# L3 Gauss' Law

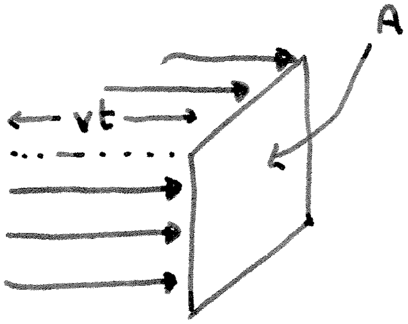
Last time, we learnt about the idea of a field - an electric field whose amplitude determined the force per unit charge on an electron, and whose direction determined the direction of that force. If we followed the electric field through space, we followed the "lines of force".

One of the interesting things we noticed last time, was that lines of force "flow" away from +ve charges & they flow towards -ve charges. When scientists notice this kind of rule, they seek to elevate it to a "law" & this is exactly what we will do today - we will learn how this idea of "flow" & "flux" is encoded in Gauss' law.

If we're going to think of an electric field as a flow of stuff, then this is going to raise some questions :—

- How do we measure the amount of "flux" of a fluid through a surface?
- In the case of the electric field – the amount of flow is presumably related to the amount of charge – how can we formulate this in math?

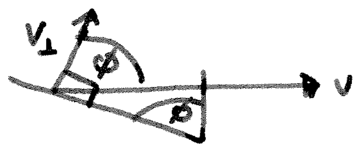
# 22.2 What is Flux?

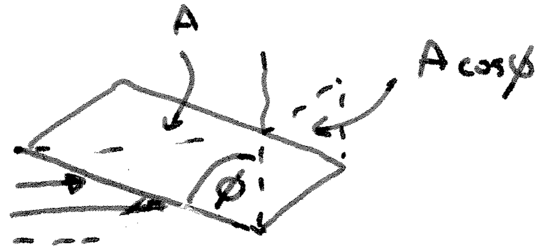



$$\text{Volume} = A \times vt$$

$$\Phi_{fluid} = \frac{\text{Volume}}{t} = vA = \frac{dV}{dt}$$

$$\Phi_{fluid} = vA\cos\phi = \overbrace{(v\cos\phi)}^{v_\perp} A$$



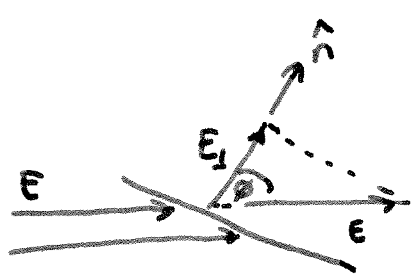




$$\Phi_{fluid} = v_\perp A$$

Vector

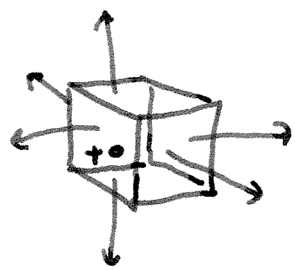
$$\left.\begin{aligned} \vec{A} &= A\hat{n} \\ v_\perp &= \vec{v}\cdot\hat{n} \end{aligned}\right\} \quad v_\perp A = (\vec{v}\cdot\hat{n})A = \underline{\underline{\vec{v}\cdot\vec{A}}}$$

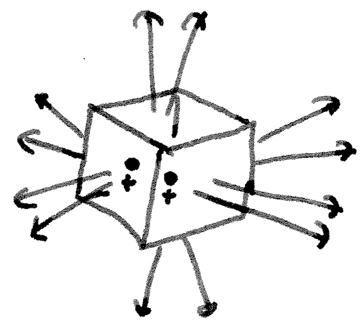
$$\boxed{\begin{aligned} \Phi_E &= E_\perp A \\ &= \vec{E}\cdot\vec{A} \end{aligned}}$$



22.1

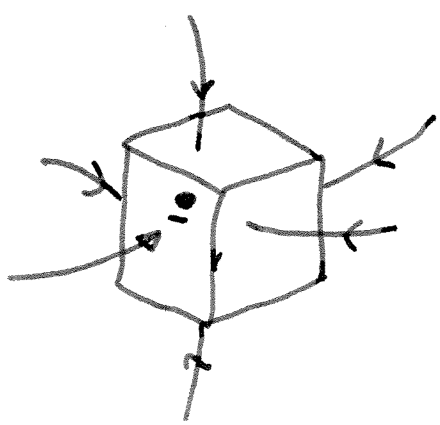
Positive charge $\equiv$ flow outwards





Twice as much charge, twice as much outwards flow $\equiv$ twice as much <u>flux</u>

$$\boxed{\text{flux outwards} = \text{constant} \times \text{amount of charge inside volume}}$$

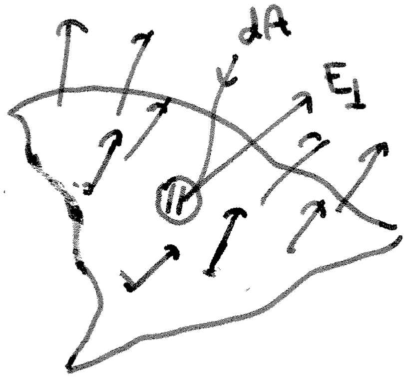


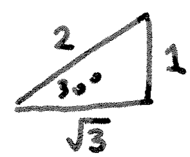
-ve charge $\Rightarrow$ flux inwards

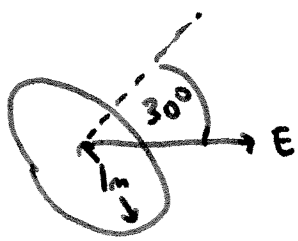
= negative flux outwards.

It seems to work.....

$$\Phi_E = \int E\cos\phi \, dA = \int \vec{E}\cdot d\vec{A}$$




$E = 20 \ N/C$

What is the flux of a field of strength $E = 20 \ N/C$ through a disk of radius 1m inclined with its normal 30° from the field direction?

$$\Phi_E = E_\perp A$$

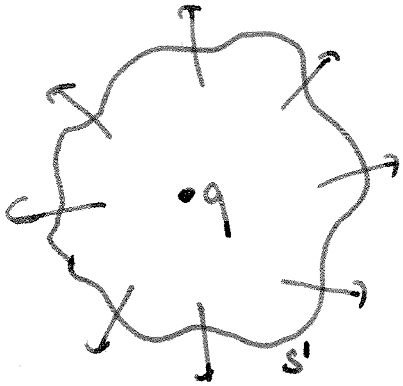
$$= (E\cos\theta)\,A$$

$$= 20 \ N/C \times \cos 30^\circ \times (3.14 \ m^2)$$

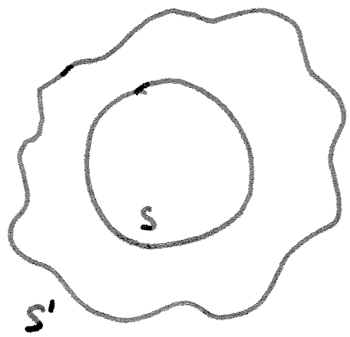
$$= 20 \ N/C \ \frac{\sqrt{3}}{2} \times \pi$$

$$= \underline{54 \ Nm^2/C}$$

Arbitrary surface enclosing a charge

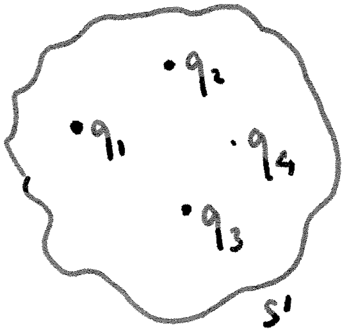



=

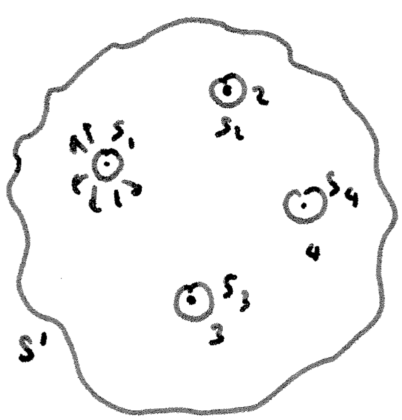



$$= \underbrace{\int_{S' \cup S} d\Phi}_{0} + \int_{S} d\Phi_E = \frac{q}{\epsilon_0}$$

Many charges



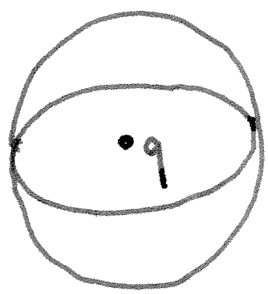

=




$$= \int_{s' \cup s_1 \cup s_2 \cup s_3 \cup s_4} d\Phi_E + \left( \int_{s_1} + \int_{s_2} + \int_{s_3} + \int_{s_4} \right) d\Phi$$

$$= (q_1 + q_2 + q_3 + q_4)/\epsilon_0$$

Electric flux through a sphere



$$\vec{E} = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r^2}\right) \cdot \hat{r}$$

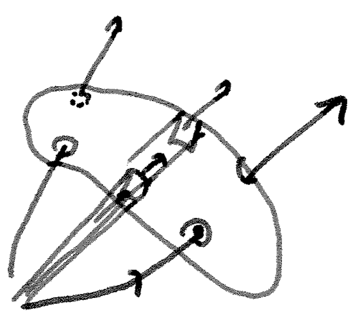
$$\vec{E}.d\vec{A} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\, dA$$

$$\Phi_E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\int dA$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \times \overbrace{\text{Area}}^{4\pi r^2}$$

$$= \frac{q}{\cancel{4\pi}\epsilon_0 \cancel{r^2}} \times \cancel{4\pi r^2} = \frac{q}{\epsilon_0}$$

If a closed surface encloses no charge, $\Phi = 0$



same field lines
enter & leave volume

$$dFlux_1 = -dFlux_2$$

$$\int d\Phi = \int dFlux_1 + \int dFlux_2$$

$$= \int (dFlux_1 + dFlux_2) = \underline{\underline{0}}$$

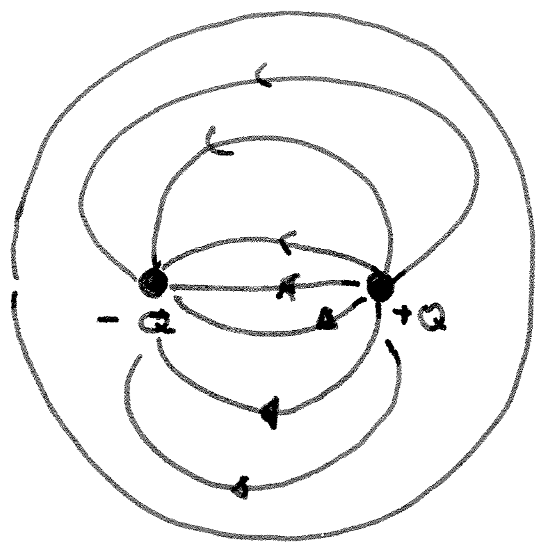
FLUX = (CHARGE ENCLOSED)/$\epsilon_0$.

$$\boxed{\int d\Phi = \frac{Q_{enclosed}}{\epsilon_0}}$$

Gauss' Law

First of four celebrated Maxwell equations.

e.g




What is the electric flux from a dipole?

- $\int d\Phi = \frac{Q_1 + Q_2}{\epsilon_0}$.

$$= \frac{Q - Q}{\epsilon_0} = 0.$$

- It depends on the distance away from the dipole.

- It depends on the size of Q

- 0