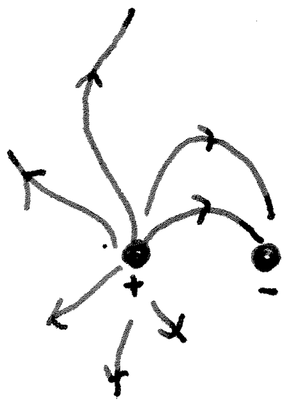
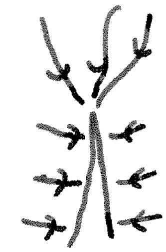
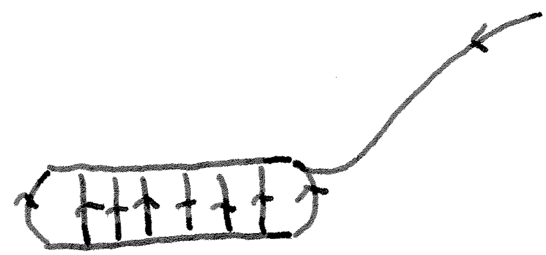
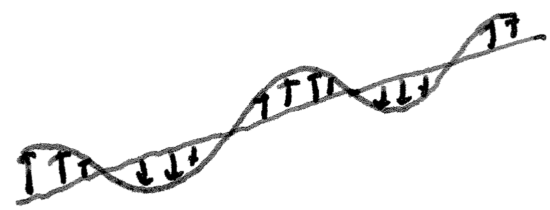
# ELECTRIC FIELDS



This week we're going to learn about electric fields. We will see that the existence of Coulomb forces tells us that space is filled with electric fields. They produce Coulomb forces on particles but they are also produced by changing magnetic fields.





Electric fields keep matter together & they carry energy – sometimes – vast amounts of it. Indeed the electric field is a kind of "elasticity" of space & when we twang the electric field we create vibrations – vibrations that are the origin of light, radiation + wireless.

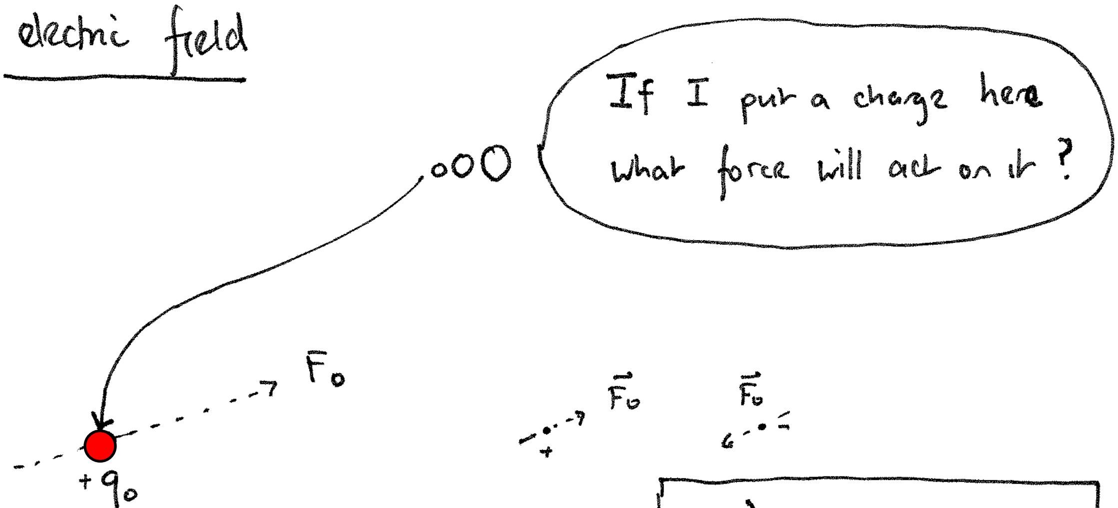
$$\vec{E} = \lim_{q_0 \to 0} \left( \frac{\vec{F}_0}{q_0} \right)$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

$$\left( \hat{r} = \frac{\vec{r}}{r} = \frac{x\hat{\imath} + y\hat{\jmath}}{\sqrt{x^2+y^2}} \right)$$

21.4 **Electric Fields**

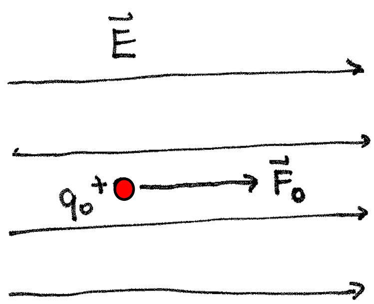
Concept : think of the electric force as a local property of space. Each charge changes space around it to produce an **electric field**

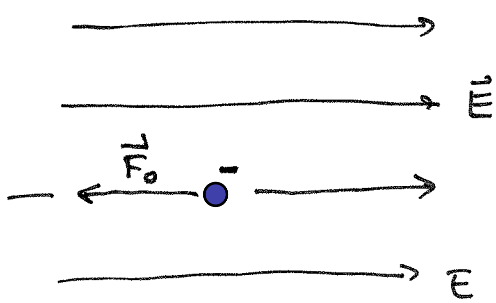



- The force will be proportional to the charge
- It will always point in the same direction

$$\boxed{\frac{\vec{F}_0}{q_0} = \vec{E} \quad \text{independent of charge}}$$

E = Force per unit charge.



$\vec{E}$ a property of each point in space — indeed we write

$\vec{E}(\vec{x})$ to show it depends on the position $\vec{x}$.

Similar idea used to explain gravity, where

$$\frac{\vec{F}_m(x)}{m} = \vec{g}(x) \qquad \text{gravitational field.}$$

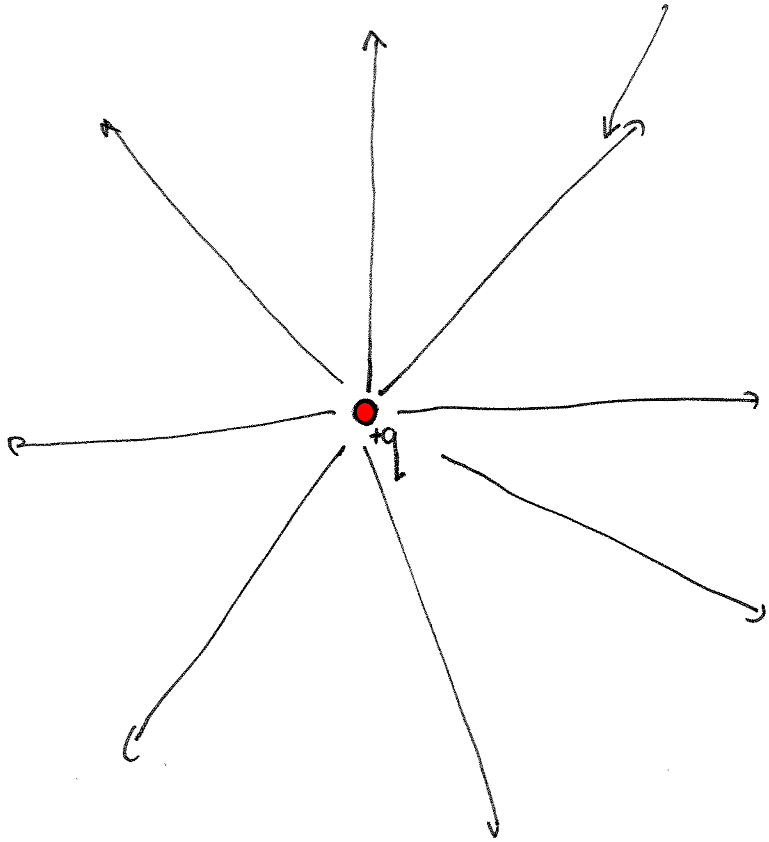
e.g. What is the field produced by a point charge $q$?

$$\vec{F}_0 = \frac{1}{4\pi\epsilon_0}\left(\frac{q\,q_0}{r^2}\right)\hat{r}$$

$$\Rightarrow \boxed{\vec{E} = \frac{\vec{F}_0}{q_0} = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r^2}\right)\hat{r}} \quad \text{or} \quad \frac{kq}{r^2}\hat{r}$$

$$\hat{r} = \cos\alpha\,\hat{\imath} + \sin\alpha\,\hat{\jmath}$$

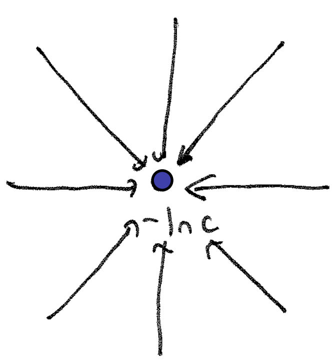
weaker field ≡ lines further apart.




Numerical example – suppose $r = 3m$ $q = -1nC$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} = 9\times10^9\, Nm/C^2 \times \frac{(-10^{-9}C)}{(3m)^2}\hat{r}$$

$$= \left(-1 \ N\right)\hat{r}$$ (points towards the charge




Notice how field lines converges on a –ve charge, emerge from a positive charge.

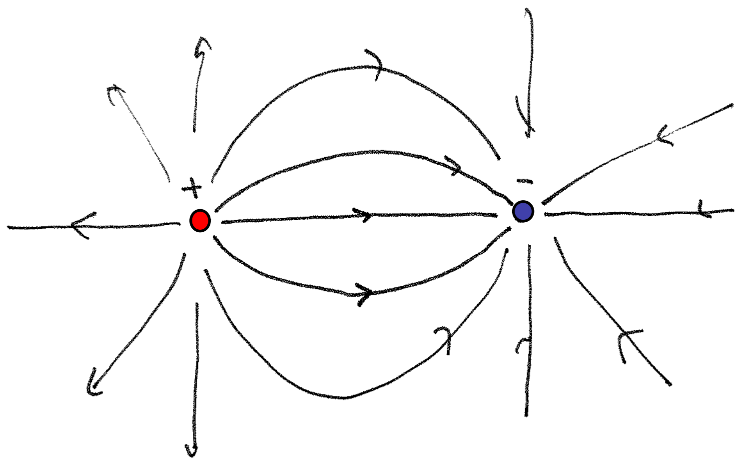
# Field distributions

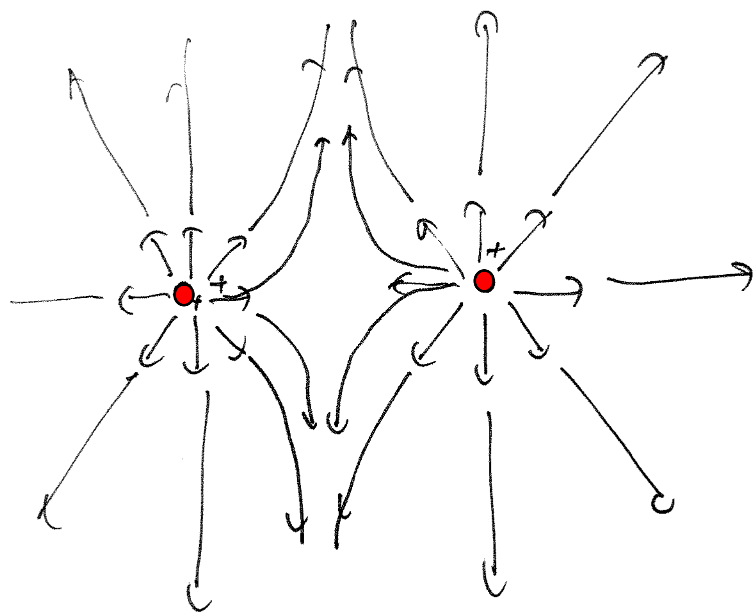
Fields emerge from +ve
converge to −ve.

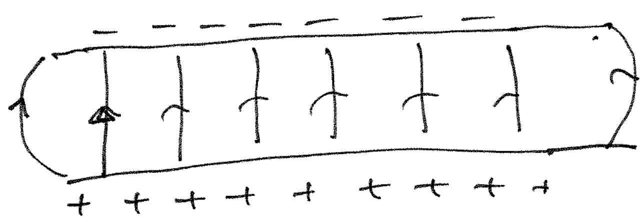
Field lines can only disappear
or reappear at a charge.

Field lines are smooth

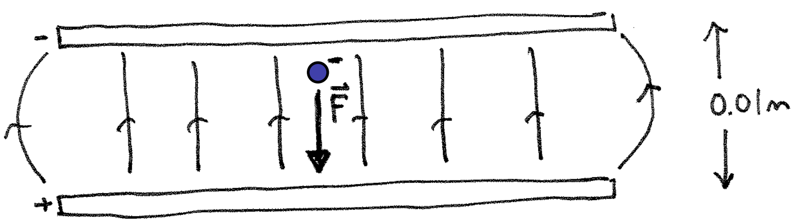
Field lines ⊥ surface of a conductor

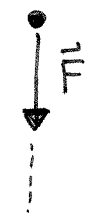


# Electron in a uniform field




$$\boxed{\begin{aligned}\vec{F} &= q\vec{E} \\ &= -e\vec{E}\end{aligned}}$$

Q. Suppose $E = 10^6$ N/C

i) what is the acceleration of the electron?

ii) how much Kinetic Energy does it gain in moving from the -ve to the positive plates?

iii) how long does it take?

i) $\vec{a} = \dfrac{\vec{F}}{m}$ (Newton)

$$a_y = -\frac{e E_y}{m} = \frac{-1.6\times 10^{-19}\,\mathrm{C} \times 10^{6}\,\mathrm{N/C}}{9.1\times 10^{-31}\,\mathrm{kg}} = \underline{-1.76\times 10^{17}\,\mathrm{m/s}}$$

Huge!

This is a method to produce high energy electrons.

ii) Work done by field = $F \times d$ = $\Delta$ kinetic energy

$$= \frac{1}{2} m v^2$$

$$F \times d = (eE)\, d = 1.6 \times 10^{-19}\,\mathrm{C} \times 10^{6}\,\mathrm{N/C} \times 0.01\,\mathrm{m}$$

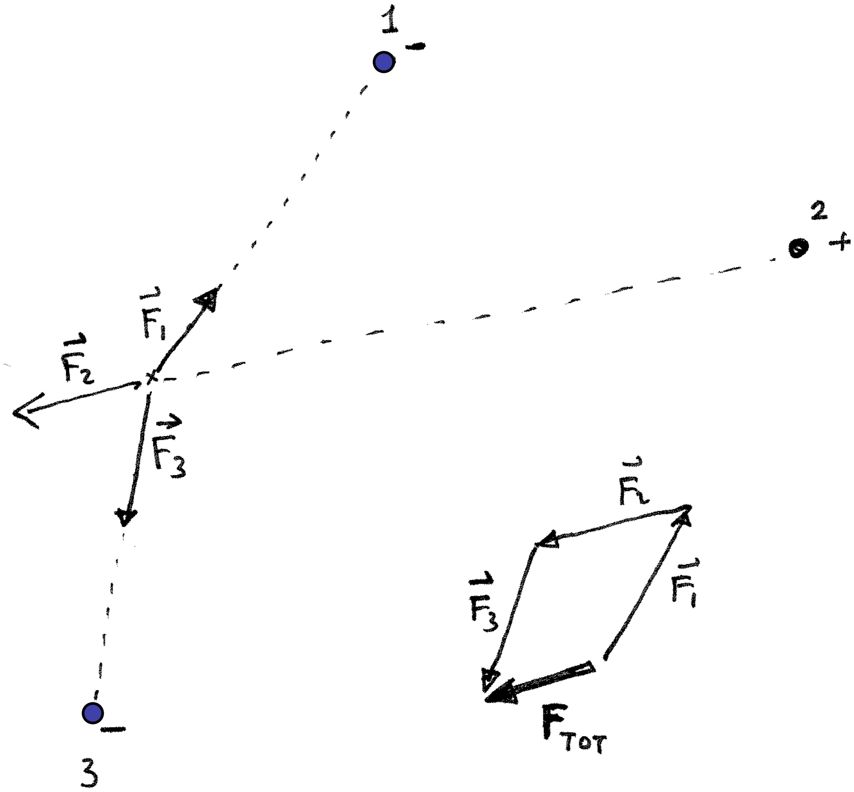
$$= 1.6 \times 10^{-15}\,\mathrm{J}$$

iii) $$d = \frac{1}{2} a t^2 \Rightarrow t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2 \times 0.01}{1.76 \times 10^{17}}}$$

$$= 3.4 \times 10^{-10}\,\mathrm{s}$$

$$= \underline{0.34\,\mathrm{ns}}$$

# 21.5 Many Charges




$$\vec{F}_{TOT} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

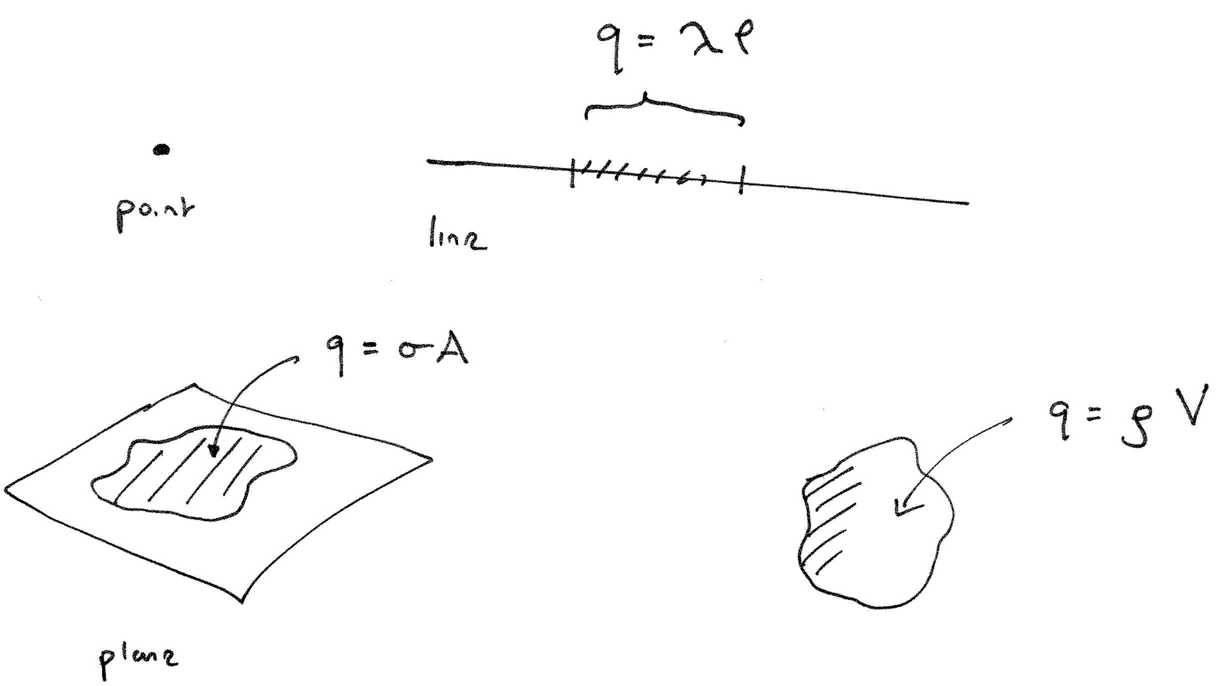
$$= q_0 \vec{E}_1 + q_0 \vec{E}_2 + q_0 E_3$$

$$\boxed{\vec{E} = \frac{\vec{F}_{TOT}}{q_0} = E_1 + E_2 + E_3}$$

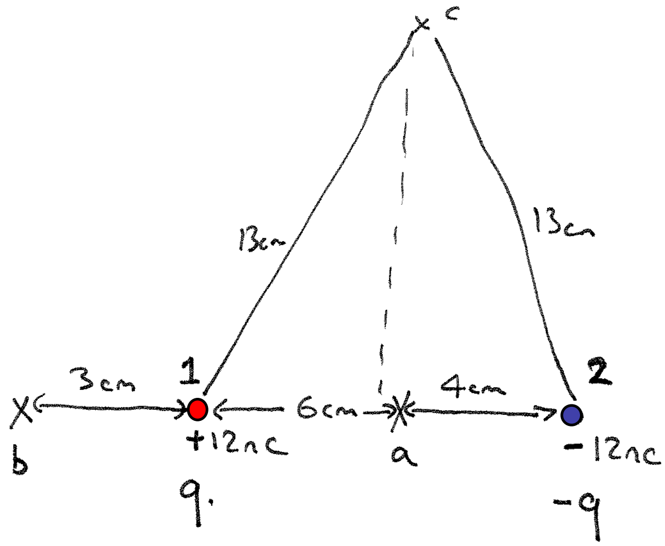
PRINCIPLE OF SUPERPOSITION

Total field = sum of fields from each charge.

We don't have to have just point charges — we can smeer the charge distribution along a line, a plane or a volume.



e.g Field near dipole




What is the field at a, b & c ?

$$\vec{E} = \vec{E}_1 + \vec{E}_2 \qquad \left\{ \begin{array}{l} \vec{E}_1 = \dfrac{k\,q}{r_1^2}\,\hat{r}_1 \\[2ex] \vec{E}_2 = -\dfrac{k\,q}{(r_2)^2}\,\hat{r}_2 \end{array} \right.$$

a) $$\vec{E}_{1a} = \frac{9\times10^{9} \times 12\times10^{-9}}{(6\times10^{-2})^2}\,\hat{\imath} = 3\times10^{4}\ \mathrm{N/c}\ \hat{\imath}$$

$$\vec{E}_{2a} = \frac{9\times10^{9} \times(-12\times10^{-9})}{(4\times10^{-2})^2}\,(-\hat{\imath}) = 6.75\times10^{4}\ \mathrm{N/c}\ \hat{\imath}$$

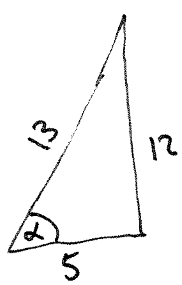
$$\vec{E} = \left(9.75\times10^{4}\ \mathrm{N/c}\right)\hat{\imath}$$

b) $\vec{E}_{1b} = -12\ N/C\ \hat{i}$

$$\vec{E}_{2b} = \frac{9\times10^9 \times (-12\times10^{-9})}{13^2}(-\hat{i}) = \underline{(0.64\ N/C)\ \hat{i}}$$

$$\vec{E}_b = (-12 + 0.64)\hat{i} = \underline{\underline{(-11.36\ N/C)\ \hat{i}}}$$

(c) $$\vec{E}_{1c} = \frac{9\times10^9 \times 12\times10^{-9}}{(13)^2}\hat{r}_1 = 0.64\ \hat{r}_1$$



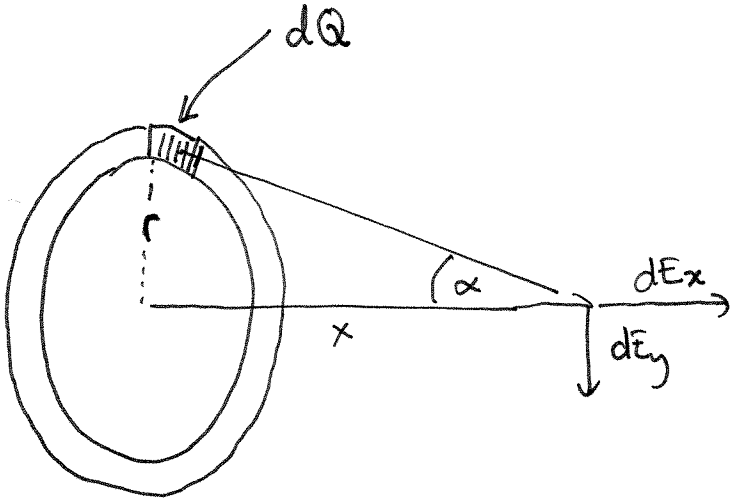

$$\hat{r}_1 = \cos\alpha\ \hat{i} + \sin\alpha\ \hat{j}$$
$$= \frac{5}{13}\hat{i} + \frac{12}{13}\hat{j}$$

$$\vec{E}_{1c} = 0.64 \times \frac{5}{13}\hat{i} + 0.64 \times \frac{12}{13}\hat{j} \quad N/C$$

$$\vec{E}_{2c} = -0.64 \times \left(-\frac{5}{13}\hat{i} + \frac{12}{13}\hat{j}\right) N/C$$

$$\underline{E_{TOT} = (0.49\ N/C)\ \hat{i}}$$

## Field near a disk




$$dE = k \frac{dQ}{x^2+r^2}$$

$$dE_x = dE\cos\alpha$$

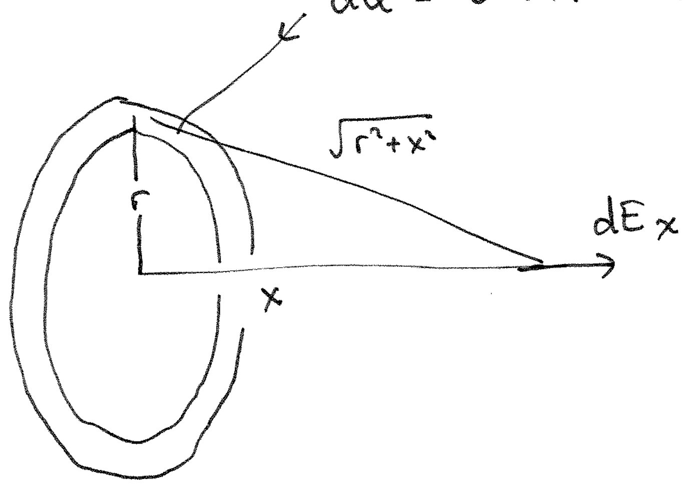
$$E_{x\,TOT} = \int dE_x = \frac{k\cos\alpha}{x^2+r^2}\int dQ$$

$$= \frac{Qk\cos\alpha}{x^2+r^2}$$

$$\boxed{E_x = \frac{Q}{4\pi\epsilon_0}\frac{x}{(x^2+r^2)^{3/2}}} \qquad E_y = 0$$

## Surface

$$dQ = \sigma\, dA = \sigma\, 2\pi r\, dr$$




$$dE_x = \frac{dQ}{4\pi\epsilon_0} \frac{x}{(r^2+x^2)^{3/2}} = \frac{\sigma x}{4\pi\epsilon_0} \frac{2\pi r\, dr}{(r^2+x^2)^{3/2}}$$

$$E_x = \frac{\sigma x}{2\epsilon_0} \int_0^R \frac{r\, dr}{(x^2+r^2)^{3/2}}$$

$z^2 = x^2 + r^2$

$2z\,dz = 2\cancel{x}\,dr$

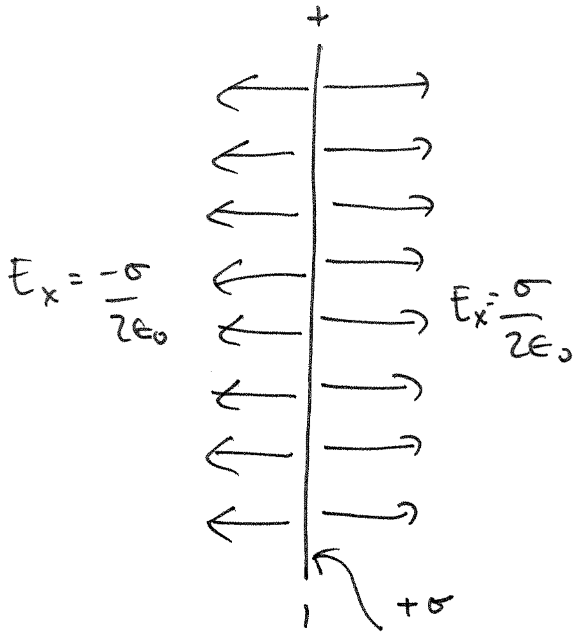
$z \in \left[x, \sqrt{R^2+x^2}\right]$

$$= \frac{\sigma x}{2\epsilon_0} \int_x^{\sqrt{R^2+x^2}} \frac{z\, dz}{z^{3/2}} = \frac{\sigma x}{2\epsilon_0} \left[ \frac{-1}{\sqrt{x^2+R^2}} + \frac{1}{x} \right]$$

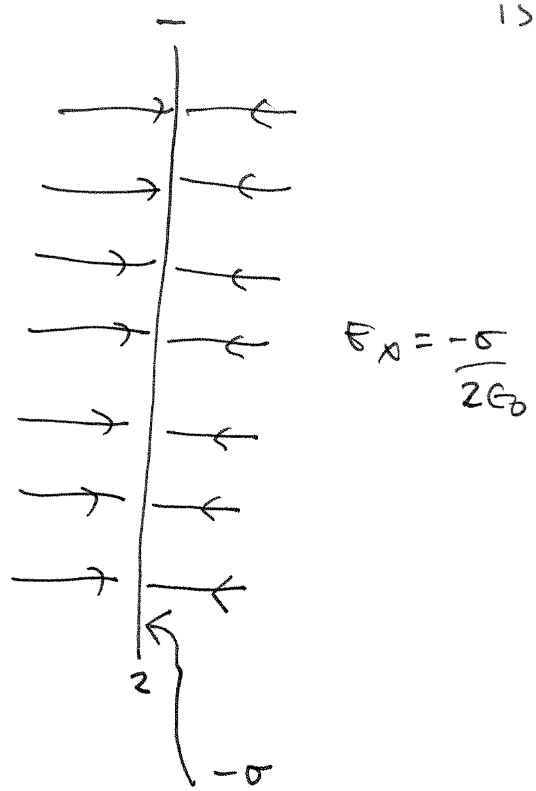
$$= \frac{\sigma}{2\epsilon_0} \left[ 1 - \frac{x}{\sqrt{x^2+R^2}} \right]$$

$R \to \infty$

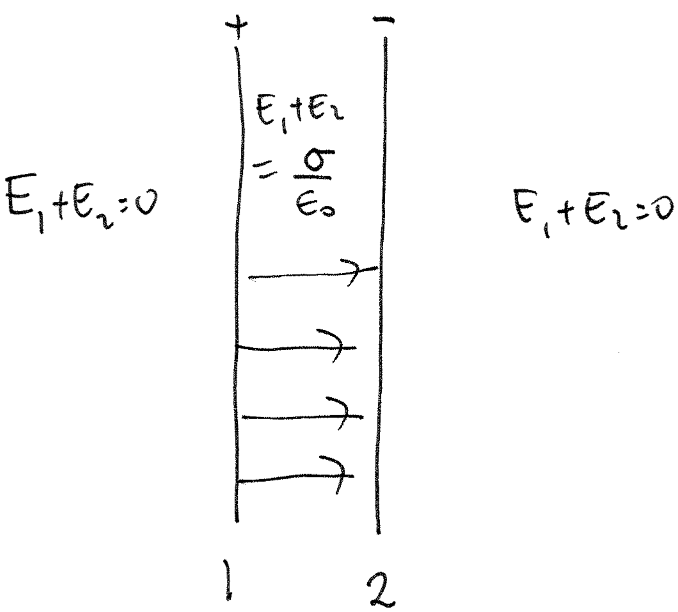
$$E = \frac{\sigma}{2\epsilon_0}$$




Infinite surface




Two oppositely charged sheets $\equiv$ capacitor




$$\boxed{E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}}$$