This week we're going to learn about electric fields. We will see that the existence of Coulomb forces tells us that space is filled with electric fields. They produce Coulomb forces on particles but they are also produced by changing magnetic fields. Electric fields keep matter together & they carry energy—sometimes vast amounts of it. Indeed the electric field is a kind of "elasticity" of space & when we throw the electric field we create vibrains—vibrains that are the origin of light, radiant & wireless.
\[ \vec{E} = \lim_{q_0 \to 0} \left( \frac{\vec{F}_0}{q_0} \right) \]

\[ \vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r} \]

\[ \left( \hat{r} = \frac{\vec{r}}{r} = \frac{x\hat{i} + y\hat{j}}{\sqrt{x^2 + y^2}} \right) \]
Concept: think of the electric force as a local property of space. Each charge changes space around it to produce an electric field.

- The force will be proportional to the charge.
- It will always point in the same direction.

\[ \frac{\vec{F}_0}{q_0} = \vec{E} \]

If I put a charge here, what force will act on it?

\[ \vec{E} = \text{force per unit charge.} \]
A property of each point in space — indeed we write \( \vec{E}(\vec{x}) \) to show it depends on the position \( \vec{x} \).

Similar idea used to explain gravity, where

\[
\frac{\vec{F}_m(\vec{x})}{m} = \vec{g}(\vec{x}) \quad \text{gravitational field.}
\]

E.g. What is the field produced by a point charge \( q \)?

\[
\vec{E} = \frac{\vec{F}_o}{q_0} = \frac{1}{4\pi\epsilon_0} \left( \frac{q}{r^2} \right) \hat{r}
\]

or \( \frac{kq}{r^2} \)

\[
\hat{r} = \cos \alpha \hat{i} + \sin \alpha \hat{j}
\]
Numerical example — suppose \( r = 3 \text{ m} \) \( q = -1 \text{ nC} \)

\[
E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} = 9 \times 10^9 \text{ Nm/C}^2 \times \frac{(-10^{-9} \text{ C})}{(3 \text{ m})^2}
\]

\[
= (-1 \times 10^{-7} \text{ N}) \hat{r} \quad \text{(points towards the charge)}
\]

Notice how field lines converge on a \(-\text{ve} \) charge, emerge from a \(+\text{ve} \) charge.
Field distributions

Fields emerge from +ve and converge to -ve.

Field lines can only disappear or reappear at a charge.

Field lines are smooth.

Field lines 1 surface of a conductor.
Electron in a uniform field

\[ \mathbf{F} = q \mathbf{E} \]
\[ = -e \mathbf{E} \]

Q. Suppose \( E = 10^6 \text{N/C} \)

i) What is the acceleration of the electron?

\[ \mathbf{a} = \frac{\mathbf{F}}{m} \quad (\text{Newton}) \]

\[ a_y = -\frac{e \mathbf{E}_y}{m} = -\frac{1.6 \times 10^{-19} \text{C} \times 10^6 \text{N/C}}{9.1 \times 10^{-31} \text{kg}} = -1.76 \times 10^{17} \text{m/s}^2 \]

This is a method to produce high energy electrons.
ii) Work done by field = $F \times d = \Delta \text{ kinetic energy} = \frac{1}{2} \ m \ v^2$

$F \times d = (eE) \ d = 1.6 \times 10^{-19} \ C \times 10^6 \ N/C \times 0.01 \ m$

$= 1.6 \times 10^{-15} \ J$

iii) $d = \frac{1}{2} \ a t^2 \implies t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2 \times 0.01}{1.76 \times 10^{13}}}$

$= 3.4 \times 10^{-10} \ s$

$= 0.34 \ ns$
21.5 Many Charges

\[ \vec{F}_{\text{Tot}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \]

\[ = q_0 \vec{E}_1 + q_0 \vec{E}_2 + q_0 \vec{E}_3 \]

\[ \vec{E} = \frac{\vec{F}_{\text{Tot}}}{q_0} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \]

**Principle of Superposition**

Total field = sum of fields from each charge.
We don't have to have just point charges—we can smear the charge distribution along a line, a plane or a volume.

\[ q = \lambda l \]

\[ q = \sigma A \]

\[ q = \rho V \]
Field near dipole

What is the field at a, b & c?

\[ \mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 \]

\[ \mathbf{E}_1 = \frac{k \mathbf{q}}{r_1^2}, \quad \mathbf{E}_2 = -\frac{k \mathbf{q}}{(r_2)^2} \]

a) \[ \mathbf{E}_{1a} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{(6 \times 10^{-2})^2} \hat{\mathbf{i}} = 3 \times 10^4 \text{ N/C} \hat{\mathbf{i}} \]

\[ \mathbf{E}_{2a} = \frac{9 \times 10^9 \times (-12 \times 10^{-9})}{(4 \times 10^{-2})^2} \hat{\mathbf{i}} = 6.75 \times 10^4 \text{ N/C} \hat{\mathbf{i}} \]

\[ \mathbf{E} = (6.75 \times 10^4 \text{ N/C}) \hat{\mathbf{i}} \]
b) \[ \vec{E}_{1b} = -12 \text{ N/C} \hat{i} \]

\[ \vec{E}_{2b} = \frac{9 \times 10^9 \times (-12 \times 10^{-9}) \hat{i}}{13^2} = (0.64 \text{ N/C}) \hat{i} \]

\[ \vec{E}_b = (-12 + 0.64) \hat{i} = (-11.36 \text{ N/C}) \hat{i} \]

(c) \[ \vec{E}_{1c} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{(13)^2} \hat{r}_1 = 0.64 \hat{r}_1 \]

\[ \hat{r}_1 = \cos \alpha \hat{i} + \sin \alpha \hat{j} \]

\[ = \frac{5}{13} \hat{i} + \frac{12}{13} \hat{j} \]

\[ \vec{E}_{1c} = 0.64 \times \frac{5}{13} \hat{i} + 0.64 \times \frac{12}{13} \hat{j} \text{ N/C} \]

\[ \vec{E}_{2c} = -0.64 \times \left( -\frac{5}{13} \hat{i} + \frac{12}{13} \hat{j} \right) \text{ N/C} \]

\[ \vec{E}_{\text{Tot}} = (0.49 \text{ N/C}) \hat{i} \]
Field near a disk

\[ dE = \frac{k \, dQ}{x^2 + r^2} \]

\[ dE_x = dE \cos \alpha \]

\[ E_{x\text{total}} = \int dE_x = \frac{k \cos \alpha}{x^2 + r^2} \int dQ \]

\[ = \frac{Q \, r \cos \alpha}{x^2 + r^2} \]

\[ E_x = \frac{Q}{4\pi \varepsilon_0} \frac{x}{(x^2 + r^2)^{3/2}} \]

\[ E_y = 0 \]
\[ dQ = \sigma dA = \sigma 2\pi r dr \]

\[ dE_x = \frac{dQ}{4\pi \varepsilon_0} \frac{x}{(r^2 + x^2)^{3/2}} = \frac{\sigma x}{4\pi \varepsilon_0} \frac{2\pi r dr}{(r^2 + x^2)^{3/2}} \]

\[ E_x = \frac{\sigma x}{2\varepsilon_0} \int_0^R \frac{r dr}{(r^2 + x^2)^{3/2}} \]

\[ = \frac{\sigma x}{2\varepsilon_0} \int_x^{\sqrt{R^2 + x^2}} \frac{z dz}{z^{3/2}} = \frac{\sigma x}{2\varepsilon_0} \left[ \frac{-1}{\sqrt{x^2 + r^2}} + \frac{1}{r} \right] \]

\[ R \to \infty \]

\[ E = \frac{\sigma}{2\varepsilon_0} \left[ 1 - \frac{x}{\sqrt{x^2 + R^2}} \right] \]
\[ E_x = \frac{-\sigma}{2\varepsilon_0} \]

\[ E_x = \frac{\sigma}{2\varepsilon_0} \]

Infinite outline

Two oppositely charged sheets \( \equiv \) capacitor

\[ E_1 + E_2 = 0 \]

\[ E_1 + E_2 = 0 \]

\[ \frac{E}{\varepsilon_0} = \frac{Q}{A \varepsilon_0} \]