L5 Electric Potential

We've learnt about the Force produced by the electric field, but we know that force produces work, and if the electric field can do work, then it must contain energy. What is this energy?

Today we will start to answer this question. We shall look at the potential energy of a charge at a position $\vec{x}$, call it $U(\vec{x})$. Just as the electric force per charge had the meaning of the electric field

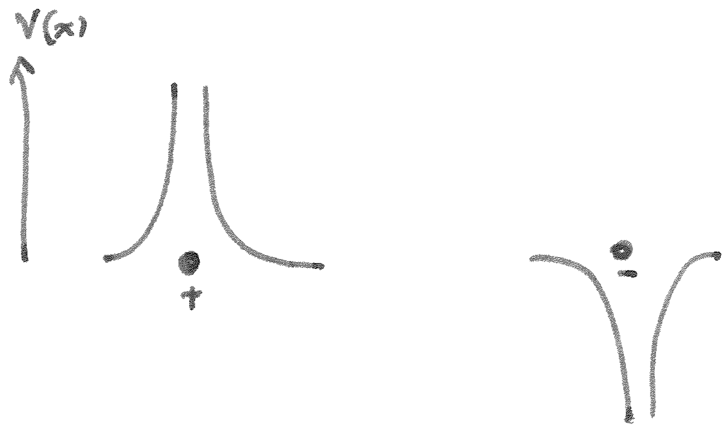
$$\frac{\vec{F}}{q_0} = \vec{E}$$

so the potential energy per test-charge q_0 has meaning — this quantity is the electri potential

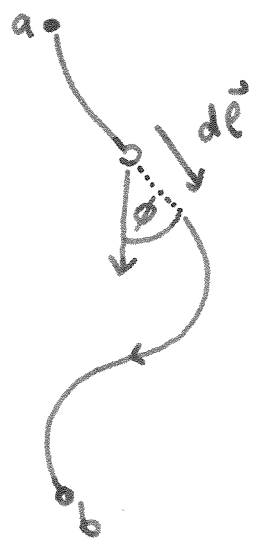
$$\boxed{\frac{U(x)}{q_0} = V(x)}$$

independent of test charge.

$$\text{Volts} = \frac{\text{Joules}}{\text{Coulomb}}$$



23.1 Electric Potential Energy.



$W_{a\to b}$ = work done by force on particle

$$= \int_a^b \vec{F}.d\vec{l} = \int_a^b F\cos\phi \, dl$$

We identify work done = loss of potential Energy

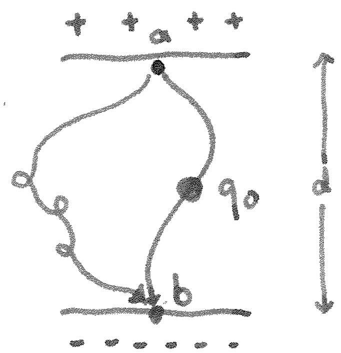
$$W_{ab} = -\Delta U = -(U_b - U_a) = U_a - U_b$$

In cases where the work is converted to K.E

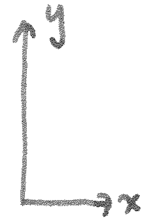
$$W_{ab} = K_b - K_a \quad \Leftrightarrow \quad \boxed{K_a + U_a = K_b + U_b}$$

conservation of mechanical energy

What happens when the force is electrical?



INDEPENDENT OF PATH – only dependent on $(y_b - y_a)$.



$$W_{a\to b} = Fd = q_0 E d$$

$$F_y = -q_0 E \quad \text{constant}$$

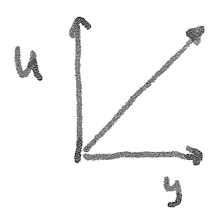
$$\int_a^b \vec{F}.d\vec{\ell} = \underbrace{\int_a^b F_y \, dy}_{-q_0E\int_{y_a}^{y_b} dy \;=\; -q_0E(y_b-y_a)} = +q_0 E d$$

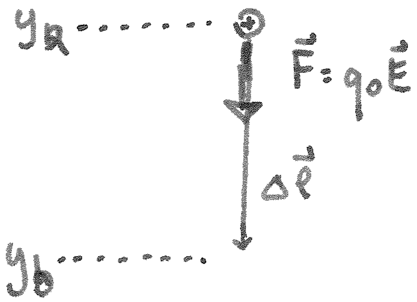
$$\underbrace{y_b - y_a}_{-d}$$

$$-\Delta U = -(U_b - U_a) = -q_0 E (y_b - y_a) = W_{a\to b}$$

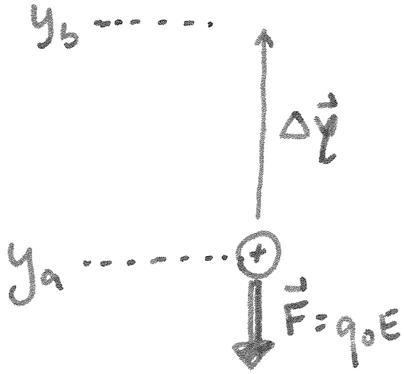
$$U_b = q_0 E y_b \qquad \left(U = -q\vec{E}.\vec{x} \right)$$

$$F_y = -\frac{\partial U}{\partial y} = -q_0 E.$$





$W_{ab} > 0$



$W_{ab} = \vec{F}.\Delta\vec{\ell} < 0$

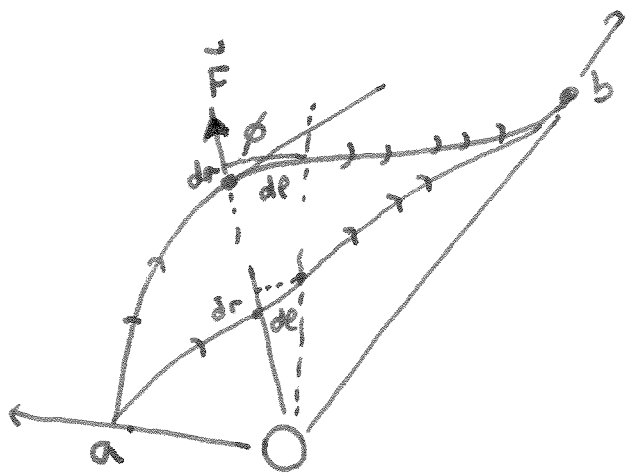
Force opposite to direction of motion: $\underline{W < 0}$

Potential Energy of 2-charges

$$\vec{F} = k \frac{q q_0}{r^2} \hat{r}$$

$$W_{a\to b} = \int_{r_a}^{r_b} F_r \, dr = \int_{r_a}^{r_b} \frac{k \, q q_0}{r^2} \, dr = k q q_0 \left[\frac{1}{r_a} - \frac{1}{r_b} \right]$$

INDEPENDENT OF PATH

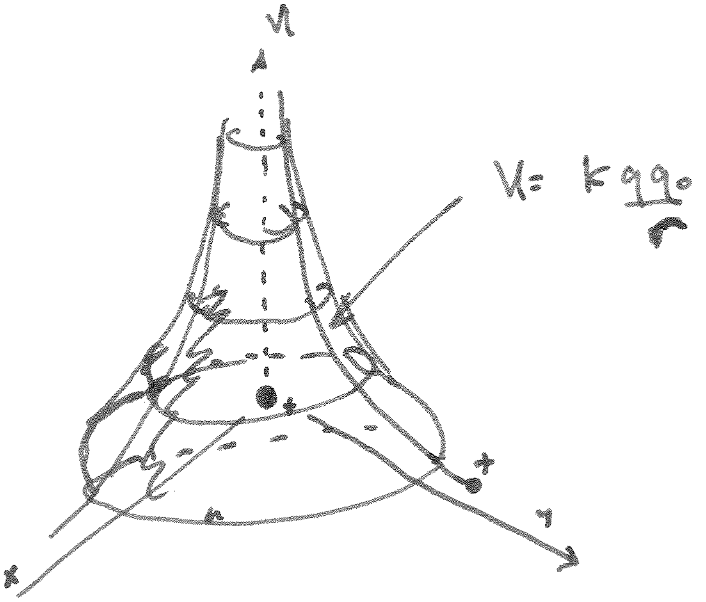


$d\ell \cos\phi = dr$ for any path

$$= U_a - U_b$$

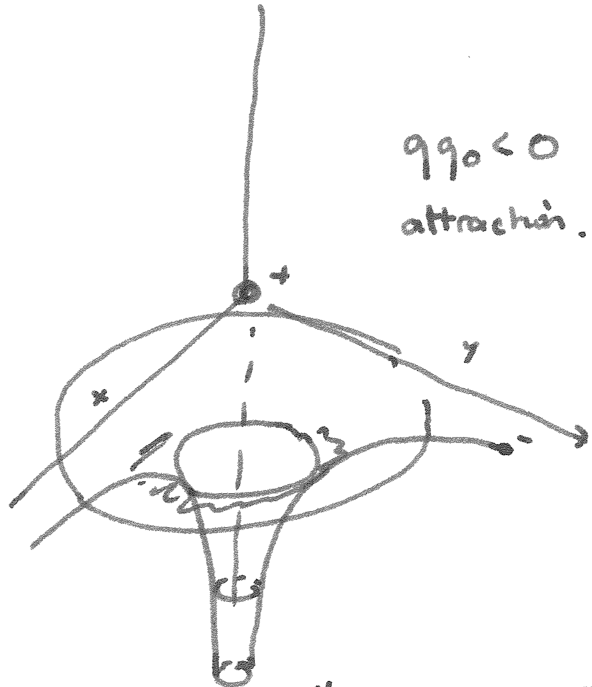
$$\boxed{U = \frac{1}{4\pi\epsilon_0} \frac{q q_0}{r}}$$

- doesn't depend on signs of q & q_0
- decays as $\frac{1}{r}$ – slowly.
- By convention take $U(\infty) = 0$.



$qq_0 > 0$

repulsion



$qq_0 < 0$

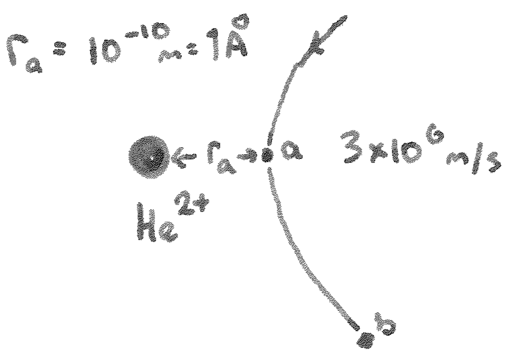
attraction.

"Potential Well"

e.g H-atom

Conservation of energy

$$K_a + U_a = K_b + U_b$$



$r_b = 2Å = 2\times10^{-10}m$

$v_b = ?$

A positron is scattered off an "α-particle" (Helium nucleus). Because $m_e/m_{He} \sim 1/6000$ we can take the He ion to be stationary.

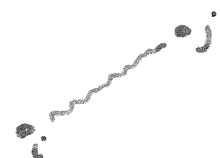
Many Charges

$$U_0 = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \cdots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_{j=1}\frac{q_j}{r_j} = q_0 V .$$

Depends uniquely on Location.

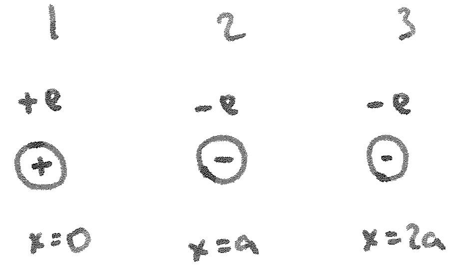
This is the P.E of the <u>test</u> charge.

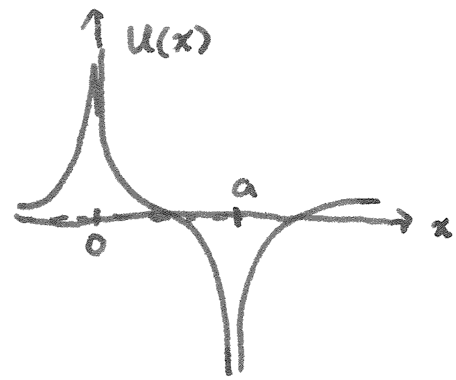
Total P.E $= \dfrac{1}{4\pi\epsilon_0}\displaystyle\sum_{i>j}\frac{q_i q_j}{r_{ij}}$



all pairs.

e.g





i) Energy to bring electron to $x = 2a$ from infinity

$$= W = U(x=2a) - U(x=\infty)$$

$$= k\, q_3 \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} \right)$$

$$= k\,(-e)\left[\frac{-e}{a} + \frac{+e}{2a} \right] = \frac{ke^2}{2a} = \frac{1}{8\pi\epsilon_0}\frac{e^2}{a}$$

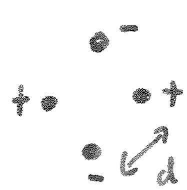
ii) Total energy

$$U = k\left(\frac{q_2 q_1}{a} + \frac{q_3 q_1}{2a} + \frac{q_3 q_2}{a} \right)$$

$$= k\left(\frac{-e^2}{a} - \frac{e^2}{2a} + \frac{e^2}{a} \right) = -\frac{ke^2}{2a}$$

$$= -\frac{1}{8\pi\epsilon_0}\frac{e^2}{a}.$$

e.g



$$U_e = k\left(\frac{-2e^2}{d} + \frac{e^2}{\sqrt{2}d} \right)$$

$$= -\frac{ke^2}{d}\left(2 - \frac{1}{\sqrt{2}} \right)$$

$$V\Big|_{TOT} = \frac{-4ke^2}{d} + \frac{\sqrt{2}ke^2}{d}.$$

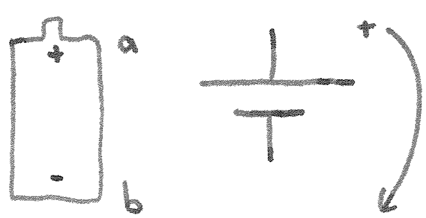
23.2 ELECTRIC POTENTIAL

$$\boxed{V = \frac{U_0}{q_0}} \iff U_0 = q_0 V$$

$1V = 1$ Joule/Coulomb $= 1J/C$

$$\frac{W_{a\to b}}{q_0} = \frac{U_a - U_b}{q_0} = V_a - V_b$$

Voltage difference = work done / charge.

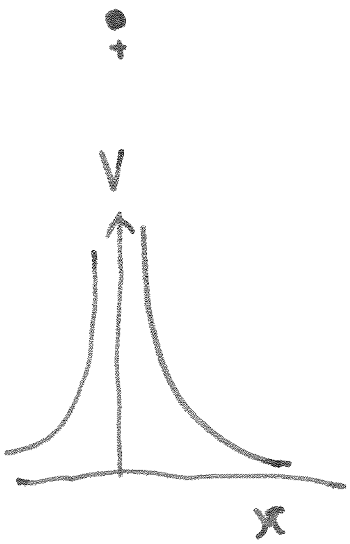


$1C \Rightarrow 1.5J$

$1e^{+} \Rightarrow 1.5eV.$

1.5V

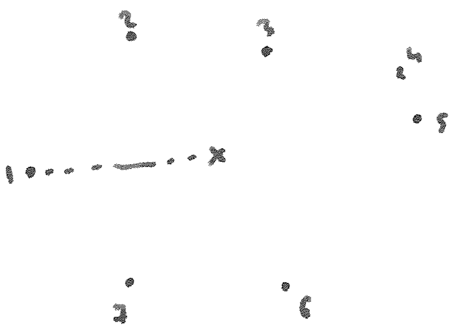
battery   symbol



$$\boxed{V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}}$$ one charge

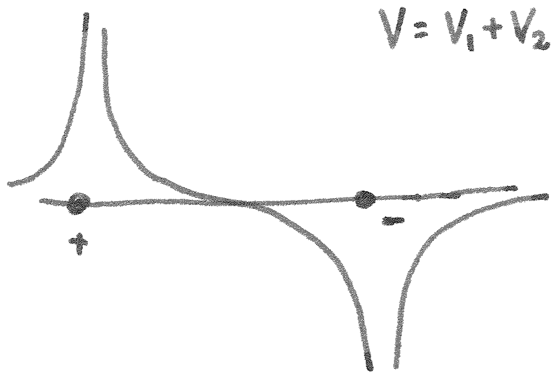
$$\boxed{V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\sum \frac{q_j}{r_j}}$$ many charges

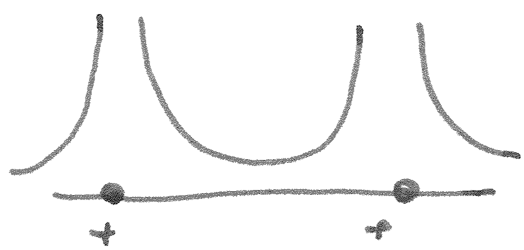
$$= \sum_j V_j$$



2 charges

$V = V_1 + V_2$





$$W_{a \to b} = \int \vec{F} \cdot d\vec{l} = q_0 \int \vec{E} \cdot d\vec{l} = U_a - U_b$$

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l} = \int_a^b E \cos\phi \, dl$$

$$\boxed{V_a - V_b = -\int_b^a \vec{E} \cdot d\vec{l}}$$

$$1 \, \frac{\text{Volt}}{\text{meter}} = 1 \, \frac{N}{C}$$

$$\underbrace{U_a - U_b}_{1\,eV} = \underbrace{q_0}_{e} \underbrace{(V_a - V_b)}_{1V}$$

"electron volt"

= energy gained by e^- in crossing one volt of P.D.

$$\boxed{1\,eV = 1.602 \times 10^{-19}\,J}$$

meV, MeV, GeV, TeV.

milli eV. Billion volts

e.g. 2000V ⇒ $\underline{\underline{2000\,eV}}$ = $2 \times 1.6 \times 10^{-16}$

$= 3.2 \times 10^{-16}\,J = 2000\,eV.$

23.5 Force & Field.

$$V_a - V_b = -\int_b^a \vec{E}\cdot d\vec{\ell}$$

$$\boxed{\begin{aligned} F &= -\nabla u \\ \vec{E} &= \frac{F}{q} = -\nabla \frac{u}{q} = -\nabla V \end{aligned}}$$

$$dV = -\vec{E}\cdot d\vec{\ell}$$

$$-dV = +\left(E_x\,dx + E_y\,dy + E_z\,dz\right)$$

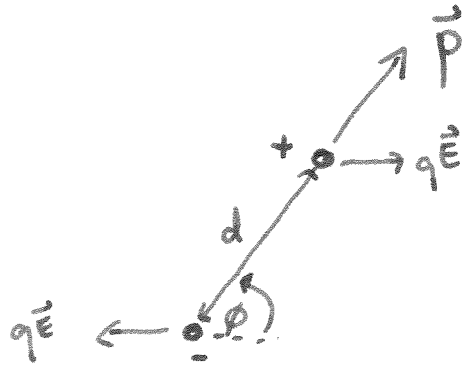
$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

$$\boxed{\begin{aligned} \vec{E} &= -\frac{\partial V}{\partial x}\hat{\imath} - \frac{\partial V}{\partial y}\hat{\jmath} - \frac{\partial V}{\partial z}\hat{k} \\ \vec{E} &= -\vec{\nabla} V \end{aligned}}$$

$$\nabla f = +\frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath} + \frac{\partial f}{\partial z}\hat{k}.$$

$$\left(\text{Radial} \qquad E_r = -\frac{\partial V}{\partial r}\right)$$

21.7 Dipoles



$$\vec{p} = q(\vec{x}_2 - \vec{x}_1)$$

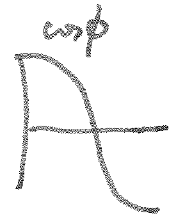
$$p = qd$$

- Torque

$$\tau = -d_\perp qE = -d\sin\phi\, qE$$

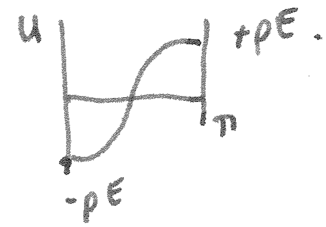
$$= -pE\sin\phi$$

(turns in $-\phi$ direction)



- Potential

$$U = U_1 + U_2$$



$$U_j = q_j V_j \qquad U_1 = -qV_1$$

$$U_2 = qV_2$$

$$U = q(V_2 - V_1)$$

$$V_2 - V_1 = -\vec{E}\cdot(\vec{x}_2 - \vec{x}_1)$$

$$U = -q(\vec{x}_2 - \vec{x}_1)\cdot\vec{E}$$

$$= -\vec{p}\cdot\vec{E} = -p\cos\phi E$$

$$\tau = -\frac{\partial U}{\partial \alpha} = -p\sin\phi E$$