L16 AMPERES LAW

In electrostatics we discovered that the calculation of electric field is simplified by Gauss' law. With Gauss' law, in symmetric situations, we can deduce the field without having to "add up" all of the electric fields from each charge.

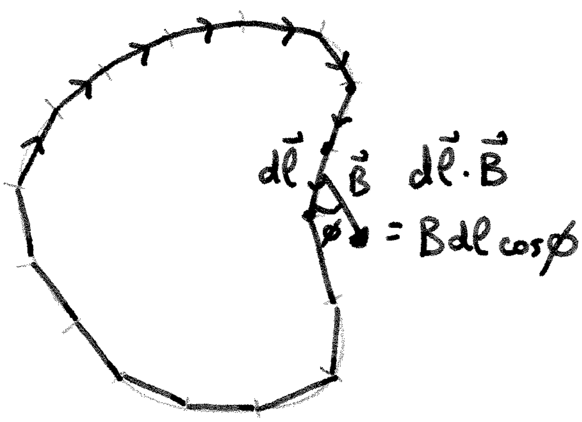
Last time we learned how to "add up" the fields produced by a wire – we found that

28.6

Just as Gauss' law required the introduction of a new kind of integral — in that case, the surface integral, Ampères law requires a new concept — the <u>line integral</u>. The line integral around a closed path is

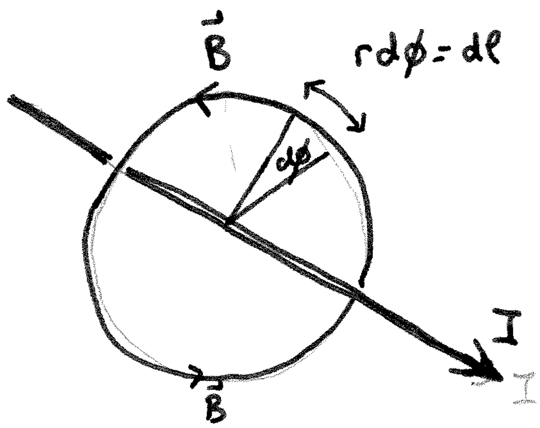
$$\oint \vec{B}\cdot d\vec{\ell}$$

This says that we divide the path up into tiny segments $d\vec{\ell}$, calculate $\vec{B}\cdot d\vec{\ell}$ for each segment, then add the contributions up.



For example, lets look at the infinite wire, where

$$B = \frac{\mu_0 I}{2\pi r}$$



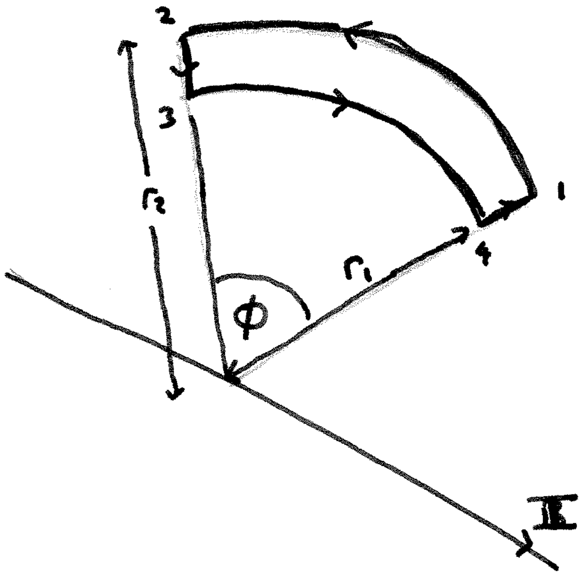
$$\vec{B}\cdot d\vec{\ell} = B(r\,d\phi)$$

$$\int \vec{B}\cdot d\vec{\ell} = Br\int d\phi$$

$$= 2\pi r B$$

$$\boxed{\int \vec{B}\cdot d\vec{\ell} = 2\pi r\left(\frac{\mu_0 I}{2\pi r}\right) = \mu_0 I}$$

What about a path that does not enclose the conductor?



$$\oint \vec{B}.d\vec{l}$$

$$= \int_1^2 + \int_2^3 + \int_3^4 + \int_4^1 \vec{B}.d\vec{l}$$

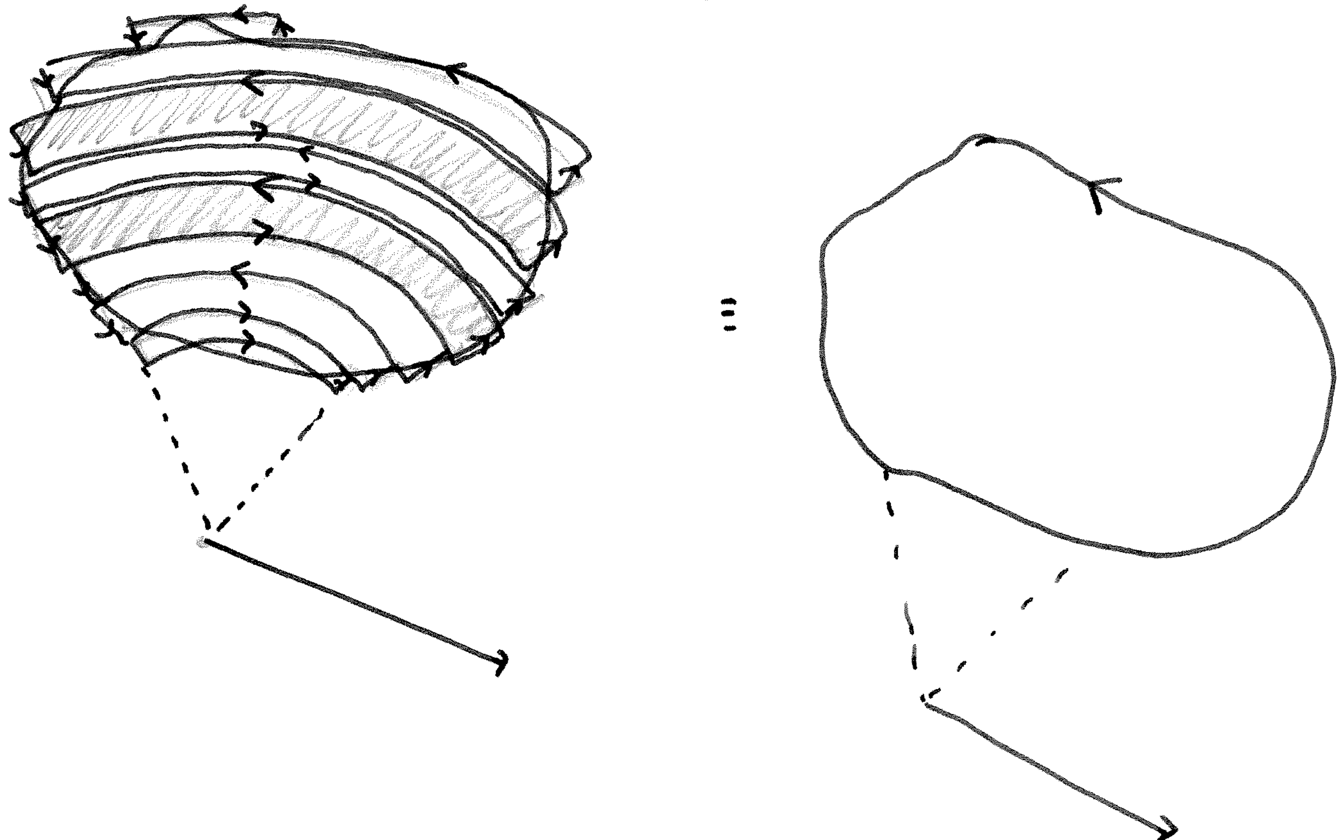
$$\int_1^2 \vec{B}.d\vec{l} = -B_2 r_2 \phi = -\frac{\mu_0 I}{2\pi r_2} r_2 \phi = -\mu_0 I \left(\frac{\phi}{2\pi}\right)$$

$$\int_2^3 \vec{B}.d\vec{l} = \int_4^1 \vec{B}.d\vec{l} = 0$$

$$\int_3^4 \vec{B}.d\vec{l} = B_1 r_1 \phi = \frac{\mu_0 I}{2\pi r_1} r_1 \phi = \mu_0 I \left(\frac{\phi}{2\pi}\right)$$

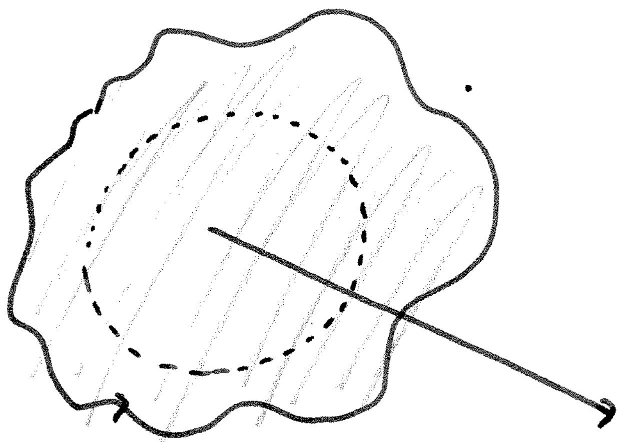
$$\oint \vec{B}.d\vec{l} = 0$$

A general loop which does not enclose a current can be broken up into such arcs.



$$\oint \vec{B}.d\vec{l} = 0$$ if loop does not enclose a current

Now this means we can always change a circular path to any other shape of path



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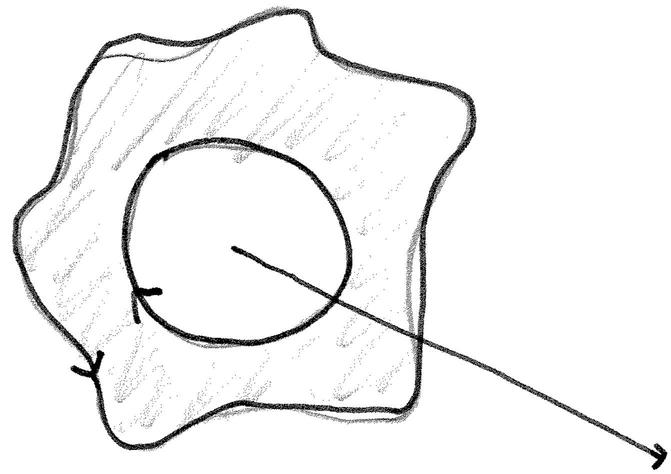
$= \int_1 - \int_2 \vec{B}.d\vec{\ell}$



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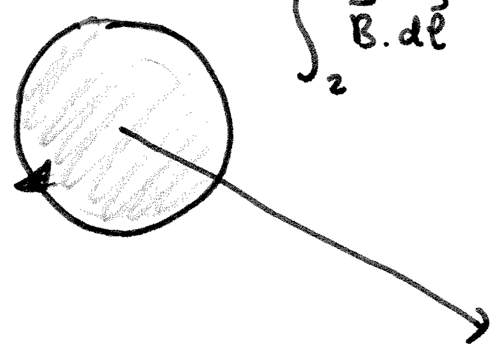
$\int_2 \vec{B}.d\vec{\ell}$



$$\int \vec{B}.d\ell = \overset{0}{\overset{\shortparallel}{\boxed{\int_1 \vec{B}.d\vec{\ell} - \int_2 \vec{B}.d\ell}}}$$

$$+ \int_2 \vec{B}.d\vec{\ell}$$

$$= \mu_0 I$$

$$\boxed{\int \vec{B}.d\vec{\ell} = \mu_0 I_{\text{enclosed}}}$$

AMPÈRE'S LAW

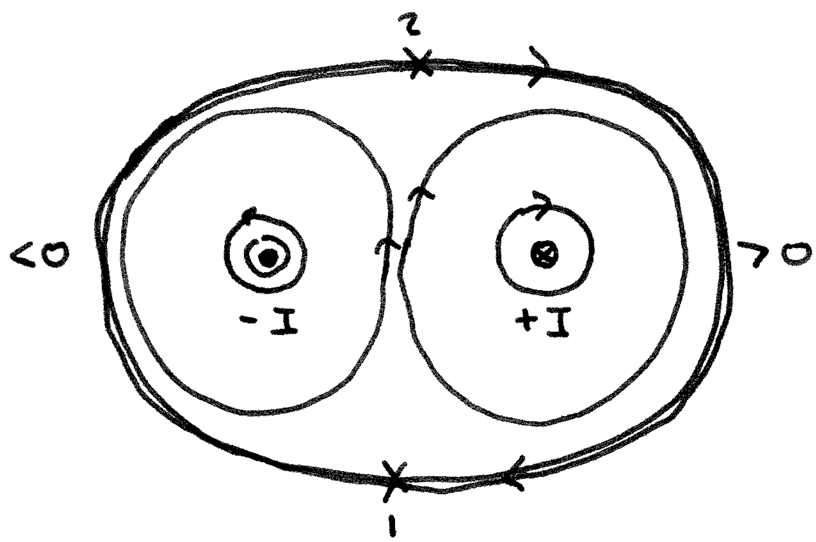
n.b.

① $\int \vec{E}.d\vec{l} = 0$ because electric forces are conservative

$\int \vec{B}.d\vec{l} = \mu_0 I$ magnetic forces are not conservative. $F = q\vec{v} \times \vec{B}$

so $\int B.dl$ is not the work done per charge either

② $\int \vec{B}.d\vec{l} = 0$ does not imply $\vec{B} = 0$ everywhere

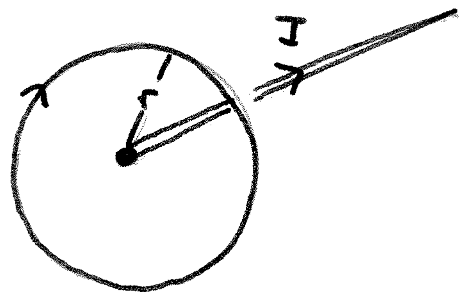


$$\oint_1^2 \vec{B}.d\vec{l} < 0$$

$$\oint_2^1 \vec{B}.d\vec{l} > 0$$

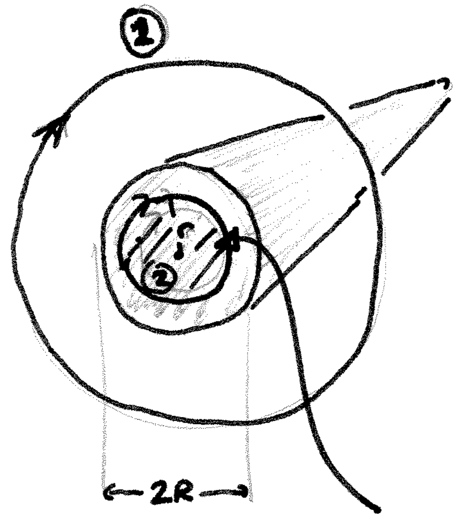
28.7 Applications of Ampère's law

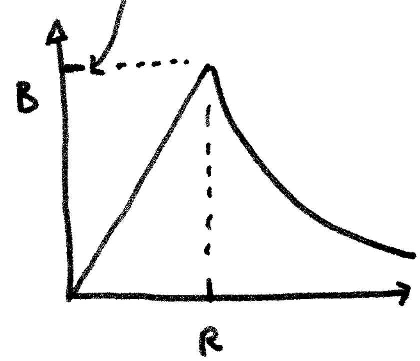
(a) • Long straight wire



$$\left.\begin{aligned}\int \vec{B}.d\vec{l} &= 2\pi r B \\ &= \mu_0 I\end{aligned}\right\} \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

(b) • Field inside a long cylindrical conductor





$$\oint_1 = 2\pi r B = \mu_0 I$$

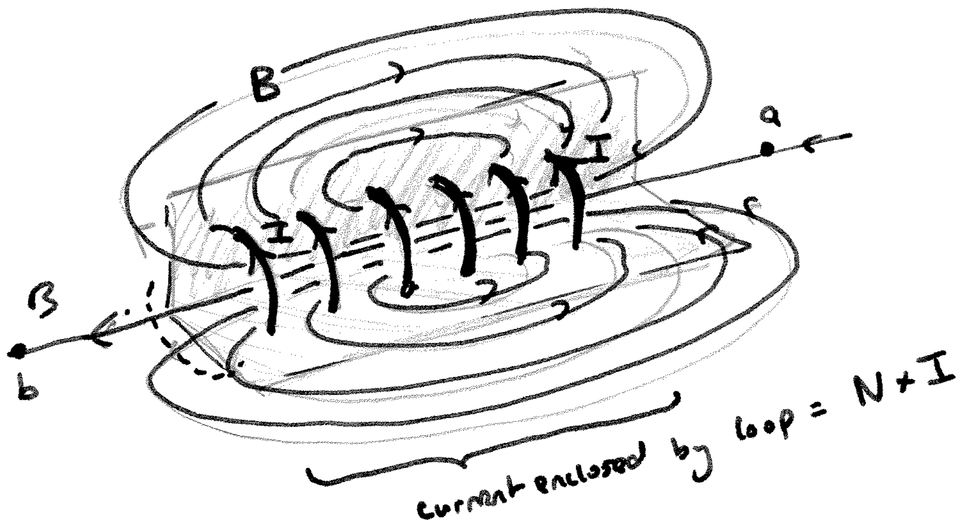
$$\boxed{r > R : \quad B = \mu_0 I/(2\pi r)}$$

$$\oint_2 = 2\pi r B = \mu_0 \times I_{\text{enclosed}}$$

$$I_{\text{enclosed}} = \frac{\pi r^2}{\pi R^2} I$$

$$= \mu_0 I \left(\frac{r^2}{R^2}\right) \Rightarrow \boxed{B(r<R) = \frac{\mu_0 I}{2\pi R}\left(\frac{r}{R}\right)}$$

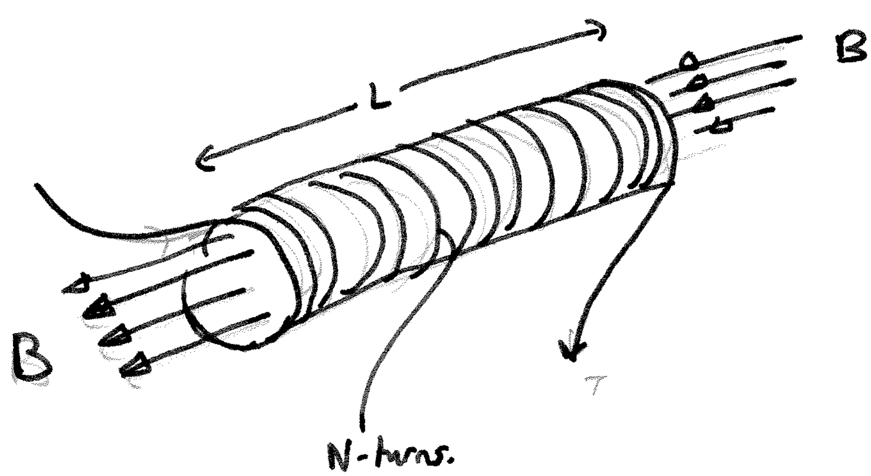
(c) Solenoid



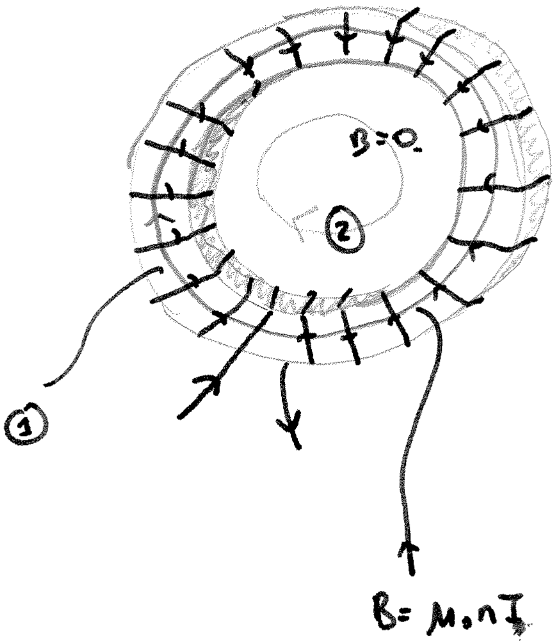
When the solenoid becomes long $(L >> r)$

$$\int_a^b \vec{B}.d\vec{\ell} = BL = \mu_0 N I$$

$\boxed{B = \mu_0 n I}$ Long solenoid $(n = N/L)$



(d) Toroidal solenoid



$$\int_{\text{interior } ①} \vec{B}.d\ell = 2\pi r\, B = \mu_0 \,(NI)$$

once again

$B = \mu_0 n I$

$$\int_{\text{exterior } ②} \vec{B}.d\vec{\ell} = 0 \qquad B = 0$$

28.8 Magnetic materials

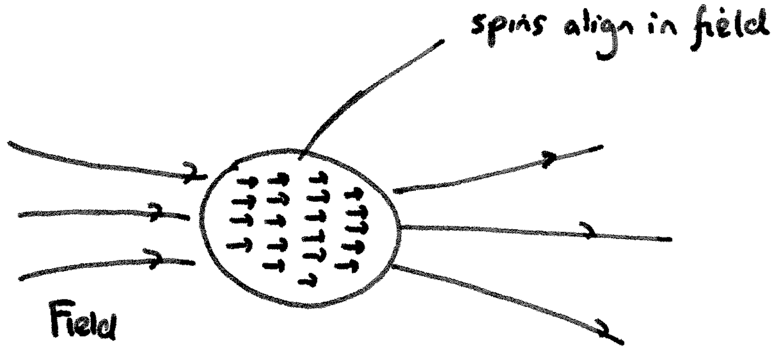
- paramagnetism
- diamagnetism (superconductors)
- ferromagnetism (disks)

Magnetic properties of matter – vital for applications

NEW STATES OF MATTER $\longrightarrow$ NEW APPLICATIONS.

We will pick this up next time – but in preparation we will have a demo showing paramagnetic behavior in liquid oxygen.

Basic idea



$$\vec{M} = \frac{\vec{\mu}}{V}$$ magnetization = mag moment / unit volume

In a paramagnet $\vec{M} = \chi \vec{B}$

Work done in increasing field $\vec{B} \rightarrow \vec{B} + d\vec{B}$ is

$$dU = -\vec{M}.d\vec{B} = -\chi B dB$$

$$\Rightarrow \quad U = -\frac{\chi B^2}{2}$$ energy / unit volume

Paramagnetic materials are <u>attracted</u> to a high field.