L22

A.C CIRCUITS

Power in our National grid comes in the form of an alternating current or "a.c" power supply.

Many electrical devices depend on this a.c supply & it is vital that we understand a.c. circuitry.

We'll see that we need to extend Ohm's law to include the concept of a frequency dependent resistance or "reactance" X. We will write

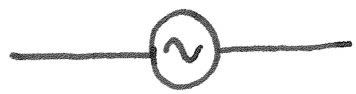
$$\boxed{V = I X}$$

(X depends on frequency).

where V & I are the amplitudes of the voltage & current.

31.1 Alternating Current

(a) Voltage



a.c source

$$\boxed{v = V \cos \omega t}$$

instantaneous voltage

AMPLITUDE

angular frequency

$\omega = 2\pi f$

$f = 60\,\text{Hz}$ N. America

$= 50\,\text{Hz}$ Europe/Asia.

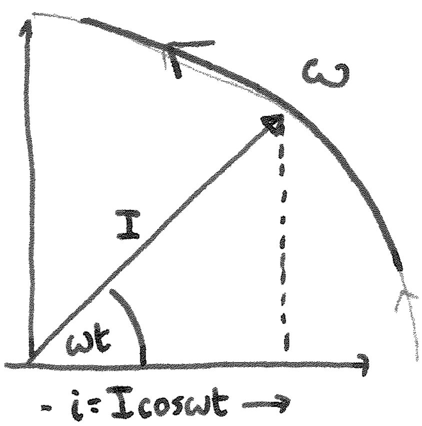
b) Current

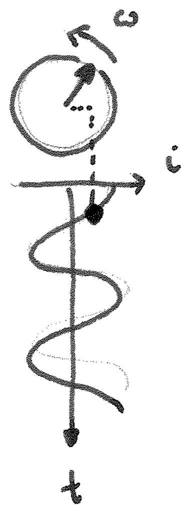
$$\boxed{i = I \cos \omega t}$$

instantaneous current

amplitude

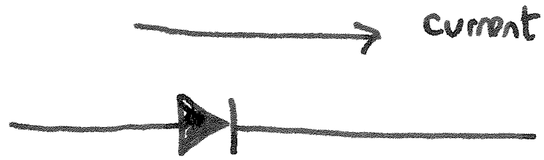
b) <u>Phasors</u> : a convenient way to represent sinusoidally varying quantities.



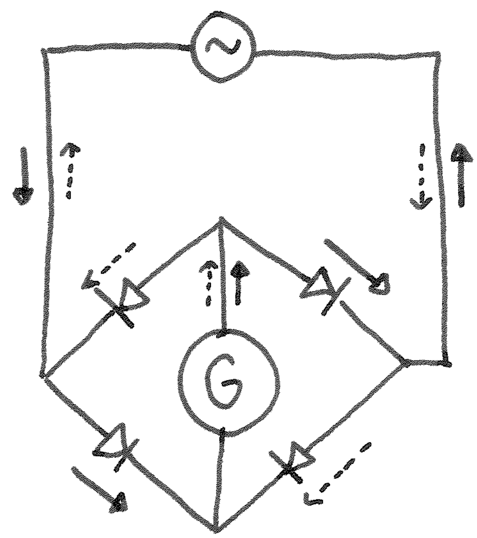


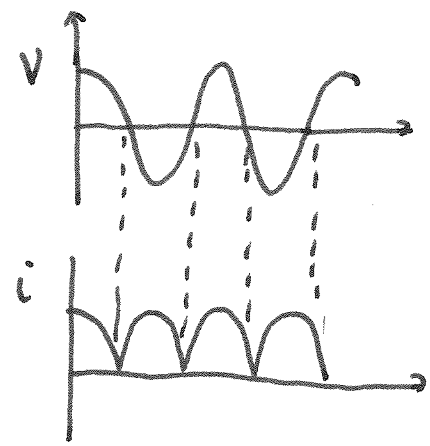
$i = I\cos\omega t$

We can measure an a.c current in various ways — with an oscilliscope — also by "rectifying" the current using a diode.



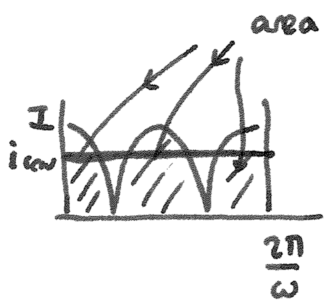
A diode only transmits a current in the direction of the arrow.





Current only flows one way through the galvanometer.

$$i_{rav} = \overline{i_{rectified}} = \frac{\int_0^{2\pi/\omega} I|\cos\omega t|\,dt}{\left(\frac{2\pi}{\omega}\right)}$$



$$= \frac{4\int_0^{\pi/2\omega} I\cos\omega t\,dt}{\left(\frac{2\pi}{\omega}\right)} = \frac{\frac{4I}{\omega}}{\frac{2\pi}{\omega}}$$

$$\boxed{i_{rav} = \left(\frac{2}{\pi}\right) I}$$

More useful concept is the r.m.s or "root-mean-square current

$$\boxed{i_{rms} = \sqrt{\overline{i^2}}}$$

$$i^2 = I^2\cos^2\omega t$$

Now remember that $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$

so

$$i^2 = I^2\left(\frac{1}{2} + \underbrace{\frac{1}{2}\cos 2A}_{\text{average} = 0}\right)$$

$$\overline{i^2} = \frac{I^2}{2}$$

$$\boxed{i_{rms} = \frac{I}{\sqrt{2}}}$$ root-mean squared current.

$$\boxed{V_{rms} = \frac{V}{\sqrt{2}}}$$ root-mean squared voltage.

Note that when we say we have a 120V a.c supply we mean $V_{rms} = 120V$, i.e $V = \sqrt{2} \times 120V = \underline{170V}$

e.g. Computer draws 2.7A from a 120V 60Hz line.

a) Average current

b) Average of the square of the current

c) Current amplitude.

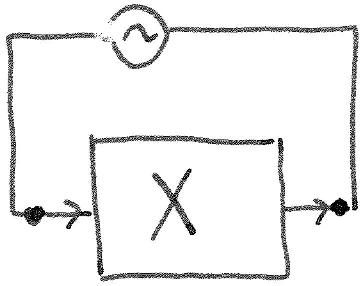
a) $i_{rms} = 2.7\ A$

Average current $= \overline{I} = 0$.

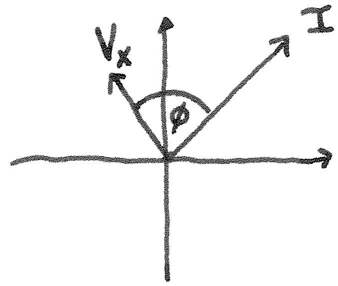
b) $i_{rms} = 2.7A$

c) $I = \sqrt{2} \times 2.7A = 3.8\ A.$

31.2 "REACTANCE"



$i = I \cos\omega t$



$i = I \cos\omega t$

$$\boxed{v_X = X I \cos(\omega t + \phi_X)}$$

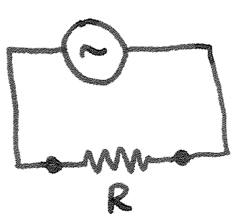
voltage amplitude

$$V_X = I X$$

reactance

X is called the "reactance" – it is a generalization of the concept of resistance.

e.g Resistor



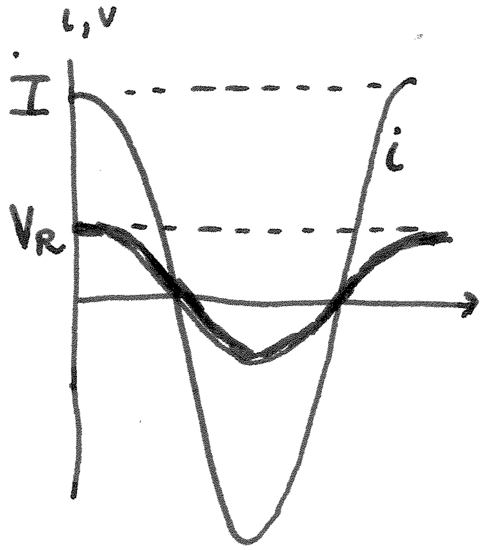
$$v_R = iR = (IR)\cos\omega t$$

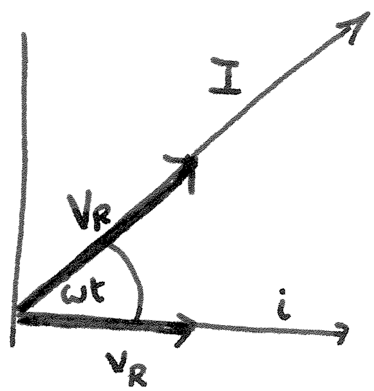
$$= V_R \cos\omega t$$

$V_R = IR$

$X = R$

$\boxed{\text{current + voltage IN PHASE}}$

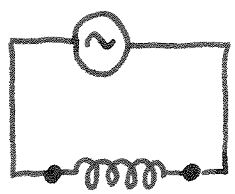




$$X_R = R$$

$$\phi_R = 0°$$

(b) Inductor

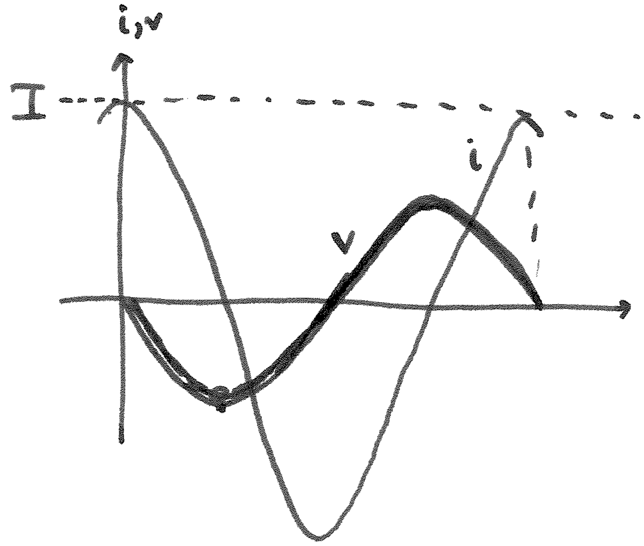


$$v = L\frac{di}{dt}$$

$$= L\frac{d}{dt}\, I\cos\omega t$$

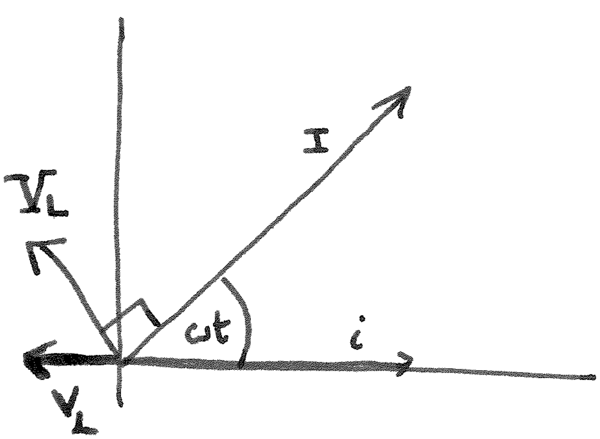
$$= -LI\omega\sin\omega t$$

$$\boxed{v = \omega L I \cos(\omega t + 90^\circ)}$$



$$\left\{\begin{array}{l} X_L = \omega L \\ \phi_L = 90^\circ \end{array}\right.$$

$$\boxed{V_L = \omega L I}$$



The amplitude of the voltage across the inductor is $(\omega L) \times I$; the voltage "leads" the current by $\frac{1}{4}$ cycle.

e.g. Require $I_L = 250\mu A$ in an inductor in a radio receiver when $V_L = 3.60V$ & $f = 1.6MHz$

a) What inductive reactance X_L is needed? What L?

b) If the voltage amplitude is constant, what will the current amplitude be at 16 MHz & 160MHz?

a) $$V_L = I X_L \Rightarrow X_L = \frac{V_L}{I_L} = \frac{3.6V}{250 \times 10^{-6} A}$$

$$= 1.44 \times 10^4 \Omega$$

$$= \underline{14.4 k\Omega}$$

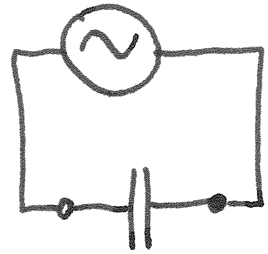
$$\omega L = X_L \Rightarrow L = \frac{1.44 \times 10^4}{2\pi \times 1.6 \times 10^6} = \underline{1.43 \times 10^{-3} H = 1.43 mH}$$

b) $$I_L = \frac{V_L}{X_L} = \frac{V_L}{\omega L}$$

$$I_L(16MHz) = \frac{3.60V}{2 \times \pi \times (16 \times 10^6) \times 1.43 \times 10^{-3}} = 25 \times 10^{-6} A = 25 \mu A$$

$$I_L(160MHz) = 2.5 \mu A$$

c) Capacitor

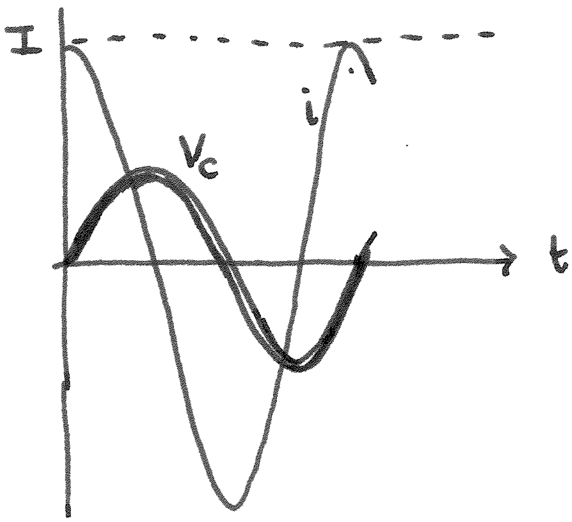


$$i = \frac{dq}{dt} = I\cos\omega t$$

$$q = \frac{I}{\omega}\sin\omega t$$

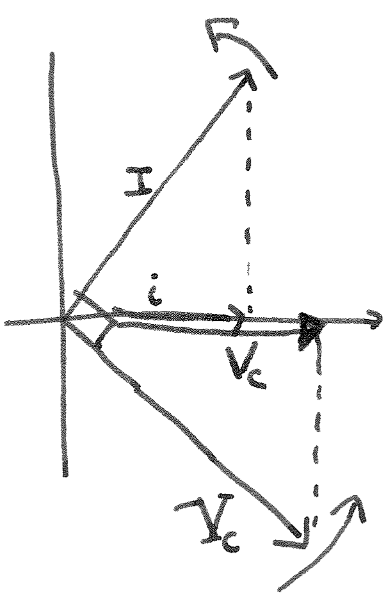
$$v = \frac{q}{C} = \frac{I}{\omega C}\sin\omega t$$

$$\boxed{v_c = \frac{I}{\omega C}\cos(\omega t - 90^\circ)}$$



$$V_c = \frac{1}{\omega C} I_c$$

$$\boxed{X_c = \frac{1}{\omega C} \qquad \phi_c = -90^\circ}$$



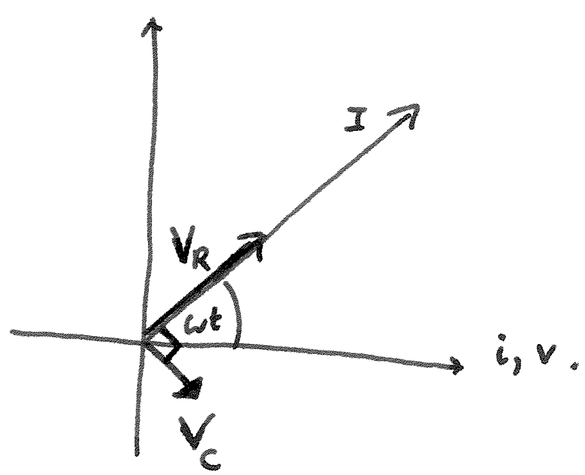
$$V_c = I_c X_c$$

c) $$v_c = V_c \cos(\omega t - \pi/2)$$

$$V_c = I X_c = 6 \times 10^{-3} \times 80 = 0.48\ \text{V}$$

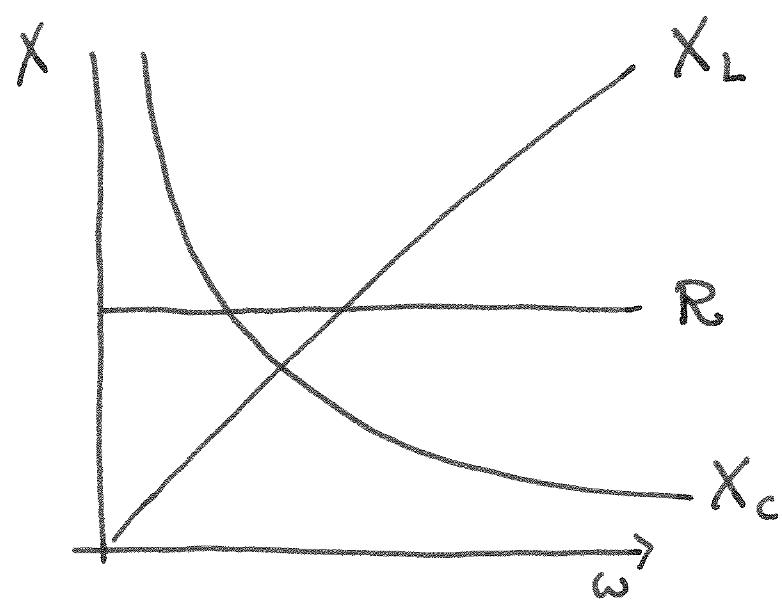
$$v_c = 0.48\text{V} \cos\left((2500\ \text{rad/s})t - \pi/2\ \text{rad}\right).$$

n.b. converted 90° to radians since argument of cosine is given in radians.



Summary

Circuit element	Amplitude relation	Circuit quantity	ϕ
Resistor	$V_R = IR$	R	0
Inductor	$V_L = IX_L$	$X_L = \omega L$	90°
Capacitor.	$V_C = IX_C$	$X_C = 1/\omega C$	-90°



$$\boxed{V = IX}$$

$X_C \to \infty$ at low frequencies (high pass)

$X_L \to \infty$ at high frequencies (low pass)