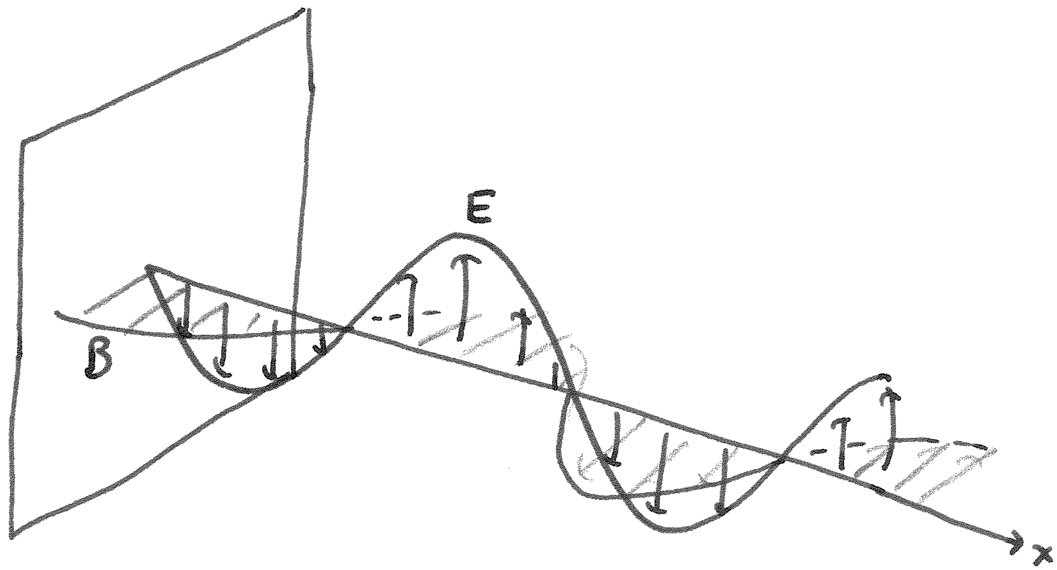
L25 STANDING WAVES, ANTENNAS + THE ELECTROMAGNETIC SPECTRUM

To complete our study of electromagnetic waves we will look at two applications: the physics of standing waves, which is important for the understanding of lasers, and the physics of antennae which are used to transmit E&M radiation. We will end with a discussion of the electromagnetic spectrum.

32.5 Standing Waves

When an EM wave reflects off a metal surface, the E field must be zero at the surface (assuming that the metal is a perfect conductor). This condition is satisfied by the generation of a "standing wave". The standing wave is a combination of incoming & reflected waves such that they precisely cancel at the metal surface.



The E field is a combination of an incoming wave travelling in the -ve x direction & an outgoing wave travelling in the +ve x direction

$$E_y(x,t) = E_{max}\left[\cos(kx+\omega t) - \cos(kx-\omega t)\right]$$

$$B_z(x,t) = B_{max}\left[-\cos(kx+\omega t) - \cos(kx-\omega t)\right]$$

When we expand using $\cos A \pm B = \cos A \cos B \mp \sin A \sin B$

$$E_y(x,t) = -2E_{max} \sin kx \sin \omega t$$

$$B_y(x,t) = -2B_{max} \cos kx \cos \omega t$$

This is like the equation for a vibrating string.

At $x=0$ $E_y(x=0,t)$ is always zero. Indeed there are

nodes at

$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2} \qquad \text{(nodal planes of } E\text{)}$$

The magnetic field is not zero at the surface & the nodal planes of B correspond to the "antinodes" of E.

We also see that when $B=0$ $(\cos\omega t=0)$ E is at its maximum magnitude & vice versa. We say

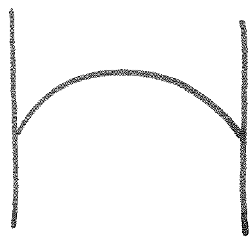
E & B are "out-of-phase".

If we wish to make a cavity to sustain such a standing wave, then we must put a second reflecting surface at a nodal plane for E, i.e

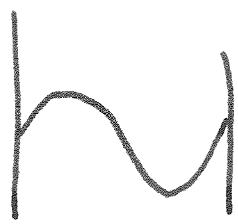
$$L = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2} \ldots = \frac{n\lambda}{2}$$

alternatively, in a cavity of length L, the standing wave modes will have wavelength

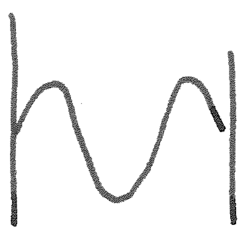
$$\boxed{\lambda_n = \frac{2L}{n} \qquad \text{NORMAL MODES}}$$



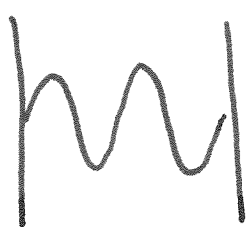
$n=1 \qquad \lambda = 2L$



$n=2 \qquad \lambda = L$

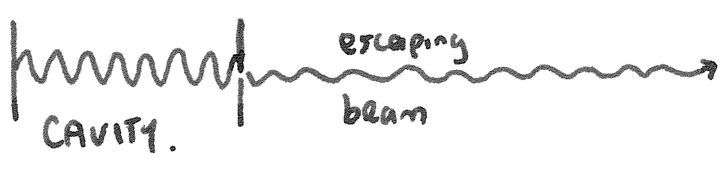


$n=3 \qquad \lambda = \frac{2L}{3}$



$n=4 \qquad \lambda = \frac{L}{2}$.

Laser:



e.g. Cavity with length $L = 1.5\,cm$. a) Calculate

λ & f for the fundamental b) where is $E=0$?

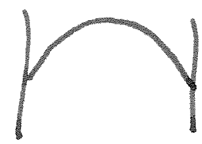
B maximum ?

$B=0$?

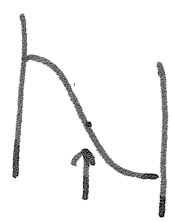
$$\lambda_1 = 2L = 3cm$$

$$f_1 = \frac{c}{\lambda_1} = \frac{3\times10^8}{3\times10^{-2}} = 10^{10}Hz = \underline{10\,GHz}.$$

- $E=0$ @ $x=0$ & $x=1.5cm$

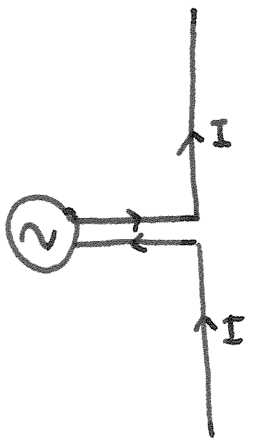


- B maximum $x=0$, $x=1.5cm$

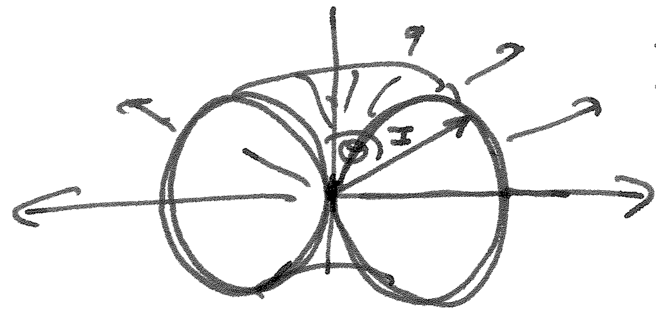


- $B=0$ @ $x=0.75cm$.

Antenna



"Dipole Antenna"



Toroidal intensity dependence

$I(\theta,\phi) \sim \sin^2\theta$.

The primary magnetic & electric fields are out-of-phase with each other & do not radiate. However these induce secondary fields which are in phase.

$$E_{max} \sim \frac{\mu_0}{4\pi} \frac{\omega^2 p}{r} \sin\theta$$

p = dipole moment

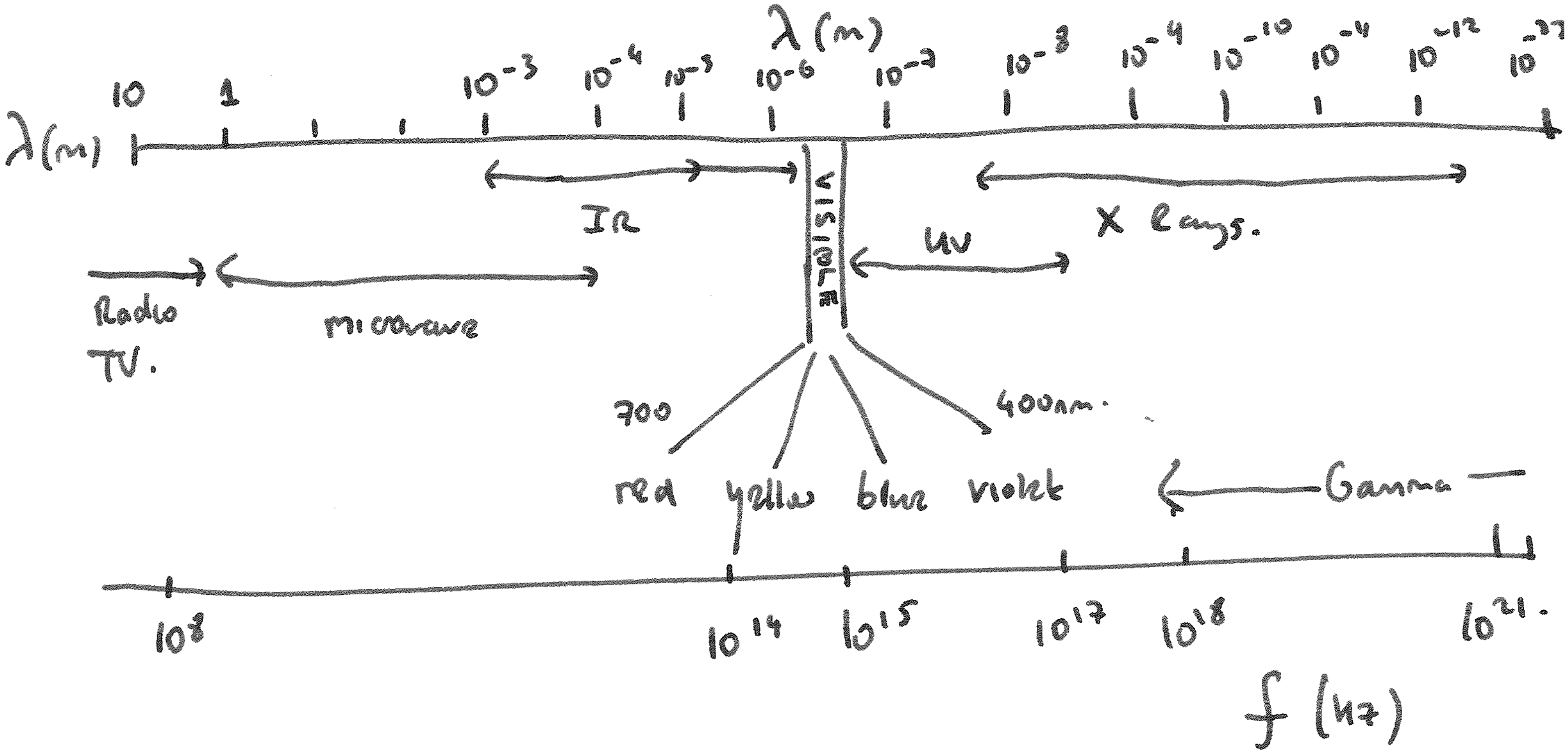
$\omega = 2\pi f$.

$$B_{max} \sim \frac{\mu_0}{4\pi c} \frac{\omega^2 p}{r} \sin\theta$$

$$I \propto \omega^4 \sin^2\theta$$

32.6 E & M Spectra

$$\lambda f = c$$



visible :
λ 400 – 700 nm
f 750 – 430 THz

AM: $5.4 \times 10^5 - 1.6 \times 10^6$ Hz.

FM: $8.8 \times 10^7 - 1.08 \times 10^8$ Hz.