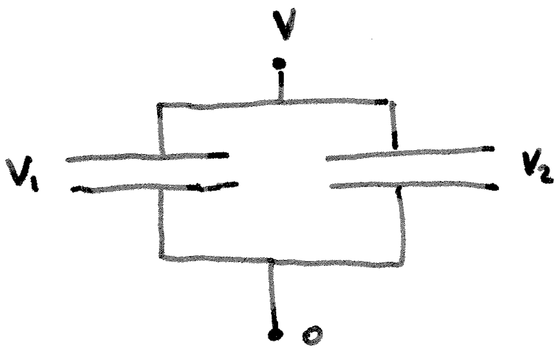


Same charge $Q_1 = Q_2 = Q$

$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} = \frac{Q}{C}$$

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}}$$

Parallel



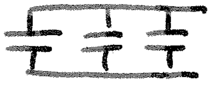
Same voltage

$V_1 = V_2 = V$

$$\begin{aligned} Q &= Q_1 + Q_2 \\ &= C_1 V + C_2 V \\ &= (C_1 + C_2) V \equiv C V \end{aligned}$$

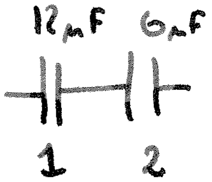
$$\boxed{C = C_1 + C_2}$$

generalizations :



$C = C_1 + C_2 + C_3 + \ldots$

24.5 12μF & 6μF in series. What is the total capacitance.



$$\frac{1}{C} = \frac{1}{12\mu F} + \frac{1}{6\mu F}$$

$$= \frac{1}{12} + \frac{2}{12} = \frac{3}{12\mu F} = \frac{1}{4\mu F}$$

$$C = 4\mu F.$$

If we apply 4V, what is the charge on each capacitor & what is the voltage across each capacitor

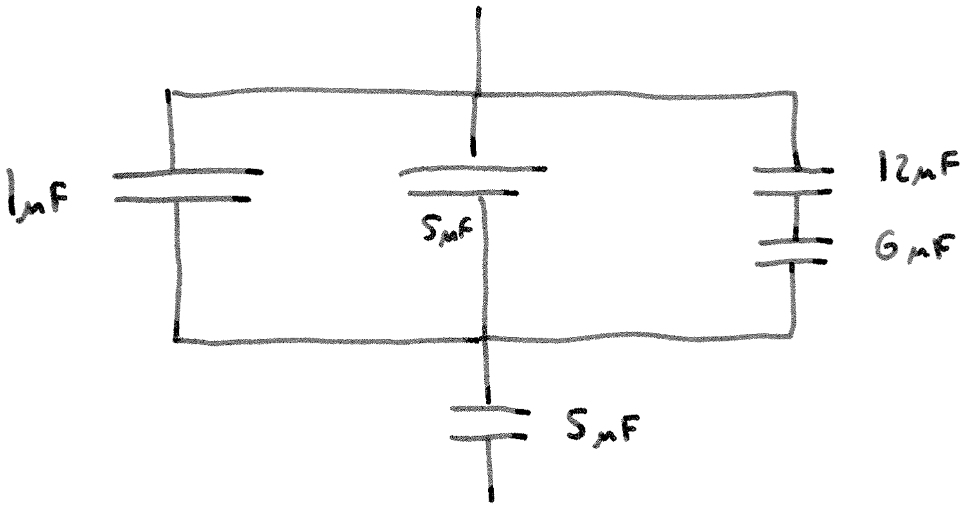
$$Q = Q_1 = Q_2 = CV = 4\times10^{-6}F \times 4V$$

$$= 1.6\times10^{-5}\,C$$

$$= 16\mu C.$$

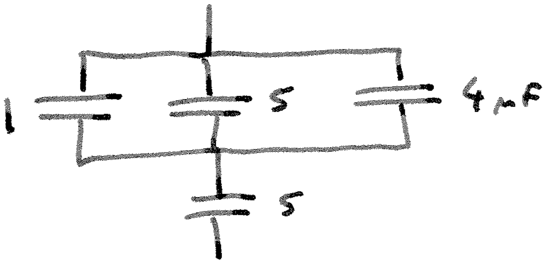
$$\left.\begin{aligned} V_1 &= \frac{Q_1}{C_1} = \frac{16\mu C}{12\mu F} = 1.\dot{3}\,V \\ V_2 &= \frac{Q_2}{C_2} = \frac{16\mu C}{6\mu F} = 2.\dot{6}\,V \end{aligned}\right\} \quad V_1+V_2 = 4V \ \checkmark$$

24.6 Networks

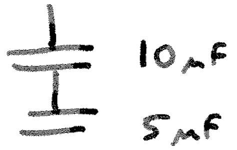


What is the equivalent capacitance

① $\frac{1}{12} + \frac{1}{6} = \frac{1}{4\mu F}$

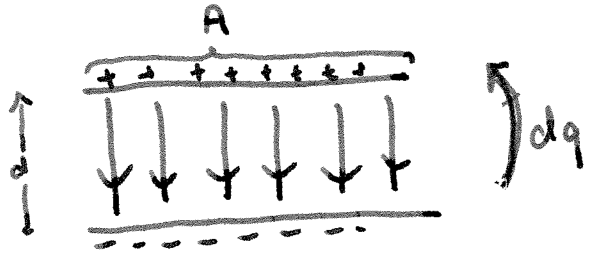


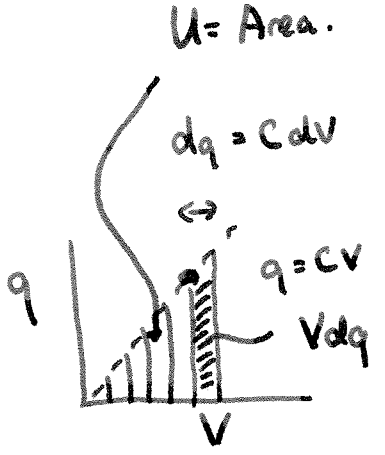
② $1 + 5 + 4\,\mu F = 10\,\mu F$



③ $\frac{1}{10\mu F} + \frac{1}{5\mu F} = \frac{3}{10\mu F} = \frac{1}{3.\dot{3}\mu F} \Rightarrow \underline{C = 3.\dot{3}\mu F}$

24.3 Energy Stored in the Electric field





$$U = \int dU = \int_0^{V_{final}} V\,dq = \int_0^{V_{final}} CV\,dV = \frac{1}{2} C V_{final}^2$$

Alternately $V = \frac{Q}{C} \Rightarrow$ $U = \frac{1}{2} C \left(\frac{Q}{C}\right)^2 = \frac{Q^2}{2C}$

$$\boxed{U = \frac{1}{2} CV^2 = \frac{1}{2} VQ = \frac{1}{2} \frac{Q^2}{2C}}$$

$$U = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)(Ed)^2 = \left(\frac{1}{2}\epsilon_0 E^2\right) \times \overbrace{Ad}^{volume}$$

$$\boxed{u = U/\text{volume} = \frac{1}{2}\epsilon_0 E^2}$$

e.g PRS How much energy is stored in a bank of 40 400μF capacitors charged up to 120V?

$$U = \frac{1}{2}\left(40 \times 4\times10^{-4}\right) \times \left(120V\right)^2$$

$$= 115J$$

50W bulb 2 seconds

$\frac{1}{1000}s \equiv 115{,}000W \equiv 2304$ bulbs.

- ~~Roughly 10~~
- Roughly 100 bulbs
- Roughly 1000 bulbs
- Roughly 10,000 bulbs