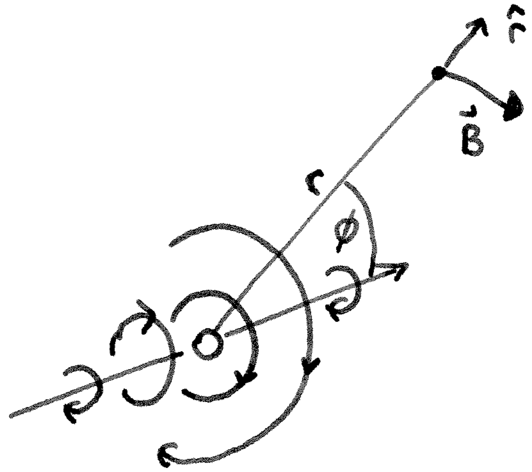
15 SOURCES OF MAGNETIC FIELD.

We've seen how the magnetic field exerts a force on moving charges. Now we need to understand how a moving charge gives rise to a magnetic field. We'll start with the case of a single charge, then we will look at the field produced by a small segment of current, & this will lead us finally to our next Maxwell equation — "Ampere's law".

28.1 Field from a moving charge.



$$\boxed{B = \frac{\mu_0}{4\pi} \frac{|q| v \sin\phi}{r^2}}$$

$(\mu_0 = 4\pi \times 10^{-7}\ \mathrm{Tm/A})$

In vector notation

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi} q \left(\frac{\vec{v} \times \hat{r}}{r^2} \right)}$$

Units : $[B] = 1\mathrm{T}$

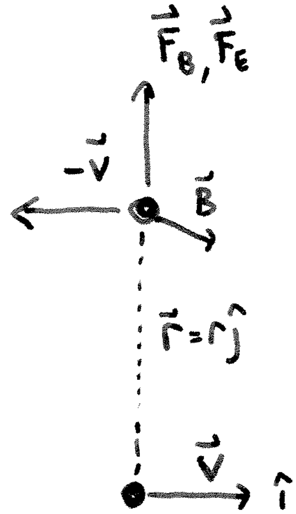
$[\mu_0] = 1\mathrm{T} / (\mathrm{C\,m^{-1}s^{-1}}) = 1\mathrm{Tm/A}$

Why is $\mu_0 = 4\pi \times 10^{-7}$ exactly ? We'll see later that this derives from the definition of the Ampere. Turns out that

$$\frac{1}{\mu_0 \epsilon_0} = c^2 \Rightarrow \frac{1}{4\pi\epsilon_0} = \frac{\mu_0 c^2}{4\pi} = 10^{-7} c^2 = 9 \times 10^9\ \mathrm{Nm^2/C}$$

✓

e.g. Comparison of electric + magnetic force between two protons



$$\vec{F}_E = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2} \hat{j}$$

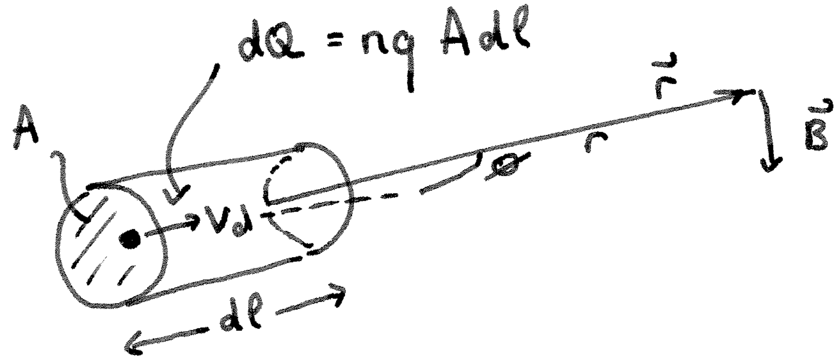
$$\vec{B} = \frac{\mu_0 q}{4\pi} \frac{\vec{v} \times \hat{r}}{r^2} = \frac{\mu_0 q v}{4\pi r^2} \overbrace{\hat{\imath} \times \hat{j}}^{\hat{k}}$$

$$\vec{F}_B = q(-v) \times \vec{B} = q(-v\hat{\imath}) \times \frac{\mu_0 q v}{4\pi r^2} \hat{k} = \frac{\mu_0 q^2 v^2}{4\pi r^2} \hat{j}$$

$$\frac{F_B}{F_E} = \mu_0 \epsilon_0 v^2 = \frac{v^2}{c^2}$$

When $v \ll c$, $F_B / F_E \ll 1$ — magnetic force only comparable with electric force when $v \sim c$, the speed of light

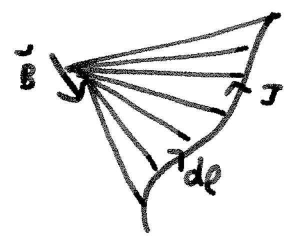
28.2 Field of a current element



$$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{\overbrace{|dQ|\,v_d}^{nqv_dA\,dl}\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{r}}{r^2}}$$

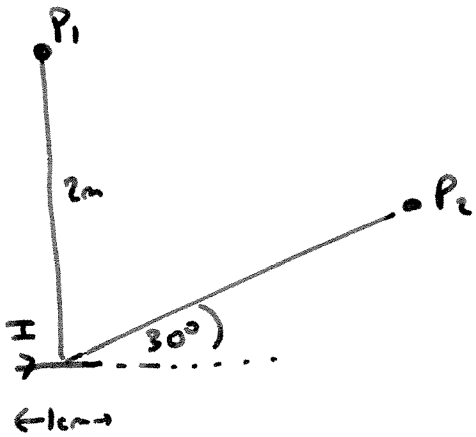
"Biot Savart law"



$$\boxed{\vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi}\int I\frac{d\vec{l} \times \hat{r}}{r^2}}$$

Can only measure <u>net</u> field from entire circuit and this can be used to indirectly verify the Biot Savart law.

e.g Calculate field produced by a 1cm segment of wire, 2m away (a) at P_1 straight to the side & (b) P_2 at 30° to the segment, if $I = 125A$



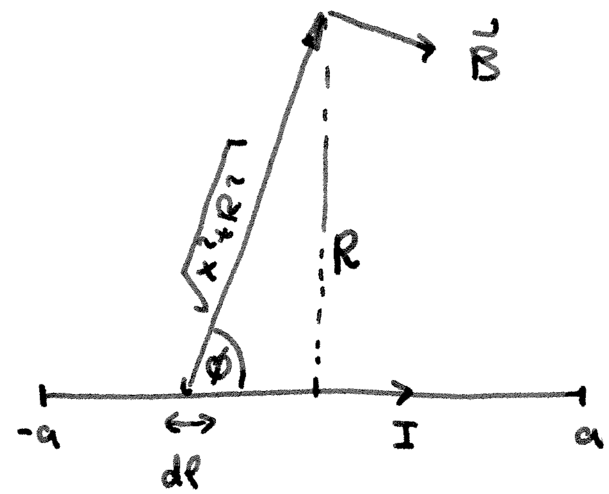
a) $$B_{P_1} = \frac{\mu_0}{4\pi} \frac{(125)}{2^2} \times \overset{dl}{(0.01)} \times \sin 90° = 10^{-7} \times \frac{125}{4} \times 10^{-2}$$

$$= 31.25 \times 10^{-9}$$

$$= 3.13 \times 10^{-8} N$$

b) $$B_{P_2} = 10^{-7} \times \frac{125}{4} \times 10^{-2} \times \sin 30° = 1.56 \times 10^{-8} N.$$

28.3

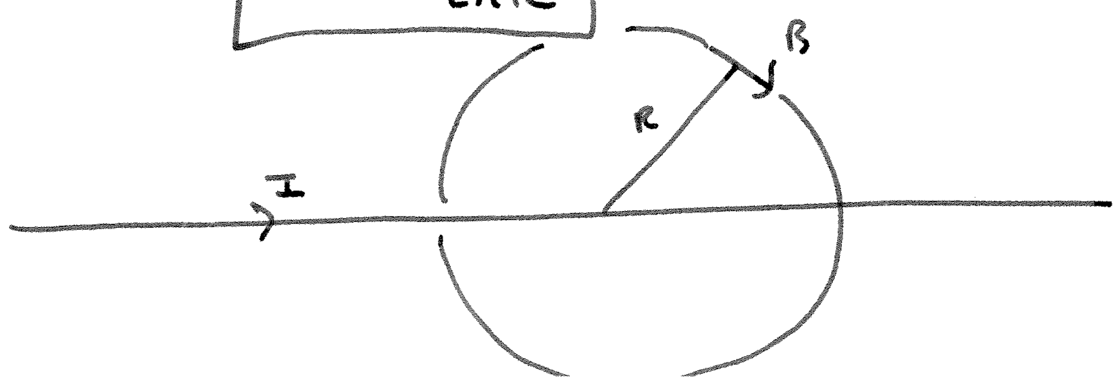


$$dB = \frac{\mu_0}{4\pi} \frac{I\, d\ell}{x^2+R^2} \sin\phi \qquad \sin\phi = \frac{R}{\sqrt{x^2+R^2}}$$

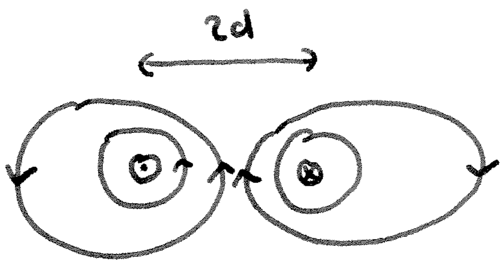
$$B = \frac{\mu_0 I R}{4\pi} \int_{-a}^{a} dx \frac{1}{(x^2+R^2)^{3/2}} = \frac{\mu_0 I R}{4\pi} \left[\frac{x}{R^2 (x^2+R^2)^{1/2}} \right]_{-a}^{a}$$

$$= \frac{\mu_0 I}{2\pi} \frac{a}{(R^2+a^2)^{1/2} R}$$

$a \to \infty$ $\boxed{B = \frac{\mu_0 I}{2\pi R}}$ infinite line



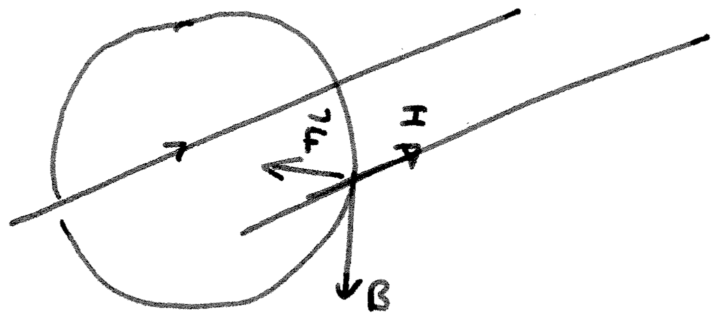
e.g. Calculate field midway between two wires carrying I in opposite directions, separation $2d$



$B_1 = \frac{\mu_0 I}{2\pi d}$ $B_2 = \frac{\mu_0 I}{2\pi d}$ (opposite currents)

$B = \frac{\mu_0 I}{\pi d}$.

28.4 Force between two wires



like currents

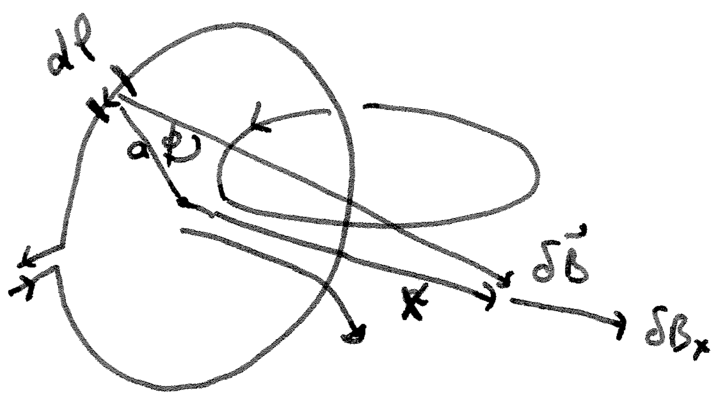
$\Rightarrow$ attractive

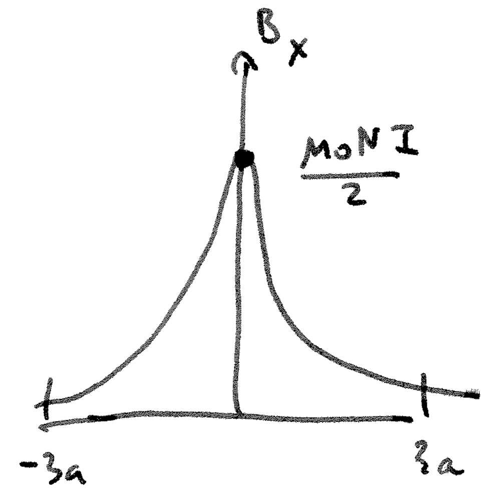
$$B = \frac{\mu_0 I_1}{2\pi r} \qquad F = I_2 \ell B = \ell \frac{\mu_0 I_1 I_2}{2\pi r}$$

1 Amp : 2×10^{-7} N/m @ 1m separation.

$$2 \times 10^{-7} = \frac{\mu_0 (1A)^2}{2\pi \times 1m} \Rightarrow \mu_0 = 4\pi \times 10^{-7} \checkmark$$

28.5 Field of circular current loop.





$$\delta \vec{B} = \frac{\mu_0 I}{4\pi} \frac{\delta \ell}{(x^2+a^2)}$$

$$\delta B_x = \delta B \cos\phi = \frac{\delta B \; a}{(x^2+a^2)^{1/2}}$$

$$B_x = \frac{\mu_0 I a}{4\pi} \int_0^{2\pi a} \frac{d\ell}{(x^2+a^2)^{3/2}} = \frac{\mu_0 I}{2} \frac{a^2}{(a^2+x^2)^{3/2}}$$

N loops: $$B_x = \frac{\mu_0 N I}{2} \frac{a^2}{(a^2+x^2)^{3/2}}.$$