L6 Calculating Potential

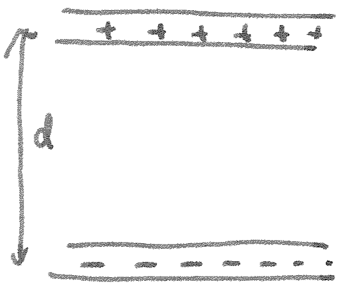
$$qV = U$$

$$V_a - V_b = -\int_b^a \vec{E} \cdot d\vec{\ell}$$

$$\vec{E} = -\nabla V = \left(-\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k} \right)$$

Today: applications

- Uniform Field



$$\vec{E} = -E\hat{j}$$

$$V_a = V_b - \int_b^a \vec{E}\cdot d\vec{\ell}$$

$$= V_b + E(y_a - y_b)$$

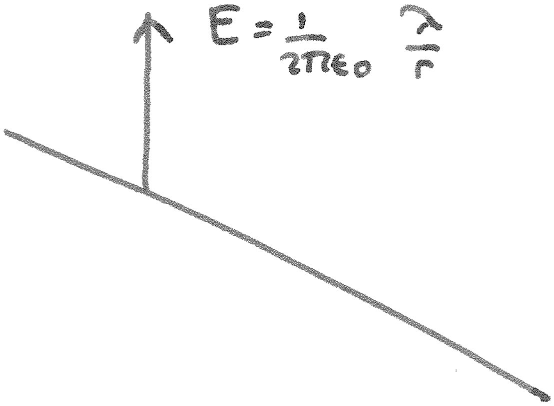
$$V_a = E y_a + c$$

$$V_a - V_b = E d \qquad (y_a = d,\ y_b = 0)$$

$$\boxed{E = \frac{V_a - V_b}{d}}$$

Charge density $\quad E = \dfrac{\sigma}{\epsilon_0} \Rightarrow \boxed{\sigma = \epsilon_0\left(\dfrac{\Delta V}{d}\right).}$

Infinite line



$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{\ell} = \frac{1}{2\pi\epsilon_0}\int_a^b \frac{\lambda}{r}\,dr$$

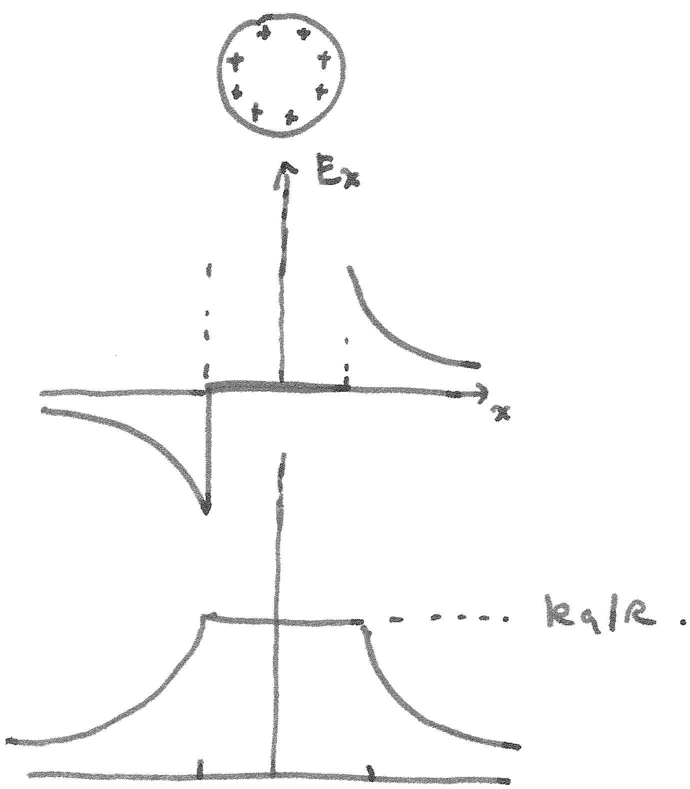
$$= \frac{1}{2\pi\epsilon_0}\ln\frac{r_b}{r_a}$$

$$V_a = -\frac{1}{2\pi\epsilon_0}\ln\frac{r_a}{r_b} + V_b$$

Can't set $V(\infty) = 0$ $\qquad V(r_0) = 0$

$$V = \frac{-1}{2\pi\epsilon_0}\ln\left(\frac{r_a}{r_0}\right).$$

- Charge Conducting Sphere



$r > R \quad V = \dfrac{kq}{r}$

$r < R \quad \vec{E} = 0 \Rightarrow V = \text{constant}$

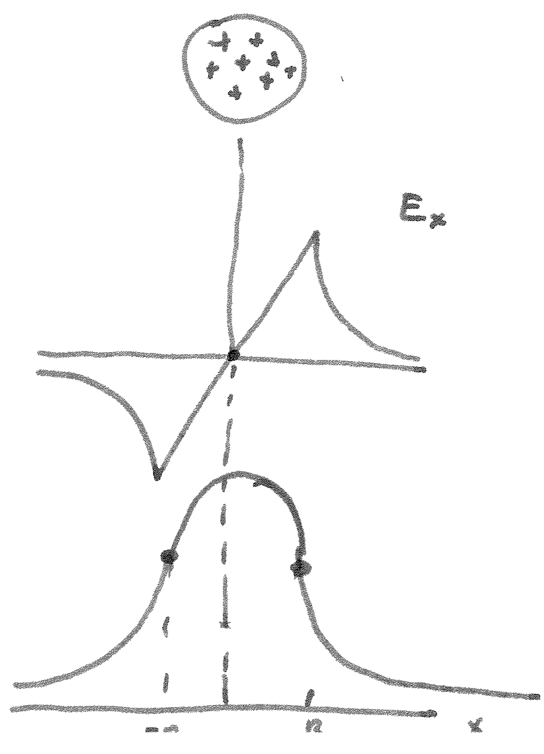
$= \dfrac{kq}{R}.$

- Uniformly charged sphere

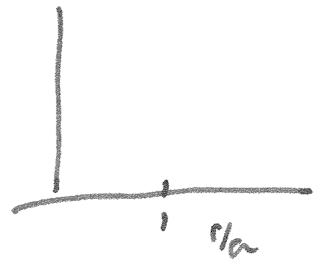
$$E = \frac{q}{4\pi\epsilon_0} \times \begin{cases} \dfrac{1}{r^2} & r > R \\ \dfrac{r}{R^3} & r < R \end{cases}$$

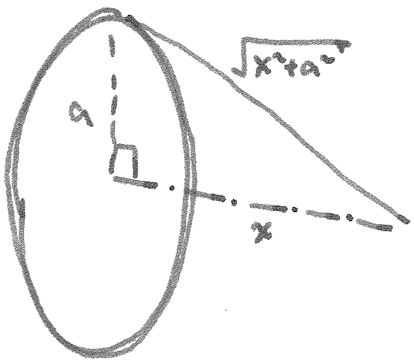
$$V = \frac{q}{4\pi\epsilon_0} \times \begin{cases} -\dfrac{r^2}{2R^3} + C & r < R \\ \dfrac{1}{r} & r > R \end{cases}$$

$C = \dfrac{3}{2R}$



Ring of Charge

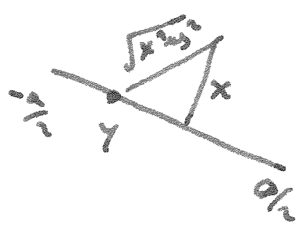




$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x^2+a^2}}$$

Line of Charge.



$$dV = \frac{1}{4\pi\epsilon_0}\overset{\lambda}{\left(\frac{Q}{2a}\right)}\frac{dy}{\sqrt{x^2+y^2}}$$

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\int_{-a/2}^{a/2}\frac{dy}{\sqrt{x^2+y^2}} = \frac{Q}{4\pi\epsilon_0}\frac{1}{2a}\ln\left[\frac{\sqrt{x^2+a^2}+a}{\sqrt{x^2+a^2}-a}\right]$$

23.4 Equipotential Surfaces

- Lines of $V =$ constant
- $\perp$ to $\vec{E}$
- Surface of conductors have constant $V \Rightarrow$ equipotential $\parallel$ to surface.

e.g