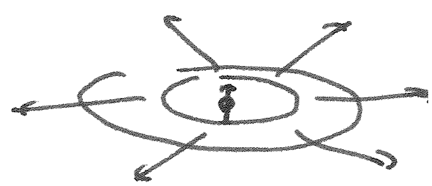
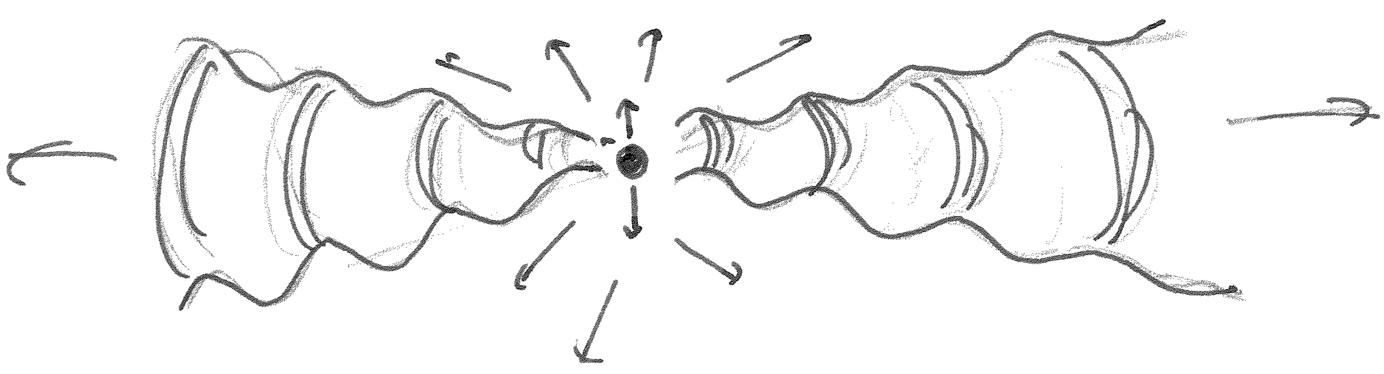
L24 Electromagnetic Waves

When electric & magnetic fields depend on time, they are no longer independent. When we "jiggle" the electric field by moving a charge up & down, we create a magnetic field. The jiggling magnetic field, in turn changes the electric field & the net effect is to produce a wave which "radiates" outwards





It was Maxwell, who in 1866 first wrote down the equations for dynamical electromagnetic fields. He predicted the existence of electromagnetic waves, but did not live to see his prediction realized in the lab. Heinrich Hertz was the first to produce electromagnetic waves with measurable wavelengths in the lab, in 1887.

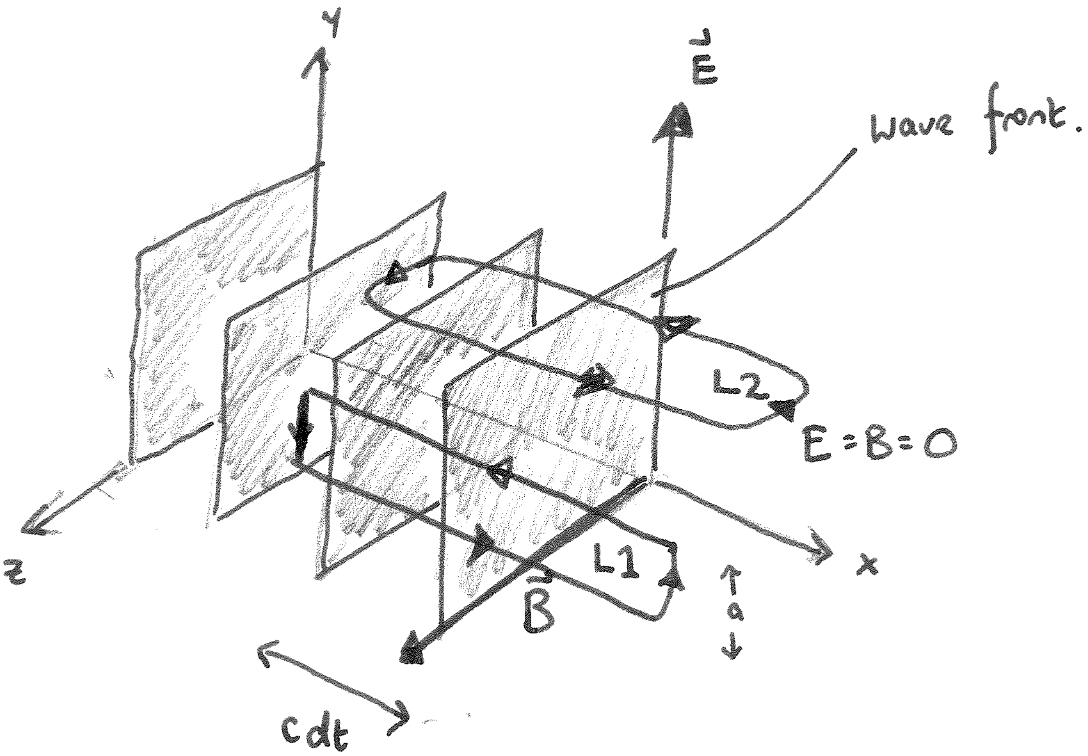
Around 1894, Guglielmo Marconi realized the potential for using "radio" waves for communication &

24.3.

in 1897 he succeeded in convincing the British Post Office to set up the first Wireless Telegraph & Signal company, the antecedent of the "BBC". The first signals were sent across the Atlantic in 1901.

(See http://www.marconiusa.org/marconi/)

32.2 Plane Waves & the Speed of Light.



$$\oint_{L2} \vec{B}.d\vec{\ell} = \frac{\partial \Phi_E}{\partial t} \times \mu_0 \epsilon_0$$

Ampere

$$\oint_{L1} \vec{E}.d\vec{\ell} = -\frac{\partial \Phi_B}{\partial t}$$

Faraday

Note that $\int E.dA = \int B.dA = 0$

$$\oint_{L1} \vec{E}.d\vec{\ell} = -Ea \qquad \frac{\partial \Phi_B}{\partial t} = \frac{Ba(cdt)}{dt} = Bac$$

Faraday: $-Ea = -Bac$

$$\Rightarrow \boxed{E = Bc}$$

Notice that B fields are much smaller in magnitude than E-fields

Ampere $\oint_{L2} \vec{B}.d\vec{\ell} = +Ba = (Eac)\mu_0\epsilon_0$

$$B = Ec\mu_0\epsilon_0 = Bc^2\mu_0\epsilon_0$$

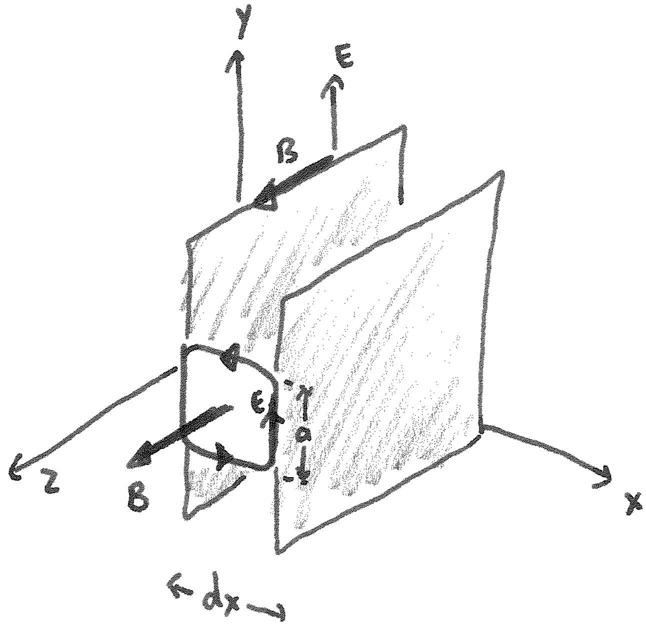
$$\Rightarrow \quad c^2\mu_0\epsilon_0 = 1$$

$$\boxed{c = \frac{1}{\sqrt{\mu_0\epsilon_0}}}$$

$$c = \frac{1}{\sqrt{(8.85\times10^{-11}\,C^2/Nm^2)(4\pi\times10^{-7}\,N/A^2)}}$$

$$= 3\times10^{8}\ m/s.$$

- Wave is <u>transverse</u>. $\vec{E}$ & $\vec{B}$ perpendicular to direction of motion. $\vec{E}\times\vec{B}$ <u>points</u> in direction of motion.
- Definite ratio between E & B $\quad E = cB$.
- Wave travels in a vacuum with a definite & unchanging speed.
- No medium required – it is the vacuum itself which transmits the EM field.



$$\int_{L1} \vec{E}\cdot d\vec{l} = a\,E_y(x+dx) - a\,E_y(x)$$

$$= a\,dx\,\frac{\partial E_y}{\partial x}$$

$$\frac{\partial \Phi_B}{\partial t} = a\,dx\,\frac{\partial B_z}{\partial t}$$

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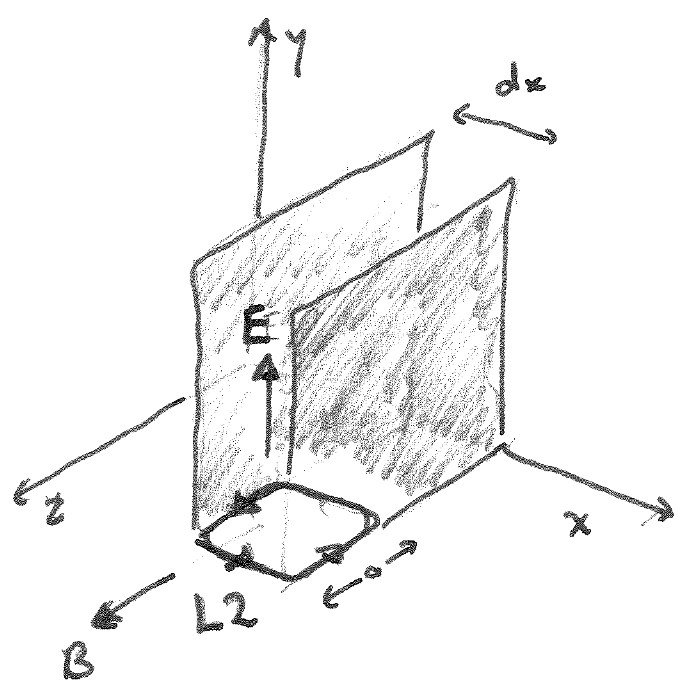
$$\boxed{\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}} \quad L1$$



$$\int_{L2} \vec{B}\cdot d\vec{l} = -a\,B_z(x+dx) + a\,B_z(x)$$

$$= -a\,dx\left(\frac{B_z(x+dx) - B_z(x)}{dx}\right)$$

$$= -a\,dx\,\frac{\partial B_z}{\partial x}$$

$$\frac{\partial \Phi_E}{\partial t} = a\,dx\,\frac{\partial E_y}{\partial t}$$

$$\boxed{\frac{\partial B_z}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E_y}{\partial t}} \quad L2$$

If we differentiate L1 with respect to x

$$\frac{\partial^2 E_y}{\partial x^2} = -\frac{\partial}{\partial x}\left(\frac{\partial B_z}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial B_z}{\partial x}\right) \qquad (3)$$

If we differentiate L2 wrt respect to t

$$\frac{\partial^2 B_z}{\partial t \partial x} = -\mu_0 \epsilon_0 \frac{\partial}{\partial t}\left(\frac{\partial E_y}{\partial t}\right) = -\mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2} \qquad (4)$$

Comparing (3) & (4)

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2} \quad \text{or} \quad \boxed{\frac{\partial^2 E_y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} = 0} \qquad (5)$$

Similarly if we differentiate L2 w.r.t x & L1 w.r.t t we get

$$\boxed{\frac{\partial^2 B_y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 B_y}{\partial t^2} = 0} \qquad (6)$$

32.3 Sinusoidal E & M waves.

Particularly important class of E & M waves are "sinusoidal waves". In fact, all E & M waves can be built up out of sinusoidal waves. These are waves in which both the x & the t dependence are sine or cosine waves.

The frequency f & wavelength of a wave are related by

$$c = \lambda f \quad \text{or} \quad \lambda = \frac{c}{f}$$

e.g $f = 60Hz$ $\lambda = \frac{3\times10^8}{60hz} = 5\times10^6 m = \underline{5000 km}$

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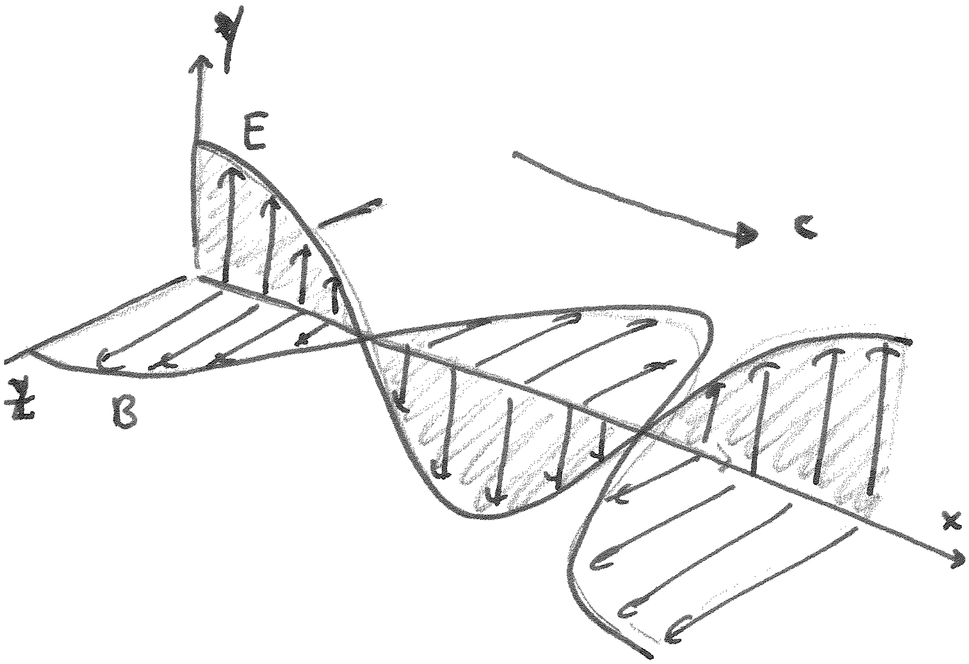
$f = 101.5 MHz$ $\lambda = \frac{3\times10^8}{101.5\times10^6 Hz} = \underline{2.45\ m}$



$$\boxed{\begin{aligned} E_y(x,t) &= E_{max}\cos(kx-\omega t) \\ B_z(x,t) &= B_{max}\cos(kx-\omega t) \end{aligned}}$$

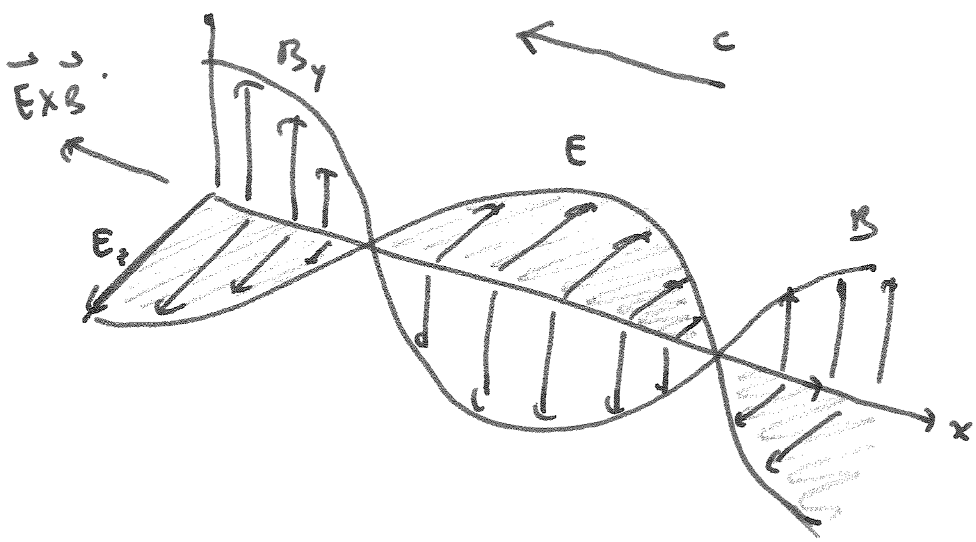
$E_{max} = c\,B_{max}$

in vectors

$$\boxed{\begin{aligned} \vec{E}(x,t) &= E_{max}\cos(kx-\omega t)\,\hat{j} \\ \vec{B}(x,t) &= B_{max}\cos(kx-\omega t)\,\hat{k} \end{aligned}}$$

Reverse direction $\cos(kx - \omega t) \rightarrow \cos(kx + \omega t)$.

e.g **FIELDS OF A LASER BEAM**



$\lambda = 10.6\,\mu m$

E = 1.5 MV/m along z axis

travelling in −x direction.

$$B_{max} = \frac{E_{max}}{c} = \frac{1.5 \times 10^6}{3 \times 10^8}$$

$$= \underline{\underline{5 \times 10^{-3}\ T}}$$

$$\vec{E} = 1.5 \times 10^6 \cos(kx + ct)\ \hat{k}$$

$$\vec{B} = 5 \times 10^{-3} \cos(kx + ct)\ \hat{y}.$$

Waves in Matter

$$\epsilon_0 \longrightarrow k\epsilon_0$$

$$\mu_0 \longrightarrow k_m\mu_0$$

because the internal fields are enhanced by the electric & magnetic polarization of the medium.

$$v = \frac{1}{\sqrt{k\epsilon_0 k_m \mu_0}} = \frac{1}{\sqrt{k k_m}} c < c$$

$$\boxed{n = \frac{1}{\sqrt{k k_m}}} = \text{index of refraction.} \qquad \boxed{v = \frac{c}{n}}$$

k & k_m are often strongly frequency dependent.

32.4 Energy + Momentum in Electromagnetic Waves

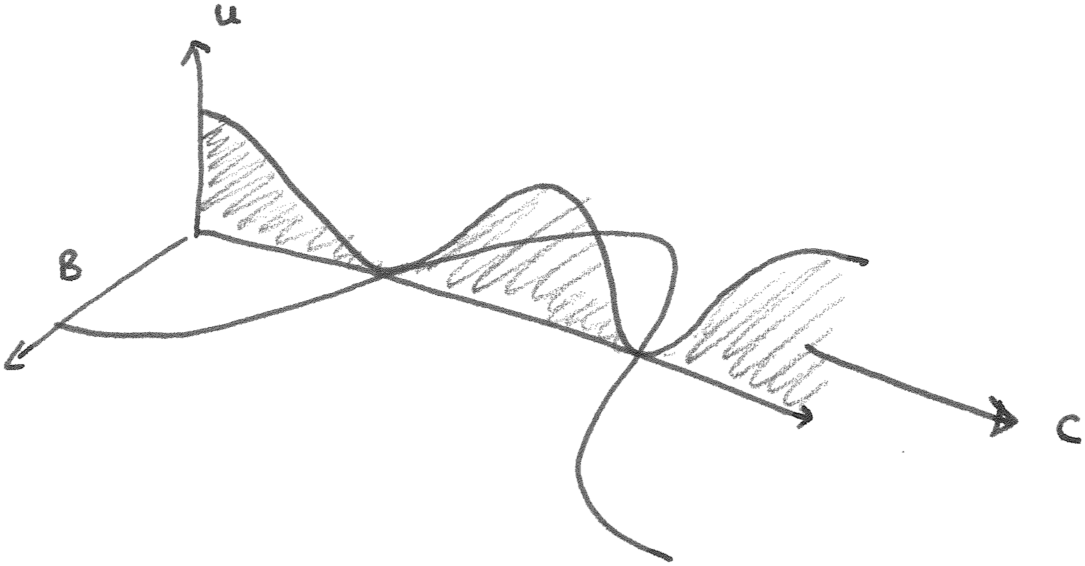
We have learnt that the energy density of the electric field is $\frac{1}{2}\epsilon_0 E^2$, that of the magnetic field is $\frac{1}{2}\frac{B^2}{\mu_0}$. The total is

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$$

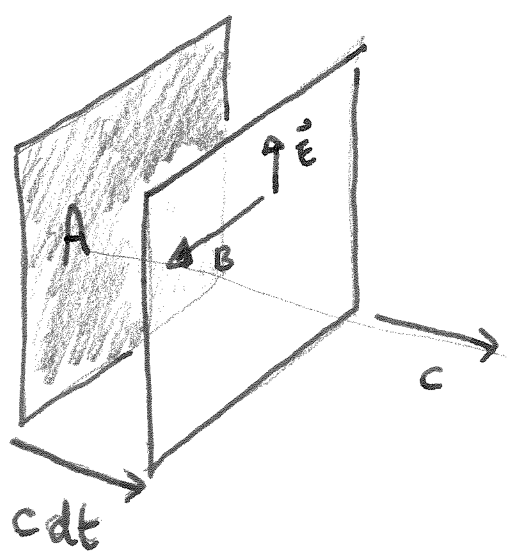
In an EM wave $B = E/c$, so

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{E^2}{\mu_0 c^2}$$

but $\mu_0\epsilon_0 = 1/c^2 \Rightarrow \epsilon_0 = \frac{1}{\mu_0 c^2} \Rightarrow$ $\boxed{u = \epsilon_0 E^2}$



The flow of energy is intimately related to the crossed electric + magnetic fields.



$$dU = u\,dV = \epsilon_0 E^2 (A c\,dt)$$

$$\boxed{S = \frac{1}{A}\frac{dU}{dt} = \epsilon_0 c E^2}$$

$$S = \frac{\epsilon_0}{\sqrt{\epsilon_0 \mu_0}} E^2 = \sqrt{\frac{\epsilon_0}{\mu_0}} E^2 = \frac{EB}{\mu_0}$$

In terms of vectors

$$\boxed{\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}}$$

"Poynting Vector"

Direction of $\vec{S}$ $\equiv$ direction of flow of radiation energy

Magnitude of S $\equiv$ intensity of radiation.

$$P = \int \vec{S} \cdot d\vec{A}$$ total power absorbed.

$$\vec{S}(\vec{x},t) = \frac{1}{\mu_0} \vec{E}(x,t) \times \vec{B}(x,t)$$

$(\hat{j} \times \hat{k} = \hat{i})$

$$= \frac{1}{\mu_0} \left[E_{max} \cos(kx - \omega t)\, \hat{j} \right] \times \left[B_{max} \cos(kx - \omega t)\, \hat{k} \right]$$

$$= \frac{1}{\mu_0} E_{max} B_{max} \cos^2(kx - \omega t)\ \hat{i}$$

or

$$\vec{S}(x,t) = \frac{1}{\mu_0} E_{max} B_{max} \left[\frac{1}{2} + \frac{1}{2}\cos[2(kx-\omega t)] \right] \hat{i}$$

$$S_{av} = \frac{1}{2\mu_0} E_{max} B_{max}$$

$$I = S_{av} = \frac{E_{max} B_{max}}{2\mu_0} = \frac{E_{max}^2}{2\mu_0 c} = \boxed{\frac{1}{2}\epsilon_0 c E_{max}^2}$$

$$\boxed{I = \epsilon_0 c E_{rms}^2}$$

e.g Laser

$$u = \epsilon_0 E_{rms}^2$$
$$= 8.85\times10^{-11}\times(1.5\times10^6)^2$$
$$= \underline{1.99\times10^2\ J/m^3}$$

$$I = uc = \underline{\underline{5.97\times10^{10}\ W/m^2}}$$

Radiation Pressure

Light carries both energy & momentum. When we go on to study modern physics, we will see that light is actually made up of tiny packets of radiation, or "photons". Each packet of radiation carries a certain amount of energy E & momentum p related by $E = pc$. This means that the energy & momentum density are related, and in particular

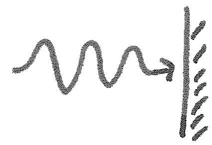
$$\text{momentum absorbed/unit time} = \frac{1}{c}\underbrace{\left(\text{energy absorbed/unit time}\right)}$$

$$pA = \frac{SA}{c}$$

where p is the "radiation pressure"

so for absorption

$$\boxed{p_A = \frac{\bar{S}}{c} = \frac{I}{c} = \frac{\epsilon_0 E_{max}^2}{2}}$$



For reflection, the reversal of each photon's momentum produces twice the change in momentum & hence twice the pressure

$$\boxed{p_{Reflection} = \frac{2\bar{S}}{c} = \frac{2I}{c} = \epsilon_0 E_{max}^2}$$