L4 APPLICATIONS OF GAUSS' LAW.

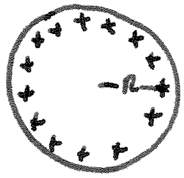
Last time, we learnt that Gauss' law

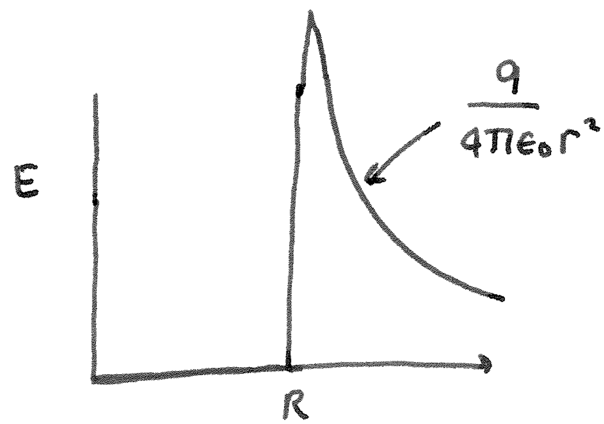
$$\text{Flux} = \frac{1}{\epsilon_0}\ \text{Charge enclosed}$$

permittivity of space
$\sim 8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$

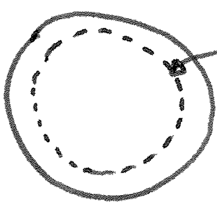
is a consequence of Coulombs law. Indeed $E \propto 1/r^2$ is the only way that the flux from a charge can be independent of distance. Today we will see that in situations of high symmetry, Gauss' law permits us to evaluate the electric field with a minimum of calculation.

a) Field of a charge conducting sphere, charge q





- $E = 0$ inside $\therefore$ all charge must be at the surface

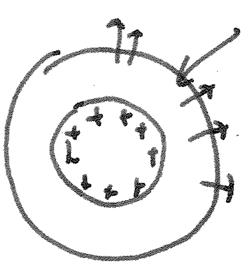


$$\int \vec{E}.dA = \int \vec{0}.d\vec{A} = 0$$

$$\Rightarrow q_{enclosed} = 0$$

- By symmetry charge is evenly distributed on surface.

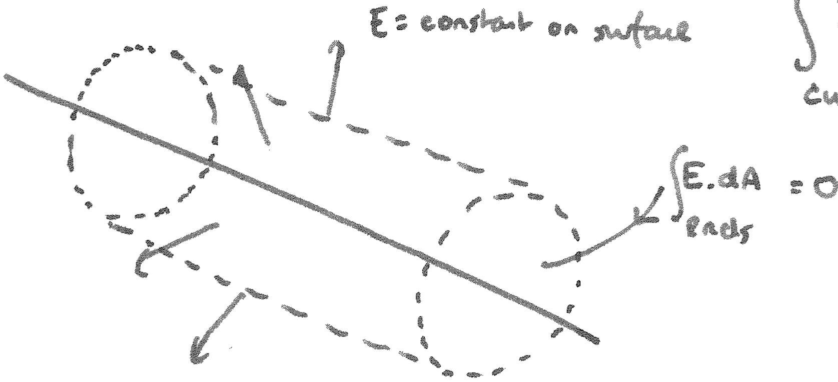
- Outside sphere $\int \vec{E}.dA = E\,4\pi r^2 = \frac{q}{\epsilon_0} \Rightarrow E = \frac{q}{4\pi\epsilon_0 r^2}$



$$\int \vec{E}.dA = \frac{q}{\epsilon_0}$$

- E @ surface $E = \frac{q}{4\pi\epsilon_0 R^2}$

4) Field around a line charge



$$\int_{curved} \vec{E}.dA = (2\pi r l) \times E$$

$$\overbrace{(2\pi r l)}^{A} E = \epsilon_0 \overbrace{\lambda l}^{q}$$

$$\Rightarrow \boxed{E = \frac{\lambda}{2\pi \epsilon_0 r}}$$

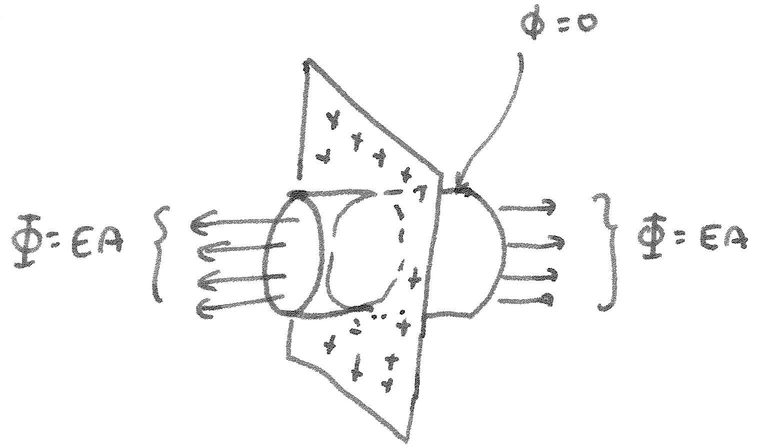
e.g. $\lambda = 2nC/m$

$r = 1cm$

$$E = \overbrace{\left(\frac{1}{2\pi\epsilon_0}\right)}^{18\times10^9} \times \frac{2\times10^{-9}}{0.01}$$

$= 3600 N/C.$

c)



$$\Phi_{TOT} = EA \times 2 = \frac{\overbrace{(\sigma A)}^{q}}{\epsilon_0}$$

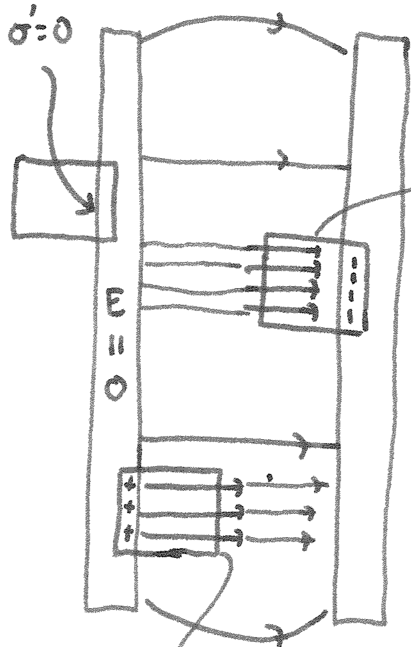
$$E = \frac{\sigma}{2\epsilon_0}$$

e.g. $\sigma = 1\,nC/m^2$

$$E = \frac{1 \times 10^{-9}}{2 \times 8.854 \times 10^{-12}}$$

$$= 56.5\ N/C.$$

4) Two conducting capacitor plates



$\Phi = -EA = -\frac{\sigma A}{\epsilon_0}$

$E = \frac{\sigma}{\epsilon_0}$

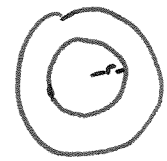
$\boxed{E = \frac{\sigma}{\epsilon_0}}$

$\Phi = +EA = +\frac{\sigma A}{\epsilon_0}$

)

e) Uniformly charged sphere

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$



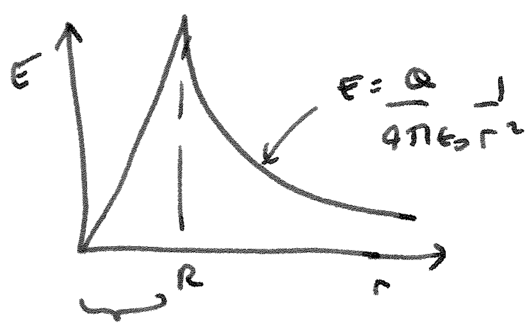
i) $r < R$

$$\left(\frac{4}{3}\pi r^3 \rho\right) = q_{enclosed} = \left(4\pi r^2 E\right)\epsilon_0$$

$$E = \left(\frac{1}{3}\frac{\rho r}{\epsilon_0}\right) = \frac{Q}{4\pi\epsilon_0}\frac{r}{R^3}$$

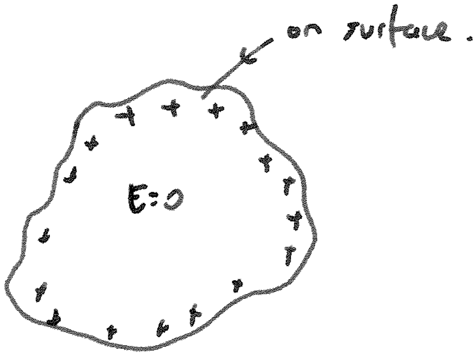
ii) $r > R$

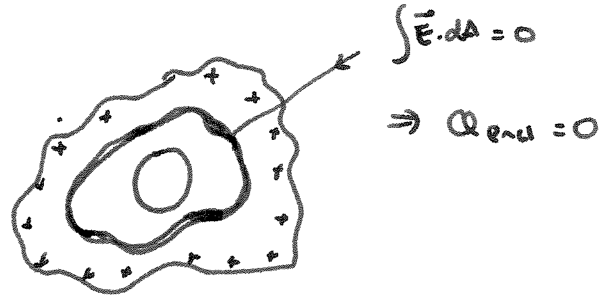
$$4\pi r^2 E \epsilon_0 = Q \qquad E = \frac{Q}{4\pi\epsilon_0}\frac{1}{r^2}.$$

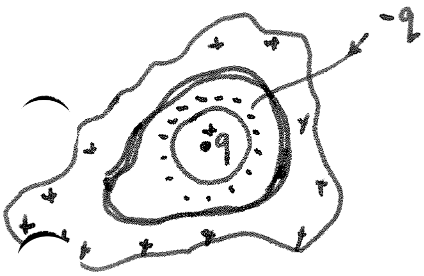


$E \quad \frac{Q}{4\pi\epsilon_0}\frac{r}{R^3}$

Charges on Conductors

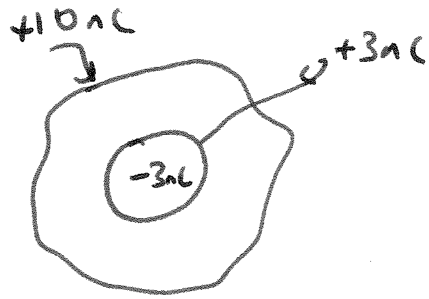




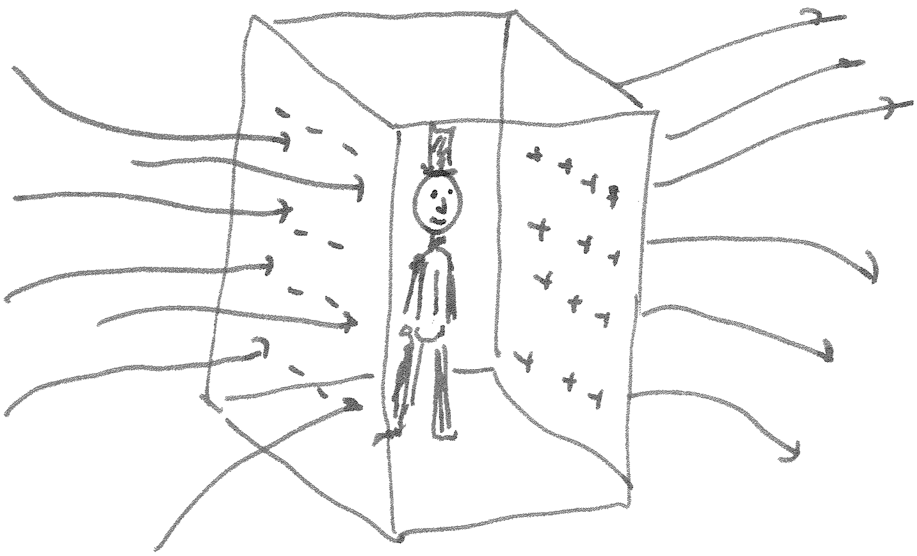


internal charge must be screened so that $Q_{enclosed} = 0$

e.g. $Q_{TOT} = +10nC$ charged conductor contains $-3nC$ in a cavity



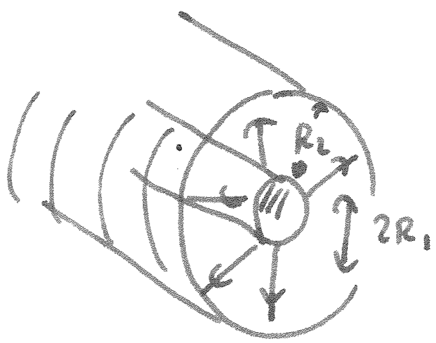
Faraday Cage



No field on interior $\Rightarrow$ charges screen external fields.

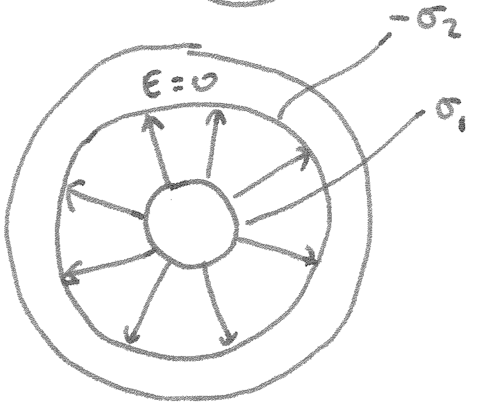
Example A coaxial cable has an internal cable of radius R_1 & the internal radius of its outer cable is R_2. σ_1 = charge density of the internal cable

a) Calculate field inside



b) $$E\,A = \frac{q_{inside}}{\epsilon_0}$$

$$= \frac{2\pi R_1 \ell \sigma_1}{\epsilon_0}$$



$A = 2\pi r \ell$

$$E = \left(\frac{R_1}{r}\right)\frac{\sigma_1}{2\pi\epsilon_0} \qquad *$$

$R_2 > r > R_1$

By Gauss' law again

$$EA = \frac{-q_{outside}}{\epsilon_0}$$

$$E = \frac{R_2}{r}\frac{\sigma_2}{2\pi\epsilon_0} \qquad **$$

$$** \Rightarrow \quad E = \frac{\sigma_1 R_1}{r\,(2\pi\epsilon_0)}$$

$\underline{\sigma_1 R_1 = \sigma_2 R_2}$.