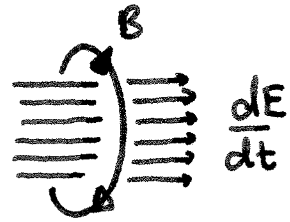
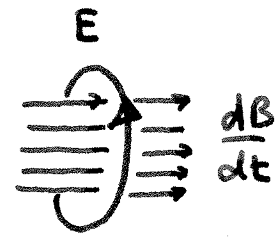
L18

ELECTROMAGNETIC INDUCTION



Displacement current.



Electromagnetic Induction.

Our world depends on our ability to convert energy into electrical form

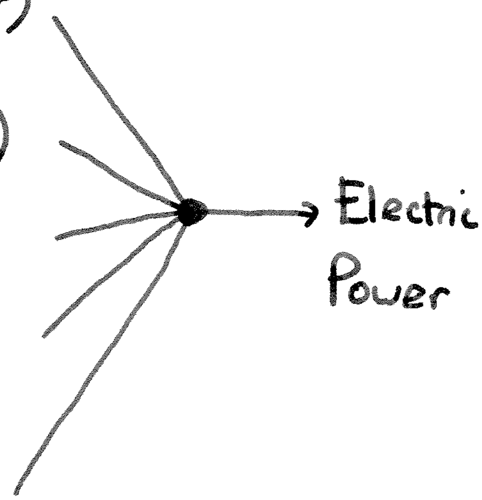
Gravitational P.E (hydroelectric power)

Chemical energy (coal, gas power)

Nuclear energy

Wind power

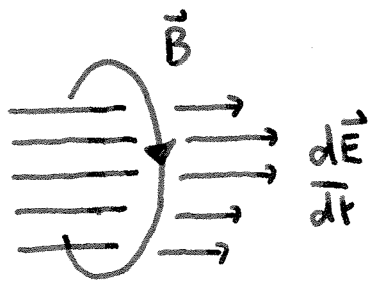
Wave power



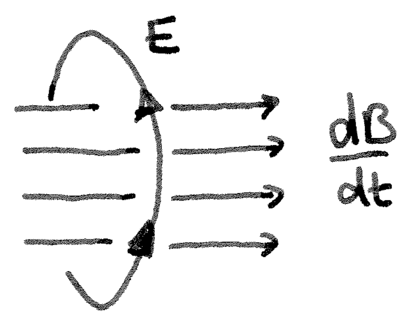
The key to each of these conversions is electromagnetic induction.

It is remarkable that this effect, at the heart of all conversion of mechanical energy to electric energy, was discovered by a young fellow in his twenties. Michael Faraday began his adult career as a bookbinder in London who would enthusiastically attend science lectures. Humphrey Davis, the first director of the Royal Institution, took him on as an assistant after having seen the exquisite notes he took. Michael Faraday was also the discoverer of the concept of fields, and he established a yearly science lecture for Children — an event which we emulate here at Rutgers with an annual Children's Physics Lecture.

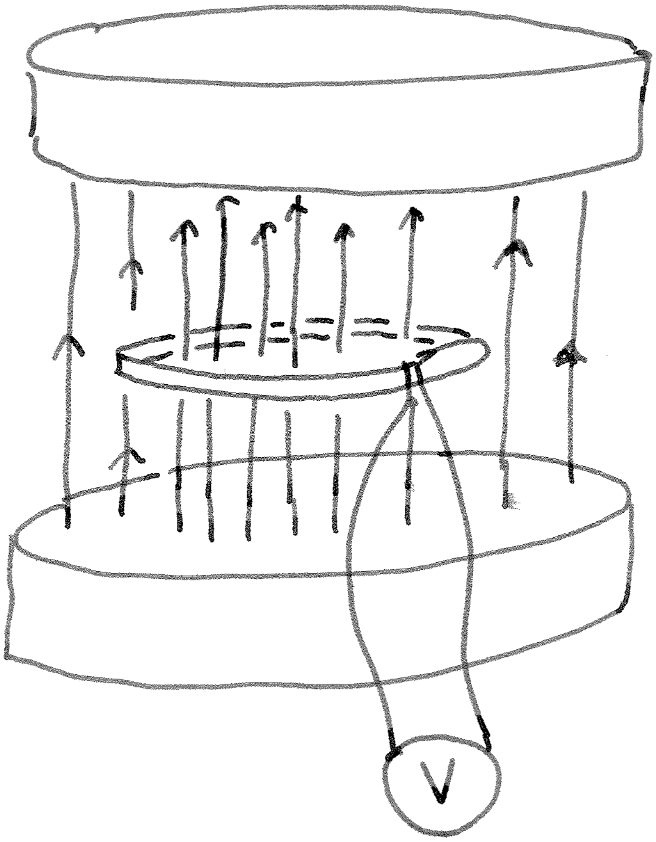
Last time we learnt that a changing electric field acts as a "displacement current" which produces a circulating magnetic field



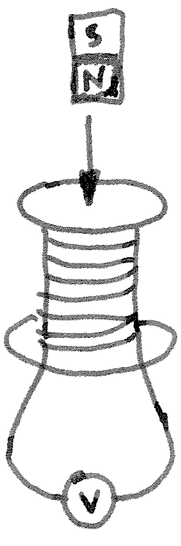
The magnetic field circulates in a <u>clockwise</u> sense about the increasing electric field. Electromagnetic induction is the analogous effect of a changing magnetic field. In this case, the electric field circulates in an <u>anticlockwise</u> sense about the increasing magnetic field



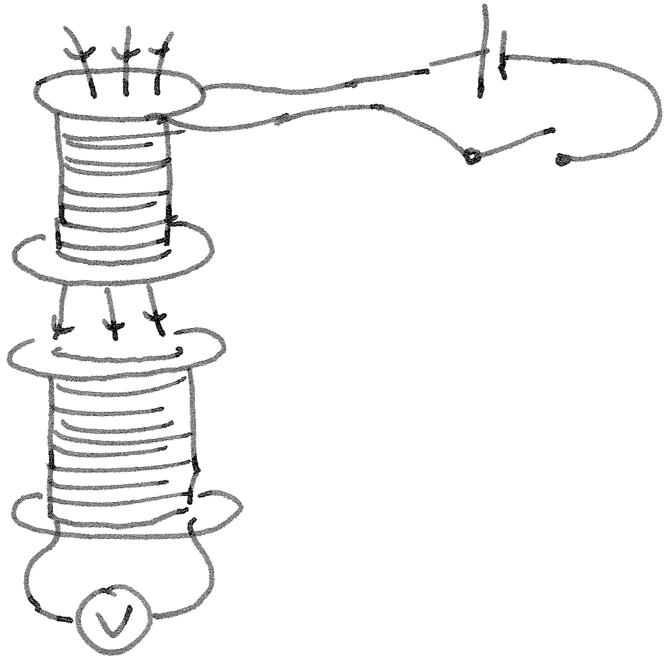
29.1 Induction Experiments



A change in the shape or orientation of the coil, or a change in the field, produces a voltage.



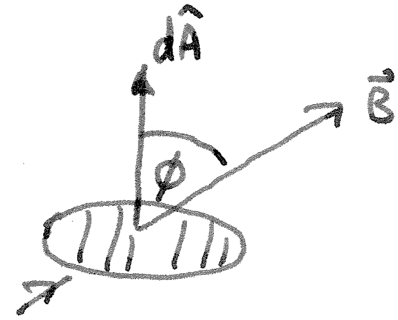
Voltage when magnet moves



Voltage induced just after switch is <u>closed</u> or <u>opened</u>.

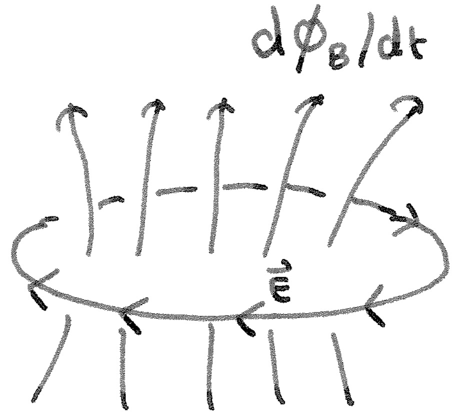
29.2

First we need the concept of magnetic flux



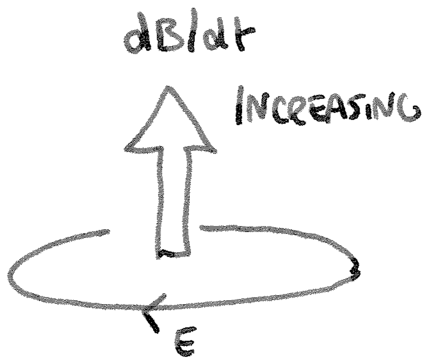
$$d\Phi_B = B\,dA\cos\phi$$
$$= \vec{B}\cdot d\vec{A}$$

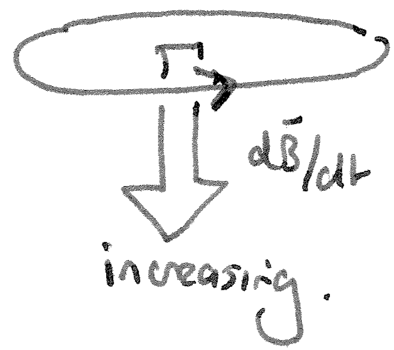
$$\Phi_B = \int \vec{B}.d\vec{A} = \int B\,dA\cos\phi$$



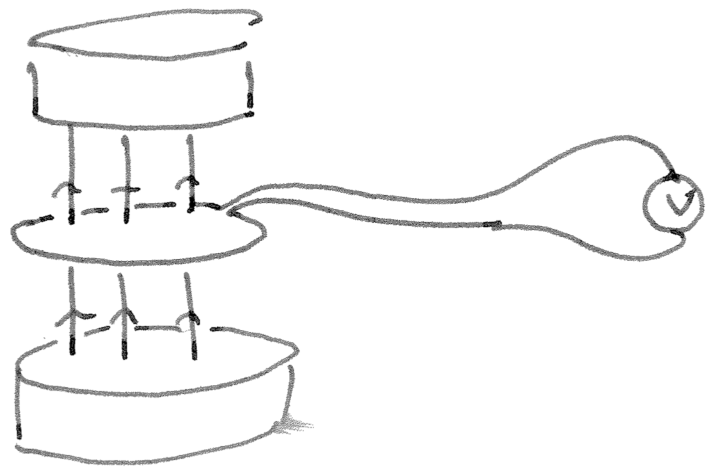
$$\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}}$$

Faraday's law of induction.





e.g



Wire loop of area $A = 120 cm^2$, resistance $R = 5\Omega$

immersed in a magnetic field which is increasing

at a rate $dB/dt = 0.02$ T/s.

a) Find EMF $\mathcal{E}$ & induced current I

b) What happens to $\mathcal{E}$ & I if the wire loop is insulating?

a) $$|\mathcal{E}| = \frac{d\Phi_B}{dt} = \frac{d BA}{dt} = \frac{dB}{dt} A = 0.02 \times (120 \times 10^{-4} m^2)$$

$$= 2.4 \times 10^{-4} \text{ V}$$

$$= \underline{0.24 \text{ mV}}.$$

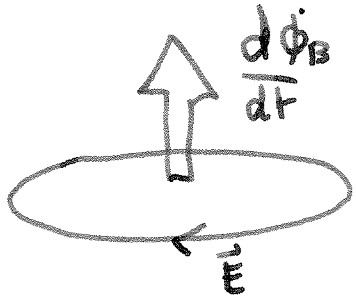
$$I = \frac{|\mathcal{E}|}{R} = \frac{2.4 \times 10^{-4} V}{5\Omega} = 4.8 \times 10^{-5} A = \underline{\underline{48 \mu A}}$$

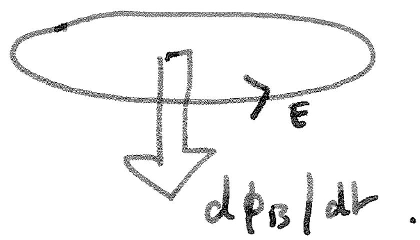
Direction of induced EMF

- First determine $\frac{d\Phi_B}{dt}$

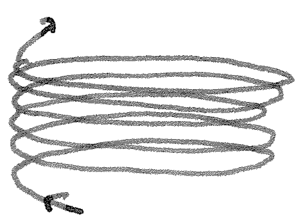
- If $d\phi_B/dt > 0 \Rightarrow \varepsilon < 0$ $\quad \vec{E}$ anticlockwise

  $d\phi_B/dt < 0 \Rightarrow \varepsilon > 0$ $\quad \vec{E}$ clockwise.



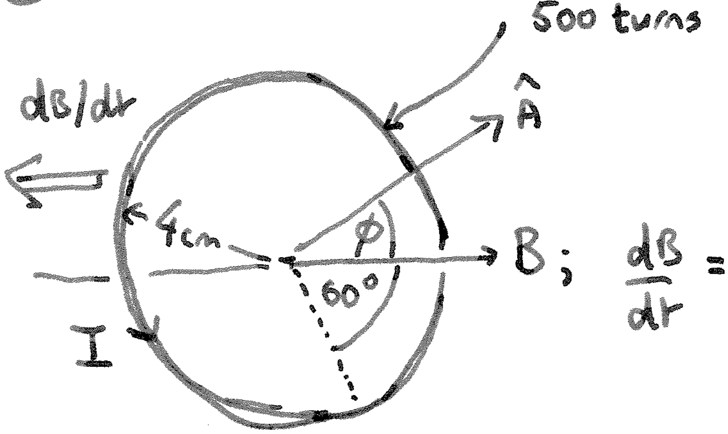


Multiturn coil



$$\boxed{\varepsilon = -N\frac{d\Phi_B}{dt}}$$

e.g



B; $\dfrac{dB}{dt} = -0.2\,T/s$ (decreasing)

a) Calculate $\mathcal{E}$

b) If $R_{coil} = 5V$, calculate I

a) $\phi = 30°$

$$\frac{d\Phi_B}{dt} = \frac{dB}{dt} A \cos\phi = (-0.2\,T/s)\left(\pi \times (4\times10^{-2})^2\right) \times \cos 30°$$

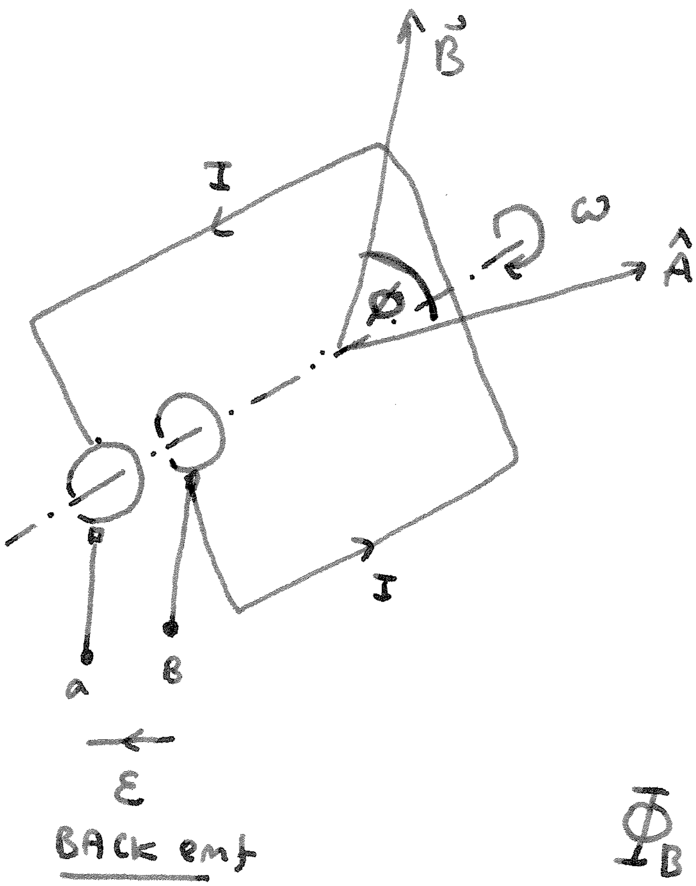
$$= \underline{-8.71 \times 10^{-4}\ Wb/s}$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -500 \times (-8.71\times10^{-4}) = \underline{0.435\ V}$$

$$I = \frac{0.435\,V}{5\,\Omega} = 87\times10^{-3}\,A = \underline{87\,mA}$$

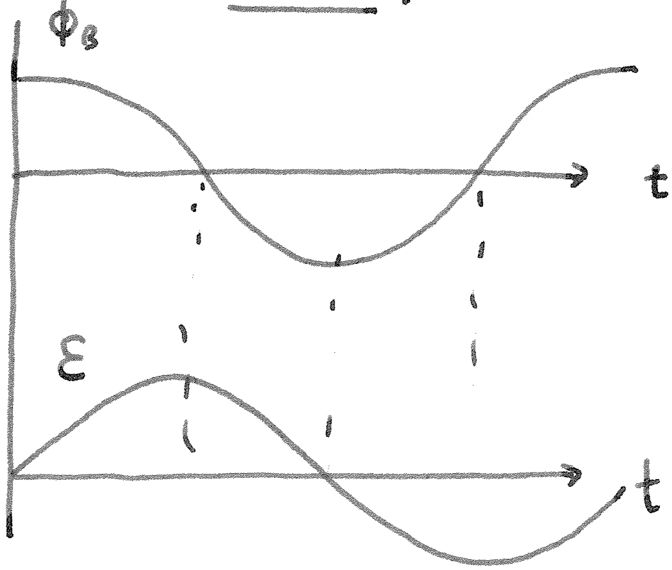
29.5/6 **ROTATING COILS + GENERATORS**

a) simple loop

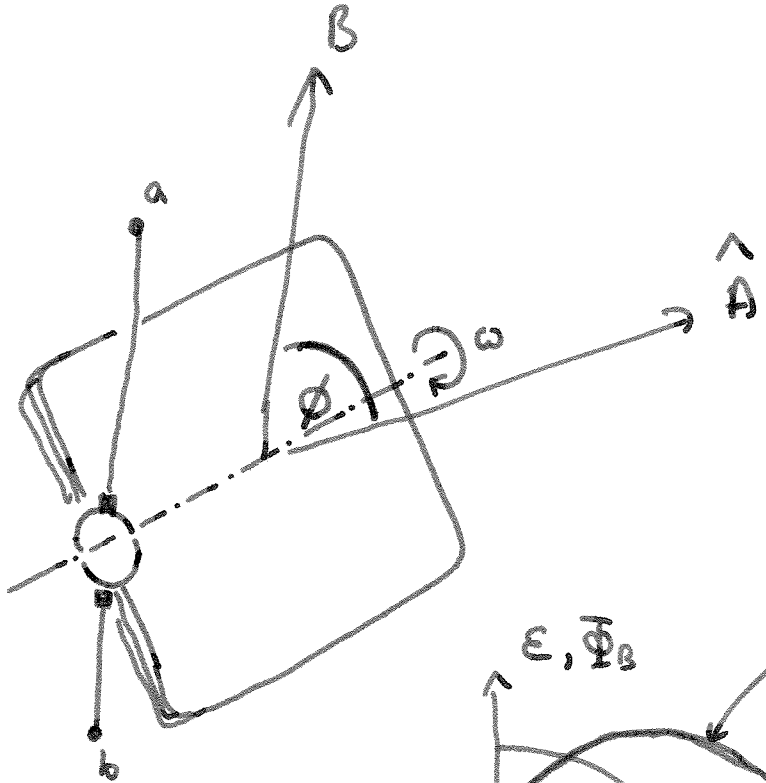


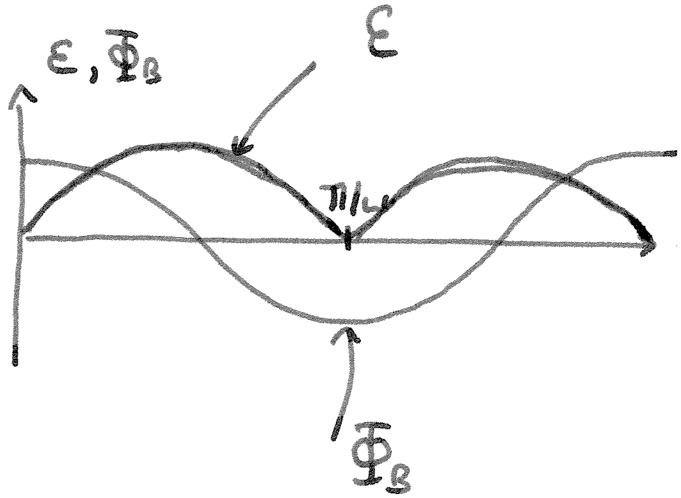
$$\Phi_B = BA\cos\omega t$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = BA\omega\sin\omega t$$



b) Multi-turn coil with split ring commutator





$$|\varepsilon| = N\,\omega BA\,|\sin\omega t|$$

$$\langle|\sin\omega t|\rangle = \frac{\int_0^{\pi/\omega} \sin\omega t\, dt}{\pi/\omega} = \frac{2/\omega}{\pi/\omega} = \frac{2}{\pi}$$

$$\langle|\varepsilon|\rangle = \frac{N\omega BA\,2}{\pi}$$ = average <u>BACK</u> emf

Suppose $B = 0.2T$ $\langle \mathcal{E}_{back} \rangle = 112V$, what is the rotation speed of a square coil with 500 turns, side length $L = 10cm$.

$$\langle |\mathcal{E}| \rangle = \frac{2 N\omega BA}{\pi} = 112V$$

$$\omega = \frac{\pi}{2} \frac{\langle |\mathcal{E}| \rangle}{NBA} = \frac{3.14}{2} \frac{\times 112V}{500 \times 0.2 \times (0.1)^2}$$

$$= 176 \text{ rad/s}$$

rotation rate f $\omega = 2\pi f$

$$f = \frac{176}{2\pi} = 28 \text{ r.p.s}$$

rotation rate / min $= 60 \times \frac{176}{2\pi} = 1680 \text{rpm}$

18.12

29.3 Lenz's Law.

$$\boxed{\text{The direction of the induced current is such as to oppose the change producing it}}$$

Really a very simple law, derived from Faraday's law.

By "opposing" the change producing it, the inductive response imposes <u>negative</u> <u>feedback</u>, so that electromagnetism tends to oscillate.

More generally $\quad d\mathcal{E} = (\vec{v} \times \vec{B}) \cdot d\vec{\ell}$

$$\boxed{\mathcal{E} = \int (\vec{v} \times \vec{B}) \cdot d\vec{\ell}}$$

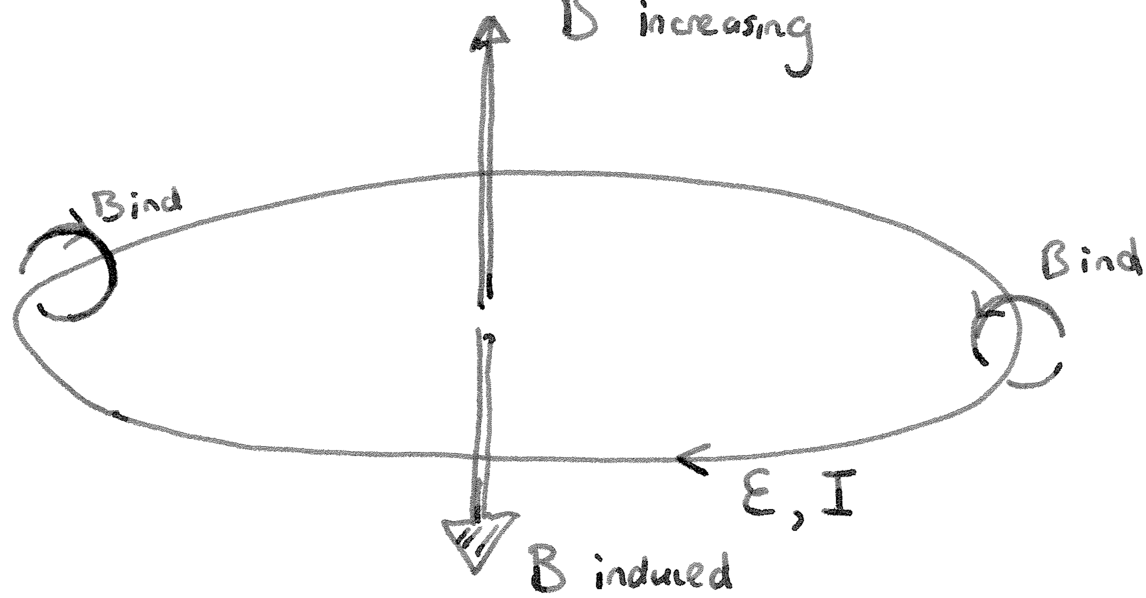
Motional EMF

e.g "Slidewire generator" — work & power

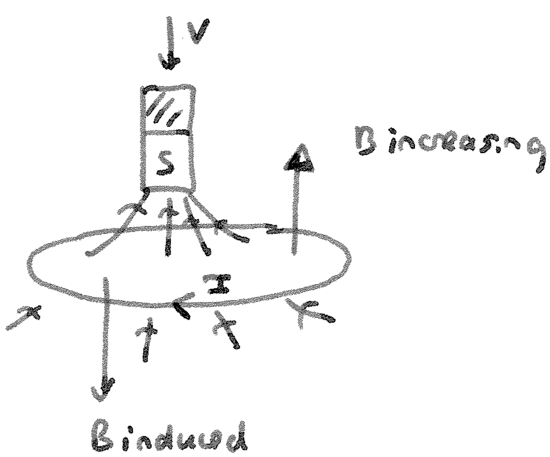
a) Calculate force on a slidewire generator & the work done on it per unit time

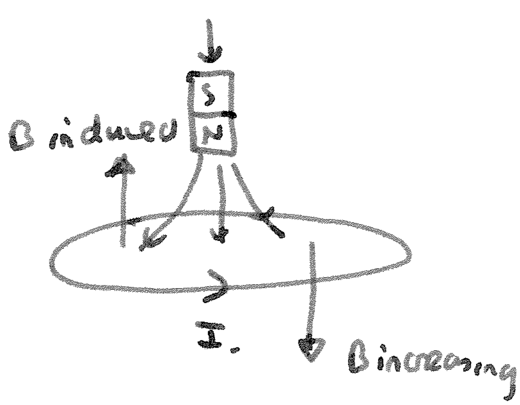
b) Calculate the induced current & the energy dissipated in the circuit / unit time

c) compare a) & b).


B increasing
Bind
Bind
E, I
B induced

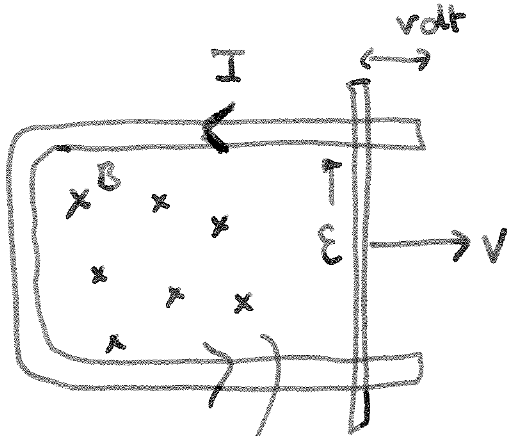
e.g


v
S
B increasing
I
B induced


S
N
B induced
I.
B increasing

29.4 Motional EMF

We've seen that changing the field & changing the coil orientation produces an EMF — what about changing the shape?



$dA = L v\, dt$

$\frac{dA}{dt} = Lv$

$\Phi_B = B A$

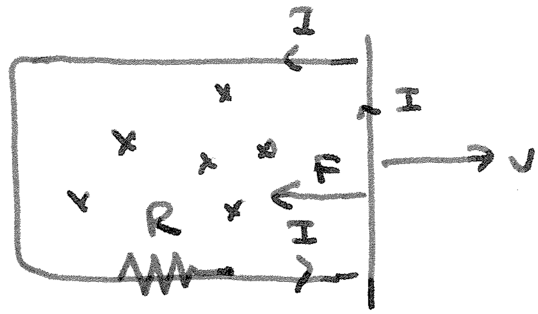
$$\frac{d\Phi_B}{dt} = B \overbrace{\frac{dA}{dt}}^{Lv} = B L v$$

$$\mathcal{E} = BLv$$

current flows counter clockwise to produce an induced field out of the paper — opposing the increasing flux into the paper.

n.b Direction of $\mathcal{E}$ is also direction of Lorentz force on +ve charges.

18.66



a) $F = BIL$ in the opposite direction to motion.

$$\mathcal{E} = BLv \Rightarrow I = \frac{\mathcal{E}}{R} = \frac{BLv}{R}$$

rate of work $= Fv = BILv$

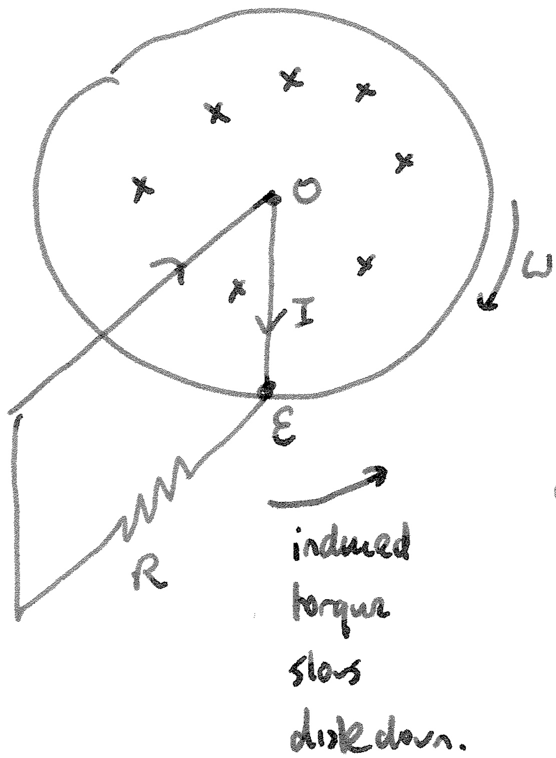
$$= \frac{B^2L^2v^2}{R}$$

b) Energy dissipated in resistor / unit time

$$= I\mathcal{E} = I^2R = \left(\frac{BLv}{R}\right)^2 R = \frac{B^2L^2v^2}{R}$$

c) Work done = energy dissipated. All of the mechanical work done is converted into heat in the resistor.

29.11 Faraday dynamo



Find the induced emf produced on the rim.

$$dE = vB\,dr \quad \text{(motional emf)}$$

$$\mathcal{E} = \int dE = \int vB\,dr$$

$$= \int_0^R \omega r B\,dr$$

$$= \frac{BR^2\omega}{2}.$$

The induced current $I = \mathcal{E}/R$

produces a torque which

will brake the wheel.