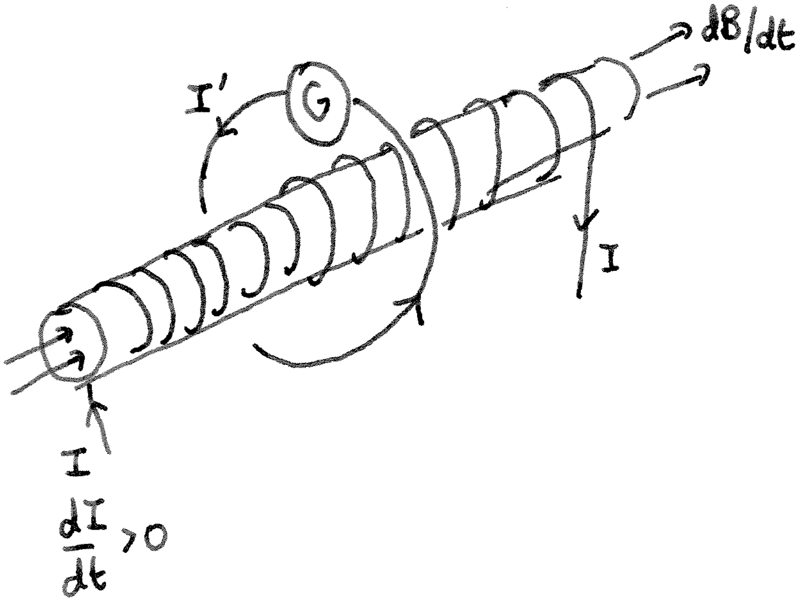
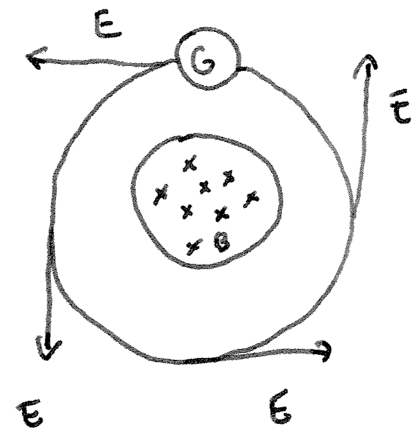
L19 MAXWELL'S EQUATIONS

Today we are going to close the last link in our quest to formulate the complete set of laws which govern the electromagnetic field. This last step will be to formulate Faraday's law as an equation linking the induced electric field to the changing magnetic flux. We will see that changing magnetic fields always induce circulating electric fields. In metals this gives rise to the phenomenon of "eddy currents".

29.5 Induced Electric Fields

We've seen that electric fields are induced inside a moving conductor, but they are also induced in a stationary conductor, if the magnetic flux is changing





The induced field is the only way we can account for the induced current in the conductor. Notice that the conductor is not even in the magnetic field, yet a current is still induced in the conductor. We saw

before that the field in the solenoid is

$$B = \mu_0 n I$$

so the flux in the solenoid is

$$\Phi_B = \mu_0 n I A$$

The induced e.m.f, by Faraday's law is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\mu_0 n A \frac{dI}{dt}$$

The induced current $I' = \mathcal{E}/R$, where R is

the resistance of the current loop.

But if there is a current, there must be an electric field to drive the current. We recall that

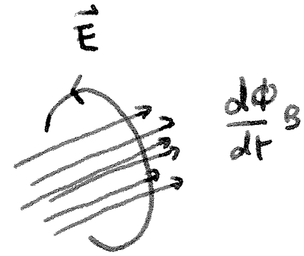
$$\mathcal{E} = V_a - V_b = \int_a^b \vec{E}.d\vec{\ell}$$

The induced e.m.f must then be the integral of the field right around the current loop, i.e

$$\mathcal{E} = \oint \vec{E}.d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

Faraday's law of induction is then written

$$\boxed{\oint_{loop.} \vec{E}.d\vec{\ell} = -\frac{d\Phi_B}{dt}}$$



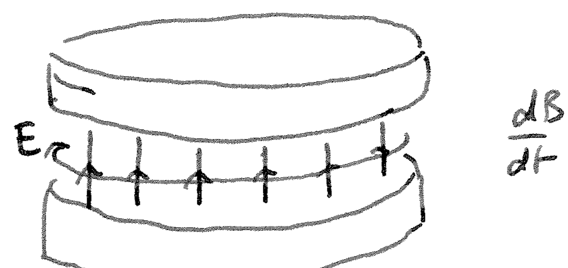
The minus sign tells us that the sense of rotation around the increasing flux is left-handed.

If the path is circular & the magnitude of the field uniform, then

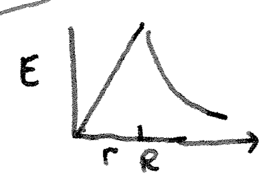
$$\int \vec{E}.d\vec{\ell} = -2\pi r \, E_{\text{anticlockwise}}$$

$$|E| = \frac{1}{2\pi r}\left|\frac{d\Phi_B}{dt}\right|$$

e.g. uniformly increasing field in z direction, calculate dependence of field on radius

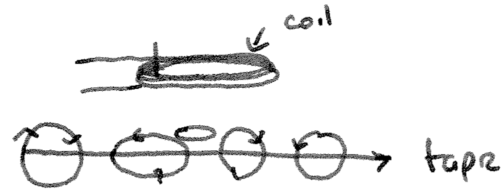


$$\frac{d\Phi_B}{dt} = \pi r^2 \frac{dB_z}{dt} \Rightarrow E = \frac{r}{2}\frac{dB_z}{dt}$$

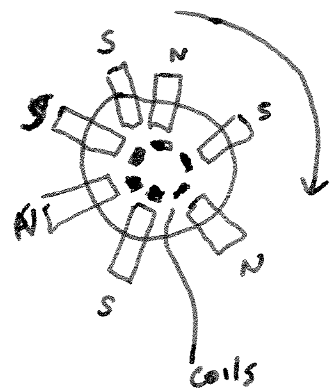


Magnetically induced electric fields important in

- playback coil of tape deck



- read head of disk drive
- pickup in electric guitar
- alternators in cars



⋮

e.g. Solenoid + $n = 500$ turns/m, $\frac{dI}{dt} = 100\ A/s$, $A = 4\text{cm}^2 = 4\times10^{-4}\ m^2$

a) Find induced emf $\mathcal{E}$ b) find the induced field, 2cm from coil axis.

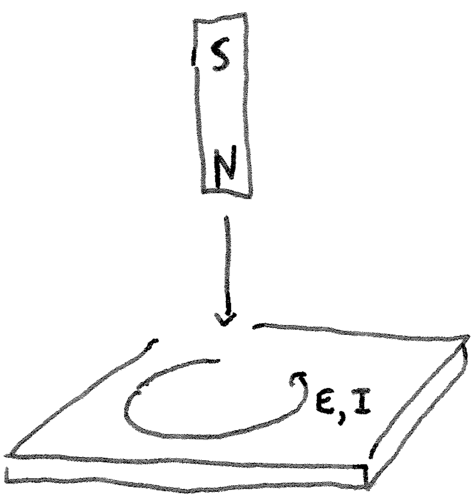
a) $$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\mu_0 n A \frac{dI}{dt} = -(4\pi\times10^{-7})(500\ \text{turns/m}) \times (4\times10^{-4}\ m^2)(100\ A/s)$$

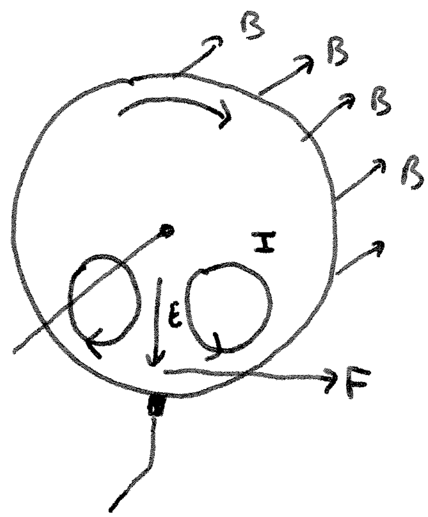
$$= -25\times10^{-6}\ Wb/s = -25\times10^{-6}\ V = -25\ \mu V.$$

$$E(2\pi r) = |\mathcal{E}| \qquad E = \frac{|\mathcal{E}|}{2\pi r} = \frac{25\times 10^{-6}\,V}{2\pi\,(0.02m)} = 2\times 10^{-4}\,V/m$$

29.6 Eddy currents

Induced field $\longrightarrow$ induced currents





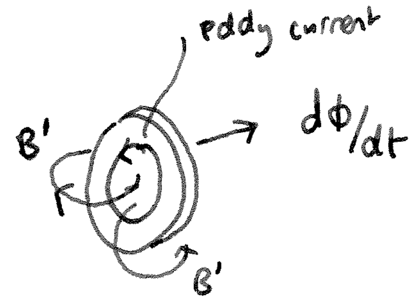
Notice in second example direction of $\vec{F} = I\vec{L}\times\vec{B}$ provides a breaking torque.

Applications

- Eddy current breaking

- Metal detectors

  (pick up magnetic field of eddy currents)

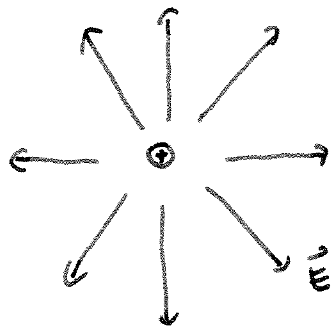


- R.F. heaters (melting metals)

Heating effect is generally wasteful — particularly in transformers.

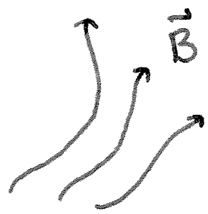
19.9.

MAXWELLS EQNS (Maxwell c. 1866)



$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

Gauss' law for electric fields.



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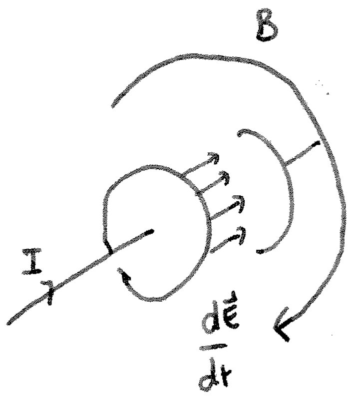
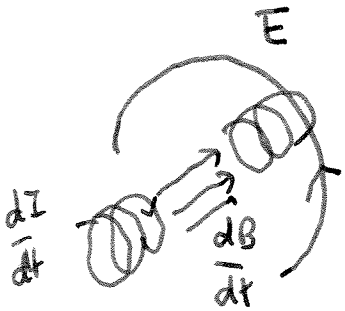
$$\int \vec{B} \cdot d\vec{A} = 0$$

Gauss' law for magnetic fields.



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I + \overbrace{\epsilon_0 \frac{d\Phi_E}{dt}}^{\text{displacement}} \right)_{encl}$$

Ampères law



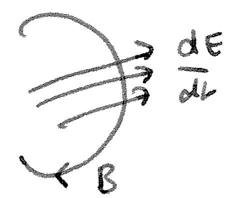
$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

Faraday's law

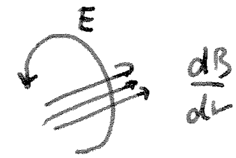
19.10.

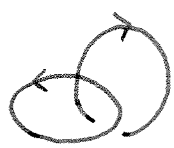
- Notice that in free space

$$\oint \vec{B}.d\vec{l} = \mu_0 \epsilon_0 \int \frac{d\vec{E}}{dt}.d\vec{A}$$



$$\oint \vec{E}.d\vec{l} = - \int \frac{dB}{dt}.d\vec{A}$$





Remarkable symmetry!

- These four equations, together with

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

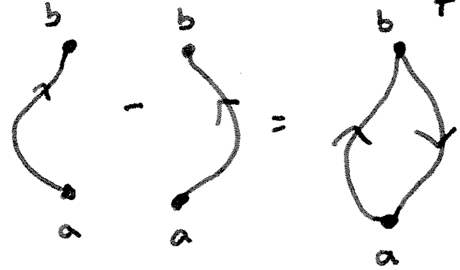
describe all of electromagnetism.

19. 10.

- A circulating electric field is not conservative

$\int_a^b E.dl$ depends on the path

$$\int_{a\,(P)}^{b} \vec{E}.d\vec{l} - \int_{a\,(P')}^{b} \vec{E}.dl = \oint \vec{E}.dl \neq 0$$



$= \oint E.dl$

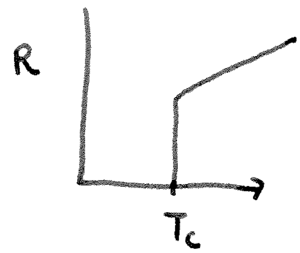
We often divide $\vec{E}$ into a conservative & an non-conservative component

$$\vec{E} = \vec{E}_c + \vec{E}_n$$

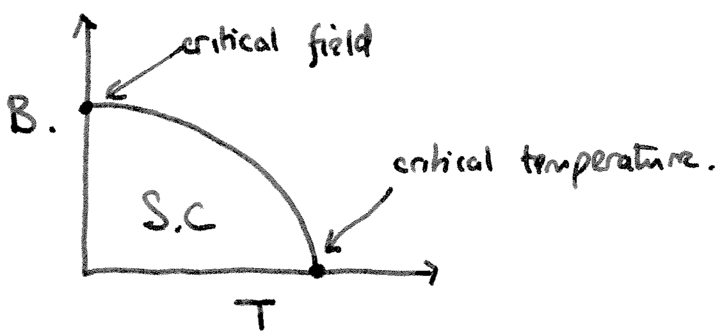
where $\int \vec{E}_c \cdot dl = 0$.

29.8 SUPERCONDUCTIVITY

Electrical resistance vanishes at T_c.

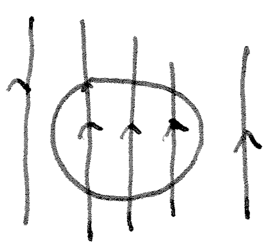


It turns out that superconductivity is intimately related to the perfect diamagnetism of superconducting matter.



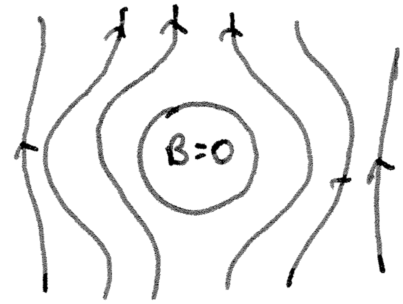
Highest T_c to date

135K.



$T > T_c$ metal

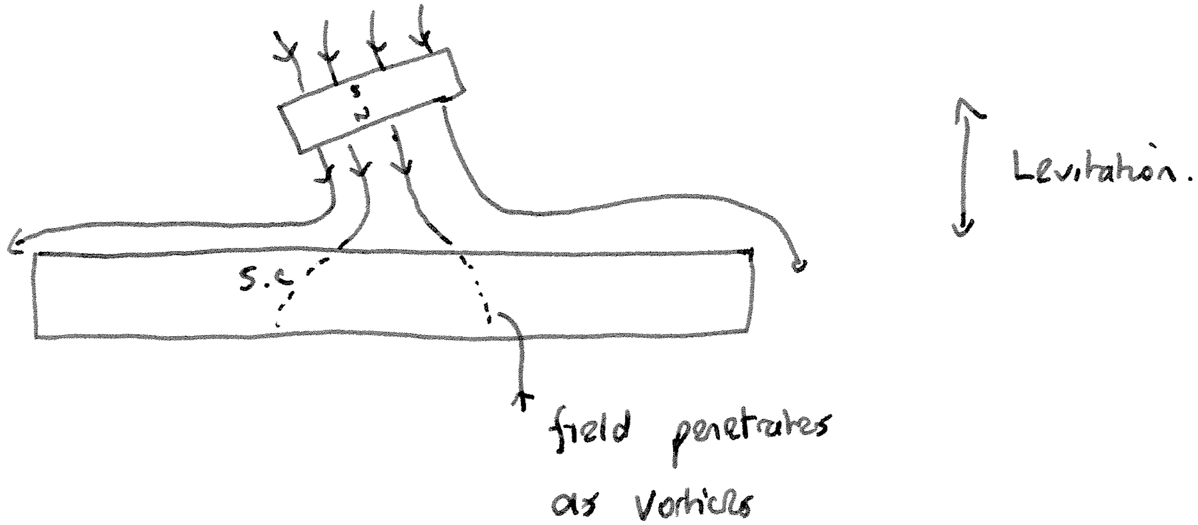
(weak paramagnet)



$T < T_c$ diamagnet

Expels flux (Meissner effect)

Repels magnets $\Leftrightarrow$ Levitation



* Applications today.

  - M.R.I (huge magnetic fields)
  - "Squids" (ultra-sensitive field measurement)
  - Cell phone microwave cavities

* Future? • Transmission lines • Maglev trains } can we find a room temperature superconductor?

* Cosmic superconductor! We live in a cosmic s.c!
  (Reason for weak force.)